

Homework: Math Background

January 22, 2021

1. Let T be the rigid transformation $T(\mathbf{x}) = W \cdot \mathbf{x} + \mathbf{b}$, where $W = (w_{ij})$ is the matrix of size 2×2

$$W = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} \quad (1)$$

and $\mathbf{b} = (-2, -2)^t$. Calculate the composition $T \circ T((2, 0)^t)$.

2. Let $f(x_1, x_2) = x_1^4 + x_2^2$. What is the gradient of f at the point $(-1, 0)$?
Note: recall that if $g(x) = x^m$, $g'(x) = mx^{m-1}$
3. Let U and V be tuples (of size n) of uniformly distributed random numbers in $(0, 1)$. Say, U is generated using the random uniform method in the **random** module and V using **numpy**. Define a python function that returns a tuple performing element-wise the function $f(u, v) = \sqrt{-2 \log u} \cos(2\pi v)$. What is the empirical mean and standard deviation of the tuple defined by f for tuples of size $n = 5000$.
4. Make a histogram of the tuple defined by the function f from the previous problem. You can use the **numpy** method `np.histogram`.