Esercitazione 7

26·10·21

Ripasso spazio $\mathbb{R}^n$

$$\mathbb{R}^n = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{pmatrix}, x_i \in \mathbb{R}, i = 1, \dots, n \right\}$$

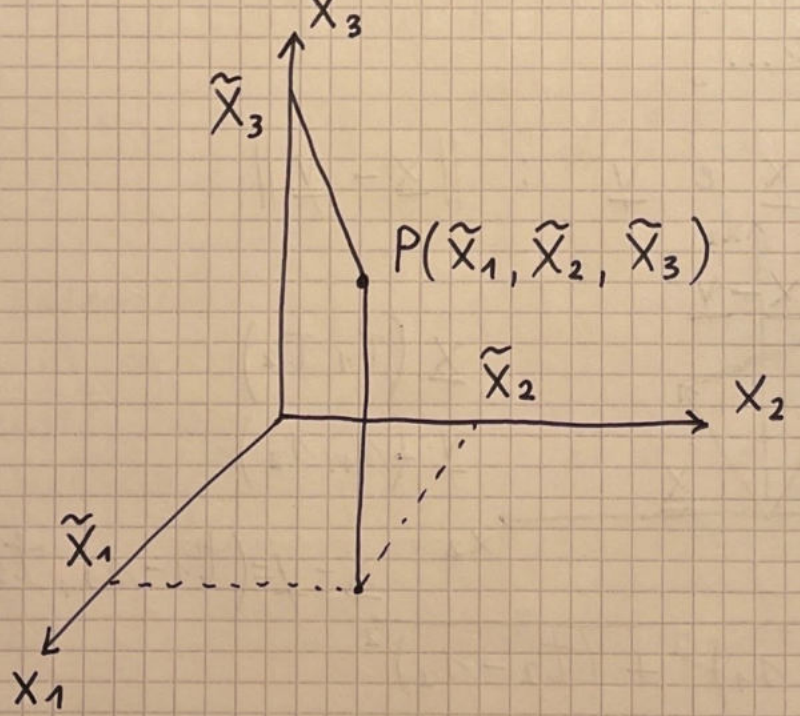
ordine rilevante
$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \neq \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\mathbb{R}^1 = \mathbb{R} = \{ x \mid x \in \mathbb{R} \}$$

$$\mathbb{R}^2 = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, x_1, x_2 \in \mathbb{R} \right\}$$

vettore a due dimensioni (bidimensionale)

$$\mathbb{R}^3 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, x_1, x_2, x_3 \in \mathbb{R} \right\}$$



PRODOTTO SCALARE $\underline{x} \in \mathbb{R}^n$, $\underline{y} \in \mathbb{R}^n$

$$\underline{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{pmatrix} \quad \underline{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_n \end{pmatrix} \quad i = 1, \dots, n$$

$$\underline{x} \cdot \underline{y} = \langle \underline{x}, \underline{y} \rangle = \underbrace{x_1 y_1 + x_2 y_2 + \ldots + x_i y_i + \ldots + x_n y_n}_{\in \mathbb{R}}$$

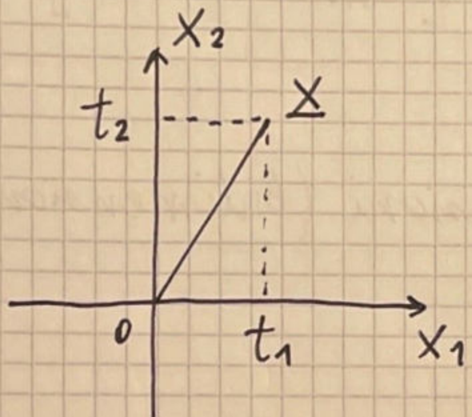
$$= \sum_{i=1}^{n} x_i y_i$$

NORMA DI UN VETTORE

dist. della punta del vettore dall'0

$\underline{x} \in \mathbb{R}^n$

$$\|\underline{x}\| = \sqrt{\langle \underline{x}, \underline{x} \rangle} = \sum_{i=1}^{n} x_i^2 = x_1^2 + x_2^2 + \ldots + x_n^2$$

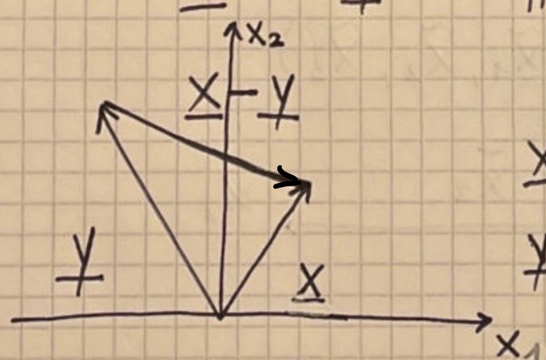


$\underline{x} = (t_1, t_2)$ $\quad \|\underline{x}\| = \sqrt{t_1^2 + t_2^2}$

$\|\underline{x}\| = \text{distanza}(\underline{x}; \underline{0})$

[in $\mathbb{R}^3$ uguale...]

→ distanza tra $\underline{x}$ e $\underline{y}$: $\|\underline{x} - \underline{y}\|$



$\underline{x}\,(t_1, t_2)$

$\underline{y}\,(s_1, s_2)$

$\underline{x} - \underline{y} = (t_1 - s_1, t_2 - s_2)$

$$\|\underline{x} - \underline{y}\| = \sqrt{(t_1 - s_1)^2 + (t_2 - s_2)^2}$$

$\underline{x} = (x_1, \ldots, x_i, \ldots, x_n) \qquad i = 1, \ldots, n$

$\underline{y} = (y_1, \ldots, y_i, \ldots, y_n)$

$\underline{x} - \underline{y} = (x_1 - y_1, \ldots, x_i - y_i, \ldots, x_n - y_n)$

$$\|\underline{x}-\underline{y}\| = \sqrt{(x_1-y_1)^2+\ldots+(x_i-y_i)^2+\ldots+(x_n-y_n)^2}$$

$$= \sqrt{\sum_{i=1}^{n}(x_i-y_i)^2}$$

essendo quadrati sono invertibili x e y

$$\text{distanza}\ (\underline{x},\underline{y}) = \|\underline{x}-\underline{y}\| = \|\underline{y}-\underline{x}\|$$

Dato $\underline{x}_0 \in \mathbb{R}^n$; $r>0$; $B_r(\underline{x}_0)=\{\underline{x}\in\mathbb{R}^n : \text{dist}(\underline{x},\underline{x}_0)<r\}$

$\|\underline{x}-\underline{x}_0\|<r$

$B_r(\underline{x}_0)$ PALLA/ SFERA centrata in $\underline{x}_0$ di raggio r

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per $n=1$ $(x_0\in\mathbb{R})$

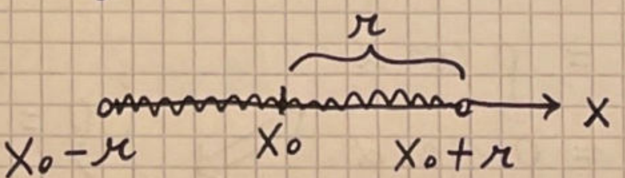
$$B_r(\underline{x}_0)=\{x\in\mathbb{R} : \text{dist}(x,x_0)<r\}$$

essendo in $\mathbb{R}$ sono punti non vettori $\leftarrow$

$$\|x-x_0\|<r$$

$$|x-x_0|<r$$

$$x_0-r<x<x_0+r$$



$\rightarrow$ insieme aperto, intorno di x_0

per $n=2$ [...] $\|\underline{x}-\underline{x}_0\|<r$

vettori

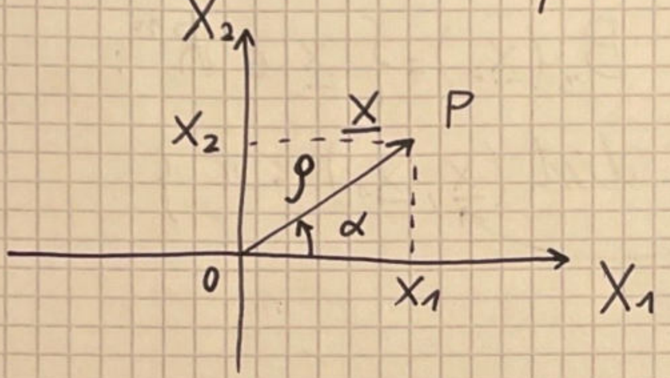
$\rightarrow$ cerchio di raggio r... privato della circonf. est.

per $n=3 \rightarrow$ sfera senza superficie esterna

$\underline{x} \in \mathbb{R}^2 \qquad \underline{x} = (x_1, x_2) \qquad x_1, x_2 \in \mathbb{R}$

$\downarrow$

coord. cartesiane del punto P del piano



$P \leftrightarrow \underline{x}\,(x_1, x_2)$

corrispondenza biunivoca

COORDINATE POLARI di P

$P(\rho, \alpha) \qquad \rho = \text{distanza } (P, \text{origine}) \geq 0$

α: angolo formato dal semiasse positivo delle ascisse e dal segmento PO

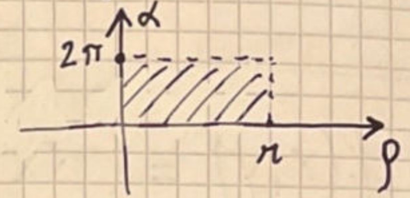
$\rho \in \mathbb{R}^+ = [0, +\infty)$

$\alpha \in [0, 2\pi) \rightsquigarrow$ giro in senso antiorario

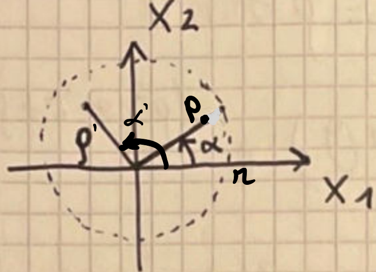
$$\begin{cases} x_1 = \rho \cos \alpha \\ x_2 = \rho \sin \alpha \end{cases} \Longleftrightarrow \begin{cases} \rho = \sqrt{x_1^2 + x_2^2} = \|\underline{x}\| \\ \alpha = \text{non univoco} \end{cases}$$

$\mathbb{R}^2 \rightarrow$ $\boxed{\text{RIASSUNTO}}$

$\left\{(\rho, \alpha) \in [0, r) \times [0, 2\pi) \; ; \; 0 \leq \rho < r \; ; \; 0 \leq \alpha < 2\pi \right\}$



$B_r(0)$ <
- coord. polari
- coord. cartesiane $= \left\{ \underline{x} \in \mathbb{R}^2 : \|\underline{x} - \underline{0}\| < r \; ; \; r > 0 \right\}$

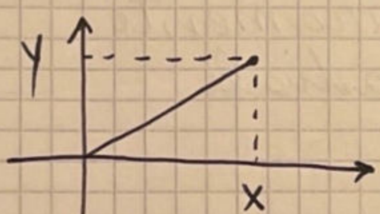


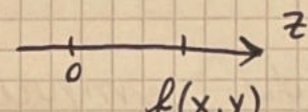
$((\alpha = \theta))$

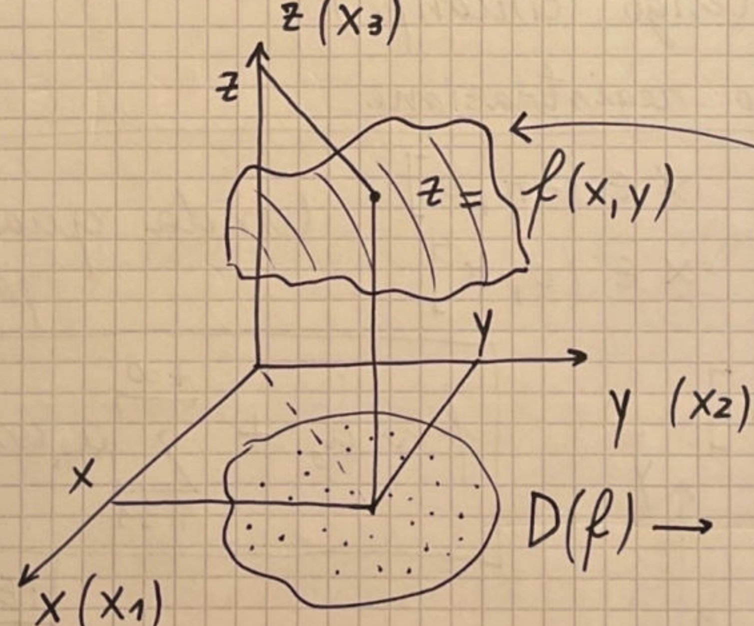
$\cdot) \; f(x, y) = (\log x)(\tan y) = z$

$\circledast$ ($D(f)$ insieme di strisce orizzontali)

$(x, y) \longrightarrow \boxed{f} \longrightarrow z$







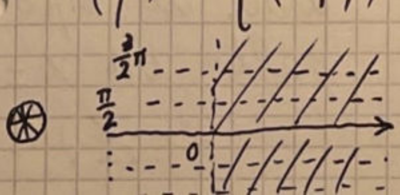
$D(f) \rightarrow$ se sollevassi tutti i punti del dominio ad altezze diverse

$\downarrow$

superficie tridimension.

$D(f) = \left\{ (x, y) \in \mathbb{R}^2 : \; x > 0, \; y \neq \frac{\pi}{2} + K\pi \right\}$

$\circledast$



$$z = f(\rho,\theta)\,(\log(\rho\cdot\cos\theta))(\tan(\rho\cdot\sin\theta))$$

• $f(x,y) = (\alpha x^2 + \beta y^2)\,e^{-(x^2+y^2)}$

$\alpha, \beta \in \mathbb{R}$ $\qquad \begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$

$$f(\rho,\theta) = (\alpha\rho^2\cos^2\theta + \beta\rho^2\sin^2\theta)\,e^{-(\rho^2\cos^2\theta+\rho^2\sin^2\theta)}$$

$$= \rho^2(\alpha\cos^2\theta + \beta\sin^2\theta)\,e^{-\rho^2(1)}$$

se $\alpha = \beta \in \mathbb{R}$

$$f(\rho,\theta) = f(\rho) = \alpha\rho^2 e^{-\rho^2}$$

funzione puramente radiale

(sezionando ~~sempre~~ la superficie con piani $\perp$ all'asse z ~~funz.~~ ottengo circonf.)

x → fin qui no registrazione

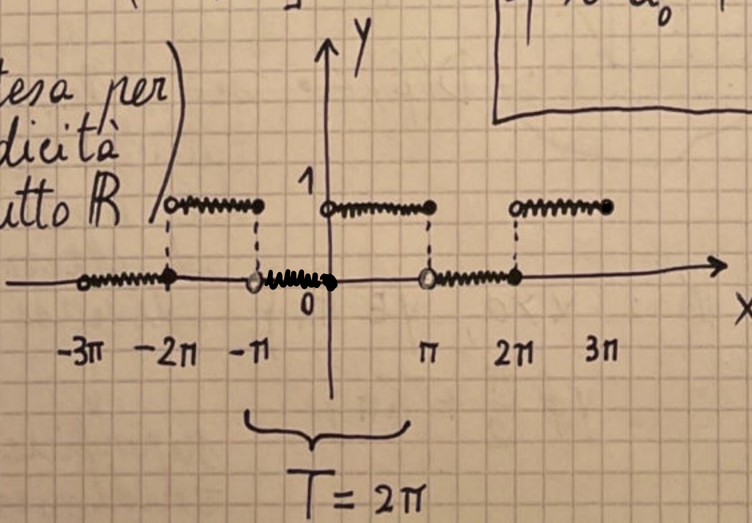
• $f(x) = \begin{cases} 0 & x \in (-\pi, 0] \\ 1 & x \in (0, \pi] \end{cases}$ (onda quadra)

$D(f) = (-\pi, \pi]$

$$\boxed{f \sim a_0 + \sum_{n=1}^{+\infty} \overset{\text{pari}}{a_n\cos nx} + \overset{\text{dispari}}{b_n\sin nx}}$$

PERIODO $2\pi = T$

(f estesa per periodicità in tutto $\mathbb{R}$)



disc. di tipo salto in $x = k\pi$ $(k \in \mathbb{Z})$