Explaining Set Theory

Using The Office

Set Theory is a mathematical theory used in the study of Semantics. This theory explains groups, or sets, and explains the relations between them. Often it is formalized in the way seen below, as done by Gensler, De Swart, and Partee.

Set Theory is useful for organizing categorically similar ideas in Semantics. For example, a set can be said to just be the members that make it up. It is almost as if saying that an Alligator is made up of the sets of scales, claws, teeth, green, etc.

First of all, it is important to explain a little more to have base to build on. As mentioned this theory groups things and properties in "sets." A set is basically a group of any variable or thing. For example, there can be a set of all things green, all plants, and all cacti. According to Georg Cantor, one of the founders of the theory, "any group of a set of M, of defined and distinguished objects of *m* of our intuition and thought. These objects are called elements of *m*." In other words, the things in a set called M, will be composed of elements like *m*. Cantor used *m* since set is *menge* in German, his native language.

As can be seen above, the sets, themselves, are expressed with capital letters while their members are expressed with lower case letters. Therefore, if someone wanted to represent The Office, or the workers at Dunder Mifflin, O could be used. Likewise, the accountants could be represented with A for the set and a, o, and k, for its members, Angela, Oscar and Kevin.

It is also worth mentioning that a set can be called a sub-set when it, itself, is a member of another set. In the words of Partee this occurs when each member of a set is also part of another set. Another one of the founders of Set Theory, Dedekind, describes it as “part of a system (set), S when each element of A is also an element (member) of S. Since this relation between system A and S occurs continuously in the following, we express it as A-S.” While a dash was used by Dedekind, the modern symbol, ⊂, is used now. However, it is also important to distinguish the members from other sets. This is usually done by using brackets {}. The symbol ∈, expresses that something is a member of a set.

While that may be a lot, we can condense it by using the accountants again in the example below.

$$a, o, k \ \in A$$

$$A \subset O$$

Before continuing, it is important to note that things need to have certain characteristics to be a member of a set. For example, we cannnot say that Cece, Jim and Pam’s daughter, is a member of the Office, because she is a little girl and cannot work at Dunder Mifflin like her parents. Therefore, we can say that one of the characteristics of the Office is that the members of the set called the Office, have to work at Dunder Mifflin. In order to formalize this example of Cece, we can just put a line through the aforementioned ∈.

$$c \notin O$$

While it is posible to list out these characteristics one by one, it is much easier to speak about them in terms like union and intersection. Union expressed, with ∪, describes the sum of all members in two or more sets. For example, Sabre, the company that acquires Dunder Mifflin, could be described as the union between the two, as can be seen below.

$$O = \{S \cup D\}$$

While Union describes the sum of the groups or sets in question, the intersection describes the things or variables shared between two or more groups. This relation is expressed with the symbol, ∩. For an example of intersection, Jim decides to distance himself from Dunder Mifflin, D, to create the company called Athlead, A, an example of the intersection of two sets.

$$J = D \cap A$$

During formalization, in terms of predicate logic, it is important to use a variable in order to define the sets, x for example. Likewise |, can be used to describe the relation between them. For example, below, The Office, O, can be described as a set of subsets, S, salesmen, T, temps, M, managers, R receptionists, and W, warehouse workers. To further disambiguate this, a the symbol def, is used to make sure that these sets are defined as such. Read literally, the example below says that the salesmen, temps, managers, recepcionists, and Warehouse workers are defined as a variable. This variable is defined as O, the Office.

$$S, T, M, R, W =_{def} \{x | x \in O\}$$

Nota all sets will have various members though. For example, in the Office, there is only one receptionist. Partee describes these types of groups as “singletons.” Below this can be read formally.

$$\{r\} \in R$$

Furthermore, Partee mentions the example of a null set, that is to say a set that contains no members. Therefore, when Pete’s ex-girlfriend comes to the Office to work for Advertising, she became sorely disappointed to find out that such a department did not exist. However, there must be a member to maintain the set, the null member. To show this necessary null, semanticists either use the symbol, ∅, or empty brackets, {}. Below we can see that there are no members in advertising as expressed below.

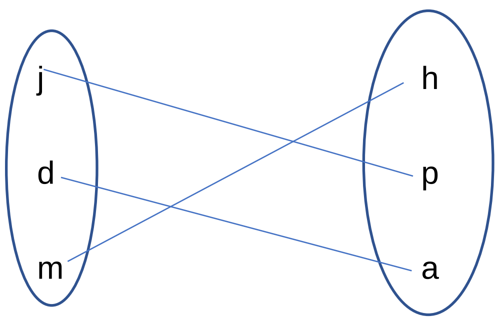
$$\{\emptyset\} \in A$$

Additionally, Weiss, shows that it is possible to show the difference between sets using the symbol, -. This describes what distinguishes one set from another. Fore example, the difference between the majority of the Office and the final seasons, is that Michael is no longer manager. To describe this, the earlier seasons may be called O1 while those afterwards are referred to as O2. The formalization below can be read as follows. O1 is different from O2 and that difference is a variable or thing called, *m*. *m* is a member of O1 but not a member of O2.

$$O1 - O2 = \quad _{def}\{m | m \in O1 \land m \notin O2\}$$

Beyond that, sets can be divided into ordinal pairs. In other words, pairs of members or variables in a set. For example, an ordinal pair can consist in two variables, such as {p,j}, where p stands for Pam and j for Jim. In this ordinal pair, Pam and Jim can be distinguished by domain and range. Domain simply refers to the variable on the right while range refers to the variable on the left so that Pam is the domain above and Jim is the range.

What’s more, one could divide all the men and women in the Office, as a set, using this to assign a domain and range value. For example, if we have the variables, j, Jim, p, Pam, h, Holly, d, Dwight, and a, Angela, they can be arranged to give the ordinal pairs {j,p}, {m,h}, and {d,a}, as further explained by the diagram below.



Cartesian products are another concept that can be derived from this information. Cartesian products are defined as all the members part of a set whose members are in ordinal pairs. For example, using the romantic sets above, a cartesian product can be developed to order them according to the aforementioned criteria. As sets the men can be defined as M and the women as W. This way we can derive the cartesian product by placing the Men in the range and the Women in the domain.

M= {<j,m,d>} W={<p,h,a>}

M X W={<j,p>,<j,h>,<j,a>,<m,p>,<m,h>,<m,a>,<d,p>,<d,h>,<d,a>}

Now we can begin to conclude this article using what we have learned. To establish the Truth conditions necessary to determine whether a member is part of the set called the

Office, we can define those members as those that work in the Scranton branch of Dunder Mifflin. In this set there are subsets such as R, recepcionists, M, managers, S, Salesmen, W, warehouse workers, A, accountants, and T, temps. The members of this subset may be represented with the first letter of their name in lower case. These subsets are in unión. However, there are also intersections with other subsets and other sets entirely. For example, Jim and Pam's family has members outside the Office.

Now the Office can be defined as follows:

Defining the Office as Union: $O =\ _{def}(x|x = R \cap G \cap V \cap A \cap C \cap T)$

Defining the subsets of the Office: $R, G, V, A, C, T \subset O$

Defining the members of the subsets : $p \in r$

$$j, d, ph, s, an, n \in V$$

$$o, a, k \in C$$

$$r, k \in T$$

$$d \in A$$

$$m \in G$$

$$p \in R$$

Defining the intersections: $O \cup F = p, j$

Defining the difference between the first and second seasons:

$$O1 - O2 = \quad_{def}\{m|m \in O1 \wedge m \notin O2\}$$

To conclude, any set of objects in the real world, or otherwise, may serve as an example to explain Set Theory. In the Office, various subsets in Union can be seen to form Dunder Mifflin. The employees are members of subsets, despite some being called “singletons.” These members have intersections with other sets, such as Jim and Pam’s family.

When Set Theory is used with examples like this, we can see that it is not too scary. And that when used with fun examples like the Office, anyone can learn about it.

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Find out more

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