Contol theory Homework #2 report. Variant F.

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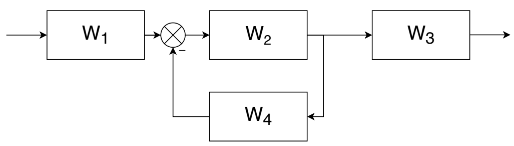
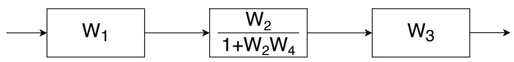
1 Task 1.

A. Calculate total transfer function

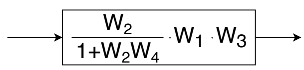
Calculate total Transfer Function of the system:



As a first step I will simplify feedback chain:



And then our system can be represented as one block:



In variant F I have:

$$W_1 = \frac{s+3}{s+1}, W_2 = \frac{1}{s+2}, W_3 = \frac{1}{s+0.1}, W_4 = \frac{1}{2s+3}$$

Substituting $W_1, ..., W_4$ to the system block: [1]

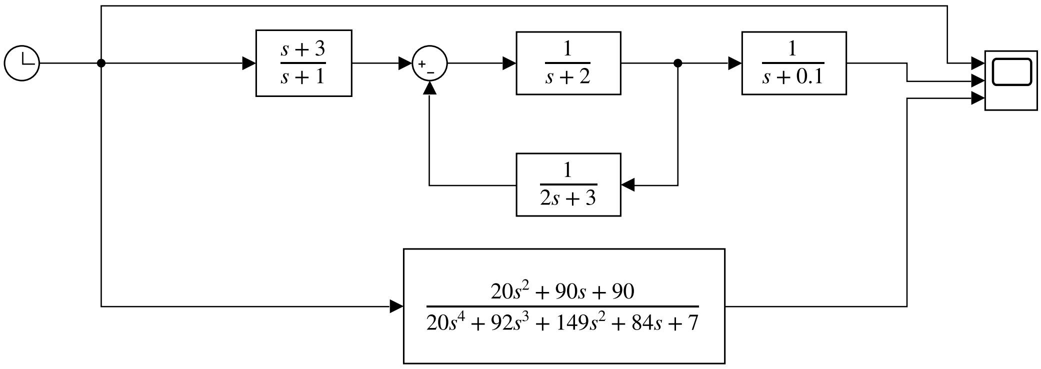
$$W_1 * W_3 * \frac{W_2}{1 + W2 * W4} = \frac{20s^2 + 90s + 90}{20s^4 + 92s^3 + 149s^2 + 84s + 7}$$

Hence, our total Transfer Function is

$$W = \frac{20s^2 + 90s + 90}{20s^4 + 92s^3 + 149s^2 + 84s + 7} \tag{1}$$

[1]This answer was get with Matlab script simplify_fraction.m in Task1 folder

B. Build initial and simplified systems and analyse responses



Simulink model for response analysis. For analysing different responses, one need to put signal source to input instead of the clock.

Step, Impulse, and Frequency responses plots. On each plot blue line is system input, red one is the initial system output, and the orange one is system output after simplification:

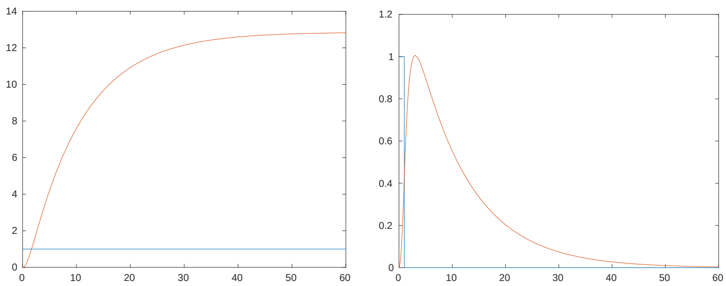


Figure 1: Step response (left) and Impulse response (right) plots

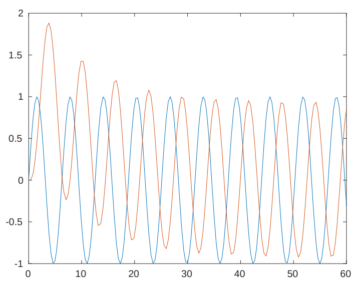
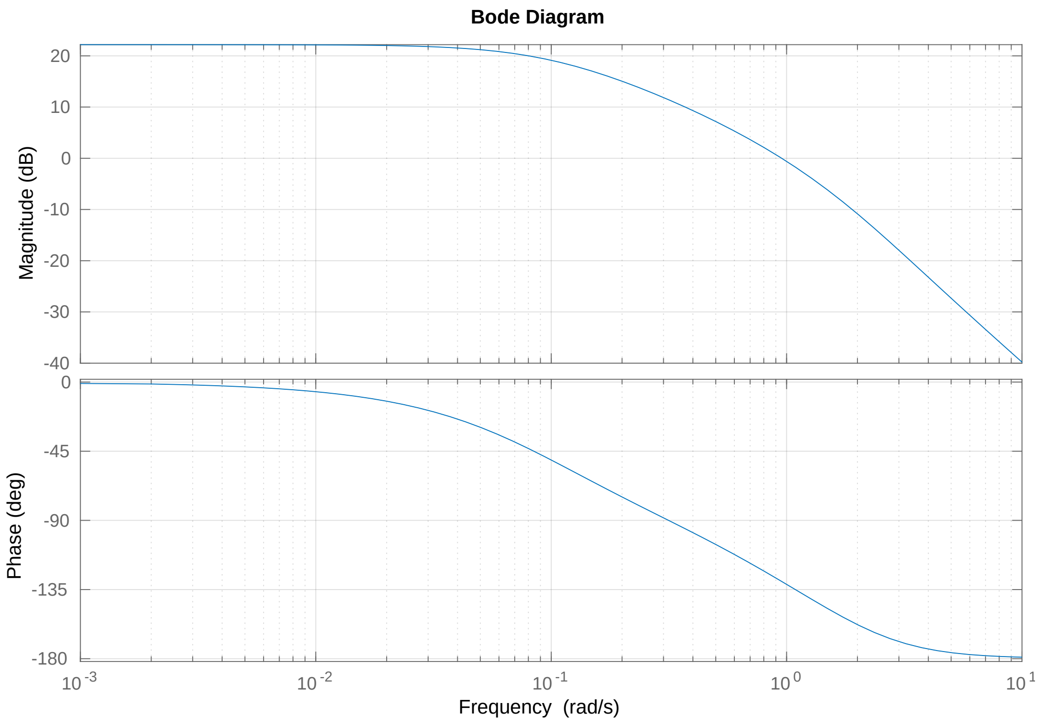


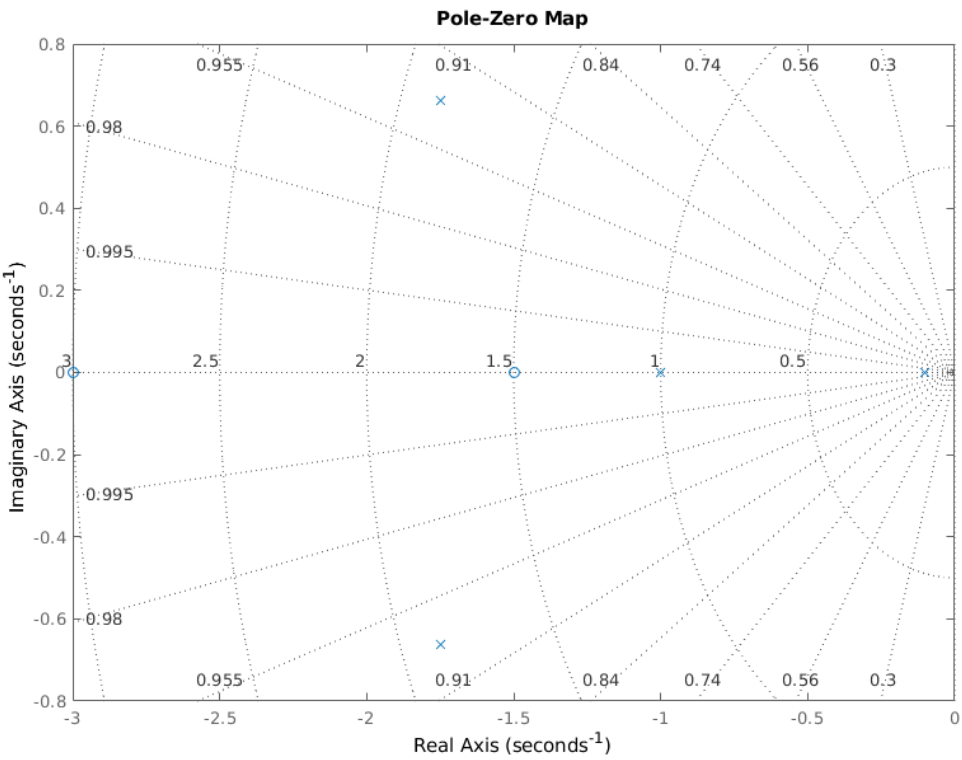
Figure 2: Frequency response plot

Note, that plots look like there is only one output, but it is not true – they are just very close to each other.

C. Bode and Pole-Zero map plots

As a input signal, for which I will generate the plots I have chosen a Frequency response (sine function).





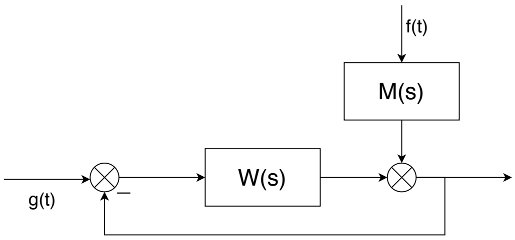
According to Matlab `pzmap()` function, our Transfer Function (1) has following poles and zeroes:

$$poles = \begin{bmatrix} -1.7500 + 0.6614i \\ -1.7500 - 0.6614i \\ -1.0000 + 0.0000i \\ -0.1000 + 0.0000i \end{bmatrix} zeroes = \begin{bmatrix} -3.0000 \\ -1.5000 \end{bmatrix}$$

As it can be seen, all our poles have negative real part, and that says us that the system is stable.

2 Task 2.

Find total Transfer Function of the system:



Firstly, let's consider case when $f(t) = 0$. In this case our Transfer Function is $\frac{W}{1+W}$. Then we'll consider case, when $g(t) = 0$, and our Transfer Function will be $M \times \frac{1}{1+W}$. Hence, total transfer function for the system in general case is

$$output = \frac{W}{1+W} \times g(t) + \frac{M}{1+W} \times f(t)$$

In variant F I have:

$$W(s) = \frac{s+1}{s^2+3s+2}, M(s) = \frac{1}{s+3}$$

After simplification of fractions with Matlab, we get

$$output = \frac{1}{s+3} \times g(t) + \frac{s+1}{(s+3)^2 \times f(t)}$$

3 Task 3.

Find transfer function of the system.
In variant F I have:

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, C = \begin{bmatrix} -1 & 4 \end{bmatrix}, D = \begin{bmatrix} 2 \end{bmatrix}$$

Using Matlab we can obtain required Transfer Function with `ss2tf()`:

```
A = [1,0;2,1];
B = [2;2];
C = [-1,4];
D = [2];
[x,y] = ss2tf(A,B,C,D);
tf(x,y);
% ans =
% 2 s^2 + 2 s + 12
% ----------------
%  s^2 - 2 s + 1
```

Hence, our Transfer Function is

$$H(s) = \frac{2s^2 + 2s + 12}{s^2 - 2s + 1}$$

Matlab's `ss2tf()` uses the same formula as we used on labs to transform state-space model to Transfer Function: $C * (sI - A)^{-1} * B + D$, where A, B, C, and D are the matrices of state-space model of the system. The other way to solve it - use the Matlab's representation of above formula, that is `C*inv(s*eye(2)-A)*B+D`, which will return vector of transfer fucntions. But to use `ss2tf()` for me is shorter and more convenient way.

4 Task 4.

Find transfer function of the system.
In variant F I have:

$$A = \begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 5 \\ 3 & 1 \end{bmatrix}, C = \begin{bmatrix} -2 & 0 \end{bmatrix}, D = \begin{bmatrix} 2 & 4 \end{bmatrix}$$

Using Matlab we can obtain required Transfer Function with `ss2tf()`:

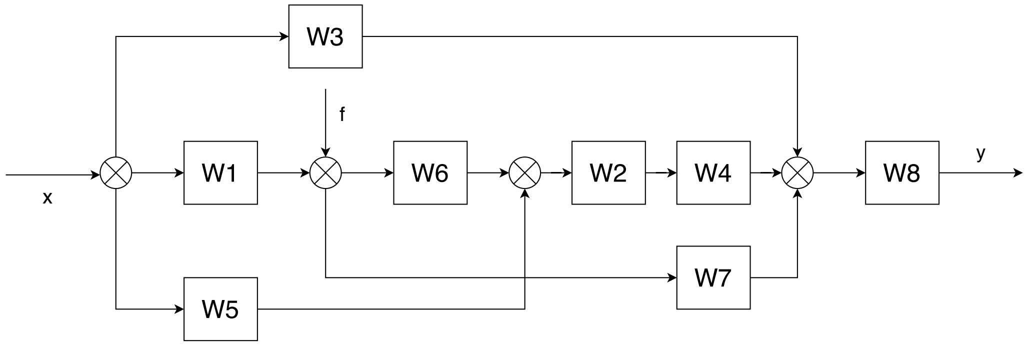
```
A = [2,1;-3,1];
B = [-1,5;3,1];
C = [-2,0];
D = [2,4];
% This system has 2 inputs, then, there will be 2
% transfer functions
[nom,denom]=ss2tf(A,B,C,D,1)
ans1 = tf(nom,denom);
% ans1 =
% 2 s^2 - 4 s + 2
% ---------------
%  s^2 - 3 s + 5
[nom,denom]=ss2tf(A,B,C,D,2)
ans2 = tf(nom,denom);
% ans2 =
% 4 s^2 - 22 s + 28
% -----------------
%   s^2 - 3 s + 5
```

Hence, our total Transfer Function is:

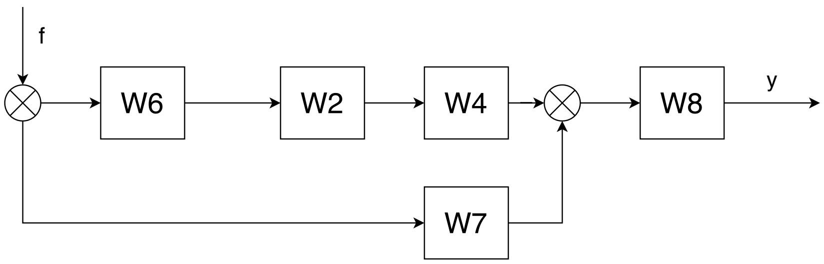
$$H(s) = \frac{2s^2 - 4s + 2}{s^2 - 3s + 5} u_1 + \frac{4s^2 - 22s + 28}{s^2 - 3s + 5} u_2$$

5 Task 5.

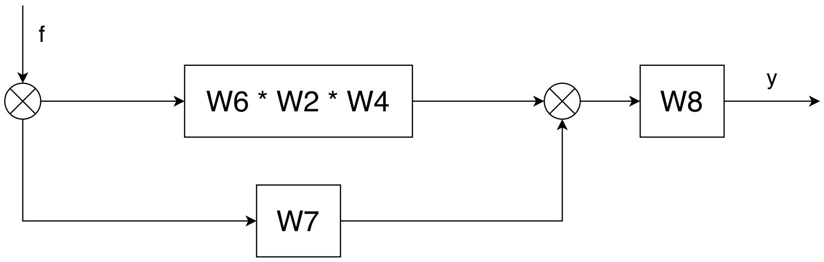
Given system, variant F:



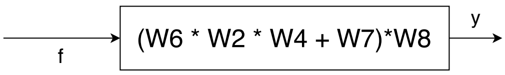
A. Assuming that x is 0:



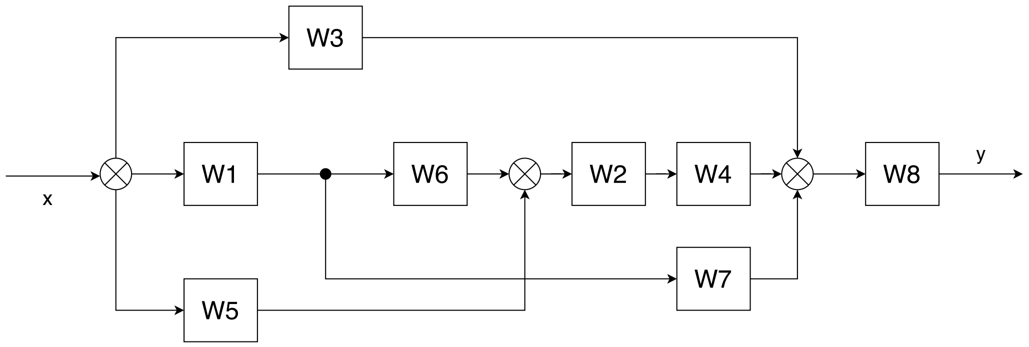
Simplifying series connection of $W6, W2, W4$ blocks:



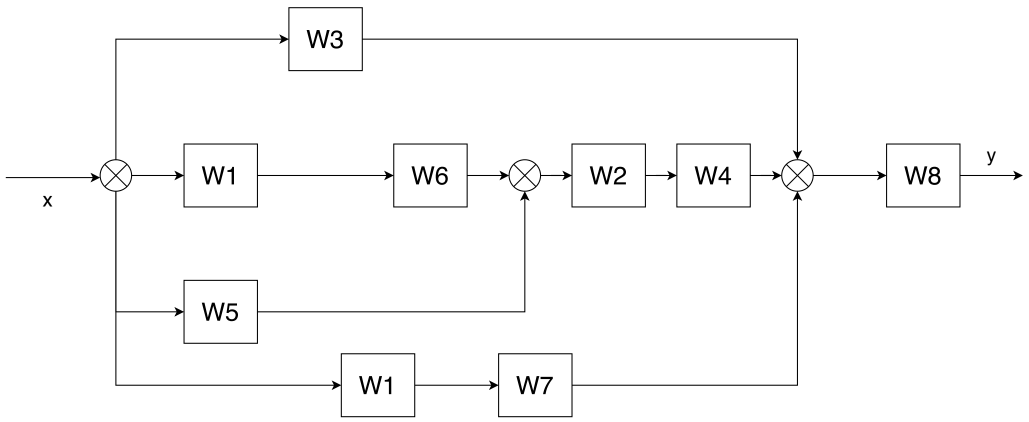
Simplifying series connection of $W6 * W2 * W4$ and $W7$ blocks, and then series connection of resulting block with block $W8$:



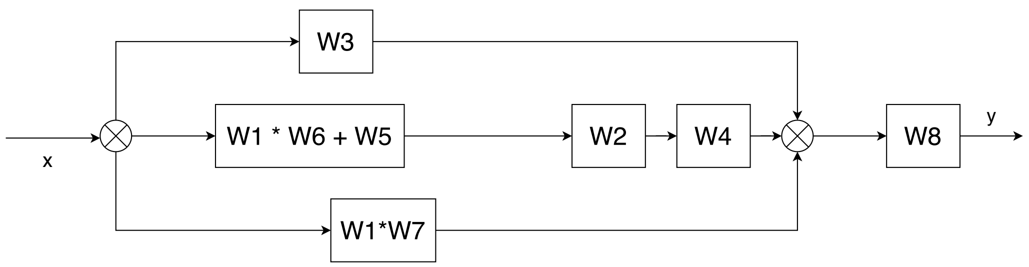
B. Assuming that f is 0:



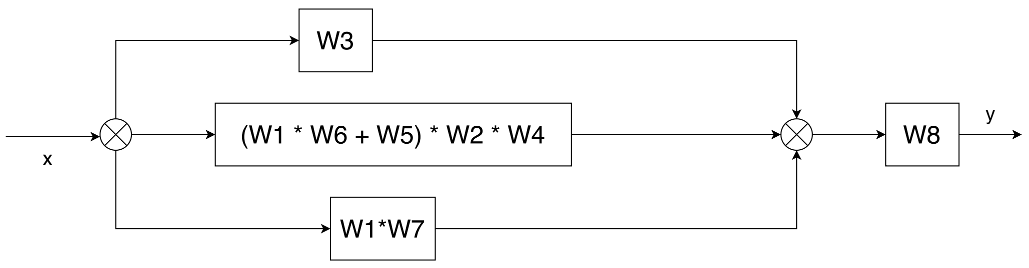
Let's examine chains $W1, W7$, and $W1, W6, W5, W2, W4$ as separate sub-chains:



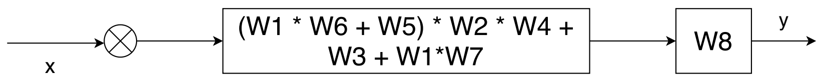
Collapsing chains $W1, W6, W5$ and $W1, W7$ to blocks $W1 * W6 + W5$, and $W1 * W7$ respectively:



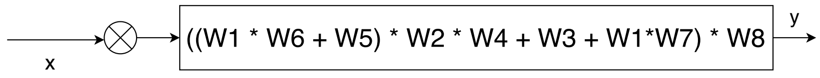
Simplifying series connection of $W1 * W6 + W5, W2, W4$ blocks:



Simplifying parallel connection:



Simplifying series connection with block $W8$:



C. Total Transfer Function

After all steps, our total Transfer Function will look like:

$$output = W_8(((W_1W_6 + W_5)W_2W_4 + W_3 + W_1W_7) \times x + (W_6W_2W_4 + W_7) \times f)$$

6 Used software.

- Python 3.8.1
- Matlab R2018b 9.5.0
- draw.io

All software was run under Manjaro Linux with 5.4.18-rt kernel