ANALISI MAT. 2

3 SETTEMBRE 2018

PROF. E. MALUTA

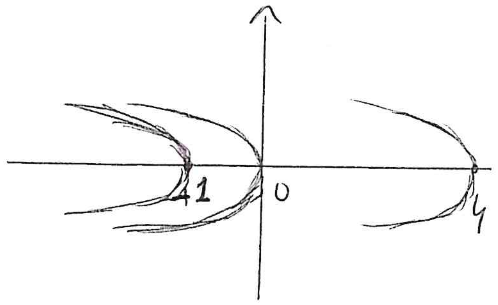
SVOLGIMENTO

1

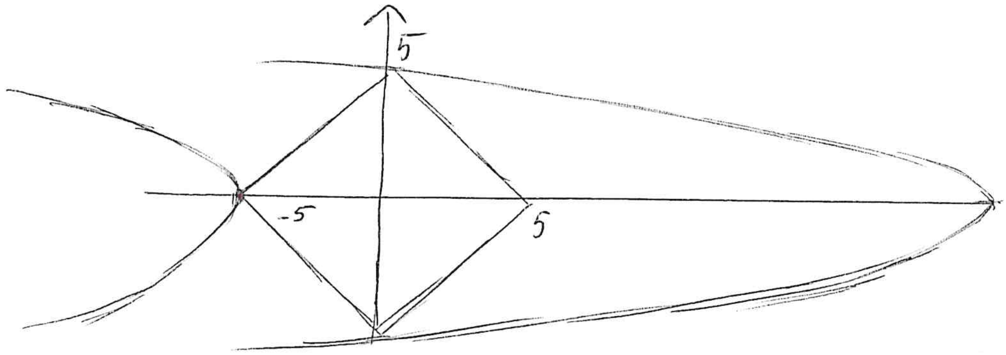
i) $k=0 \quad x=-y^2$

$k=1 \quad x=-y^2-1$

$k=-4 \quad x=-y^2+4$



ii)



curve di livello $x=-y^2-k$ passa per

$(-5,0)$ per $k=5$

$(0,5)$ e $(0,-5)$ per $k=-25$ quindi

curve di livello intersecano $D \iff -25 \le k \le 5$.

iii) $\max_D f(x,y) = 5 \ \wedge \ \min_D f(x,y) = -25$

(2) $F_\alpha(x,y,z) = (2xz, (\alpha^2-2\alpha)z^2, 6yz-\alpha x^2)$

$F_\alpha \in \mathcal{C}^\infty(R^3)$ - R^3 sempl connesso $\Rightarrow$

F_α conservativo $\Leftrightarrow$ F_α irrotazionale -

$$\frac{\partial F_\alpha^1}{\partial y}(x,y,z) = 0 \qquad \frac{\partial F_\alpha^2}{\partial x}(x,y,z) = 0$$

$$\frac{\partial F_\alpha^1}{\partial z}(x,y,z) = 2x \qquad \frac{\partial F_\alpha^3}{\partial x}(x,y,z) = -2\alpha x$$

$$\frac{\partial F_\alpha^2}{\partial z}(x,y,z) = 2(\alpha^2-2\alpha)z \qquad \frac{\partial F_\alpha^3}{\partial y}(x,y,z) = 6z$$

Deve essere $\begin{cases} \alpha = -1 \\ \alpha^2-2\alpha = 3 \end{cases}$ soddisfatta per $\alpha = -1$

$\exists\, U : R^3 \to R \; : \; \nabla U = F_{-1}$

Calcolo di U:

$$\int 2xz\, dx = x^2 z + g(y,z)$$

$dg(y,z) + \cancel{x^2 dz} = 3z^2 dy + (6yz + \cancel{x^2})dz$

$$\int 3z^2\, dy = 3z^2 y + h(z)$$ da cui $6zy + h'(z) = 6yz$

$h'(z) = 0 \Rightarrow h(z) = \text{cost.}$

$$U(x,y,z) = x^2 z + 3z^2 y + c$$

$$\tilde{U}(0,0,0) = 3 \Rightarrow c = 3$$

$$\tilde{U}(x,y,z) = x^2 z + 3z^2 y + 3$$

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(3) Eq. a variab. separabili $y' = f(t)\, g(y)$

$$f(t) = \frac{1}{t} \quad ; \quad g(y) = \frac{1+y^2}{y} \qquad f \in C(\mathbb{R} \setminus \{0\}) \quad g \in C^1(\mathbb{R} \setminus \{0\})$$

il teorema di esistenza e unicità locale vale $\forall (x_0, y_0)$

$\in \mathbb{R}^2 \setminus (\{(0,y)\} \cup \{(x,0)\})$

b) $y(t) =$ costante $\Rightarrow y'(t) = 0$ impossibile : $\nexists$ soluz. costanti

c) calcolo integrale generale

$$\int \frac{y}{1+y^2}\, dy = \int \frac{1}{t}\, dt \qquad \frac{1}{2} \log(1+y^2) = \log|t| + c$$

$$1+y^2 = e^{2c} t^2 \qquad 1+y^2 = K t^2 \quad K > 0$$

$$y(t) = \pm\sqrt{Kt^2 - 1}$$

$$y(1) = -1 \qquad -1 = -\sqrt{K-1} \Rightarrow K = 2$$

soluz $\varphi(t) = -\sqrt{2t^2-1}$ def per $t \geq \frac{1}{\sqrt{2}}$

d) $y(-1) = 1 \qquad 1 = \sqrt{K-1} \Rightarrow K = 2$

soluz $\tilde{\varphi}(t) = \sqrt{2t^2-1}$ def per $t \geq \frac{1}{\sqrt{2}}$