Molecular Dynamics Assignment 1.

Disclaimer: I completed about 1/3 of the questions in each chapter. The questions were relatively time consuming so I didn't do more, but I of course covered all the content of the chapters.

Chapter

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Chapter 2.

2.16 :

$$1 = \int \wp(\Lambda)\, d\Lambda$$

$$1 = A \int \Lambda^{n}\, d\Lambda$$

$$1 = \frac{A}{n+1} \Lambda^{n+1} \Big]_{\Lambda=0}^{\Lambda=1}$$

$$A = \frac{1}{n+1}$$

$$\langle \Lambda \rangle = \int \Lambda \wp(\Lambda)\, d\Lambda$$

$$= \frac{1}{n+1} \int_0^1 \Lambda^{n+1}\, d\Lambda$$

$$= \left[(n+1)(n+2) \right]^{-1}$$

$$\langle \Lambda^2 \rangle = \int \Lambda^2 \wp(\Lambda)\, d\Lambda$$

$$= \frac{1}{n+1} \int_0^1 \Lambda^{n+2}\, d\Lambda$$

$$= \left[(n+1)(n+3) \right]^{-1}$$

2.17 a) $\mathcal{P}(v) = Av^2 \exp(-mv^2/2k_BT)$

$1 = \int \mathcal{P}(v)\,dv$

$1 = A \int v^2 \exp(-mv^2/2k_BT)$

let $x^2 = mv^2/2k_BT$

$v^2 = 2k_BTx^2/m \qquad 2v\,dv = 2k_BT/m \cdot 2x\,dx$

$1 = A(2k_BT/m) \int x^2 \exp(-x^2)\,dx$

$1 = A(2k_BT/m)\ \frac{\sqrt{\pi}}{4}$

$A = 2m/\sqrt{\pi}k_BT$

b) $\langle v^2 \rangle = \int v^2 \mathcal{P}(v)\,dv$

$\left(\frac{2m}{\sqrt{\pi}k_BT}\right) \int v^4 \exp(-mv^2/2k_BT)\,dv$

let $x^2 = mv^2/2k_BT$

$v^2 = 2k_BTx^2/m$

$= \left(\frac{2m}{\sqrt{\pi}k_BT}\right)\left(\frac{4k_B^2T^2}{m^2}\right) \int x^4 \exp(-x^2)\,dx$

$= \frac{8k_BT}{m} \cdot 3\frac{\sqrt{\pi}}{8}$ 　　 Then $\langle K \rangle = \frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}k_BT$

2.18 a) $x(t) = (mv_0/c_1) \cdot [1 - \exp(-c_1 t/m)]$

$x'(t) = (mv_0/c_1)[0 + c_1/m \cdot \exp(-c_1 t/m)$

$= v_0 \exp(-c_1 t/m) = v_0 \exp(t/t_{visc})$

b) $x(t) = (v_0/k) \cdot \ln(1+kt)$

$$x'(t) = v_0/k \cdot \frac{1}{1+kt} \cdot k$$

$= v_0/(1+kt)$

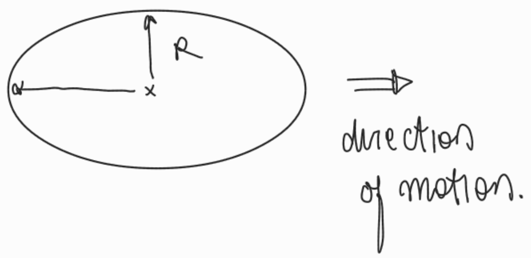
substituting $k = c_2 v_0/m$ returns expected result.

x reaching a limiting value means $v_0 \to 0$ as $t \to 0$:

$\lim_{t\to 0} v(t) = \lim_{t\to 0} v_0/(1+kt) = v_0$

Unless v_0 is zero, x will NOT reach a limiting value.

2.19.



let l x be the length of the ellipsoid.

we have to compute the prefactor to: $F = c_1 v$.

to do so, we have to compute the contribution of all the normal vectors in the direction of $\vec{v}$ (direction of motion)

2.20 $F = a v^{1/2}$, initial speed $= v_0$

$m \dot{v} = a v^{1/2}$

$m \frac{dv}{dt} = a v^{1/2}$

$\int m v^{-1/2} \, dv = \int a \, dt$

$2 m v^{\frac{1}{2}} = a t + c$

$v^{\frac{1}{2}} = \frac{a}{2m} t + c$

Initial conditions:

$v_0^{1/2} = \frac{a}{2m}(0) + c$

$c = v_0^{1/2}$

Now integrate to get $x(t)$:

$\int v(t) \, dt = \int \left(\frac{a}{2m} t + v_0^{1/2} \right)^2 dt$

$= \int \frac{a^2}{4m^2} t^2 \, dt + \int v_0^{1/2} \frac{a}{m} t \, dt + \int v_0 \, dt$

$= \frac{3}{4} \frac{a^2}{m^2} t^3 + \frac{2a}{m} v_0^{1/2} t^2 + v_0 t$ (assuming $x_0 = 0$)

2.21 $F = cv^{3/2}$

$m\dot{v} = cv^{3/2}$

$\int m v^{-\frac{3}{2}} dv = \int c\, dt$

$-2mv^{-\frac{1}{2}} = ct + c_0$

ln

Chapter 3

3.1 The height varies via θ: $z = \frac{\theta}{2\pi} \cdot H$ where $H = 3\mu m$

This is just an exercise to compute the path length:

for each $d\theta$, the length is:

and $dz = \frac{H\,d\theta}{2\pi}$

Hence, the distance is: $\sqrt{(dz)^2 + (r\,d\theta)^2}$

$$= \sqrt{\left(\frac{H\,d\theta}{2\pi}\right)^2 + (r\,d\theta)^2}$$

$$= d\theta\sqrt{(H/2\pi)^2 + r^2}$$

Therefore: $L = \int_0^{2\pi} \sqrt{(H/2\pi)^2 + r^2}\; d\theta$

$$= 2\pi\sqrt{(H/2\pi)^2 + r^2} =$$

The angle it makes is: $\tan\left(\frac{dz}{r\,d\theta}\right)$

$$= \tan\left(\frac{H\,d\theta}{2\pi}(r\,d\theta)^{-1}\right)$$

$$= \tan\left(\frac{H}{2\pi r}\right) =$$

3.2. persistence length: $\xi = 2mm$, where $\kappa_f = \beta\xi$

what is the energy to bend microtubule of length 20cm into an arc of radius 10 cm? Using Eq 3.15:

$$E = \kappa_f \frac{\theta^2}{2L_c} \quad \text{where } R_c = \frac{L_c}{\theta} \Rightarrow \theta = \frac{L_c}{R_c}$$

Thus we can compute the energy:

$$E = \frac{1}{k_B T}\xi \cdot \frac{L_c}{2R_c^2}$$

$$=$$

$L_c = 200$ mm

$R_c = 100$ mm

$T = 300\,K$

$\xi = 2$

3.3 Determine $\langle\theta^2\rangle^{1/2}$

We have that: $\langle\theta^2\rangle = \frac{2s}{\beta\kappa_f}$

3.4 What is ξ of spaghetti @ 300K? $\kappa_f = \beta\xi$ and $\kappa_f = Y I$

Additionally, spaghetti is a solid cylinder w/ $I = \pi R^4/4$

Therefore we can compute $\xi = \kappa_f/\beta = YI/\beta = Y(\pi R^4/4)k_B T$

$R = 1\times10^{-3}$ m, $Y = 1\times10^{8}\ J/m^3$, $T = 300K$

3.5 flagella are hollow cylinder. Inner radius: 0.07 μm

outer radius: 0.10 μm

$T = 300K$ $\quad Y = 1 \times 10^8 \, J/m^3$

follow same formula as above, but I is different:

$I = \pi(R^4 - R_i^4)/4$ for hollow cylinder.

3.6 mass : "rigidity"

we can consider rigidity as energy required to bend.

mass of cylinder of length l:

$\rho \cdot \pi r^2 L$ for solid cylinder

$\rho(\pi r^2 - \pi r_i^2) L$ for hollow cylinder.

$E = Y I L_c / 2R_c^2$, where

$I = \pi r^4/4$ for solid cylinder

$I = \pi(r^4 - r_i^4)/4$ for hollow cylinder

it is the same material so Y is the same.

ratio solid: $\dfrac{\rho \pi r^2 L}{Y(\pi r^4/4)L/2R_c} = \dfrac{8R_c\rho}{Y r^2}$

ratio hollow: $\dfrac{\rho\pi(r^2 - r_i^2)L}{Y(\pi(r^4 - r_i^4)/4)L/2R_c} = \dfrac{8R_c\rho}{Y(r^2 + r_i^2)}$

Now divide solid by hollow:

$$\frac{\frac{8R_c\rho}{\gamma r^2}}{\frac{8R_c\rho}{\gamma(r^2+r_i^2)}} = \frac{\gamma(r^2+r_i^2)}{\gamma r^2} = 1 + \frac{r_i^2}{r^2}$$

we see that solid has a better ratio.

→ One solid one is more efficient than any number of hollow ones: the ratios will be the same regardless.

i.e: $\frac{Nm}{NE} = \frac{m}{E}$.

But of course many hollow ones will have better rigidity compared to one.

3.7. Contour length = 32,980 nm.

DNA persistence length = 53 ± 2 nm

If DNA is 2mm in diameter, we can use that information and treat it like a self avoiding chain.

For a random chain, we treat persistence length as the step size: $\langle r^2 \rangle = Nb^2$ (b is step size)

3.12 $\langle \theta^2 \rangle^{1/2} \sim 0.1$

$l = 100\,\mu m$

$d = 5\,\mu m.$

$Y = 10^9\ J/m^3 = \frac{10^9}{10^{18}} J/\mu m = 10^{-9}\ J/\mu m$

(a) We can use the small oscillation approximation:

$\langle \theta^2 \rangle \simeq 2s/\beta \kappa_f$ where $\kappa_f = Y I$ and $I = \pi r^4/4$

$$= \frac{2(100)}{\frac{1}{k_B T}(10^{-9}\,\mu m)\pi(2.5)^4/4}$$

$$= \frac{2 \cdot 100 \cdot 4 \cdot k_B T \cdot 10^9}{\pi(2.5)^4}$$

(b) $F_{buckle} = \pi^2 Y I / L_c^2$

$$= \pi^2(10^{-9})(\pi(2.5)^4/4)/(100)^2$$

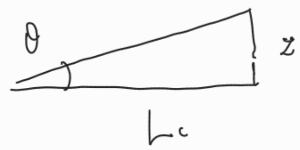
$$= \frac{\pi^3(2.5)^4}{10^9 \cdot 4 \cdot 100^2}$$

3.13 $z(x) = (F/\varphi \mathcal{I}) \cdot (Lx^2/2 - x^3/6)$ $\xi = 2mm$

substitute $\xi = \frac{\varphi I}{\beta} \Rightarrow \varphi \mathcal{I} = \frac{\xi}{k_B T}$

(a) $z(5 \times 10^{-9}) = (0.5 \times 10^{-12} / (2 \times 10^{-3} / (k_B \cdot 294))) \cdot 2 (5 \times 10^{-9})^3 / 6$

(b) angular displacement :



we are assuming here that L_c is equal to the horizontal length.

$\tan\theta = \frac{z}{L_c}$

$\theta = \tan^{-1}(z/L_c)$

(c) Using eq for $\langle\theta^2\rangle$:

$\langle\theta^2\rangle = 2s/\beta k_f$ $\xi = \beta k_f \Rightarrow k_f = \frac{\xi}{\beta}$

$= 2(5 \times 10^{-9}) / 0.5 \times 10^{-12}$

Then $\langle\theta^2\rangle^{1/2} =$

3.31 We can frame this in terms of bending energy.

First: what is bending energy of the massless rod?

$$E = YIL_c / 2R_c^2$$

the force is applied at x, so we substitute x for L:

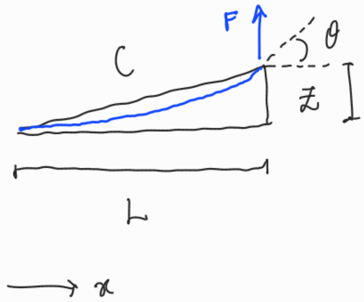
$$E = YIx / 2R_c^2$$

Now we need to compute the dependence on z. but we can also put it in terms of the arc length:

$$L_c = R_c\theta \Rightarrow R_c = \frac{L_c}{\theta} \Rightarrow R_c = \frac{x}{\theta}$$

then: $$E = YI\theta^2 / 2x$$

Now we need to compute z



C is "chord length".

$$C^2 = x^2 + z^2 \Rightarrow z^2 = C^2 - x^2$$

With the chord length, we can get the expression for energy and from that obtain force. Pulling the rod puts potential energy into the system equal to the bending energy:

$$F = \vec{\nabla} U = \frac{\partial}{\partial x} U = \frac{\partial}{\partial x} E_{bend}.$$

Then we need to rearrange for z in E_{bend}

Chapter 4.

pitch $p = 5\,\mu m$ $\qquad$ $a = 2\pi p / (4\pi^2 r^2 + p^2)$

radius $r = 0.5\,\mu m$ $\qquad$ $C = 4\pi^2 r / (4\pi^2 r^2 + p^2)$

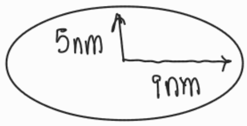
(a) substituting p and r, we obtain

$$a = 0.901 \ \mu m^{-1}$$

$$C = 0.566 \ \mu m^{-1}$$

(b) comparing to $2\pi/5 = 1.256$, the value of α from part (a) is smaller because the helix is ascending the cylinder. That means the twist is more "slow", i.e. smaller value, as it goes around than for a flat circle, which is what the value $2\pi/5$.

4.2.



← cross section of the elliptical rod.

The ellipse twists around its central axis.

(a) torsional rigidity $(\mu = 10^8\ J/m^3)$

$$K_{tor} = \mu \pi a^3 b^3 / (a^2 + b^2)$$

$$= \frac{(10^8)\pi (5\times10^{-6})^3 (9\times10^{-6})}{(5\times10^{-6})^2 + (9\times10^{-6})^2}$$

$$= 2.70 \times 10^{-13}\ Jm$$

(b) The distance covered is the pitch length for a circle w/ radius equal to semi-major axis:

height = 74 nm. (full rotation)

$\phi = 2\pi$.

i.e. the length of the cylinder is 74 nm.

$$\alpha = 2\pi / (74 \times 10^{-6})$$
$$= 8.49 \times 10^{4}$$

Then:

$$\frac{2\pi}{74 \times 10^{-6}} = 2\pi p / (4\pi^2 r^2 + p^2)$$

$$\frac{10^6}{74} \times (4\pi^2 r^2 + p^2) = 2\pi p$$

$$\frac{10^6}{74} 4\pi^2 r^2 + \frac{10^6}{74} p^2 = 2\pi p$$

$$0 = -\frac{10^6}{74} p^2 + 2\pi p - \frac{10^6}{74} 4\pi^2 r^2$$

solving for p yields:

$$p = 6.98 \times 10^{-6} \text{ or } 4.58 \times 10^{-4} \text{ m}.$$

(c) The same calculation as for (b) but using $r = 5$ nm

$p = 2.13 \times 10^{-6}$ or $p = 4.63 \times 10^{-4}$ m

4.3 $\tau = 10^{-20}$ Nm is applied to free end of 10 µm long filament

(a) solid filament w/ radius 5×10^{-12} m

$$K_{tor} = \mu \pi R^4 / 2 \quad \text{and} \quad \alpha = \tau / K_{tor}$$

$$= 10^8 \cdot \pi \cdot (5 \times 10^{-9})^4 / 2$$

$$= 9.82 \times 10^{-26}$$

(b) angle of rotation?

Then: $\alpha = \tau / K_{tor}$

$$= 10^{-20} / 9.82 \times 10^{-26}$$

$$= 1.02 \times 10^{6}$$

computing finally: ϕ from: $\alpha = \phi / L$

$$\phi = \alpha \cdot 10 \times 10^{-9}$$

$$\phi = 1.02 \times 10^{-2}$$

A very small angle.

4.6 .

Chapter 5

5.1 Eq 5.33 :

(a) $$\sigma_p = \frac{1 - 5\tau/(\sqrt{3}\,k_{sp})}{3 + \tau/(\sqrt{3}\,k_{sp})}$$ ← where is this negative?

will be negative when $1 < \dfrac{5\tau}{\sqrt{3}\,k_{sp}}$

$$\sqrt{3}\,k_{sp} < 5\tau$$

Thus the range is $\tau > \dfrac{\sqrt{3}}{5} k_{sp}$

(b) Range of the area: substitute τ into A:

area per vertex: $A = \sqrt{3}\, s^2/2$

Then the value of $s(\tau)$ is: $s_\tau = \dfrac{s_0}{1 - \tau/(\sqrt{3}\,k_{sp})}$

$$A = \frac{\sqrt{3}}{2}\left[\frac{s_0}{1-\tau/(\sqrt{3}\,k_{sp})}\right]^2$$

$$A > \frac{\sqrt{3}}{2}\left[\frac{s_0}{1 - \frac{\sqrt{3}}{5}k_{sp}/\sqrt{3}\,k_{sp}}\right]^2$$

$$A > \frac{\sqrt{3}}{2}\left[\frac{s_0}{4/5}\right]^2$$

$$A > \sqrt{3}\,s_0 \cdot 25\, s_0 / 8$$

5.2. Fig 5.18 ↙ "reduced temperature"

$$k_B T / k_{sp} s_0^2 = 0.2 \rightarrow k_B T = 0.2\, k_{sp} s_0^2$$

Is this temperature <u>above</u> a system w/ parameters:

$\mu = 5 \times 10^{-6}$ J/m² (the stream)

$s_0 = 75$ nm.

where $\mu = (\sqrt{3}\, k_{sp} / 4) \cdot (1 + \sqrt{3}\, \tau / k_{sp})$

Unsure how to relate given parameters to thermal exp.

5.3 shear modulus = 5×10^{-6} J/m²

what is K_A? (assuming 6-fold network).

$$\langle \Delta A^2 \rangle^{1/2} / A_0 \quad @ \; T = 300K$$

and use $(\beta K_A)^{-1} = \langle \Delta A^2 \rangle / \langle A \rangle$

(i) $s_0 = 75$ nm $\quad A_0 = \sqrt{3} s_0 / 2$

$$K_A = (\sqrt{3}\, k_{sp} / 2)(1 - \tau / [\sqrt{3}\, k_{sp}])$$

and $K_A = -\frac{\partial A}{\partial \tau} \frac{1}{A}$ shear modulus is:

$\mu = 5 \times 10^{-6}$

$$\mu = (\sqrt{3}\, k_{sp}/4)\cdot(1 + \sqrt{3}\,\tau/k_{sp})$$

First calculate K_A using μ...

$$1 + \sqrt{3}\,\tau/k_{sp} = 4\mu/(\sqrt{3}\, k_{sp})$$

$$\frac{1}{3} + \tau/[\sqrt{3}\, k_{sp}] = 4\mu/(3\sqrt{3}\, k_{sp})$$

$$1 - \tau/[\sqrt{3}\, k_{sp}] = -4\mu/(3\sqrt{3}\, k_{sp}) + \frac{4}{3}$$

so:

$$K_A = (\sqrt{3}\, k_{sp}/2)(-4\mu/(3\sqrt{3}\, k_{sp}) + \tfrac{4}{3})$$

$$= (\sqrt{3}\, k_{sp}/8)(-\mu/(3\sqrt{3}\, k_{sp}) + 1/3)$$

5.16

$\Pi A = N k_B T$. (2 dimensional analogue)

Recall: $K_A^{-1} = -\frac{\partial A}{\partial \Pi}\frac{1}{A}$

$\Rightarrow \quad \frac{A}{K_A} = -\frac{\partial A}{\partial \Pi}$

From eqn of state: $A = \frac{N k_B T}{\Pi}$

Then: $\frac{\partial A}{\partial \Pi} = N k_B \frac{\partial}{\partial \Pi}\left(\frac{T}{\Pi}\right)$

$= N k_B \left(-\frac{T}{\Pi^2} + \frac{T_\pi}{\Pi}\right)$

and: $\frac{\partial A}{\partial \Pi} \cdot \frac{1}{A} = N k_B \left(-\frac{T}{\Pi^2} + \frac{T_\pi}{\pi}\right) \frac{\Pi}{N k_B T}$

$= (-T/\pi + T_\pi)(1/T)$

Moreover, we know that $\partial T/\partial \Pi = T_\pi = 0$ since T is constant w/ changing pressure, as we are assuming an isothermal process.

Thus: $K_A^{-1} = -\frac{1}{A}\frac{\partial A}{\partial \Pi} = -\left[-\frac{T}{\Pi}\cdot\frac{1}{T}\right] = \frac{1}{\Pi}$

5.19 use s_t to obtain area compression by differentiation:

$$K_A^{-1} = (\partial A / \partial \tau) / A$$

stress dependent? Stress is defined: $\sigma_{ij} = \frac{\partial F}{\partial u_{ij}}$

$$s_\tau = s_0 / (1 - \tau / [\sqrt{3}\, k_{sp}])$$

s_τ is calculated by minimizing the enthalpy:

$$H = E - \tau A$$

Let's differentiate the area per vertex by τ:

$$\rightarrow \frac{\partial}{\partial \tau} A$$

$$= \frac{\partial}{\partial \tau} \sqrt{3}\, s_\tau^2 / 2 \qquad \text{where } s_\tau = s_0 / (1 - \tau / \sqrt{3}\, k_{sp})$$

$$= \frac{\sqrt{3}}{2} \cdot 2 s_\tau \cdot \frac{\partial}{\partial \tau} s_\tau$$

$$= \sqrt{3}\, s_\tau \frac{\partial}{\partial \tau} \left[\frac{s_0}{1 - \tau / \sqrt{3}\, k_{sp}} \right]$$

$$= \sqrt{3}\, s_\tau \frac{- s_0 / \sqrt{3}\, k_{sp}}{(1 - \tau / \sqrt{3}\, k_{sp})^2}$$

$$= -\sqrt{3}\, s_\tau \frac{1}{\sqrt{3}\, k_{sp}} \frac{s_0}{1 - \tau / \sqrt{3}\, k_{sp}} \cdot \frac{1}{1 - \tau / \sqrt{3}\, k_{sp}}$$

$$= -\frac{1}{k_{sp}} s_\tau \cdot s_\tau \cdot \frac{1}{1-\tau/\sqrt{3}\,k_{sp}}$$

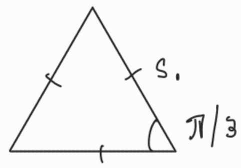
Now we can substitute this into into definition for K_A

$$K_A = -\frac{\partial A}{\partial \tau} \cdot \frac{1}{A} \qquad A = \sqrt{3}\, s_\tau^2/2$$

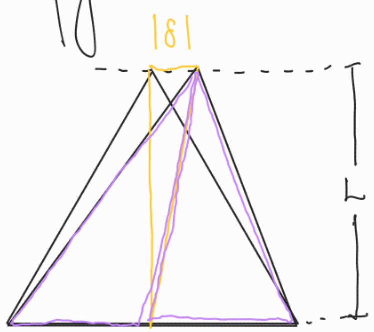
$$= \frac{1}{k_{sp}} \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{1-\tau/\sqrt{3}\,k_{sp}}$$

which matches the original expression derived for K_A

5.20. (a) In plaquette at rest, the sides are length s_0.

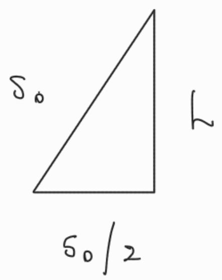


Then a small displacement by $|\delta|$ leads to the following configuration:



assume height L is fixed.

Height of an equilateral triangle:

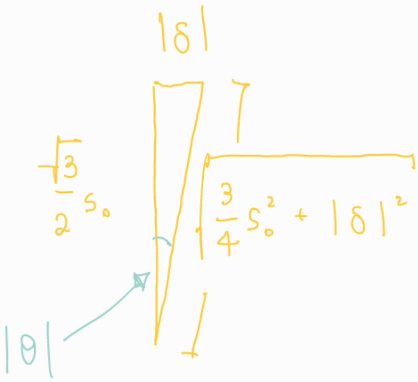


$$L^2 + \frac{S_0^2}{4} = S_0^2$$

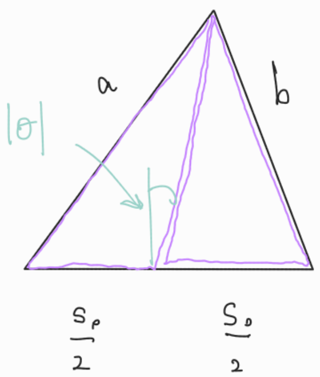
Hence: $$L^2 = \frac{3S_0^2}{4}$$

$$L = \frac{\sqrt{3}}{2} S_0$$

Thus:



Now we can start computing lengths of the sides:



we are just missing the change in angle though.
Recall cosine law:

$$c^2 = a^2 + b^2 - 2ab\cos\theta:$$

$$|\delta|^2 = \frac{3}{4} S_0^2 + \frac{3}{4} S_0^2 + |\delta|^2 - 2 \frac{\sqrt{3}}{2} S_0 \sqrt{\frac{3}{4} S_0^2 + |\delta|^2} \cos|\theta|$$

$$\frac{3}{2} s_0^2 = \sqrt{3}\, s_0 \sqrt{\frac{3}{4} s_0^2 + |\delta|^2}\, \cos|\theta|$$

$$\frac{\sqrt{3}}{2} s_0 \left(\frac{3}{4} s_0^2 + |\delta|^2\right)^{-\frac{1}{2}} = \cos|\theta|$$

$$|\theta| = \cos^{-1}\left[\frac{\sqrt{3}/2 \cdot s_0}{\sqrt{\frac{3}{4} s_0^2 + |\delta|^2}}\right]$$

$$a^2 = \frac{s_0^2}{4} + \frac{3}{4} s_0^2 + |\delta|^2 - 2\frac{s_0}{2}\sqrt{\frac{3}{4} s_0^2 + |\delta|^2}\, \cos\left(\frac{\pi}{2} + |\theta|\right)$$

we are throwing away all 2^{nd} order terms in $|\delta|$

$$a^2 = s_0^2 - s_0\sqrt{\frac{3}{4} s_0^2 + |\delta|^2}\, \cos\left(\frac{\pi}{2} + |\theta|\right)$$

A similar equation is for the other side, but the angle is $\cos\left(\frac{\pi}{4} - |\theta|\right)$.

Chapter 6.

6.1 $\rho^* = 3\,mg/ml$

$r = 5\ \mu m.$

(a) $T = 300\,K$

$\rho^* k_B T = 1.242 \times 10^{-20}$ J % density of water.

need to convert ρ^* to ρ: 3% density of water.

(b) eq D35: $1/\beta K_V = \langle (\Delta V)^2 \rangle / V_0$

where $K_V = \rho k_B T$.

also: $\beta = 1/k_B T$ so D35 is simply:

$$1/\rho = 1/(3\ mg/ml)$$

6.2. compression modulus @ 0 tension:

$$K_V = k_{sp}/8b.$$

(a) $s = 4\,nm$ find k_{sp} ...

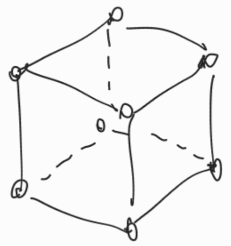
$a = 1.3\ nm.$

we have that $V_{\text{per vertex}} = a^2 b$

$$K_v = \frac{k_{sp}}{8b} \cdot (1 - 4\tau b / k_{sp})$$

6.29. We are given volume of a section is:

$4a^2b.$



← one section.

Chapter 7

7.1 Total length of a saturated hydrocarbon:

$\theta_{bond} = 109.5°$

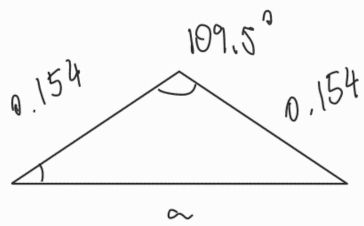
w/ linear zig-zag configuration.

$C-C = 0.154$

$C-H = 0.107$

all: C-C chains.

so I just need the base length of the isosceles triangle.



$$a^2 = 0.154^2 + 0.154^2 + 2(0.154)(0.154)\cos(109.5)$$

$$= 2 \cdot 0.154^2 (1 + \cos 109.5)$$

$$= 0.0316$$

Thus for $n = 14$, length is $0.0316 \times 7 = 0.221$

$n = 20$, length is $0.0316 \times 10 = 0.316$

7.2 S_{gas}/k_B for ideal gas: $PV = nRT$

$$\left.\begin{aligned} T &= 273\text{ K} \\ P &= 1\text{ atm} \end{aligned}\right\} \text{STP}$$

$$S_{gas} = k_B \left\{ \frac{5}{2} - \ln\left[\rho \left(\frac{h}{\sqrt{2\pi m k_B T}} \right)^3 \right] \right\}$$

we are dealing w/ H_2 molecules, so we can get m.

This is compared to free energy: $\frac{PV}{k_B T} = \frac{nRT}{k_B T}$

$$= nR/k_B$$

7.3 Eq. 7.6:

$$p_{agg} = \left[\frac{\sqrt{2\pi m k_B T}}{h} \right]^3 e^{5/2} \, e^{-\frac{2\pi n_c R_{hc} l_{cc} \gamma}{k_B T}}$$

$T = 300\text{ K}$ and lipid mass $= 500$ Da

7.5 How much energy to bend flat membrane into a cylinder?

Assuming $\kappa_b = \kappa_G = 10 k_B T$

Energy density is: $\mathcal{F} = \left(\frac{\kappa_b}{2}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^2 + \frac{\kappa_G}{R_1 R_2}$

we substitute the parameters directly:

$$R_1 = 3\,\mu m$$
$$R_2 = 1\,\mu m$$

this eq is applicable because a "sphero-cylinder" is pretty much an ellipsoid.

$$\mathcal{F} = \left(\frac{10 k_B T}{2}\right)\left(\frac{1}{3} + \frac{1}{1}\right)^2 + \frac{10 k_B T}{(3)(1)}$$

$$= 10 k_B T \left[\frac{16}{9} + \frac{1}{3}\right] = \frac{190}{9} k_B T$$

Hence energy density equals $\frac{190}{9}$ per $k_B T$

7.16 Thin & solid sheet of thickness t.

$$u_{xx} = u_{yy} = -\frac{P}{Y}(1-\sigma_p)$$

$$u_{zz} = \frac{2P\sigma_p}{Y}$$

Prove 3D energy density is

$$\Delta\mathcal{F}_{3D} = P^2(1-\sigma_p)/Y.$$

I will use $\mu = \frac{1}{2}\frac{Y}{1+\sigma_p}$ and $K_v = \frac{Y}{3(1-2\sigma_p)}$

$$\Delta\mathcal{F} = \frac{K_v}{2}\cdot(\text{tr}\, u)^2 + \mu\sum(u_{ij} - \frac{\delta_{ij}}{3}\text{tr}\, u)^2$$

substituting in the values from above. Additionally, we know $\text{tr}\, u = u_{xx} + u_{yy} + u_{zz}$, and

$\sum_{ij} \cdots = (u_{xx} - \text{tr}\, u)^2 + \cdots$ since $\delta_{ii} = 1$

$$\Delta\mathcal{F} = \frac{1}{2}\frac{Y}{3(1-2\sigma_p)}(u_{xx} + u_{yy} + u_{zz})^2$$

$$+ \frac{1}{2}\frac{Y}{1+\sigma_p}\sum_{ij}(u_{ij} - \delta_{ij}\text{tr}\, u)^2$$

The expression for $(u_{xx} + u_{yy} + u_{zz})^2$:

$$= \left(-\frac{2P}{Y}(1-\sigma_p) + \frac{2P\sigma_p}{Y}\right)^2$$

$$= \frac{4P^2}{Y^2}(1-\sigma_p)^2 - \frac{8P^2}{Y^2}\sigma_p(1-\sigma_p) + \frac{4P^2\sigma_p^2}{Y^2}$$

$$= \frac{4P^2}{Y^2}\left[1 - 2\sigma_p + \sigma_p^2 - 2\sigma_p + 2\sigma_p^2 + \sigma_p^2\right]$$

$$= \frac{4P^2}{Y^2}\left[1 - 4\sigma_p + 4\sigma_p^2\right] = \frac{4P^2}{Y^2}(1-2\sigma_p)^2$$

substituting this back... the first term becomes.

$$\frac{1}{3}\frac{2P^2}{Y}(1-2\sigma_p)^2$$

Then the next term, we have $\sum(u_{ij} - \delta_{ij}\,\mathrm{tr}\,u)^2$

$$(u_{xx} - \mathrm{tr}\,u)^2 = u_{xx}^2 - 2u_{xx}\mathrm{tr}\,u + (\mathrm{tr}\,u)^2$$

$$= \frac{P^2}{Y^2}(1-\sigma_p)^2 + \frac{2P}{Y}(1-\sigma)\frac{2P}{Y}(1-2\sigma_p) + \frac{4P^2}{Y^2}(1-2\sigma_p)^2$$

$$= \frac{P^2}{Y^2}\left(1 - 2\sigma_p + \sigma_p^2 + 4 - 12\sigma_p + 8\sigma_p^2 + 4 - 16\sigma_p + 16\sigma_p^2\right)$$

$$= \frac{P^2}{Y^2}\left(9 - 30\sigma_p + 25\sigma_p^2\right)$$

The expression is the same for u_{yy} then, u_{zz}:

$$= \frac{4P^2\sigma_p^2}{Y^2} - \frac{4P}{Y}\sigma_p\frac{2P}{Y}(1-2\sigma_p) + \frac{4P^2}{Y^2}(1-2\sigma_p)^2$$

$$= \frac{4p^2}{\gamma^2}\left(\sigma_p^2 - 2\sigma_p + 4\sigma_p^2 + 1 - 4\sigma_p + 4\sigma_p^2\right)$$

$$= \frac{p^2}{\gamma^2}\left(1 - 28\sigma_p + 920\sigma_p^2\right)$$

Adding it all together:

$$= \frac{p^2}{\gamma^2}\left(22 - 88\sigma_p + 82\sigma_p^2\right)$$

This is not a factor of $(1+\sigma_p)$ so the equation doesn't reduce nicely.

Chapter 8

8.1 $h(x, y) = h_0 e^{-\frac{x^2+y^2}{2w^2}}$

approximate deformation energy:

$$E = \pi \kappa_b \left(\frac{h_0}{w}\right)^2$$

(a) substitute $E = k_B T$, $w = 1$ and $\kappa_b = 10 k_B T$

$$k_B T = \pi (10 k_B T) \left(\frac{h_0}{1}\right)^2$$

$$1 = 10\pi h_0^2$$

$$h_0 = \sqrt{\frac{1}{10\pi}}$$

deformation energy is $k_B T$ when $h_0 = \sqrt{(10\pi)^{-1}}$

(b) bending contribution to deformation energy?

$$E = \pi (30 k_B T) \left(\frac{0.05}{1}\right)^2 = \pi \frac{30}{400} k_B T$$

Hence, the answer is $\pi \frac{30}{400}$

8.2.

$$E = 2\pi^2 K_b \left(\frac{2\pi h_0}{\lambda} \right)^2$$

(a) assume $E = k_B T$, $\lambda = 1$ and $\kappa_b = 20 k_B T$

substituting: $k_B T = 2\pi^2 (20 k_B T) \left(\frac{2\pi h_0}{1} \right)^2$

$$1 = 8\pi^4 \cdot 20 \cdot h_0^2$$

$$h_0 = \sqrt{(160\pi^4)^{-1}}$$

Thus the value of h in this scenario is

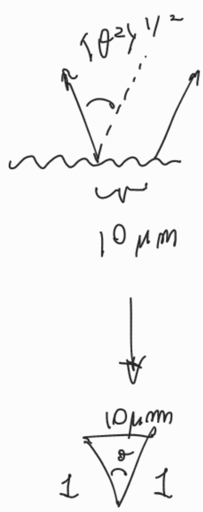
$$h_0 = \sqrt{(160\pi^4)^{-1}}$$

(b)

8.3. Suppose we observe RMS:

$$\langle \theta^2 \rangle^{1/2} = 1/10 \quad , \quad b = 1.$$

for a distance of 10 μm



$$\cos\theta = 1 - \theta^2/2$$

$\xi_p = ?$

$$\xi_p \sim e^{\frac{2\pi \kappa_b}{k_B T}}$$

$\dfrac{\kappa_b}{k_B T} = ?$

8.4. The persistence length is:

$$\xi_p \sim e^{\frac{2\pi \kappa_b}{k_B T}}$$

dividing by different factors:

$$\frac{e^{2\pi}}{e^{2\pi \cdot 10}} = e^{2\pi(1-10)}$$

Hence the persistence length would change by a factor of $e^{-\frac{18\pi}{10}}$

8.11 Eq 8.3 $$\hat{n} = \frac{(-h_x, -h_y, 1)}{(1+h_x^2+h_y^2)^{1/2}}$$

Eq 8.15 $$\frac{\partial \hat{n}}{\partial x}, \quad \frac{\partial \hat{n}}{\partial y}$$

where: $\vec{r} = [x, y, h(x,y)]$, $h_x = \frac{\partial h}{\partial x}$, $h_y = \frac{\partial h}{\partial y}$

$$\frac{\partial \hat{n}}{\partial x} = \frac{\partial}{\partial x}\left[\frac{(-h_x, -h_y, 1)}{(1+h_x^2+h_y^2)^{1/2}}\right]$$

$$= \frac{\frac{\partial}{\partial x}(-h_x, -h_y, 1)}{(1+h_x^2+h_y^2)^{1/2}} - \frac{1}{2}\frac{(-h_x, -h_y, 1)}{(1+h_x^2+h_y^2)^{3/2}} \times \frac{1}{(2h_{xx}+2h_{yx})}$$

$$= \frac{-h_{xx}(1+h_x^2+h_y^2) \times (2h_{xx}+2h_{yx}) + \frac{1}{2}h_x, \; -h_{xy}(1+h_x^2+h_y^2) \times (2h_{xx}+2h_{xy}) + \frac{1}{2}h_y, \; -\frac{1}{2}}{(1+h_x^2+h_y^2)^{3/2}(2h_{xx}+2h_{yx})}$$

Likewise for $\frac{\partial \vec{n}}{\partial y}$

Thus achieving the desired form

8.14 $x^2 + y^2 + z^2 - R^2 = 0$ w/ $z > 0$

(hemispherical surface)

(a) $h(x,y) = ?$ and $\vec{n}(x,y) = ?$ using Eq. 8.3

Eq 8.3:

$$\vec{n} \equiv \frac{\partial_x \vec{r} \times \partial_y \vec{r}}{|\partial_x \vec{r} \times \partial_y \vec{r}|} = \frac{(-h_x, -h_y, 1)}{\sqrt{1 + h_x^2 + h_y^2}}$$

where $\partial_x \vec{r} = (1, 0, h_x)$

$\partial_y \vec{r} = (0, 1, h_y)$

and z is the "height" from xy-plane

For the hemisphere, h is simply z. We can isolate for x and y:

$$h(x,y) = z(x,y) = R^2 - x^2 - y^2$$

Then $h_x = z_x = \frac{\partial}{\partial x} z(x,y)$

$$= \frac{\partial}{\partial x}(R^2 - x^2 - y^2)$$

$$= 2x$$

similarly: $h_y = z_y = 2y$

Then we can substitute to get the expression for $\hat{n}$:

$$\hat{n}(x,y) = \frac{(2x,\ 2y,\ 1)}{\sqrt{(2x)^2 + (2y)^2 + 1}}$$

$$= \frac{(2x,\ 2y,\ 1)}{2\sqrt{x^2 + y^2 + 1/4}}$$

(b) Mean & Gaussian curvatures from Eq 8.11 and Eq. 8.12

$b_{\alpha\beta}$, $g_{\alpha\beta}$?

Eq. 8.11: $$\frac{C_1 + C_2}{2} = \frac{g_{xx} b_{yy} + g_{yy} b_{xx} - 2 g_{xx} b_{xy}}{2g}$$

8.12: $$C_1 C_2 = \frac{b_{xx} b_{yy} - b_{xy}^2}{g}$$

g is the metric: $g = 1 + h_x^2 + h_y^2 = 1 + 4x^2 + 4y^2$

$= (\partial_x \vec{r}) \cdot (\partial_y \vec{r})$

$\partial_x \vec{r} = (1,\ 0,\ 2x)$

$\partial_y \vec{r} = (0,\ 1,\ 2y)$

$$g_{xx} = (1, 0, 2x) \cdot (1, 0, 2x) = 1 + 4x^2$$

$$g_{yy} = (0, 1, 2y) \cdot (0, 1, 2y) = 1 + 4y^2$$

$$g_{xy} = (1, 0, 2x) \cdot (0, 1, 2y) = 4xy$$

$$b_{\alpha\beta} = \hat{n} \cdot (\partial_\alpha \partial_y \hat{r}) = -(\partial_\alpha \vec{r}) \cdot (\partial_\beta \hat{n})$$

$$\partial_x \hat{n} = \frac{2(x^2 + y^2 + 1/4) - 2x^2}{2(x^2 + y^2 + 1/4)^{3/2}} = \frac{y^2 + 1/4}{(x^2 + y^2 + 1/4)^{3/2}}$$

(only x-component is non zero)

$$\partial_y \hat{n} = \frac{x^2 + 1/4}{(x^2 + y^2 + 1/4)^{3/2}}$$

(only y-component is nonzero)

$$b_{xx} = -\partial_x \hat{n}$$

$$b_{yy} = -\partial_y \hat{n}$$

$$\left.\begin{aligned} b_{xy} &= 0 \\ b_{yx} &= 0 \end{aligned}\right\} \text{ cross components are zero}$$

finally, we can do substitutions:

$$\frac{C_1 + C_2}{2} = \frac{g_{xx} b_{yy} + g_{yy} b_{xx} - 2 g_{xx} b_{xy}}{2g}$$

$$C_1 C_2 = \frac{b_{xx} b_{yy} - b_{xy}^2}{g}$$

$$\frac{g_{xx} b_{yy} + g_{yy} b_{xx} - 2 g_{xy} b_{xy}}{2g}$$

$$= \frac{(y^2 + 1/4)(1 + x^2) + (x^2 + 1/4)(1 + y^2)}{2(1 + 4x^2 + 4y^2)(x^2 + y^2 + 1/4)^{3/2}}$$

$$= \frac{5y^2 + 10x^2 y^2 + 8}{32(x^2 + y^2 + 1/4)^{5/2}}$$

and

$$\frac{b_{xx} b_{yy} - b_{xy}^2}{g} = \frac{(x^2 + 1/4)(y^2 + 1/4)}{(1 + 4x^2 + 4y^2)(x^2 + y^2 + 1/4)^3}$$

Chapter 9

9.1 Negatively charged plate w/ $\sigma_s = 0.3\ C/m^2$

$\varepsilon = 80\,\varepsilon_0$

$T = 300K$

$q = +e$

(a) l_B , χ, $\rho(z=0)$

spatial distribution of counterions lying distance z to one side:

$$\rho(z) = [2\pi l_B (z+\chi)^2]^{-1}$$

and $l_B = q^2 / 4\pi\varepsilon k_B T$

$\chi = 2\varepsilon k_B T / (-q\sigma_s)$

The parameters can be substituted to get l_B and χ

$$l_B = \frac{e^2}{4\pi(80\varepsilon_0)k_B(300)} = \frac{e^2}{4.163\times10^{-18}}$$

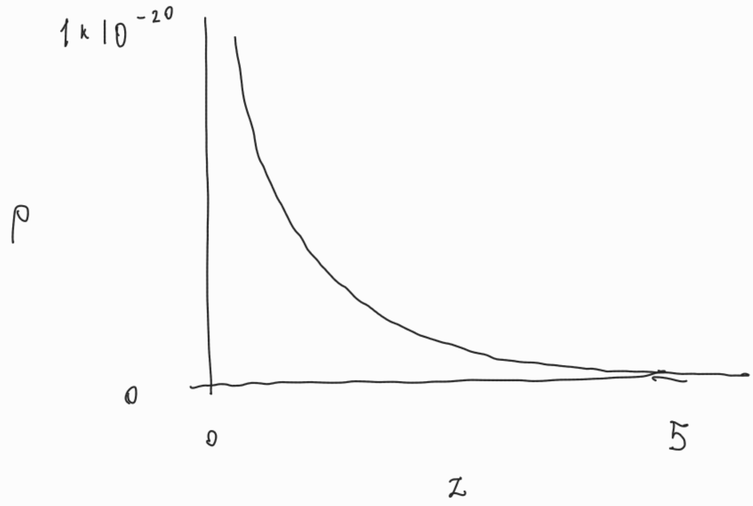
$\varepsilon_0 = 8.854\times10^{-12}\ Fm^{-1}$

$e = 1.602\times10^{-19}\ C$

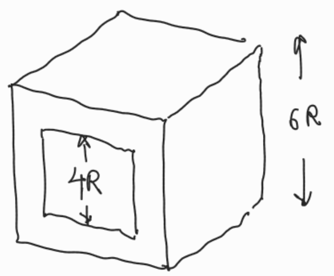
$k_B = 1.3806\times10^{-23}$

$$\chi = \frac{2(80\varepsilon_0)k_B(300)}{-e\cdot 0.3} = \frac{2.20896\times10^{-18}}{-e}$$

(b)



9.7



Deformation energy compared to a sphere?

from 9.18: deformation energy $\Delta E / \kappa_b$ is 5/2 of a sphere

This result is used w/o proof.

Compare w/ adhesion energy per cell:

$K_b = 10^{-19}$ J

$W_{ad} = 10^{-5}$ J/m^2

$R = 3\,\mu m$

9.11 $\quad \psi(z) = \psi(z)\, e^{-z/l_D}$ where $l_D = \dfrac{1}{\sqrt{8\pi l_B \rho_s}}$

$l_B = q^2/4\pi\varepsilon k_B T$

$$\rho(z) = \frac{1}{2\pi l_B (z+\chi)^2} \qquad \chi = \frac{2\varepsilon k_B T}{-q\sigma_s}$$

(a) $$\rho(z=0) = \frac{1}{2\pi l_B \chi^2} = \frac{4\pi\varepsilon k_B T}{2\pi q^2 \chi^2}$$

$$= \frac{q^2 \sigma_s^2\, 4\pi\varepsilon k_B T}{2\pi q^2\, 4\varepsilon^2 k_B^2 T^2}$$

$$= \frac{\sigma_s^2}{2\varepsilon k_B T}$$

(b) The fraction of free counterions can be determined from evaluating the integral:

$$P_{free} = \int_0^{\chi} \rho(z) \Big/ \int_0^{\infty} \rho(z)$$

indeterminate integral:

$$\int \rho(z)\,dz = \int \frac{1}{2\pi l_B (z+x)^2}\,dz = \frac{1}{2\pi l_B} \int (z+x)^{-2}\,dz$$

$$= -\frac{1}{2\pi l_B}(z+x)^{-1}$$

so: $$\int_0^\infty \rho(z)\,dz = \left[-\frac{1}{2\pi l_B}(z+x)^{-1}\right]_{z=0}^{z=\infty}$$

$$= \frac{1}{2\pi l_B x}$$

and: $$\int_0^x \rho(z)\,dz = \left[-\frac{1}{2\pi l_B}(z+x)^{-1}\right]_{z=0}^{z=x}$$

$$= -\frac{1}{4\pi l_B x} + \frac{1}{2\pi l_B x}$$

$$= \frac{1}{4\pi l_B x}$$

dividing these: $$\int_0^x \rho(z)\,dz \Big/ \int_0^\infty \rho(z)\,dz$$

$$= \frac{1}{2}$$ Hence: half is in $0 < z < x$

Chapter 10

10.1. $K_V = 3 \times 10^9 \, J/m^3$

(i) spherical cell of radius $10 \, \mu m$.

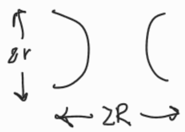
(ii) spherical weather balloon of radius $10 \, m$.

K_A and K_V: $K_A \sim K_V d_p$

10.2. Bending energy:

$\kappa_b = 20 k_B T \qquad \kappa_g = 0$

From table 10.1, the shape conforms to III:



$$E = \pi \kappa_b (-8 + \pi p) - 4\pi \kappa_g$$

$$= \pi (20 k_B T)(-8 + \pi p)$$

Therefore, the total bending energy would be

$$3500 \times \pi (20 k_B T)(-8 \pi p)$$

10.3 4 pancakes to and bottom w/ radius 10r

and middle w/ radius 30r

Results from table 10.1:

$$C_0 = 0$$
$$\kappa_b = 20 k_B T$$
$$\kappa_G = 0$$
$$T = 300\ K$$

We can use the first, since a pancake is flat w/ rounded edges.

$$I: \quad E = \pi \kappa_b \left(8 + \pi \frac{R}{r} \right) + 4\pi \kappa_G$$

$$\frac{R}{r} >> 1$$

$$A = 2\pi r^2 \left(\pi \frac{R}{r} + 2 \right)$$

$$V = 2\pi r^3 \left(\frac{\pi R}{2r} + \frac{2}{3} \right)$$

The necks b/w the pancakes have a different geometry:

But we dont have any geometry with this specific shape so we can just reuse I.

The bending energy is the sum of bending energies of all the components:

$\frac{R}{r}$ for top $= 10$

for middle $= 30$

for connection $= 5$

$$E_{top} = \pi K_b (8 + \pi \cdot 10) + 4\pi K_G$$

$$E_{md} = \pi K_b (8 + \pi 30) + 4\pi K_G$$

$$E_{con} = \pi K_b (8 + \pi 5) + 4\pi K_G$$

add up all contributions:

$$E_{total} = 4E_{top} + 3E_{md} + 4E_{conn}$$

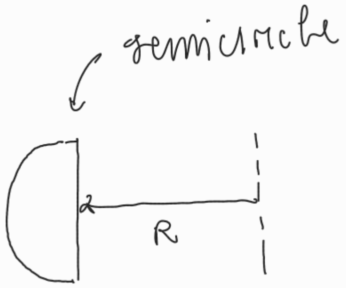
$$= 48\pi K_G + 88\pi K_b + 150\pi^2 K_b$$

10.10 Prove that pancake shape surface area and volume are:

$$A = 2\pi r^2 [(R/r)^2 + \pi R/r + 2]$$

$$V = 2\pi r^3 [(R/r)^2 + (\pi/2) R/r + \tfrac{2}{3}]$$

These can both be solved via integration: the edges will be determined individually and then added.

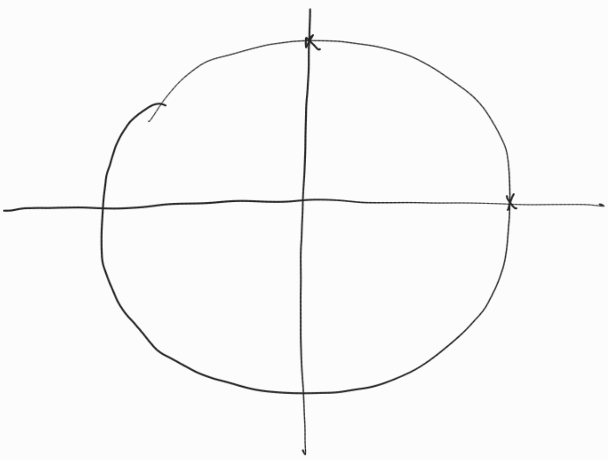


for area, we want to compute dA all the way around.

easiest way to compute it is vertically:

eq of circle perimeter at bottom: $2\pi R$

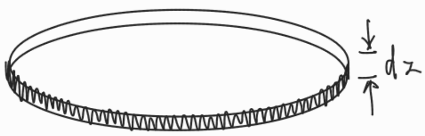
Then this increases w/ z to $2\pi(R+r)$. The equation of this change is from eq of a circle:



$$x^2 + y^2 = r^2$$

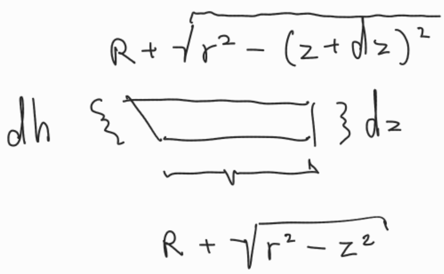
$$x = \sqrt{r^2 - y^2}$$

so we have: $dA = 2\pi\left(R + \sqrt{r^2 - z^2}\right) dz$



circumference:

$2\pi r$



determine dh.

$$\Downarrow$$

$$\sqrt{r^2-(z+dz)^2}-\sqrt{r^2-z^2}$$

$$dh^2 = dz^2 + \left[\sqrt{r^2-(z+dz)^2} - \sqrt{r^2-z^2}\right]^2$$

$$= dz^2 + r^2 - (z+dz)^2$$
$$+ 2\sqrt{r^2-(z-dz)^2}\sqrt{r^2-z^2} + r^2 - z^2$$

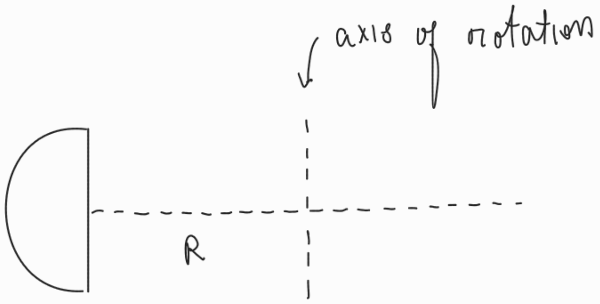
$$= dz^2 + r^2 - z^2 + 2zdz - dz^2$$
$$+ 2\sqrt{r^2-(z-dz)^2}\sqrt{r^2-z^2} + r^2 - z^2$$

$$= 2r^2 - 2z^2 + 2zdz$$
$$+ 2\sqrt{(r^2-(z-dz)^2)(r^2-z^2)}$$

$$= 2r^2 - 2z^2 + 2zdz$$
$$+ 2\sqrt{r^4 - r^2(z-dz)^2 - r^2z^2 + (z-dz)^2z^2}$$

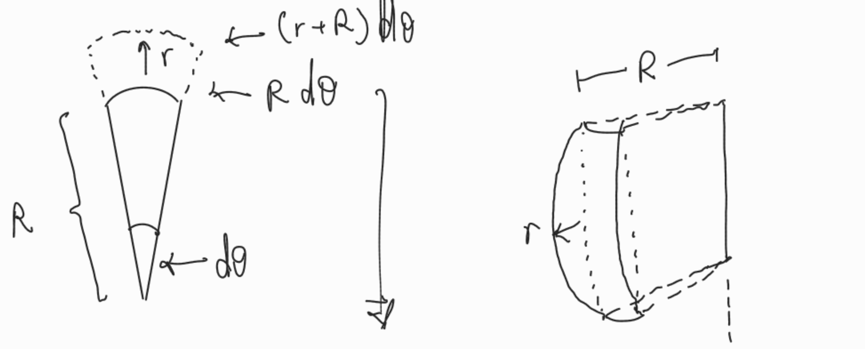
$$= 2r^2 - 2z^2 + 2zdz$$
$$+ 2\sqrt{\begin{matrix} r^4 - r^2z^2 + 2r^2zdz - r^2dz^2 \\ - r^2z^2 + z^4 - 2z^3dz + z^2dz^2 \end{matrix}}$$

This form is not correct though, and there is potentially another approach to constructing the integral



take dA to be the semicircle perimeter: πr

Then integrate this around the entire pancake. Must determine dA with respect to θ.



$(R+\sqrt{r^2-z^2})\,d\theta$

But this is also quite challenging to construct.

Therefore, we will do the straightforward approach.

then: $A = 2 \cdot \int_0^r 2\pi\left(R + \sqrt{r^2 - z^2}\right) dz$

$$= 4\pi \Big[\int_0^r R\, dz + \underbrace{\int_0^r \sqrt{r^2 - z^2}\, dz}_{\text{using wolfram alpha to solve}} \Big]$$

$$= 4\pi \left\{ Rr + \frac{1}{2}\left[z\sqrt{r^2 - z^2} + r^2 \operatorname{atan}\left(\frac{z}{\sqrt{r^2 - z^2}} \right) \right]_{z=0}^{z=r} \right\}$$

$$= 4\pi \left(Rr + \frac{\pi r^2}{4} \right)$$

$$= 4\pi Rr + \pi^2 r^2$$

Now we just add the center: $2\pi R^2$

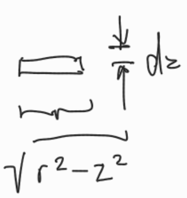
$$A = 4\pi Rr + \pi^2 r^2 + 2\pi R^2$$

$$= 2\pi r^2 \left[\frac{2R}{r} + \frac{\pi}{2} + \frac{R^2}{r^2} \right]$$

which matches the expression

For volume we follow a very similar process.

$$dV = dA \cdot \sqrt{r^2 - z^2}$$



So we have:

$$V = 2\int_0^r 2\pi \left(R + \sqrt{r^2 - z^2}\right)\sqrt{r^2 - z^2}\, dz$$

$$= 4\pi \int_0^r R\sqrt{r^2 - z^2} + (r^2 - z^2)\, dz$$

$$= 4\pi R \frac{\pi r^2}{4} + 4\pi\left(\frac{2r^3}{3}\right)$$

$$= \pi^2 R r^2 + \frac{8}{3}\pi r^3$$

Then add the center:

$$V = \pi^2 R r^2 + \frac{8}{3}\pi r^2 + \pi R^2 (2r)$$

$$= 2\pi r^3 \left[\frac{\pi}{2}\frac{R}{r} + \frac{4}{3} + \frac{R^2}{r^2} \right]$$

which also matches the expression for volume.

Chapter 11

11.1. 5 μm.

$[M] = 10\ \mu M$ length $= 8 nm$.

$$\frac{dn}{dt} = +k_{on}[M] - k_{off}$$

$k_{on} = 8.9 \pm 0.3$

$k_{off} = 44 \pm 14$

so solving this:

$$dn = (10(+k_{on}) - k_{off})\, dt$$

$$n = (10(+k_{on}) - k_{off})t$$

we have $n = \dfrac{5\ \mu m}{8\ nm} = \dfrac{5000}{8} = 625$

thus: $t = \dfrac{625}{10(+k_{on}) - k_{off}}$

$= 4.6\ s$

when it collapses. then we can use the same eq with different constants:

$k_{on} = 0 \qquad k_{off} = 733$

$$\Rightarrow \quad t = \frac{625}{0 - 733} = 0.85 \text{ s.}$$

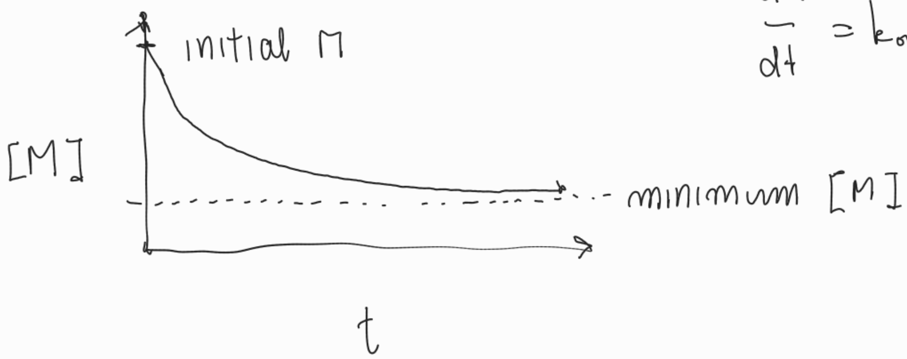
11.11. Actually should be Fig. 11.12.

(a) The intersection means that the microtubule is neither growing nor shrinking. It is the transition b/w shrinking and growing regimes. Filament length is constant at intersection.

(b) When the two lines are parallel, the -ve end is growing at the same rate +ve is shrinking. or vice versa.

(c) If $dn/dt > 0$, this means that material is accumulating. Therefore, if this is the case at the same point of an intersection, then filament must be widening

11.12. (a)



$$\frac{dn}{dt} = k_{on}[M] - k_{off}$$

(b) The equation at $t=0$ is the initial monomer value, M_0.

(c) The asymptotic value is $[M]_{t\to\infty} = \frac{k_{off}}{k_{on}}$

since here $\frac{dn}{dt} = 0$.