COGNOME - NOME CODICE PERSONA

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1

$$\int_{\tilde{Q}} \frac{1}{\rho^2} \rho \, d\rho \, d\vartheta \qquad \tilde{Q} = \{(\rho, \vartheta) : 1 \le \rho \le \sqrt{2} \ \wedge \ \frac{\pi}{6} \le \vartheta \le \frac{\pi}{2}\}$$

$$= \int_{\pi/6}^{\pi/2} d\vartheta \int_1^{\sqrt{2}} \frac{1}{\rho} \, d\rho = \frac{\pi}{3} \cdot \frac{1}{2} \log 2 = \frac{\pi}{6} \log 2$$

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2

Integrabile per serie su $[0,1]$ - $\sin x^2 = \sum_0^{+\infty} (-1)^k \frac{(x^2)^{2k+1}}{(2k+1)!}$ conv tot su $[0,1]$

$$\int_0^1 \sin x^2 \, dx = \sum_0^{+\infty} (-1)^k \int_0^1 \frac{x^{4k+2}}{(2k+1)!} = \sum_0^{+\infty} (-1)^k \frac{1}{(4k+3)(2k+1)!}$$ segni alterni per Leibniz

$E < |a_k| < \frac{1}{1000}$ per $\frac{1}{(4k+3)(2k+1)!} < \frac{1}{1000}$ per $k=2$ $\left(\frac{1}{11 \cdot 5!} < \frac{1}{1000}\right)$

$$\sum_{k=0}^{1} (-1)^k \frac{1}{(4k+3)(2k+1)!} = \frac{13}{42}$$

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3

Int generale $\varphi(t) = (c_1 + c_2 t) e^{-2t}$ $\quad \varphi(0) = c_1$ quindi

$c_1 = 3 \quad c_2 \in \mathbb{R}$ qualunque

infinite soluzioni

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4

Eq. di Bernoulli (ma anche eq. a variab. separabili)

Bernoulli $z(t) = \frac{1}{y(t)}$ $\quad z' = -10z + 1 \quad z(t) = c e^{-10t} + \frac{1}{10}$

$$y(t) = \frac{1}{z(t)} = \frac{10}{1 + 10 c e^{-10t}} \quad \vee \quad y(t) \equiv 0$$

5)

$\det(A - \lambda I) = (3-\lambda)(-1-\lambda) = 0 \qquad \lambda_1 = 3 \quad \lambda_2 = -1$

$\underline{\varphi}(t) = c_1 \underline{h}\, e^{-t} \to \underline{0}$ per $t \to +\infty$ infinite soluzioni

$\underline{h}$ autovettore $\quad 4h_1 + 2h_2 = 0 \qquad \underline{h} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \qquad \underline{\varphi}(t) = c_1 \begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix}$

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6)

$f$ è un polinomio quindi $f \in \mathcal{C}^\infty(\mathbb{R}^2)$

$\nabla f(2,1) = (0,2) \qquad f(2,1) = -15$

$z = 2y - 17$

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7)

$\nabla f(x,y) = (0,0) \qquad P_1 = (0,0) \qquad P_{2,3} = (\pm 2, 0)$

$H_f(x,y) = \begin{bmatrix} 12x^2 - 16 & 0 \\ 0 & 2 \end{bmatrix} \qquad |H_f(0,0)| = -32$ punto di sella

$|H_f(\pm 2, 0)| = 64 \qquad \frac{\partial^2 f}{\partial x^2}(\pm 2, 0) > 0 \qquad (\pm 2, 0)$ punti di minimo locale

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8)

$\lim_{\|(x,y)\| \to +\infty} f(x,y) = +\infty \qquad \sup_{\mathbb{R}^2} f = +\infty \qquad \Rightarrow \exists \operatorname{Min}_{\mathbb{R}^2} f \quad \Rightarrow$

$\operatorname{Min}_{\mathbb{R}^2} f = f(\pm 2, 0) = -16$

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9)

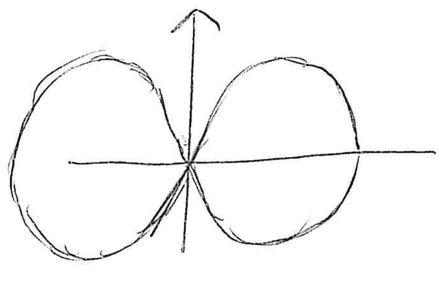
$y^2 - 8x^2 + x^4 = 0$ simmetrica rispetto agli assi.

per $x, y \geq 0$

$y = +\sqrt{x^2(8 - x^2)}$

C.E. $\quad 0 \leq x \leq \sqrt{8}$

$y(x) \sim \sqrt{8}\, x$ per $x \to 0$



**10|** $\underline{r}(t) = (t, 1-t^2) \quad t \in [0,2] \qquad \underline{r}'(t) = (1, -2t)$

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**11|** $$\int_\gamma \underline{F} \cdot (dx, dy) = \int_0^2 \underline{F}(\underline{r}(t)) \cdot \underline{r}'(t)\, dt = \int_0^2 \Big(2t(1-t^2)\cdot 1 + (1+t^2)(-2t)\Big)\, dt$$

$$= \int_0^2 (-4t^3)\, dt \left(\cancel{= -16}\right) = -16$$

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**12|** $U(x,y) = (1+x^2)y + c$

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**13|** $$(*)\begin{cases} y'' + a(t)y' + b(t)y = f(t) \\ y(t_0) = y_0 \\ y'(t_0) = y_1 \end{cases}$$

Teorema - dato il probl. di Cauchy $(*)$ se $a, b, f \in \mathcal{C}(I)$, $I$ intervallo, $\forall t_0 \in I$ $\forall y_0, y_1 \in \mathbb{R}$ esiste una e una sola soluzione di $(*)$, definita su tutto $I$.

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