COGNOME - NOME   CODICE PERSONA

1] $\lim_{\rho \to 0} \frac{\rho^5 \cos^2\vartheta \sin^3\vartheta}{\rho^{2\alpha}} = \lim_{\rho \to 0} \rho^{5-2\alpha} \cos^2\vartheta \sin^3\vartheta =$

$= \begin{cases} 0 & \alpha < \frac{5}{2} \\ \nexists & \alpha \geq \frac{5}{2} \end{cases}$

2] $\frac{\partial f}{\partial x}(0,0) = \lim_{t \to 0} \frac{f(t,0) - f(0,0)}{t} = 0$

$\frac{\partial f}{\partial y}(0,0) = \lim_{t \to 0} \frac{f(0,t) - f(0,0)}{t} = \lim_{t \to 0} \frac{t - 0}{t} = 1$

3] f certamente differenziabile in $R^2 \setminus \{(0,y);\, y \in R\}$

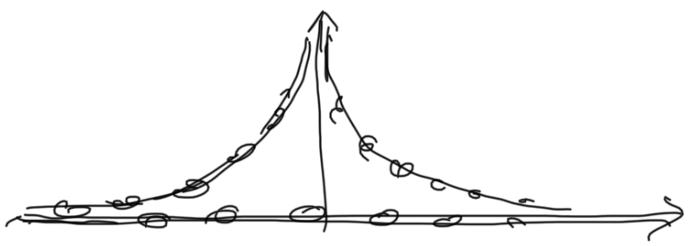
su $\{(0,y)\}$: $f(0,y) = y \Rightarrow \nexists$ max o min.

per $x \neq 0$ $\nabla f(x,y) = \left(-\operatorname{sign} x \cdot y^2,\ 1 - 2|x|y\right)$ -

$\frac{\partial f}{\partial x}(x,y) = 0 \iff y = 0$ ma $y = 0 \Rightarrow \frac{\partial f}{\partial y}(x,0) = 1$

$\nexists$ punti stazionari

4] $y - |x|y^2 = 0 \iff y = 0 \ \vee\ y = \frac{1}{|x|}$



5) $$\frac{\partial F_1}{\partial y}(x,y) = 3ax^2 + 2bxy = 2xy - 3x^2 = \frac{\partial F_2}{\partial x}(x,y)$$

$\iff a = -1 \wedge b = 1$ (R^2 sempl. connesso)

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6) $$U(x,y) = \int F_1(x,y)\,dx = \frac{x^5}{5} - x^3y + \frac{x^2y^2}{2} + k(y)$$

$k'(y) = -y^4 \Rightarrow U(x,y) = \frac{x^5}{5} - x^3y + \frac{x^2y^2}{2} - \frac{y^5}{5}$ quindi

$$\int_\gamma F(x,y)\cdot d\underline{r} = U(2,1) - U(0,0) = \frac{1}{5}$$

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7) $\lim_{n\to+\infty} \sqrt[n]{\frac{2^n-1}{3^n+1}} = \frac{2}{3} \Rightarrow R = \frac{3}{2}$ - per $x = 1-\frac{3}{2}$, $x = 1+\frac{3}{2}$

si ha risp $\sum_0^{+\infty} \frac{2^n-1}{3^n+1}\left(-\frac{3}{2}\right)^n$ e $\sum_0^{+\infty} \frac{2^n-1}{3^n+1}\left(\frac{3}{2}\right)^n$ - NON convergenti perché $|a_n| \to 1$ - ins. conv $\left(-\frac{1}{2}, \frac{5}{2}\right)$ aperto

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8) 

$$\int_0^1 \int_0^{1-x} (x+2y)\,dy\,dx = \int_0^1 \left(x(1-x) + (1-x)^2\right)dx = \int_0^1 (1-x)\,dx$$

$$= \frac{1}{2}$$

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9) Variabili separabili $f(t) = 2\sqrt[3]{t}$ $f \in \mathcal{C}(R)$

$g(y) = \sqrt[3]{y}$ $g \in \mathcal{C}^1(R \setminus \{0\})$ quindi

esiste soluz locale unica $\forall (x_0, y_0)$ con $y_0 \neq 0$

10] $\dfrac{dy}{\sqrt[3]{y}} = 2\sqrt[3]{t}\,dt \qquad \dfrac{3}{2}y^{\frac{2}{3}} = 2\cdot\dfrac{3}{4}t^{\frac{4}{3}} + c$

$y(0) = 0 \Rightarrow c = 0$ quindi $y = t^2$

11] $\lambda^2 - 2\lambda - 3 = 0 \qquad \lambda = -1 \vee \lambda = 3 \qquad \varphi(t) = c_1 e^{-t} + c_2 e^{3t}$

$\psi_0(t) = At + B \qquad -2A - 3At - 3B = t$

$A = -\frac{1}{3} \wedge B = \frac{2}{9} \qquad \psi(t) = c_1 e^{-t} + c_2 e^{3t} - \frac{t}{3} + \frac{2}{9}$

12] $|\cos x|$ pari $\Rightarrow b_1 = 0$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |\cos x|\,dx = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} \cos x\,dx = \frac{4}{\pi}$$

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} |\cos x| \cos x\,dx = \frac{1}{\pi}\left(\int_0^{\frac{\pi}{2}} \cos^2 x\,dx - \int_{\frac{\pi}{2}}^{\pi} \cos^2 x\,dx\right) = 0$$

13] f regolare a tratti (continua a tratti ed esistono in ogni punto almeno pseudoderivata destra e sinistra) – Σ converge puntualmente a $\frac{1}{2}\left(f^+(x) + f^-(x)\right)$, quindi a $f(x)$ dove f è continua.

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