Institute of Technology of Cambodia
Department of Industrial and Mechanical Engineering

Construction Mechanics

Homework 3

Topic: Universal Robot (UR10)

Group: I4- Mechanical (3)

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I. Introduction

In this assignment, we leaned forward and inverse kinematics on the UR10 robot. Forward kinematics is the calculation of points with given joint values. Inverse kinematics is the finding of these values for a given point.

II. Forward Kinematics.

We first begin by giving the forward kinematics, describing the position of the end effector as a function of joint angles:

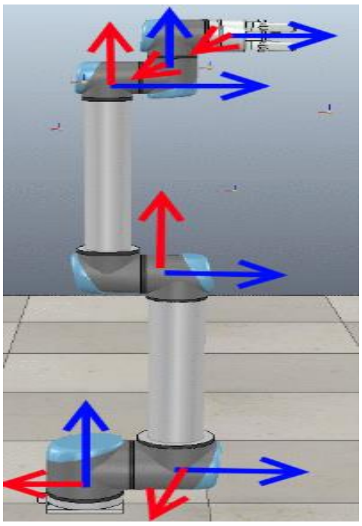
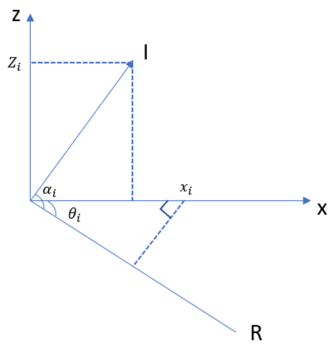


Figure01: Universal Robot Arm

Apply the Cylindric Coordinate on Robot Arm UR10:



$$\boxed{M_1^T \times M_1^R \times M_2^T \times M_2^R \times \dots\dots\dots \times M_6^T \times M_6^R}$$

Figure02: Cylindric Coordinate

Denavit-Hartenberg Parameters

4 parameters of Denavit-Hartenberg: $\theta, d, a\ and\ \alpha$:

a_i: link length (displacement along x_{i-1} from $z_{i-1}$to z_i)

α_i: link twist (rotation around x_{i-1} from $z_{i-1}$to z_i)

d_i: link offset (displacement along z_i from $x_{i-1}$to x_i)

θ_i: joint angle (rotation around z_i from $x_{i-1}$to x_i)

$$\begin{aligned}DH = {} & Tans(x_1) \times Rot(x_1) \times Trans(z_1) \times Rot(z_1) \times Tans(x_2) \times Rot(x_2) \times Trans(z_2) \\ & \times Rot(z_2) \times Tans(x_3) \times Rot(x_3) \times Trans(z_3) \times Rot(z_3) \times Tans(x_4) \times Rot(x_4) \\ & \times Trans(z_4) \times Rot(z_4) \times Tans(x_5) \times Rot(x_5) \times Trans(z_5) \times Rot(z_5) \\ & \times Tans(x_6) \times Rot(x_6) \times Trans(z_6) \times Rot(z_6)\end{aligned}$$

Where, $Trans_x(a_i) = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$Rot_x(\alpha_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\ \alpha_i & -sin\ \alpha_i & 0 \\ 0 & sin\ \alpha_i & -cos\ \alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Tans_z(d_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot_z(\theta_i) = \begin{bmatrix} cos\ \theta_i & -sin\ \theta_i & 0 & 0 \\ sin\ \theta_i & cos\ \theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence, $a = M_1(\theta_1) = M_1(\theta_1)$

$b = M_1(\alpha_2) = M_1(\theta_2)$

$c = M_1(\alpha_3) = M_1(\theta_3)$

$d = M_1(\alpha_4) = M_1(\theta_4)$

$e = M_1(\theta_5) = M_1(\theta_5)$

$f = M_1(\alpha_6) = M_1(\theta_6)$

$[P] = \prod[M_i] \quad , i = 1,2,3 \ldots ,6$

Denavit-Hartenberg Parameters

link	d_i	a_i	α_i	θ_i
0	0	0	0	0
1	d_1	0	0	θ_1
2	0	a_2	α_2	0
3	0	a_3	α_3	0
4	d_4	0	α_4	0
5	d_5	0	0	θ_5
6	d_6	0	α_6	0

III. Inverse Kinematic

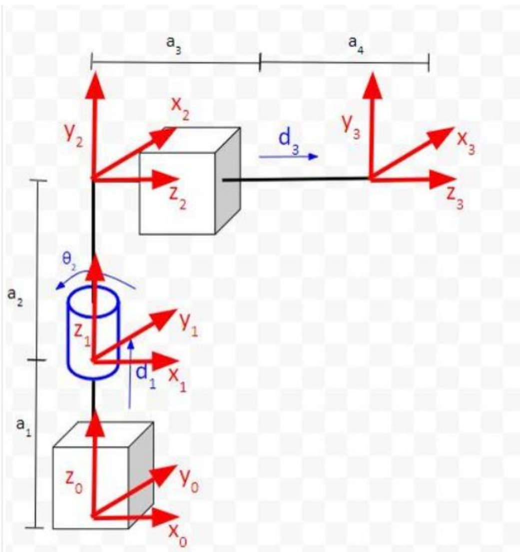
Inverse kinematics is the use of kinematic equations to determine the motion of a robot to reach a desired position.

Inverse Kinematics for a 6DOF UR10 Robot Using an Analytical Approach:

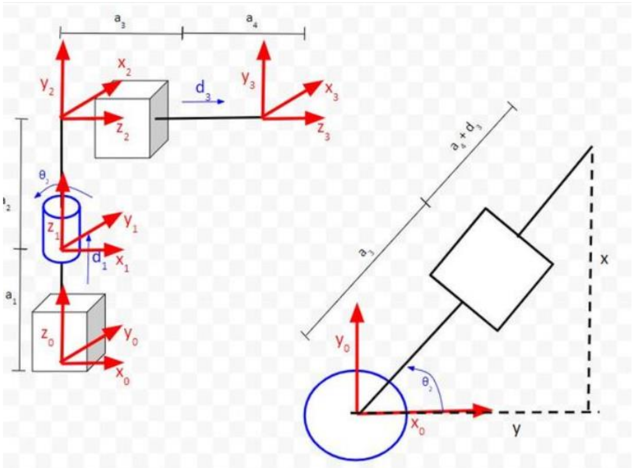
Description: Given a desired end position (x, y, z) of the end effector of a robot, and a desired orientation of the end effector (relative to the base frame), calculate the joint angles.

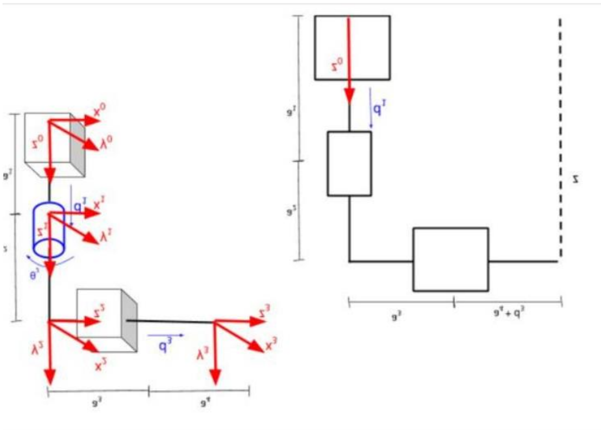
Here are the steps for calculating inverse kinematics for a six degree of freedom UR10:

Step 1: Draw the kinematic diagram of just the first three joints, and perform inverse kinematics using the graphical approach.



Step 2: Compute the forward kinematics on the first three joints to get the rotation of joint 3 relative to the global (i.e. base) coordinate frame. The outcome of this step will yield a matrix R_0^3, which means the rotation of frame 3 relative to frame 0 (i.e. the global coordinate frame).





$$\theta_2 = \tan^{-1}\left(\frac{y}{x}\right)$$

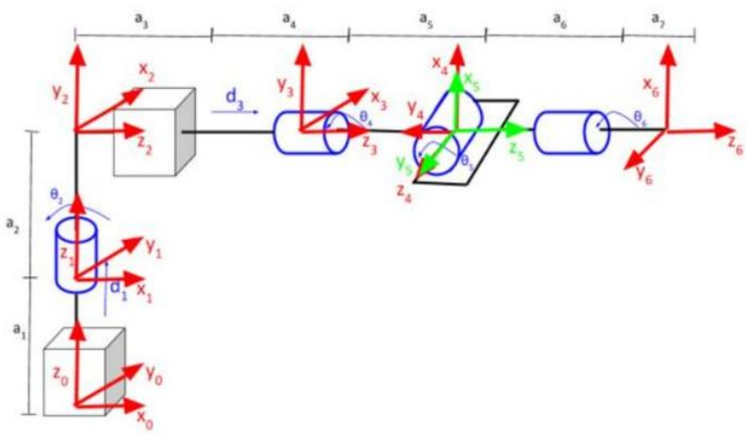
$$d_3 = \sqrt{x^2 + y^2} - a_3$$

- $d_1 = z - a_1 - a_2$

Step 3: Calculate the inverse of R_0^3

$$R_0^3 = \begin{bmatrix} -sin\theta_2 & 0 & cos\theta_2 \\ cos\theta_2 & 0 & sin\theta_2 \\ 0 & 1 & 0 \end{bmatrix}$$

Step 4: Compute the forward kinematics on the last three joints, and extract the part that governs the rotation. This rotation matrix will be denoted as R_3^6.



$$R_6^3 = \begin{bmatrix} -\sin\theta_4\cos\theta_5\cos\theta_6 - \cos\theta_4\sin\theta_6 & \sin\theta_4\cos\theta_5\sin\theta_6 - \cos\theta_4\cos\theta_6 & -\sin\theta_4\sin\theta_5 \\ \cos\theta_4\cos\theta_5\cos\theta_6 - \sin\theta_4\sin\theta_6 & -\cos\theta_4\cos\theta_5\sin\theta_6 - \sin\theta_4\cos\theta_6 & \cos\theta_4\sin\theta_5 \\ -\sin\theta_5\cos\theta_6 & \sin\theta_5\sin\theta_6 & \cos\theta_5 \end{bmatrix}$$

Step 5: Determine the rotation matrix from frame 0 to 6 (i.e. R_0^6).

Imagine you want the end effector to point upwards towards the sky. In this case, z6 will point in the same direction as z0. Accordingly, if you can imagine z6 pointing straight upwards, x6 would be oriented in the opposite direction as x0, and y6 would be oriented in the opposite direction as y0. Our rotation matrix is thus:

$$R_0^6 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 6: Taking our desired x, y, and z coordinates as input, use the inverse kinematics equations from Step 1 to calculate the angles for the first three joints.

Step 7: Given the joint angles from Step 6, use the rotation matrix to calculate the values for the last three joints of the robotic arm.

$$R_6^0 = R_3^0 R_6^3$$

So,

$$R_6^3 = [R_3^0]^{-1}\ R_6^0 \Rightarrow [R_3^0]^T\ R_6^0$$

Since R_3^0 **depends on the first three joints variable**, we can find its value by **solving the forward kinematics** after knowing the three joints values.

Finally use all this step to write the code on Python.

IV. Conclusion

I learned to use how to use its inverse and forward kinematics. So, by defining a concrete robot chain I can now control most of the non-mobile robots.