SC9502B – Scientific Computing. Winter 2024

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Assignment 5. Due: Tue. Apr. 9, 2024 by 23:59.
See the template(s) for presenting the results + all answers should be returned using GitHub.
Each problem is of equal value.
Use the lecture notes on random numbers.

1. Construct an interpolating polynomial for the function $f(x) = e^{-x^2}$ with 11 nodes ($n = 10$) using the SciPy `barycentric_interpolate` function for each of the choice of nodes given below on the interval $[-1, 1]$. For each case, plot the original 11 points sampled from $f(x)$ (as symbols) and at least 100 points of the interpolating function. Also, plot a separate graph of the error ($|f(x) - p_{10}(x)|$) using at least 100 points.

   a) Evenly spaced points
   b) Chebyshev nodes

   $$x_i = \cos\left(\frac{2i+1}{2n+2}\pi\right)$$

   where $n = 0, 1, \ldots 10$.

2. Write your own function to construct a natural cubic spline. It should return arrays or lists of the coefficients a_j, b_j, c_j, and d_j for the splines given an input set of points x_j and function values $f(x_j)$. You can use the SciPy `scipy.linalg.solve_banded` function. Test your function by comparing it to the SciPy `interp1d` function for the function $f(x) = e^{-x^2}$.