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ALIYEVA JAMILA RAYIMJONOVNA

TUTASH MUHITLAR MEXANIKASI

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Aliyeva Jamila Rayimjonovna

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Ushbu o’quv qo’llanma tutash muhitlar mexanikasi faniga bag’ishlangan. Qo’llanmada tutash muhitlar mexanikasi fanining mazmun mohiyati ochib berilgan hamda unda mavzularga doir masalalar va ularni yechishga na’munalar keltirilgan. Ushbu qo’llanmadan tutash muhitlar mexanikasi fani o’qitiladigan bacha oliy o’quv yurtining professor-o’qituvchilari va talabalari foydalanishlari mumkin.

Taqrizchilar:

R.V.Mullajonov – Andijon davlat universiteti o’quv-uslubiy boshqarma boshlig’i, fizika matematika fanlari nomzodi, dotsent;

L.O.Olimov – Andijon Mashinasozlik instituti “Materialshunoslik va yangi materiallar texnologiyasi” kafedrasi dotsenti, fizika-matematika fanlari doktori.

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SO’Z BOSHI

Respublikamizda ta’lim va tarbiya sohasidagi islohotlar bugungi dolzarb, ertangi taqdirimizni hal qiluvchi muammoga aylanmoqda. Jamiyatimizning yangilanishi, hayotimiz taraqqiyoti va istiqboli, amalga oshirilayotgan islohotlar rejasining samarasi taqdiri — bularning barchasi, avvalombor, zamon talablariga javob beradigan yuqori malakali, ongli mutaxassis kadrlar tayyorlash muammosi bilan chambarchas bog‘liq. Bunday mutaxassislar tayyorlashda asosiy ma’suliyat oliy ta’lim bosqichiga bog’liq bo’ladi. Oliy ta’lim bosqichida yetuk malakali mutaxassislarni tayyorlashda o’quv adabiyotlarining, ayniqsa o’zbek tilida yozilgan adabiyotlarning o’rni juda muhim hisoblanadi.

Ma’lumki, hozirda matematika va mehanika yo’nalishlarida tutash muhitlar mehanikasi fani o’qitiladi. Bu fan doirasida bugungi kunga qadar juda katta ilmiy ma'lumotlar to'plangan, nazariyalar ishlab chiqilgan va fizik maydonlarning harakati, gazlarning harakati va muvozanati, suyuqliklar va qattiq deformatsiyalanadigan jismlar to'g'risida ma'lumotlar to'plangan. Ammo bu ma’lumotlarning katta qismi o’zbek tilida emas. Shunning uchun men bu qo’llanmani yozishga qaror qildim.

Ushbu qo’llanmada turli xil aniq masalalarni maxsus o'rganish uchun yetarli va zarur bo'lgan tutash muhitlar mehanikasi asoslari bayon qilingan. Bu qo’llanma kirish qismi va 4ta bobdan iborat bo’lib, unda tutash muhitlar mehanikasi fanining muammolari yoritilgan. Har bir bobning oxirida bobga doir masalalar va ularni yechishga na’munalar keltirilgan. Qo’llanmada tenzor hisob elementlari, tenzorlar ustida amallar, deformatsiyalanuvchi muhit kinematikasi va deformatsiyalar nazariyasi haqida ma’lumotlar berilgan. Keyingi mavzular qo’llanmaning 2-qismiga rejalashtirilgan.

Ushbu qo’llanmadan tutash muhitlar mexanikasi fani o’qitiladigan bacha oliy o’quv yurtining professor-o’qituvchilari va talabalari foydalanishlari mumkin. Bu qo’llanmaning afzalligi shundaki undan ham ma’ruza, ham amaliyot darslarida foydalanish mumkin. Bu qo’llanma ayrim kamchiliklardan holi bo’lmasligi mumkin.

KIRISH. TUTASH MUHITLAR MEXANIKASI FANI PREDMETI. TUTASH MUHITLAR TUSHUNCHASI.

Tutash muhitlar mexanikasi mexanikaning gazlar, suyuqliklar, plazma va deformatsiyalanuvchi qattiq jismlarning harakati hamda muvozanatini oʻrganuvchi boʻlimidir.

Tutash muhitlar mexanikasida qattiq, suyuq va gaz holatidagi jismlarning makroskopik nuqtai nazardan harakatlari o’rganiladi. Bu fan fundamental tushunchalar asosida, farazlar vositasida yaratilgan. Mexanik nuqtai nazardan o’rganilayotgan qattiq, suyuq va gaz holatidagi jismlar fazoning biror chekli yoki cheksiz bo’lagini to’la to’kis tutash holda egallagan, deb faraz qilinadi. Fazoda olingan istalgan geometrik nuqtada tutash muhit «zarrasi» mavjud degan tushunchani kiritamiz.

Tabiiy hodisalarni kuzatish natijasida va ko’plab texnik muammolarni xal qilishda biz duch keladigan suyuqliklar, gazlar va qattiq deformatsiyalanadigan jismlarning turli xil harakatlarini belgilashimiz mumkin.

Kundalik shaxsiy tajribaga tayanib, deformatsiyalangan jismlarning ko’p harakatlarini kerakli darajada boshqarishimiz mumkin. Kundalik hayotdagi tajribalar bizda haqiqat va “umumiy tushuncha” tuyg’usini yuzaga keltiradi. Bu esa ko’pincha bizga kerakli mexanik ta’sirni to’g’ri taxmin qilish va yaratishga imkon beradi.

Biroq, murakkab holatlarda buni amalga oshirib bo’lmaydi. Buning uchun nazariy va tajribalarni o’rganishning maxsus usullari talab qilinadi. Bunday izlanishlar fan sifatida tutash muhitlar mexanikasining yaratilishiga va rivojlanishiga olib keldi. Bu fanning yaratilishi quyidagi muammolarga javob topishga yo’l ochdi. Masalan, gaz siqilgan holatda bo’lgan silindrdagi teshikdan chiqadigan gazning tezligi qancha; havo oqimining atmosferada qanday harakatlanishi; samolyot yoki kema suvga chidamliligini qanday kamaytirish mumkin; 500 metr balandlikdagi televizion metal minorani, eng yaqin ikki tayanch orasida ikki kilometrdan oshiq ko’prikni qanday qurish kerak; samolyotda pervanel diametrining oshishi yoki kamayishi bilan nima sodir bo’ladi; bosimlarning tarqalishi va bomba portlashi paytida havo harakati haqida nima deyish mumkin va hokazo.

Shuni ta'kidlashimiz kerakki, ko'pgina savollar va muammolar mavjud bo'lib, ularga hanuzgacha tajribalardan foydalanib, kerakli qoniqarli javob bera olmaymiz. Yangi murakkab muammolarni hal qilish, ilmiy va amaliy ahamiyatga ega bo'lgan vazifalar, shuningdek, ilm-fanning oldingi rivojlanishi tomonidan ishlab chiqilgan vazifalarni bugungi kunda ilmiy tadqiqot mavzusi hisoblanadi. Shoshilinch yangi muammolarga misollar: suvning 100 m/s tezlikda harakatlanishining pasayishi; millionlab daraja haroratga ega bo'lgan plazmani yaratish va ishlab turishi; yuqori yuk va yuqori haroratlarda materiallarning egiluvchanlik va boshqa hodisalarni hisobga olgan holda portlash paytida tuzilmalarga ta'sir etuvchi kuchlarni aniqlash; uzoq masofaga yo'lovchilarning parvozlari uchun yuqori darajadagi samolyotni yaratish; atmosferadagi umumiy havo aylanishini tushuntirish; ob-havo bashorati; o'simliklar va tirik organizmdagi mexanik jarayonlarni o'rganish; yulduzlar evolyutsiyasi muammolari, quyoshda ro'y beradigan hodisalar va boshqalar.

Biz tutash muhitlar mexanikasining qanday muammolar bilan shug'ullanishini aytib o'tamiz. Ular elastiklik nazariyasi masalalari, plastiklik nazariyalari va uning masalalari, gidrostatika masalalari, suyuqlikda harakatlanuvchi jismga suyuqlikning ta'siri masalalari, filtratsiya masalalari, to'lqin harakati, suyuqlik va qattiq jismlarda to'lqinning tarqalish masalalari, gazlarning kimyoviy o'zgarishlari bo'layotgan holatdagi, portlash va yonish jarayonlari bilan bo'lgan harakatlarini o'rganish masalalari, qattiq jismlarning atmosfera qalin qatlamlariga kirganida yonish va erib ketishdan saqlash masalalari, suyuqlikning turbulent harakati masalalari, magnit gidrodinamikasi masalalari, ob-havoning o'zgarishini tekshirish yo'nalishi, biologik mexanika hamda boshqa masalalardan iboratdir. Masalan suyuqlik va gazning quvurlar orqali va umuman, turli xil mashinalar ichidagi harakatini ko'radigan bo'lsak. Ushbu masalalarda suyuqlikning oqim chegaralari bilan o'zaro ta'siri qonunlari, xususan, harakatlanuvchi va qo'zg'almas qattiq devorlarning qarshiligi hamda tezlikni taqsimlashda notekislik hodisalari va boshqalar muhim ahamiyatga ega hisoblanadi. Ushbu vazifalar gaz quvurlari, neft quvurlari, nasoslar, trubinalar va boshqa gidravlik mashinalarni loyihalashda bevosita ahamiyatlidir.

So'nggi paytlarda biologik mexanika sohasida ham juda ko'p izlanishlar olib borilmoqda, xususan, tirik organizmlardagi qonning harakatini va mushaklarning qisqarishi hodisasini tasvirlashga imkon beradigan mexanik modellar yaratilmoqda.

Shuni alohida ta’kidlash kerakki, yaqinda kimyo korxonalarida ishlab chiqarish tehnologiyasi masalalari tutash muhitlar harakatini mehanik o’rganishga asoslangan.

Tutash muhitlar mexanikasida deformatsiyalanadigan jismlarning harakatini o'rganish uchun matematik usullar ko'rib chiqiladi. Matematik usullar turli mexanik hodisalarni o’rganishda muhim rol o’ynaydi, xususan, koordinatalar sistemasi tushunchasidan keng foydalaniladi. Koordinatalar sistemasi mexanik hodisalarni o’rganishda vosita sifatida xizmat qiladi.

Tutash muhitlar mexanikasida mehanik muammolarni matematik muammolar, ya’ni turli hil matematik usullardan foydalangan holda ma’lum sonlarni yoki sonli funksiyalarni topish usullari ishlab chiqilgan.

Bundan tashqari, tutash muhitlar mexanikasining eng muhim maqsadi deformatsiyalangan jismlarning umumiy xususiyatlari va harakat qonuniyatlarini yaratishdir.

Shuni ta’kidlash kerakki, matematik usullar yordamida tutash muhitlar mexanikasining o’ziga xos muammolarini hal qilish mumkin.

Tutash muhitlar mexanikasi ko’p o’zgaruvchili funksiyalar nazariyasining ba’zi qismlarini, differensial tenglamalar, integral tenglamalar va boshqalarni rivojlantirishga katta ta’sir ko’rsatdi.

Tutash muhit mexanikasining asosiy usullaridan biri - matematik tahlil usulidir. TMM(Tutash muhit mexanikasi)da mexanik masala yechilishi kerak bo’lgan ma’lum matematik masalaga keltiriladi va u o’z navbatida matematikaning ham rivojida katta rol o’ynab keladi. TMMda tajriba asosida olingan yechimlar ham muhim rol o’ynaydi. Tajriba matematik usullar asosida olingan yechimlarninggina emas, balki matematik tenglamalarga keltirilgandagi tutash muhit xarakati va holatini akslantiruvchi munosabatlarning maqsadga muvofoqlik darajasini ham ko’rsatuvchi omil bo’lib xizmat qiladi. Bundan tashqari, tutash muhit mexanikasining turli xil muammolari va ularni o'rganishning matematik usullari ko'p hollarda bir-biri bilan chambarchas bog'liq ekanligi ma'lum bo'ldi.

Yuqoridagi muammolarning fizik, geometrik va boshqa ichki tub xususiyatlarini aniqlovchi natijalar qaysi koordinatalar sistemasi yordamida olinishiga bog’liq emas. Shuning uchun mexanik hodisalarning koordinatalar sistemasi tanlanishiga bog’liq bo’lmagan ichki fizik xususiyatlarini va ular orasidagi munosabatlarni o’rganish muhim hisoblanadi. Bunda esa tenzor va tenzor funksiyalar muhim rol o’ynaydi.

Tenzorlarning xossalarini va ular ustidagi amallarni o’rganish tenzor hisob deb ataladi.

Ushbu hisob geometriyada, nisbiylik nazariyasida, mexanikada, ayniqsa tutash muhit mexanikasida keng qo’llaniladi.

I BOB. TENZOR HISOB ELEMENTLARI

1 §. Miqdorlarni indeksli belgilash.

Biz bilamizki, agar uchta erkli o’zgaruvchi berilgan bo’lsa, ularni uchta harf yordamida, masalan, x, y, z deb belgilash mumkin. Tenzor hisobda esa ushbu erkli o’zgaruvchilarni indeksli bir dona harf yordamida belgilash mumkin:

$$x^1,\ x^2,\ x^3 \quad \text{yoki} \quad x^\alpha \ \left(\alpha = 1, 2, 3\right) \qquad (1.1.1)$$

Bu belgilash odatda erkli o’zgaruvchilar uchun qabul qilingan. Ammo bu belgilashni x_α ko’rinishda ham kiritish mumkin. Masalan, (1.1.1) o’zgaruvchilarning bir jinsli chiziqli funksiyasini quyidagi ko’rinishda yozish mumkin;

$$\sum_{\alpha=1}^{3} a_\alpha x^\alpha = a_1 x^1 + a_2 x^2 + a_3 x^3 \qquad (1.1.2)$$

Ta’rif: Muayyan bir harfni turli indekslar yordamida belgilab, hosil qilingan miqdorlar to’plami ob’yekt deb ataladi.

Ta’rif: Bir indeks yordamida x^α va a_α kabi belgilangan ob’yektlar birinchi rang ob’yekt deb ataladi. Indeksli ayrim harflar (x^1, x^2, x^3 va a_1, a_2, a_3) esa mazkur ob’yektning komponentalari (tashkil etuvchilari) deb ataladi.

Birinchi rang ob’yektlar uchun ikki xil komponentalar mavjud:

a) quyi indeksli (kovariant) - $a_\alpha \quad \left(\alpha = 1, 2, 3\right)$

b) yuqori indeksli (kontravariant) -$a^\alpha \quad \left(a = 1, 2, 3\right)$.

Demak, birinchi rang ob’yektning uchta kovariant va uchta kontravariant komponentalari bo’ladi.

Tenzor hisobda ikkinchi, uchinchi va hokazo n - rang ob’yektlar ham ishlatiladi.

Ta’rif: Ikki indeksli ob’yektlar 2-rang ob’yektlar deb ataladi.

Masalan, uch o’zgaruvchili funksiyaning kvadratik funksiyasi ushbu ko’rinishga ega;

$$\sum_{\alpha,\beta=1}^{3} a_{\alpha\beta}x^{\alpha}x^{\beta} = a_{11}x^1x^1 + a_{12}x^1x^2 + a_{13}x^1x^3 + a_{21}x^2x^1 + \\ + a_{22}x^2x^2 + a_{23}x^2x^3 + a_{31}x^3x^1 + a_{32}x^3x^2 + a_{33}x^3x^3 \tag{1.1.3}$$

bu yerda $a_{\alpha\beta}$ $(\alpha, \beta = 1, 2, 3)$ kvadratik formaning o'zgarmas koeffitsientlaridir. Ularning har bir indeksini yuqori va quyi indeks sifatida yozish mumkin;

$$a^{\alpha\beta},\ \ a_{\alpha\beta},\ \ a^{\alpha\cdot}_{\cdot\beta},\ \ a^{\cdot\beta}_{\alpha\cdot} \qquad (\alpha, \beta = 1,\ 2,\ 3).$$

Yani, ikkinchi rang ob'yektlarning to'rt xil ko'rinishidagi komponentalari mavjud bo'ladi. Ular $a_{\alpha\beta}$ - kovariant, $a^{\alpha\beta}$ -kontrvariant, $a^{\alpha\cdot}_{\cdot\beta}$ - birinchi xil aralash, $a^{\cdot\beta}_{\alpha\cdot}$ - ikkinchi xil aralash komponentalar deb ataladi. Bu yerda nuqta yordamida indeksning o'rni ko'rsatiladi. Ikkinchi rang ob'yektning bir xil ko'rinishdagi komponentalarining soni to'qqizta bo'ladi. Qaralayotgan birinchi, ikkinchi va hokazo n - rang ob'yektlar ketma-ketligi to'la bo'lishi uchun indekssiz miqdorlar nolinchi rangli ob'yekt sifatida qabul qilinadi. Yuqori rangli ob'yektlarni belgilash uchun nomerli indekslardan ham foydalaniladi. Masalan, n - rang ob'yektning kovariant komponentasini $a_{\alpha_1\alpha_2\ \ldots\alpha_n}$ ko'rinishda belgilash mumkin.

Indeksli harflarning yig'indisi. Indeksli harflarning yig'indisi A.Eynshteyn taklif qilgan qoidaga ko'ra bajariladi. Agar bir hadli ifodada biror indeks ikki marta - bir gal yuqori va ikkinchi gal quyi indeks sifatida uchrasa, u holda, ushbu indeksga 1, 2, 3 qiymat berilib, yig'indi hisoblanadi. Bu qoidaga binoan quidagi tenglik hosil bo'ladi;

$$a_{\alpha}x^{\alpha} = a_1x^1 + a_2x^2 + a_3x^3$$
$$a^{\alpha\cdot}_{\cdot\alpha} = a^{1\cdot}_{\cdot1} + a^{2\cdot}_{\cdot2} + a^{3\cdot}_{\cdot3}$$

ushbu $a_{\alpha\alpha}$, $a^{\alpha\alpha}$, $a^{\cdot\cdot}_{\alpha\alpha}$ ifodalarda esa yig'indi hisoblanmaydi, chunki ularda α indeksi bo'yicha yig'indi hisoblash qoidasi buziladi.

Ta'rif: Yig'indini hisoblash amali bajariladigan indeks befarq yoki soqov (gung) indeks deb ataladi. Undan boshqa indekslar esa erkin indekslar deb ataladi.

Masalan, $a_{\alpha\beta}b^{\beta}$ ifodada β - befarq, α esa erkin indekslardir. Erkin indeks 1, 2, 3 sonlardan birini qabul qilishi mumkin. Yani, $a_{\alpha\beta}b^{\beta}$ ifoda ushbu $a_{1\beta}b^{\beta}$, $a_{2\beta}b^{\beta}$, $a_{3\beta}b^{\beta}$ ifodalardan biri bo'lishi mumkin.

Agar biror indeks harfli tenglikning ikkala tomonida uchrasa va tenglikning bir tomonida erkin bo'lsa, u holda ushbu indeks tenglikning ikkinchi tomonida ham erkin bo'ladi. Masalan,

$a_{\alpha} = b_{\alpha}c^{\alpha}$ tenglikda α - erkin indeks bo'ladi.

A.Enshteyn qoidasidan ko'p karrali yig'indilarni yozishda ham foydalanish mumkin. Masalan,

$$\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3} a_{\alpha\beta}x^{\alpha}x^{\beta} = a_{\alpha\beta}x^{\alpha}x^{\beta}$$

$$\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}\sum_{\gamma=1}^{3} a^{\cdots\,\alpha\beta\gamma}_{\alpha\beta\gamma\,\cdots} = a^{\cdots\,\alpha\beta\gamma}_{\alpha\beta\gamma\,\cdots}$$

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2 §. To'g'ri burchakli Dekart koordinatalar sistemasi.

TMMda koordinatalar sistemasi ham muhim ahamiyatga ega. TMMda ikki xil, ya'ni to'g'ri burchakli dekart koordinatalar sistemasi va egri chiziqli koordinatalar sistemasidan foydalaniladi.

To'g'ri burchakli dekart koordinatalar sistemasi berilgan bo'lsin. Markazi O nuqta, koordinata chiziqlari esa y^1, y^2, y^3. Ixtiyoriy M nuqtaning vaziyati, koordinata tekisliklarining tenglamalarini beruvchi $y^{\alpha} = c^{\alpha}$ $(\alpha = 1, 2, 3)$ munosabatlar bilan aniqlanadi.

Fazoda nuqtaning vaziyati mazkur uchta tekisliklarning kesishishi natijasida aniqlanadi. Tekisliklardan ikkitasi kesishganda hosil bo'ladigan to'g'ri chiziqlar koordinata chizig'i deb ataladi. U y^{α} koordinata chizig'i bo'ylab musbat tomonga yo'nalgan. $y^{\alpha} = c^{\alpha}$ koordinata tekisligiga perpendikulyar bo'lgan birlik vektor $\bar{k}_{\alpha}$ deb belgilanadi.

Quyidagi birlik vektorlar koordinata bazislari deb ataladi. Va ular fazoning barcha nuqtalarida bir xil yo'nalishga ega. Ular uchun quidagi tenglik o'rinli bo'ladi;

$$\bar{k}_\alpha \cdot \bar{k}_\beta = \delta_{\alpha\beta} = \begin{cases} 1, & \alpha = \beta \\ 0, & \alpha \neq \beta \end{cases} \qquad (\alpha,\ \beta = 1, 2, 3) \tag{1.2.1}$$

bu yerdagi $\delta_{\alpha\beta}$ - Kronekker belgisi (deltasi) deb ataladi.

Demak, to'g'ri burchakli dekart koordinata sistemasida bir xil nomli koordinata tekisliklari ham, bir xil nomli koordinata chiziqlari ham bir-biriga parallel bo'ladi. Har bir nuqtada esa uchchala koordinata tekisliklari ham, uchchala koordinata chiziqlari ham o'zaro ortogonal bo'ladi. Undan tashqari, bir xil nomli koordinata shu nomli koordinata tekisliklariga ortogonal bo'ladi.

Koordinatalari y^α va $y^\alpha + d\,y^\alpha$ $(\alpha = 1,\ 2,\ 3)$ sonlar bilan berilgan *M* va M' nuqtalar orasidagi masofa quyidagi tenglik bilan aniqlanadi;

$$ds^2 = \left(dy^1\right)^2 + \left(dy^2\right)^2 + \left(dy^3\right)^2 = \delta_{\alpha\beta} dy^\alpha dy^\beta \tag{1.2.2}$$

M nuqtaning radius vektori $\bar{r}$ quyidagiga teng bo'ladi

$$\bar{r} = y^1 \bar{k}_1 + y^2 \bar{k}_2 + y^3 \bar{k}_3 = y^\alpha \bar{k}_\alpha \ . \tag{1.2.3}$$

Ortogonal dekart koordinatalar sistemasida nuqta koordinatalari quyi indeks (y_α, x_α) yordamida ham belgilanishi mumkin .

3 §. Egri chiziqli koordinatalar sistemasi.

Uchta x^α $(\alpha = 1, 2, 3)$ miqdor dekart koordinatalari y^β $(\beta = 1, 2, 3)$ larning funksiyasi sifatida berilgan bo'lsin;

$$x^\alpha = x^\alpha\left(y^1,\ y^2,\ y^3\right), \quad \left(\alpha = 1, 2, 3\right) \tag{1.3.1}$$

x^α funksiyalar uzluksiz differensiallanuvchi funksiyalardir. Bu funksiyalarning Vronskiy determinanti ya'ni yakobiani noldan farqli bo'lsin;

$$J=\left|\frac{\partial x^{\alpha}}{\partial y^{\alpha}}\right|=\begin{vmatrix}\frac{\partial x^1}{\partial y^1} & \frac{\partial x^1}{\partial y^2} & \frac{\partial x^1}{\partial y^3}\\ \frac{\partial x^2}{\partial y^1} & \frac{\partial x^2}{\partial y^2} & \frac{\partial x^2}{\partial y^3}\\ \frac{\partial x^3}{\partial y^1} & \frac{\partial x^3}{\partial y^2} & \frac{\partial x^3}{\partial y^3}\end{vmatrix}\neq 0, \tag{1.3.2}$$

yani, (1.3.2) o'zaro bir qiymatli akslanish bo'lib, undan

$$y^{\beta}=y^{\beta}\left(x^1,\ x^2,\ x^3\right), \qquad (\beta=1,\ 2,\ 3) \tag{1.3.3}$$

lokal munosabatlarni olish mumkin bo'lsin.

U holda, uchta x^1, x^2, x^3 sonlar berilsa, (1.3.3) tenglikka binoan uchta y^1, y^2, y^3 sonlar (dekart koordinatalari) ham berilgan bo'ladi, yani fazoda muayyan nuqta aniqlanadi. Demak, x^{α} $(\alpha=1,\ 2,\ 3)$ sonlarni fazodagi nuqtaning koordinatalari deb qarash mumkin va $x^{\alpha}=b^{\alpha}$ $(\alpha=1,\ 2,\ 3)$ (b^{α} – o'zgarmas sonlar) munosabatlar yordamida hosil qilingan egri chiziqli sirtlarni koordinata sirtlari deyishimiz mumkin. U holda mazkur uchta sirtlarning kesishuvi fazoda biror $M\left(x^1,\ x^2,\ x^3\right)$ nuqtani beradi. Ulardan istalgan ikkitasining kesishuvi esa egri chiziqni ya'ni koordinata chizig'ini bildiradi.

Egri chiziqli koordinata sirtlari va chiziqlarining ko'rinishi qaralayotgan nuqtaning holatiga bog'liqdir.

Koordinata chizig'i bo'ylab faqat x^{α} o'zgaradi va (1.2.3), (1.3.3) formulalarga ko'ra ushbu x^{α} koordinata chizig'ining tenglamasi quyidagi ko'rinishda bo'ladi:

$$\bar{r}_{\alpha}=\bar{r}_{\alpha}\left(x^{\alpha}\right) \tag{1.3.4}$$

4 §. Kovariant koordinata bazislari.

Ixtiyoriy ikkita bir-biriga yaqin koordinatalari x^{α} va $x^{\alpha}+d\,x^{\alpha}$ $(\alpha=1,2,3)$ bo'lgan M va M' nuqtalar fazoda berilgan bo'lsin. Bu ikki nuqta yordamida aniqlangan $MM' = \bar{d}\,r$ yo'naltirilgan kesma koordinatalar sistemasining tanlanishiga bog'liq emas. Ular invariant miqdordir.

Ta'rif: Ushbu yo'naltirilgan $\bar{d}r$ kesma elementar ko'chish vektori, uning uzunligi ds, yani M va M' nuqtalar orasidagi masofa esa $\bar{d}r$ ning moduli deb ataladi.

Agar yuqoridagi mulohazani uzunligi bir birlik bo'lgan kesma uchun qo'llanilsa, moduli birga teng bo'lgan elementar ko'chish vektori $\overline{Э}$ hosil qilinadi. Ushbu birlik vektor yordamida $\overline{d}r$ ni quyidagi ko'rinishda yozish mumkin.

$$dr = ds\vec{Э} \tag{1.4.1}$$

Agar M va M' nuqtalar x^α koordinata chizig'ida yotsa, elementar ko'chish vektori $\overline{dr_\alpha}$ deb belgilanadi.

Elementar ko'chish vektori $\bar{d}\,r\left(x^1,\, x^2,\, x^3\right)$ni quyidagi ko'rinishida yozish mumkin.

$$\bar{d}r = \frac{\partial \bar{r}}{\partial x^1}dx^1 + \frac{\partial \bar{r}}{\partial x^2}dx^2 + \frac{\partial \bar{r}}{\partial x^3}dx^3 \equiv \frac{\partial \bar{r}}{\partial x^\alpha} = \overline{Э}_\alpha dx^\alpha \tag{1.4.2}$$

Bundan tashqari (1.3.4) tenglamadan

$$\overline{dr_\alpha} = \frac{\partial \bar{r}}{\partial x^\alpha}dx^\alpha = \overline{Э}_\alpha dx^\alpha \quad (\alpha\text{-erkin indeks}) \tag{1.4.3}$$

va $\bar{d}r$ ning (1.4.2) ko'rinishidan quyidagi munosabatlar o'rinli bo'ladi.

$$d\bar{r} = d\bar{r}_1 + d\bar{r}_2 + d\bar{r}_3$$

Demak,

$$\overline{Э}_\alpha\left(x^1,\; x^2,\; x^3\right) = \frac{\partial \bar{r}}{\partial x^\alpha} = \frac{\partial y^\beta}{\partial x^\alpha}\bar{k}_\beta \tag{1.4.4}$$

vektor x_α koordinata chizig'iga urinma bo'ylab musbat tomonga yo'nalgan va $d\bar{r}_\alpha$ ga kollinear bo'lgan vektordir.

Bundan tashqari, $\overline{Э}_\alpha\left(\alpha = 1, 2, 3\right)$ lar nokomplanar vektorlar. Ular uchun quyidagi tengliklar o'rinli bo'ladi.

$$\left(\overline{\mathrm{Э}}_1\overline{\mathrm{Э}}_2\overline{\mathrm{Э}}_3\right)=\overline{\mathrm{Э}}_1\left(\overline{\mathrm{Э}}_2\times\overline{\mathrm{Э}}_3\right)=\frac{\partial\bar{r}}{\partial x^1}\left(\frac{\partial\bar{r}}{\partial x^2}\times\frac{\partial\bar{r}}{\partial x^3}\right)=$$

$$=\begin{vmatrix}\frac{\partial y^1}{\partial x^1} & \frac{\partial y^2}{\partial x^1} & \frac{\partial y^3}{\partial x^1}\\ \frac{\partial y^1}{\partial x^2} & \frac{\partial y^2}{\partial x^2} & \frac{\partial y^3}{\partial x^2}\\ \frac{\partial y^1}{\partial x^3} & \frac{\partial y^2}{\partial x^3} & \frac{\partial y^3}{\partial x^3}\end{vmatrix}=J^{-1}\neq 0 \qquad (1.4.5)$$

Ular (1.3.1) va (1.3.2) munosabatlarning o’zaro bir qiymatli akslanishni ifodalashidan kelib chiqadi.

Ta’rif: Fazoning har bir nuqtasida (1.4.4) formula bilan aniqlangan uchta $\overline{\mathrm{Э}}_\alpha$ vektorlar kovariant bazis vektorlar deb ataladi. Ular egri chiziqli koordinatalar sistemasining koordinata bazislari sifatida qabul qilinadi.

Elementar ko’chish vektori modulining kvadratini (1.4.2) yoyilma yordamida

$$ds^2=\left(d\bar{r}\cdot d\bar{r}\right)=\left(dx^\alpha\overline{\mathrm{Э}}_\alpha\right)\cdot\left(dx^\beta\overline{\mathrm{Э}}_\beta\right)=\left(\overline{\mathrm{Э}}_\alpha\cdot\overline{\mathrm{Э}}_\beta\right)dx^\alpha dx^\beta \qquad (1.4.6)$$

yoki

$$g_{\alpha\beta}=\overline{\mathrm{Э}}_\alpha\cdot\overline{\mathrm{Э}}_\beta \qquad (1.4.7)$$

belgilash yordamida

$$ds^2=g_{\alpha\beta}\,dx^\alpha dx^\beta \qquad (1.4.8)$$

ko’rinishida yozish mumkin.

Ta’rif: $ds^2=g_{\alpha\beta}\,dx^\alpha dx^\beta$ ifoda asosiy kvadratik forma deb ataladi.

Asosiy kvadratik formaning koeffitsientlari (1.4.7) formula bilan aniqlanganligi sababli $g_{\alpha\beta}=g_{\beta\alpha}$ tenglik o’rinli bo’ladi. Ya’ni ushbu koeffitsientlarning matritsasi $\left\|g_{\alpha\beta}\right\|$ simmetrik bo’ladi. Bundan tashqari (1.4.5) ga ko’ra $g_{\alpha\beta}$ koeffitsientlarning determinanti uchun quyidagi munosabat o’rinli bo’ladi;

$$g=\left|g_{\alpha\beta}\right|=\left(\overline{\mathrm{Э}}_1\,\overline{\mathrm{Э}}_2\,\overline{\mathrm{Э}}_3\right)^2=J^{-2}\neq 0 \qquad (1.4.9)$$

bundan $\left\|g_{\alpha\beta}\right\|$ ning xosmas matrisa ekanligi kelib chiqadi.

Ta’rif: Kovariant bazis vektorlarning moduli

$$Э_\alpha = \left|\bar{Э}_\alpha\right| = \sqrt{\bar{Э}_\alpha \cdot \bar{Э}_\alpha} = \sqrt{g_{\alpha\alpha}} \qquad (1.4.10)$$

Lame koeffitsientlari deb ataladi.

Lame koeffitsientlari yordamida normalangan kovariant bazis vektorlarlarni kiritish mumkin:

$$\bar{e}_\alpha = \frac{\bar{Э}_\alpha}{\left|\bar{Э}_\alpha\right|} = \frac{\bar{Э}_\alpha}{\sqrt{g_{\alpha\alpha}}} \qquad (1.4.11)$$

bu yerdagi $\bar{e}_\alpha$-normalangan kovariant bazis vektorlar.

5 §. Kontravariant koordinata bazislari

Endi kontravariant koordinata bazislari tushunchalarini kiritamiz. Buning uchun fazoda koordinata bazisi sifatida ushbu sirtlarning normallari bo’ylab yo’nalgan vektorlarni tanlaymiz. Mazkur sirtlar

$$x^\alpha = const \qquad (\alpha = 1,\ 2,\ 3)$$

tenglamalar bilan aniqlanadi va shu tufayli

$$dx^\alpha = \frac{\partial x^\alpha}{\partial y^\beta} dy^\beta = \left(grad\ x^\alpha,\ d\bar{r}\right) \equiv \left(\nabla x^\alpha,\ d\bar{r}\right) = 0$$

munosabatlar o’rinli bo’ladi. Bu yerda quyidagi ko’rinishda belgilangan

$$\nabla x^\alpha \equiv grad\ x^\alpha = \frac{\partial x^\alpha}{\partial y^\beta} \bar{k}_\beta \qquad \left(\alpha = \overline{1,3}\right)$$

va koordinata sirtlariga normal bo’ylab yo’nalgan miqdorlar nokomplanar vektorlar uchligini tashkil etadi:

$$\nabla x^1 \cdot \left(\nabla x^2 \times \nabla x^3\right) = \begin{vmatrix} \frac{\partial x^1}{\partial y^1} & \frac{\partial x^1}{\partial y^2} & \frac{\partial x^1}{\partial y^3} \\ \frac{\partial x^2}{\partial y^1} & \frac{\partial x^2}{\partial y^2} & \frac{\partial x^2}{\partial y^3} \\ \frac{\partial x^3}{\partial y^1} & \frac{\partial x^3}{\partial y^2} & \frac{\partial x^3}{\partial y^3} \end{vmatrix} = J \neq 0\ . \qquad (1.5.1)$$

Demak, ushbu vektorlar uchligini ham koordinata bazisi sifatida qabul qilish mumkin.

Ta’rif: Quyidagi ko’rinishda belgilangan koordinata bazisi kontravariant koordinata bazisi deb ataladi.

$$\bar{Э}^{\alpha}\left(x^{1},\, x^{2},\, x^{3}\right) = grad\, x^{\alpha} \equiv \nabla x^{\alpha} \equiv \frac{\partial x^{\alpha}}{\partial \bar{r}} = \frac{\partial x^{\alpha}}{\partial y^{\beta}} \bar{k}_{\beta} \qquad (1.5.2)$$

Elementar ko’chish vektorining kontravariant bazisi orqali yoyilmasi quyidagi tenglik bilan aniqlanadi:

$$\overline{dr} = dx_{\beta} \bar{Э}^{\beta}$$

bu yerdagi dx_{β} koeffitsientlar $\overline{dr}$ vektorning kovariant tashkil etuvchilari (komponentalari) deb ataladi. Ushbu dx_{β} miqdorlar x_{β} koordinatalarning differensiali bo’lmaydi.

Agar

$$g^{\alpha\beta} = \bar{Э}^{\alpha} \cdot \bar{Э}^{\beta} = g^{\alpha\beta} \qquad (1.5.3)$$

deb belgilansa, u holda asosiy kvadratik forma quyidagi ko’rinishda yoziladi.

$$ds^{2} = \left(\bar{d}r \cdot \bar{d}r\right) = \left(\bar{Э}^{\alpha} dx_{\alpha}\right) \cdot \left(\bar{Э}^{\beta} dx_{\beta}\right) = g^{\alpha\beta} dx_{\alpha} dx_{\beta} \qquad (1.5.4)$$

Yuqoridagi forma koeffitsientlarining matritsasi $\left\| g^{\alpha\beta} \right\|$ simmetrik va xosmas bo’ladi. Bu xossalar (1.5.3) dan va osonlik bilan tekshirish mumkin bo’lgan quyidagi munosabatdan kelib chiqadi.

$$\left(\bar{Э}^{1} \bar{Э}^{2} \bar{Э}^{3}\right)^{2} = g_{1} = J^{2} \neq 0 \qquad (1.5.5)$$

Kontravariant bazis vektorining moduli quyidagiga teng

$$Э^{\alpha} = \left|\bar{Э}^{\alpha}\right| = \sqrt{\bar{Э}^{\alpha} \cdot \bar{Э}^{\alpha}} = \sqrt{g^{\alpha\alpha}} \quad .$$

Ushbu

$$\bar{e}^{\alpha} = \frac{\bar{Э}^{\alpha}}{\left|\bar{Э}^{\alpha}\right|} = \frac{\bar{Э}^{\alpha}}{\sqrt{g^{\alpha\alpha}}}$$

normalangan, yani moduli birga teng bo’lgan bazis vektor kiritiladi.

Kovariant va kontravariant bazis vektorlarning (1.4.4) va (1.5.2) formulalar bilan aniqlanganligi e'tiborga olinsa, ularning skalyar ko'paytmasi uchun quyidagi o'zarolik munosabatlari o'rinli ekanligini ko'rishimiz mumkin.

$$\overline{Э}_{\alpha} \cdot \overline{Э}^{\beta} = \frac{\partial \bar{r}}{\partial x^{\alpha}} \cdot \frac{\partial x^{\beta}}{\partial \bar{r}} = \frac{\partial x^{\beta}}{\partial x^{\alpha}} = \delta_{\alpha\cdot}^{\cdot\beta} \; ; \qquad \overline{Э}^{\alpha} \cdot \overline{Э}_{\beta} = \delta_{\cdot\beta}^{\alpha\cdot} \qquad (1.5.7)$$

Misol uchun, $\alpha = 1$ va $\beta = 1,\ 2,\ 3$ qiymatlariga teng bo'lsa

$$\overline{Э}^{1} \cdot \overline{Э}_{1} = Э^{1} \cdot Э_{1} \cos\left(\overline{Э}^{1}, \overline{Э}_{1}\right) = 1,\ \overline{Э}^{1} \cdot \overline{Э}_{2} = 0,\ \overline{Э}^{1} \cdot \overline{Э}_{3} = 0$$

tengliklar hosil bo'ladi.

Demak, $\overline{Э}^{1}$ vektori $\overline{Э}_{2}$ va $\overline{Э}_{3}$ vektorlarga perpendikulyar bo'ladi, $\overline{Э}^{1}$ va $\overline{Э}_{1}$ vektorlar orasidagi burchak nolga teng emas .

O'zarolik munosabati bo'lmish (1.5.7) formulada, formal bajarilgan amalning to'g'ri ekanligini koordinata bazislarini aniqlovchi (1.4.4) va (1.5.2) formulalardan foydalanib tekshirish mumkin:

$$\overline{Э}_{\alpha} \cdot \overline{Э}^{\beta} = \frac{\partial \bar{r}}{\partial x^{\alpha}} \cdot \frac{\partial x^{\beta}}{\partial \bar{r}} = \frac{\partial y^{i}}{\partial x^{\alpha}} \bar{k}_{i} \cdot \frac{\partial x^{\beta}}{\partial y^{j}} \bar{k}_{j} = \frac{\partial x^{\beta}}{\partial x^{\alpha}} = \delta_{\alpha\cdot}^{\cdot\beta}$$

Ko'pincha kontravariant koordinata bazisi o'zarolik munosabatlari yordamida kiritiladi.

Agar (1.4.5) formulada

$$\overline{Э}_{2} \times \overline{Э}_{3} = \overline{Э}^{1} J^{-1},\ \left(\overline{Э}_{3} \times \overline{Э}_{1}\right) = \overline{Э}^{2} J^{-1},\ \left(\overline{Э}_{1} \times \overline{Э}_{2}\right) = \overline{Э}^{3} J^{-1}$$

ko'rinishdagi belgilashlarni kiritsak, $Э^{\beta} \qquad \left(\beta = 1,\ 2,\ 3\right)$ vektorlar quyidagi shartni qanoatlantiradi;

$$\overline{Э}_{\alpha} \cdot \overline{Э}^{\beta} = \delta_{\alpha\cdot}^{\cdot\beta}$$

Demak $\overline{Э}^{\beta} \quad \left(\beta = 1, 2, 3\right)$ vektorlar kovariant koordinata bazisiga qo'shma bo'lgan nokomplonar vektorlar uchligini tashkil etadi. Va ularni koordinata bazisi sifatida qabul qilish mumkin.

6 §. Indekslarni ko'tarish va tushurish amallari.

Yuqorida berilgan (1.5.7) ifodani mos ravishda $g^{\alpha\gamma}$ va $g_{\alpha\gamma}$ ga ko'paytirib, α indeksi bo'yicha yig'indi amalini bajarsak, quyidagi tengliklar hosil bo'ladi:

$$g^{\alpha\gamma}\left(\overline{Э}_{\alpha}\cdot\overline{Э}^{\beta}\right)=g^{\alpha\gamma}\delta_{\alpha\cdot}^{\cdot\beta}=g^{\beta\gamma}=\overline{Э}^{\beta}\cdot\overline{Э}^{\gamma}$$
$$\Rightarrow\left(g^{\alpha\gamma}\overline{Э}_{\alpha}-\overline{Э}^{\gamma}\right)\cdot\overline{Э}_{\beta}=0$$
$$g_{\alpha\gamma}\left(\overline{Э}^{\alpha}\cdot\overline{Э}_{\beta}\right)=g_{\alpha\gamma}\delta_{\cdot\beta}^{\alpha\cdot}=g_{\beta\gamma}=\overline{Э}_{\beta}\cdot\overline{Э}_{\gamma}$$
$$\Rightarrow\left(g_{\alpha\gamma}\overline{Э}^{\alpha}-\overline{Э}_{\gamma}\right)\cdot\overline{Э}_{\beta}=0$$

bu yerda $\overline{Э}_{\beta}$ va $\overline{Э}^{\beta}$ nokomplanar vektorlar bo’lganligidan

$$\overline{Э}^{\gamma}=g^{\gamma\alpha}\overline{Э}_{\alpha};\qquad \overline{Э}_{\gamma}=g_{\gamma\alpha}\overline{Э}^{\alpha} \tag{1.6.1}$$

munosabatlar-kontravariant (kovariant) bazis vektorlarning kovariant (kontravariant) bazis vektorlar orqali yoyilmasi kelib chiqadi.

O’zarolik munosabatlarini e’tiborga olib, indekslarni ko’tarish va tushirish qoidasini beruvchi (4.22) tengliklarning ikkala tomonini mos ravishda $\overline{Э}_{\beta}$ va $\overline{Э}^{\beta}$ vektoriga ko’paytiramiz:

$$\overline{Э}^{\beta}\cdot\overline{Э}_{\gamma}=g_{\alpha\gamma}\overline{Э}^{\alpha}\cdot\overline{Э}^{\beta}=g_{\alpha\gamma}g^{\alpha\beta}=\delta_{\gamma\cdot}^{\cdot\beta}$$
$$\overline{Э}_{\beta}\cdot\overline{Э}^{\gamma}=\overline{Э}_{\beta}g^{\gamma\alpha}\cdot\overline{Э}_{\alpha}=g_{\beta\alpha}g^{\gamma\alpha}=\delta_{\beta\cdot}^{\cdot\gamma}$$

bu yerda matris’alarni ko’paytirish qoidasidan foydalanib, quyidagi munosabatlarga ega bo’lamiz. Ya’ni bular o’zaro teskari matritsa va determinantlardir.

$$\begin{gathered}g^{\alpha\gamma}g_{\alpha\beta}=\delta_{\cdot\beta}^{\gamma\cdot}\Rightarrow\\ \left\|g^{\alpha\gamma}\right\|\cdot\left\|g_{\alpha\beta}\right\|=1,\qquad \left|g^{\alpha\gamma}\right|\cdot\left|g_{\alpha\beta}\right|=1\end{gathered} \tag{1.6.2}$$

Elementar ko’chish vektorining tashkil etuvchilari uchun ham (1.6.1)ga o’xshash indeksni ko’tarish va tushirish qoidalari mavjud. Elementar ko’chish vektori uchun o’rinli bo’lgan

$$d\bar{r}=dx^{\alpha}\overline{Э}_{\alpha}=dx_{\beta}\overline{Э}^{\beta}$$

yoyilmalarda (1.6.11) dan foydalanib, $\overline{Э}_{\alpha}$ vektori $\overline{Э}^{\beta}$ orqali ifodalansa, u holda quyidagi tenglik hosil bo’ladi.

$$\left(g_{\alpha\beta}dx^{\alpha}-dx_{\beta}\right)\overline{Э}^{\beta}=0$$

$\overline{Э}^{\beta}$ ni $\overline{Э}_{\alpha}$ orqali ifodalasak esa ushbu

$$\left(dx^{\alpha}-g^{\alpha\beta}dx_{\beta}\right)\overline{Э}_{\alpha}=0$$

tenglik hosil bo'ladi. Yuqoridagilardan, $\overline{Э}^{\beta}$ va $\overline{Э}_{\alpha}$ lar erkli vektorlar bo'lgani uchun ushbu

$$\begin{aligned} dx_{\beta} &= g_{\alpha\beta}dx^{\alpha} \\ dx^{\alpha} &= g^{\alpha\beta}dx_{\beta} \end{aligned} \quad . \tag{1.6.3}$$

munosabatlar o'rinli boladi. Bu munosabatlar indekslarini ko'tarish va tushirish qoidasini beradi. Bundan tashqari elementar ko'chish vektorining kovariant (kontravariant) komponentalarini kontravariant (kovariant) komponentalari orqali yoyilmasini bildiradi

Yuqorida elementar ko'chish vektorining kovariant komponentalarini, umuman olganda, $x_{\beta}\left(x^{1},x^{2},x^{3}\right)$ - fazo nuqtalari koordinatasi funksiyasining to'la differensiali deb qarash mumkin emas deyilgan edi. Haqiqatan quyidagi tenglamaning

$$dx_{\beta}=\frac{\partial x_{\beta}}{\partial x^{\alpha}}dx^{\alpha}=g_{\alpha\beta}dx^{\alpha} \quad \Rightarrow \quad g_{\alpha\beta}=\frac{\partial x_{\beta}}{\partial x^{\alpha}}$$

integrallanish sharti

$$\frac{\partial g_{\beta\alpha}}{\partial x^{\gamma}}=\frac{\partial g_{\beta\gamma}}{\partial x^{\alpha}}$$

bajarilmaydi. $\beta=2,\quad \alpha=1,\quad \gamma=2$ qiymatlarni qabul qilganda integrallanish sharti bajarilmaydi. Masalan, silindrik koordinata sistemasida. quyidagilar o'rinli

$$g_{11}=g_{33}=1, \qquad g_{22}=r^{2}, \qquad g_{12}=g_{13}=g_{25}=0.$$

Ortogonal dekart koordinatalar sistemasida y^{α} koordinata chiziqlarining urunmasi va $y^{\alpha}=c^{\alpha}$ koordinata tekisligining normali bir xil yo'nalgan bo'ladi.

Ushbu

$$\overline{Э}_{\alpha}=\overline{Э}^{\alpha}=\bar{k}_{\alpha}; \qquad g^{\alpha\beta}=g_{\alpha\beta}=\delta_{\alpha\beta}=0, \quad \alpha\neq\beta; \qquad g_{\alpha\alpha}=1$$

munosabatlar o'rinli bo'lgani tufayli yagona bazis mavjud bo'ladi. Shu sababli, bu sistemada indekslarni yuqorida yoki quyida joylashtirishning farqi yo'q bo'lib, erkli o'zgaruvchilarni ham x_α, y_α ko'rinishda belgilash mumkin.

7 §. Koordinata bazisi elementlari ustida amallar

Koordinata bazisi elementlari ustida quyidagi amallar bajariladi.

1) Qo'shish amali ikkita ixtiyoriy oddiy ob'yekt uchun aniqlangan. Ikkita oddiy ob'yektning yig'indisi, mazkur vektorlar yordamida yasalgan parallelogramm diagonaliga teng. Bu amal kommutativlik va distributivlik xossasiga ega:

$$\overline{Э}_\alpha + \overline{Э}_\beta = \overline{Э}_\beta + \overline{Э}_\alpha \;; \qquad \rho\left(\overline{Э}_\alpha + \overline{Э}_\beta\right) = \rho\,\overline{Э}_\alpha + \rho\,\overline{Э}_\beta$$

2) Skalyarga ko'paytirish amali har qanday skalyar va ixtiyoriy ob'yektga nisbatan aniqlangan. Biror ρ skalyar va $\overline{Э}_\alpha$ oddiy ob'yekt uchun $\rho\overline{Э}_\alpha$ ko'paytmaning moduli $|\rho|\left|\overline{Э}_\alpha\right|$ bo'lgan va $\overline{Э}_\alpha$ bo'ylab yo'nalgan ob'yektni bildiradi.

3) Skalyar ko'paytirish amali ixtiyoriy ikkita oddiy ob'yekt uchun quyidagicha aniqlangan:

$$\overline{Э}_\alpha \cdot \overline{Э}_\beta = g_{\alpha\beta}\,, \qquad \overline{Э}^\alpha \cdot \overline{Э}^\beta = g^{\alpha\beta}$$
$$\overline{Э}_\alpha \cdot \overline{Э}^\beta = \delta_{\alpha\cdot}^{\cdot\beta}\,, \qquad \overline{Э}^\alpha \cdot \overline{Э}_\beta = \delta_{\cdot\beta}^{\alpha\cdot}$$

Bu amal quyidagi xossalarga ega:

$$\text{a) } \overline{Э}_\alpha \cdot \overline{Э}_\beta = \overline{Э}_\beta \cdot \overline{Э}_\alpha\,, \qquad \overline{Э}_\alpha \cdot \overline{Э}^\beta = \overline{Э}^\beta \cdot \overline{Э}_\alpha$$

$$\text{b) } \rho\left(\overline{Э}_\alpha \cdot \overline{Э}_\beta\right) = \left(\rho\,\overline{Э}_\alpha \cdot \overline{Э}_\beta\right) = \left(\overline{Э}_\alpha \cdot \rho\,\overline{Э}_\beta\right)$$

4) Vektor ko'paytirish amali ikkita ixtiyoriy oddiy ob'yekt uchun aniqlangan bo'lib, ushbu amal natijasida yana vektor hosil bo'ladi. Bu vektorning yoyilmasini koordinata bazislarining birortasi orqali yozish mumkin:

$$\begin{aligned} &\overline{Э}_\alpha \times \overline{Э}_\beta = e_{\alpha\beta\gamma}\,\overline{Э}^\gamma\,, \qquad \overline{Э}^\alpha \times \overline{Э}^\beta = e^{\alpha\beta\gamma}\,\overline{Э}_\gamma \\ &\overline{Э}_\alpha \times \overline{Э}^\beta = e_{\alpha\cdot\gamma}^{\cdot\beta}\,\overline{Э}^\gamma = e_{\alpha\cdot}^{\cdot\beta\gamma}\,\overline{Э}_\gamma \end{aligned} \qquad (1.7.1)$$

bu amal uchun ushbu xossa o'rinli,

$$\rho\left(\overline{\ni}_{\alpha}\times\overline{\ni}_{\beta}\right)=\left(\rho\,\overline{\ni}_{\alpha}\times\overline{\ni}_{\beta}\right)=\left(\overline{\ni}_{\alpha}\times\rho\,\overline{\ni}_{\beta}\right)$$

ammo kommutativlik xossasi o'rinli emas:

$$\overline{\ni}_{\alpha}\times\overline{\ni}_{\beta}\neq\overline{\ni}_{\beta}\times\overline{\ni}_{\alpha}.$$

5) Noaniq (indefenit) ko'paytirish amali chekli sondagi oddiy ob'yektlar uchun aniqlangan bo'lib, ko'paytirish natijasida yangi murakkab, poliada deb ataluvchi, ob'yekt hosil bo'ladi:

$$\overline{\ni}_{\alpha}\overline{\ni}_{\beta},\quad \overline{\ni}^{\alpha}\overline{\ni}^{\beta},\quad \overline{\ni}_{\alpha}\overline{\ni}^{\beta},\quad \overline{\ni}^{\alpha}\overline{\ni}_{\beta},$$

$$\overline{\ni}_{\alpha}\overline{\ni}_{\beta}\overline{\ni}_{\gamma},\quad \overline{\ni}_{\alpha}\overline{\ni}_{\beta}\overline{\ni}^{\gamma},\quad \overline{\ni}_{\alpha}\overline{\ni}^{\beta}\overline{\ni}^{\gamma},\quad \overline{\ni}^{\alpha}\overline{\ni}_{\beta}\overline{\ni}^{\gamma}$$

va hokazo.

Bu amal uchun ham ushbu

$$\rho\left(\overline{\ni}_{\alpha}\overline{\ni}_{\beta}\overline{\ni}_{\gamma}\right)=\left(\left(\rho\overline{\ni}_{\alpha}\right)\overline{\ni}_{\beta}\overline{\ni}_{\gamma}\right)=\left(\overline{\ni}_{\alpha}\left(\rho\overline{\ni}_{\beta}\right)\overline{\ni}_{\gamma}\right)=\left(\overline{\ni}_{\alpha}\overline{\ni}_{\beta}\left(\rho\overline{\ni}_{\gamma}\right)\right)$$

xossa o'rinli. Ammo kommutativlik xossasi

$$\overline{\ni}_{\alpha}\overline{\ni}_{\beta}\neq\overline{\ni}_{\beta}\overline{\ni}_{\alpha}$$

o'rinli emas.

Ta'rif: Ikkita oddiy ob'yektni ko'paytirish natijasida hosil qilingan poliada diada deb ataladi. Uchta obektni ko'paytirish natijasi esa triada deb ataladi.

Ta'rif: Poliada indekslarining soniga teng bo'lgan ob'yektlarining soni poliadaning rangi deb ataladi.

Diada 2-rang, triada 3-rang poliada bo'ladi. n-rang poliadalar odatda nomerli indekslar yordamida quyidagicha ifodalanadi:

$$\overline{\ni}^{\alpha_1}\overline{\ni}^{\alpha_2}\ldots\overline{\ni}^{\alpha_n},\qquad \overline{\ni}^{\alpha_1}\overline{\ni}_{\alpha_2}\ldots\overline{\ni}_{\alpha_n}\quad \text{va hokazo.}$$

8 §. Ortogonal egri chiziqli koordinatalar sistemasi.

Ta'rif: Fazoning har bir nuqtasida koordinata chiziqlari o'zaro ortogonal bo'lgan egri chiziqli koordinatalar sistemasi ortogonal koordinatalar sistemasi deb ataladi. Koordinatalar sistemasining ortogonallik sharti

$$\overline{\ni}_{\alpha}\cdot\overline{\ni}_{\beta}=0,\quad \alpha\neq\beta\ \Rightarrow\ g_{\alpha\beta}=0,\quad \alpha\neq\beta \qquad (1.8.1)$$

ko'rinishida yoziladi. Shuning uchun ortogonal sistemada

$$\left\| g_{\alpha\beta} \right\| = \left\| \begin{matrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{matrix} \right\| , \qquad \left| g_{\alpha\beta} \right| = g_{11}\, g_{22}\, g_{33} \tag{1.8.2}$$

va (1.6.2) ga asosan

$$\left\| g^{\alpha\beta} \right\| = \left\| \begin{matrix} g_{11}^{-1} & 0 & 0 \\ 0 & g_{22}^{-1} & 0 \\ 0 & 0 & g_{33}^{-1} \end{matrix} \right\| , \qquad \left| g^{\alpha\beta} \right| = \frac{1}{g_{11}\, g_{22}\, g_{33}} \tag{1.8.3}$$

formulalar o'rinli bo'ladi.

Ortogonallik shartini almashtirish formulalari yordamida ham yozish mumkin.

To'g'ri burchakli dekart koordinatalar sistemasi $y^{\alpha} = x^{\alpha}$ berilgan bo'lsin, ya'ni

$$g_{\alpha\beta} = g^{\alpha\beta} = \delta_{\alpha\cdot}^{\cdot\beta} = \delta_{\alpha\beta} , \qquad \left| g_{\alpha\beta} \right| = \left| g^{\alpha\beta} \right| = 1 \tag{1.8.4}$$

formulalar o'rinli bo'lsin. «Yangi» $\hat{C}$ sistema $\hat{x}^{\alpha}$ koordinatalar bilan aniqlansin. U holda (1.8.4) formula e'tiborga olinsa,

$$\hat{g}_{ij} = g_{\alpha\beta} \frac{\partial \xi^{\alpha}}{\partial \eta^{i}} \frac{\partial \xi^{\beta}}{\partial \eta^{j}}, \quad g_{\alpha\beta} = \hat{g}_{ij} \frac{\partial \eta^{i}}{\partial \xi^{\alpha}} \frac{\partial \eta^{j}}{\partial \xi^{\beta}}$$

$$\hat{g}^{ij} = g^{\alpha\beta} \frac{\partial \eta^{i}}{\partial \xi^{\alpha}} \frac{\partial \eta^{j}}{\partial \xi^{\beta}}, \quad g^{\alpha\beta} = \hat{g}^{ij} \frac{\partial \xi^{\alpha}}{\partial \eta^{i}} \frac{\partial \xi^{\beta}}{\partial \eta^{j}}$$

almashtirish qoidalari

$$\hat{g}_{\alpha\beta} = \frac{\partial x^{i}}{\partial \hat{x}^{\alpha}} \frac{\partial x^{j}}{\partial \hat{x}^{\beta}} \delta_{ij} = \sum_{i=1}^{3} \frac{\partial x^{i}}{\partial \hat{x}^{\alpha}} \frac{\partial x^{i}}{\partial \hat{x}^{\beta}}$$

ko'rinishga keladi.

Demak, koordinatalar sistemasini almashtirish matrisasi

$$\left\| \frac{\partial x^{i}}{\partial \hat{x}^{\alpha}} \right\|$$

ning ixtiyoriy ikki ustuni elementlari ko'paytmasining yig'indisi nolga teng bo'lsa, bu koordinata sistemasi ortogonal sistema bo'ladi.

Agar koordinatalar sistemasi bazislari normallangan bo'lsa, u holda ortogonallik shartini

$$\delta_{ij}\frac{\partial x^i}{\partial \hat{x}^\alpha}\frac{\partial x^j}{\partial \hat{x}^\beta}=\delta_{\alpha\beta}\Rightarrow \cos\left(x^i,\hat{x}^\alpha\right)\cos\left(x^i,\hat{x}^\beta\right)=\delta_{\alpha\beta} \qquad (1.8.5)$$

ko'rinishida yozish mumkin. Bu yerda

$$\frac{\partial x^i}{\partial \hat{x}^\alpha}=\cos\left(x^i,\hat{x}^\alpha\right),$$

chunki, normallangan bazisli sistemalarda

$$\cos\left(x^i,\hat{x}^\alpha\right)=\bar{e}_i\cdot\hat{e}_\alpha=\overline{K}_i\cdot\frac{\partial \bar{r}}{\partial \hat{x}^\alpha}=\overline{K}_i\cdot\overline{K}_\beta\frac{\partial x^\beta}{\partial \hat{x}^\alpha}=\frac{\partial x^i}{\partial \hat{x}^\alpha} \qquad (1.8.6)$$

munosabatlar o'rinli bo'ladi.

Ortogonal egri chiziqli koordinatalar sistemasiga misol tariqasida mexanikada ko'p qo'llaniladigan silindrik va sferik koordinatalar sistemalarini ko'ramiz. Bizga y^α - to'g'ri burchakli dekart va ξ^i - egri chiziqli koordinatalar sistemalari berilgan bo'lsin. Ushbu sistemalarning bazis vektorlari (1.4.4) formulalar

$$\overline{Э}_\alpha=\frac{\partial \bar{r}}{\partial y^\alpha}=\overline{K}_\alpha,\ \hat{Э}_i=\frac{\partial \bar{r}}{\partial \xi^i}=\overline{K}_\alpha\frac{\partial y^\alpha}{\partial \xi^i},\ \hat{Э}^j=\frac{\partial \xi^i}{\partial y^\alpha}\overline{K}_\alpha \qquad (1.8.7)$$

bilan va ularning metrikasi (1.4.7) esa

$$\hat{g}_{ij}=\hat{Э}_i\cdot\hat{Э}_j=\overline{K}_\alpha\cdot\overline{K}_\beta\frac{\partial y^\alpha}{\partial \xi^i}\frac{\partial y^\beta}{\partial \xi^j}=\delta_{\alpha\beta}\frac{\partial y^\alpha}{\partial \xi^i}\frac{\partial y^\beta}{\partial \xi^j}=\sum_{\alpha=1}^{3}\frac{\partial y^\alpha}{\partial \xi^i}\frac{\partial y^\alpha}{\partial \xi^j} \qquad (1.8.8)$$

formulalar bilan aniqlanadi.

Bu yerda ixtiyoriy egri chiziqli koordinatalar sistemasining ortogonallik sharti

$$\hat{g}_{ij}=\sum_{\alpha=1}^{3}\frac{\partial y^\alpha}{\partial \xi^i}\frac{\partial y^\alpha}{\partial \xi^j}=0,\ i\neq j \qquad (1.8.9)$$

ko'rinishida yoziladi.

Agar dekart koordinatalarini x, y, z orqali, yani $y^1=x,\ y^2=y,\ y^3=z$ deb belgilasak, ortogonal sistemaning metrikasi (1.8.8) va (1.8.9) larga ko'ra

$$\hat{g}_{ij} = \frac{\partial x}{\partial \xi^i}\frac{\partial x}{\partial \xi^j} + \frac{\partial y}{\partial \xi^i}\frac{\partial y}{\partial \xi^j} + \frac{\partial z}{\partial \xi^i}\frac{\partial z}{\partial \xi^j} = 0, \quad i \neq j \tag{1.8.10}$$

$$\hat{g}_{ii} = \left(\frac{\partial x}{\partial \xi^i}\right)^2 + \left(\frac{\partial y}{\partial \xi^i}\right)^2 + \left(\frac{\partial z}{\partial \xi^i}\right)^2, \quad (i = 1,2,3) \tag{1.8.11}$$

formulalar bilan aniqlanadi.

9 §.Silindrik koordinatalar sistemasi

Agar sistema koordinatalarini ρ, θ, z orqali, ya'ni $\xi^1 = \rho, \quad \xi^2 = \theta, \quad \xi^3 = z$ deb belgilasak, ushbu silindrik va y^α sistemalari orasidagi bog'lanish

$$\begin{gathered} x = \rho\cos\theta, \quad y = \rho\sin\theta, \quad z = z \\ \xi^1 = \rho = \sqrt{x^2 + y^2}, \quad \xi^2 = \theta = arctg\frac{y}{x}, \quad \xi^3 = z = z \end{gathered} \tag{1.9.1}$$

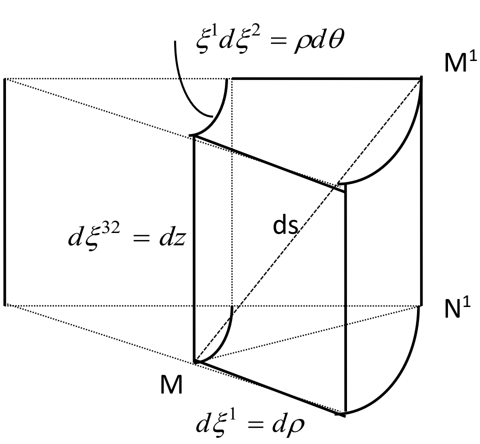
formulalar bilan beriladi. Ushbu bog'lanishlar va (1.8.10), (1.8.11) formulalardan foydalanib, osonlik bilan silindrik koordinatalar sistemasining metrikasini topamiz:

$$g_{11} = 1, \quad g_{22} = \rho^2, \quad g_{33} = 1, \quad g_{12} = g_{13} = g_{23} = 0. \tag{1.9.2}$$

Sistemaning bazislari esa (1.8.7) formulalardan aniqlanadi:

$$\begin{gathered} \hat{э}_1 = \bar{k}_1\cos\theta + \bar{k}_2\sin\theta, \ \hat{э}_2 = -\bar{k}_1\rho\sin\theta + \bar{k}_2\rho\cos\theta, \ \hat{э}_3 = \bar{k}_3 \\ \hat{э}^1 = \frac{x}{\rho}\bar{k}_1 + \frac{y}{\rho}\bar{k}_2, \ \hat{э}^2 = -\frac{y}{\rho^2}\bar{k}_1 + \frac{x}{\rho^2}\bar{k}_2, \ \hat{э}^3 = \bar{k}_3 \end{gathered} \tag{1.9.3}$$

Sistemaning metrikasini quyidagi 1-rasm va asosiy kvadratik formadan topishimiz mumkin.



1-rasm

Bu rasmdan quyidagi tenglikning o'rinliligi kelib chiqadi;

$$\left|\overline{MM'}\right|^2 = \left|\overline{MN'}\right|^2 + \left|\overline{M'N'}\right|^2, \qquad \left|\overline{MN'}\right|^2 = (d\rho)^2 + \rho^2 (d\theta)^2, \qquad \left|\overline{M'N'}\right| = dz$$

demak, $ds = |MM'|$ kesma uzunligining kvadrati

$$ds^2 = (d\rho)^2 + \rho^2 (d\theta)^2 + (dz)^2$$

bo'ladi.

Ikkinchi tomondan asosiy kvadratik forma

$$ds^2 = g_{ij}\, d\xi^i\, d\xi^j \tag{1.9.4}$$

formula bilan aniqlanadi. Ushbu munosabatlardan sistemaning metrikasi haqiqatan ham (1.9.2) formulalar bilan aniqlanishini ko'ramiz.

10 §. Sferik koordinatalar sistemasi

Biz sferik koodinatalar sistemasini analitik geometriya fanida ko'rib o'tganmiz. To'g'ri burchakli dekart va sferik koordinatalar sistemalari orasidagi bog'lanish ushbu formulalar bilan berilgan bo'lsin,

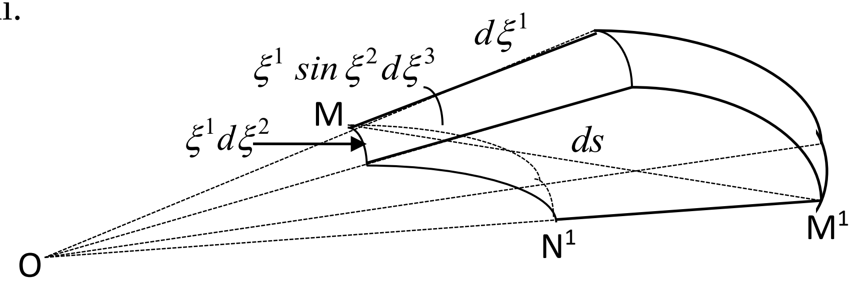
$$x = r\cos\lambda\sin\theta, \qquad y = r\sin\theta\sin\lambda, \qquad z = r\cos\theta$$

$$r = \sqrt{x^2 + y^2 + z^2}, \qquad \theta = arctg\frac{\sqrt{x^2 + y^2}}{z}, \qquad \lambda = arctg\frac{y}{x}$$

bu yerda $\xi^1 = r$, $\xi^2 = \theta$, $\xi^3 = \lambda$ ekanligi nazarda tutilsa, (1.8.7), (1.8.10) va (1.8.11) formulalar sistema bazislarini va metrikasini aniqlash imkonini beradi. Buni 2-rasm va asosiy kvadratik formadan foydalanib ham aniqlashimiz mumkin. Haqiqatan $ds = |MM'|$ kesmaning kvadrati uchun o'rinli bo'lgan

$$ds^2 = (dr)^2 + (rd\theta)^2 + (r\sin\theta\, d\lambda)^2$$

munosabatdan va (1.9.4) dan $g_{11} = 1$, $g_{22} = r^2$, $g_{33} = (r\sin\theta)^2$ formulalar kelib chiqadi.



2-rasm

11 §. Koordinatalarni almashtirish

Ikkita-«eski» ξ^1, ξ^2, ξ^3 va «yangi» η^1, η^2, η^3 koordinatalar sistemasi berilgan bo'lsin va ular orasida o'zaro bir qiymatli moslik o'natigan bo'lsin, yani «eski» C sistemadan «yangi» $\hat{C}$ sistemaga o'tish qoidasini beruvchi

$$\eta^\alpha = \eta^\alpha\left(\xi^1, \xi^2, \xi^3\right) \qquad \alpha = 1, 2, 3 \tag{1.11.1}$$

funksiyalar uzluksiz differensiallanuvchi va almashtirish yakobiani

$$J = \left|\frac{\partial \eta^\alpha}{\partial \xi^\beta}\right| \neq 0 \tag{1.11.2}$$

bo'lsin. Demak qaralayotgan sohaning har bir nuqtasi atrofida (1.11.1) formulalarni ξ^1, ξ^2, ξ^3 larga nisbatan yechib, $\hat{C}$ sistemadan C sistemaga o'tish qoidalarini beruvchi

$$\xi^\alpha = \xi^\alpha\left(\eta^1, \eta^2, \eta^3\right) \tag{1.11.3}$$

formulalarni yozish mumkin.

Koordinata bazisi elementlarini almashtirish uchun quyidagicha ish ko'riladi.

$\overline{Э}_\alpha$ va $\overline{Э}^\beta$ lar C sistemaning, $\hat{Э}_i$ va $\hat{Э}^i$ lar esa $\hat{C}$ sistemaning koordinata bazislari bo'lsin. Bazis vektorlarning tarifiga ko'ra;

$$\begin{aligned} \hat{Э}_i &= \frac{\partial \bar{r}}{\partial \eta^i} = \frac{\partial \bar{r}}{\partial \xi^\alpha}\frac{\partial \xi^\alpha}{\partial \eta^i}, \quad \hat{Э}^i = \frac{\partial \eta^i}{\partial \bar{r}} = \frac{\partial \eta^i}{\partial \xi^\beta}\frac{\partial \xi^\beta}{\partial \bar{r}} \\ \overline{Э}_\alpha &= \frac{\partial \bar{r}}{\partial \xi^\alpha} = \frac{\partial \bar{r}}{\partial \eta^i}\frac{\partial \eta^i}{\partial \xi^\alpha}, \quad \overline{Э}^\beta = \frac{\partial \xi^\beta}{\partial \bar{r}} = \frac{\partial \xi^\beta}{\partial \eta^j}\frac{\partial \eta^j}{\partial \bar{r}} \end{aligned} \tag{1.11.4}$$

formulalar yoki

$$\hat{Э}_i = \overline{Э}_\alpha \frac{\partial \xi^\alpha}{\partial \eta^i}, \quad \overline{Э}_\alpha = \hat{Э}_i \frac{\partial \eta^i}{\partial \xi^\alpha} \tag{1.11.5}$$

$$\hat{Э}^i = \overline{Э}^\beta \frac{\partial \eta^i}{\partial \xi^\beta}, \quad \overline{Э}^\beta = \hat{Э}^i \frac{\partial \xi^\beta}{\partial \eta^i} \tag{1.11.6}$$

formulalar o'rinli bo'ladi.

Ta'rif: Kovariant bazis vektorlarini almashtirish qoidasini beruvchi (1.11.5) formulalar kovariant almashtirish qonuni deb ataladi.

Ta’rif: Kontravariant bazis vektorlarni almashtirish qoidasini beruvchi (1.11..6) formulalar kontravariant almashtirish qonuni deb ataladi.

Elementar ko’chish vektori komponentalarini almashtirish qoidasi esa quyidagicha bo’ladi.

Invariant miqdor bo’lgan elementar ko’chish vektori C va $\hat{C}$ sistemalarning bazislari orqali quyidagi ko’rinishlarda ifodalanadi:

$$\overline{dr} = d\xi_\beta \overline{\ni}^\beta = d\eta_j \cdot \hat{\ni}^j, \ \overline{dr} = d\xi^\alpha \overline{\ni}_\alpha = d\eta^i \cdot \hat{\ni}_i$$

bu yerdagi ifodalarni (1.11.5) va (1.11.6) qoidalar yordamida bir xil sistemadagi bazis vektorlar orqali yozilsa, u holda quyidagi tengliklar kelib chiqadi.

$$\begin{aligned} &\left(d\xi_\beta - d\eta_j \frac{\partial \eta^j}{\partial \xi^\beta}\right)\overline{\ni}^\beta = 0, \ \left(d\xi_\beta \frac{\partial \xi^\beta}{\partial \eta^j} - d\eta_j\right)\hat{\ni}^j = 0 \\ &\left(d\xi^\alpha - d\eta^i \frac{\partial \xi^\alpha}{\partial \eta^i}\right)\overline{\ni}_\alpha = 0, \ \left(d\xi^\alpha \frac{\partial \eta^i}{\partial \xi^\alpha} - d\eta^i\right)\hat{\ni}_i = 0 \end{aligned} \qquad (1.11.9)$$

Bazis vektorlar erkli bo’lganligi sababli, (1.11.9) dan mazkur $d\bar{r}$ vektori komponentalarini almashtirish qoidalari deb ataluvchi quyidagi munosabatlar o’rinli bo’ladi:

$$d\xi_\beta = d\eta_i \frac{\partial \eta^i}{\partial \xi^\beta}, \qquad d\eta_j = d\xi_\beta \frac{\partial \xi^\beta}{\partial \eta^j} \qquad (1.11.10)$$

$$d\xi^\alpha = d\eta^i \frac{\partial \xi^\alpha}{\partial \eta^i}, \qquad d\eta^i = d\xi^\alpha \frac{\partial \eta^i}{\partial \xi^\alpha} \qquad (1.11.11)$$

12 §. Skalyar va vektor miqdorlar.

Ta’rif: U yoki bu matematik ob’yekt bilan bog’liq bo’lgan va bu ob’yektlar bilan yoki ular qaralayotgan hisoblash sistemalari bilan ma’lum almashtirishlar bajarilganda o’zgarmaydigan sonlar, algebraik ifodalar va boshqa miqdorlarga invariantlar deb ataladi.

Biror geometrik ob’yekt yoki fizik hodisani o’rganish uchun koordinatalar sistemasini kiritishga to’g’ri keladi. Muayyan $\xi^1,\ \xi^2,\ \xi^3$ koordinatalar sistemasida berilgan $f\left(\xi^1,\ \xi^2,\ \xi^3\right)$ ifodaning qiymatlari p biror ob’yektning ichki xususiyatlarini aks ettirsa, u hisoblash sistemasining tanlanishiga bog'liq bo'lmaydi, yani boshqa η^1, η^2, η^3 sistema kiritilsa,

$$\begin{aligned} & f\left(\xi^1,\xi^2,\xi^3\right)=f\left(\xi^1\left(\eta^1,\eta^2,\eta^3\right),\xi^2\left(\eta^1,\eta^2,\eta^3\right),\xi^3\left(\eta^1,\eta^2,\eta^3\right)\right)= \\ & =f\left(\eta^1,\eta^2,\eta^3\right)=p \end{aligned} \tag{1.12.1}$$

munosabat o'rinli bo'ladi. (1.12.1) munosabatni qanoatlantiruvchi hamma ifodalar invariantlar deb ataladi. Masalan, $M_1\left(\xi_1^1,\xi_1^2,\xi_1^3\right)$ va $M_2\left(\xi_2^1,\xi_2^2,\xi_2^3\right)$ nuqtalar η^1, η^2, η^3 koordinatalar sistemasida boshqa koordinatalar bilan aniqlanadi, lekin ular orasidagi masofa o'zgarmasdan qoladi.

Invariant miqdorlarning eng sodda misoli skalyar va vektor miqdorlardir.

Ta'rif:_Koordinatalar sistemasining tanlanishiga bog'liq bo'lmagan va muayyan koordinatalar sistemasida bir son bilan beriladigan miqdor skalyar deb ataladi.

Misol uchun ikki nuqta orasidagi masofa, bosim, zichlik, harorat va boshqa ko'p miqdorlar skalyar bo'ladi.

Ta'rif: Koordinatalar sistemasini tanlanishiga bog'liq bo'lmagan va oddiy ob'yektlarning quyidagi chiziqli ifodasi ko'rinishida beriladigan miqdor vektor deb ataladi:

$$\bar{a}=a^{\alpha}\bar{Э}_{\alpha}=a_{\beta}\bar{Э}^{\beta} \tag{1.12.2}$$

Vektorning tashkil etuvchilari mazkur vektorni bazis vektorlarga skalyar ko'paytirish natijasida hosil qilinadi:

$$\begin{aligned} \bar{a}\cdot\bar{Э}^{\beta}&=a^{\alpha}\bar{Э}_{\alpha}\cdot\bar{Э}^{\beta}=a^{\alpha}\,\delta_{\alpha\cdot}^{\cdot\beta}=a^{\beta} \\ \bar{a}\cdot\bar{Э}_{\alpha}&=a_{\beta}\bar{Э}^{\beta}\cdot\bar{Э}_{\alpha}=a_{\beta}\,\delta_{\cdot\alpha}^{\beta\cdot}=a_{\alpha} \end{aligned} \tag{1.12.3}$$

Koordinatalar sistemasini almashtirganda vektor komponentalarining o'zgarish qoidasi (1.12.3) va bazis vektorlarni almashtirish qoidalari esa (1.11.5), (1.11.6) formulalar yordamida hosil qilinadi. Natijada quyidagilar hosil bo'ladi:

$$\hat{a}^j = \bar{a} \cdot \hat{Э}^j = \bar{a}\frac{\partial \eta^j}{\partial \xi^\beta}\bar{Э}^\beta = a^\beta \frac{\partial \eta^j}{\partial \xi^\beta}$$
$$\hat{a}_i = \bar{a} \cdot \hat{Э}_i = \bar{a} \cdot \bar{Э}_\alpha \frac{\partial \xi^\alpha}{\partial \eta^i} = a_\alpha \frac{\partial \xi^\alpha}{\partial \eta^i} \tag{1.12.4}$$

$$a^\beta = \bar{a} \cdot \bar{Э}^\beta = \bar{a}\frac{\partial \xi^\beta}{\partial \eta^j}\hat{Э}^j = \hat{a}^j \frac{\partial \xi^\beta}{\partial \eta^j}$$
$$a_\alpha = \bar{a} \cdot \bar{Э}_\alpha = \bar{a} \cdot \hat{Э}_i \frac{\partial \eta_i}{\partial \xi^\alpha} = \hat{a}_i \frac{\partial \eta^i}{\partial \xi^\alpha} \tag{1.12.5}$$

Vektorning ta’rifidan va skalyar ko’paytirish qoidasidan kelib chiqadigan quyidagi munosabatlardan:

$$\bar{a} \cdot \bar{a} = a^\alpha \bar{Э}_\alpha \cdot a_\beta \bar{Э}^\beta = a^\alpha a_\beta \bar{Э}_\alpha \cdot \bar{Э}^\beta = a^\alpha a_\beta \delta_{\alpha\cdot}^{\cdot\beta} = a^\alpha a_\alpha$$
$$\bar{a} \cdot \bar{a} = a^\alpha \bar{Э}_\alpha \cdot a^\beta \bar{Э}_\beta = a^\alpha a^\beta \bar{Э}_\alpha \cdot \bar{Э}_\beta = a^\alpha a^\beta g_{\alpha\beta}$$
$$\bar{a} \cdot \bar{a} = a_\beta \bar{Э}^\beta \cdot a_\alpha \bar{Э}^\alpha = a_\beta a_\alpha \bar{Э}^\beta \cdot \bar{Э}^\alpha = a_\beta a_\alpha g^{\beta\alpha}$$

vektor modulini hisoblash va indekslarni ko’tarish va tushirish qoidalari kelib chiqadi:

$$|\bar{a}| = \sqrt{a^\alpha a_\alpha}, \qquad a_\alpha = g_{\alpha\beta} a^\beta, \qquad a^\alpha = g^{\beta\alpha} a_\beta$$

Vektorlarning yoyilmasini quyidagicha normallangan bazislar orqali ham yozish mumkin:

$$\bar{a} = a^\alpha \bar{Э}_\alpha = a^\alpha \frac{\bar{Э}_\alpha}{\sqrt{g_{\alpha\alpha}}}\sqrt{g_{\alpha\alpha}} = a^\alpha \sqrt{g_{\alpha\alpha}}\bar{e}_\alpha = \overset{*}{a}{}^\alpha \bar{e}_\alpha \Rightarrow$$
$$\overset{*}{a}{}^\alpha = \sqrt{g_{\alpha\alpha}} a^\alpha = a^\alpha Э_\alpha \;\; (!)$$
$$\bar{a} = a_\beta \bar{Э}^\beta = a_\beta \sqrt{g^{\beta\beta}} \frac{\bar{Э}^\beta}{\sqrt{g^{\beta\beta}}} = \overset{*}{a}_\beta \bar{e}^\beta \;\; \Rightarrow \tag{1.12.7}$$
$$\overset{*}{a}_\beta = a_\beta \sqrt{g^{\beta\beta}} = a_\beta Э^\beta \;\; (!)$$

Bu yerda $\overset{*}{a}{}^\alpha$ va $\overset{*}{a}_\beta$ lar $\bar{a}$ vektorning kontravariant va kovariant fizik komponentalari deb ataladi.

Masalalar yechishga doir namuna

1. $Ox_1'x_2'x_3'$ va $Ox_1x_2x_3$ koordinatalar sistemasini bog'lovchi koordinat almashtirishlar quyidagi jadval shaklida berilgan. Bu jadvaldan foydalanib ortoganallik sharti bajarilishini, A(1,2,4) nuqtaning shtrixli koordinatalar sistemasidagi koordinatalarini aniqlang va $a(a_1,\ a_2,\ a_3)$ vektorni shtrixli koordinatalar sistemasida hamda $Ax+By+Cz+D=0$ tekislik tenglamasini shtrixli koordinatalar sistemasida ifodalang.

	x_1	x_2	x_3
x_1'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$

Yechish.

Berilgan jadvaldan foydalanib ortoganallik sharti bajarilishini aniqlaymiz. Buning uchun ixtiyoriy satr(ustun)ning komponentalarini boshqa ixtiyoriy satr(ustun)ning mos komponentalariga ko'paytmalari yig'indisi nolga teng bo'lishi kerak:

$$-\frac{3}{5\sqrt{2}}\cdot\frac{1}{2}+\frac{4}{5}\cdot 0+\frac{3}{5\sqrt{2}}\cdot\frac{1}{\sqrt{2}}=0;$$

$$-\frac{3}{5\sqrt{2}}\cdot\left(-\frac{4}{5\sqrt{2}}\right)+\frac{4}{5}\cdot\left(-\frac{3}{5}\right)+\frac{3}{5\sqrt{2}}\cdot\frac{4}{\sqrt{2}}=0;$$

$$-\frac{3}{5\sqrt{2}}\cdot\left(-\frac{4}{5}\right)+\frac{4}{5}\cdot 0+\frac{3}{5\sqrt{2}}\cdot\left(-\frac{3}{5}\right)=0;$$

$$-\frac{3}{5\sqrt{2}}\cdot\frac{3}{5\sqrt{2}}+\left(\frac{1}{\sqrt{2}}\right)^2-\frac{4}{5\sqrt{2}}\cdot\frac{4}{5\sqrt{2}}=0.$$

Demak ortoganallik sharti bajarilar ekan.

Shtrixli koordinatalar sistemasida A nuqtaning koordinatalari quyidagicha aniqlanadi:

$$x_1'=-\frac{3}{2\sqrt{5}}\cdot 1+\frac{1}{\sqrt{2}}\cdot 2-\frac{4}{5\sqrt{2}}\cdot 4=-\frac{23}{5\sqrt{2}};$$

$$x_2' = -\frac{4}{5}\cdot 1 + 0\cdot 2 - \frac{3}{5}\cdot 4 = -\frac{16}{5};$$

$$x_3' = \frac{3}{2\sqrt{5}}\cdot 1 + \frac{1}{\sqrt{2}}\cdot 2 + \frac{4}{5\sqrt{2}}\cdot 4 = \frac{19+\sqrt{5}}{5\sqrt{2}}.$$

Demak, shtrixli koordinatalar sistemasida A$\left(-\frac{3}{2\sqrt{5}}, -\frac{16}{5}, \frac{19+\sqrt{5}}{5\sqrt{2}}\right)$ ga teng bo'ladi.

Tekislik tenglamasi shtrixli koordinatalarda quyidagicha ifodalanadi:

$$Ax_1 + Bx_2 + Cx_3 + D =$$

$$= A\left(-\frac{3}{2\sqrt{5}}x_1' + \frac{1}{\sqrt{2}}x_2' - \frac{4}{5\sqrt{2}}x_3'\right) + B\left(-\frac{4}{5}x_1' - \frac{3}{5}x_3'\right)$$

$$+ C\left(\frac{3}{5\sqrt{2}}x_1' + \frac{1}{\sqrt{2}}x_2' + \frac{4}{5\sqrt{2}}x_3'\right) + D = 0 \ \Rightarrow$$

$$\left(-\frac{3A}{2\sqrt{5}} + \frac{4B}{5} + \frac{3C}{5\sqrt{2}}\right)x_1' + \left(-\frac{A}{\sqrt{2}} + \frac{C}{\sqrt{2}}\right)x_2' + \left(-\frac{4A}{5\sqrt{2}} - \frac{3B}{5} + \frac{4C}{5\sqrt{2}}\right)x_3' + D = 0$$

I bobga doir masalalar

1. Indekslarni sonli qiymatlaridan foydalanib, quyidagi ifodalarni yoyib yozing. O'zaro teng ifodalarni ko'rsating.

a) t_{ii};

b) $p_{ij}u_j$, u_jp_{ij}, $p_{ij}u_i$, u_ip_{ij};

c) $q_{ij}a_ib_j$, $q_{ij}b_ja_i$, $b_jq_{ij}a_i$, $a_iq_{ij}b_j$, $a_ib_jq_{ij}$, $b_ja_iq_{ij}$, $q_{ij}a_jb_j$;

d) $a_{ij}b_{ij}$, $a_{ji}b_{ji}$, $a_{ij}b_{ji}$, $b_{ij}a_{ji}$.

2. a) δ_{ii}, $\delta_{ij}\delta_{ji}$, $\delta_{ij}\delta_{jk}\delta_{ki}$ ifodalarni yigindisini hisoblang;

b) agar barcha indekslar 1,2,…,n qiymatlarni qabul qilsa, ularning yigindisini hisoblang.

3. a) Quyidagi munosabatni qanoatlantiruvchi bazis e_i berilgan, e^k birlik bazis bo'lishini ko'rsating:

$$e^i = g^{ik}e^k, \; e_j = g_{jk}e^k,$$

bu yerda g^{ik}- $\|g^{ij}\|$ matrisaning komponentalari, $\|g_{ij}\|$ teskari matrisa, $g_{ij} = e_ie_j$.

b) $e^ie^j = g^{ij}$ tenglikni tekshiring.

v) $uv = u^iv_i$, $v^i = e^iv$, $v_i = e_iv$ tenglikni tekshiring

g) e_i va e^j bazislarni o'zaro bir-biri orqali quyidagi formulalar bilan ifodalanishini tekshiring,

$$e^1 = \frac{e_2 \times e_3}{V_*},\ e^2 = \frac{e_3 \times e_1}{V_*},\ e^3 = \frac{e_1 \times e_2}{V_*},\ V_* = e_1(e_2 \times e_3),$$

$$e_1 = \frac{e^2 \times e^3}{V^*},\ e_2 = \frac{e^3 \times e^1}{V^*},\ e_3 = \frac{e^1 \times e^2}{V^*},\ V^* = e^1(e^2 \times e^3).$$

Bu yerda $e_2 \times e_3$- e_2, e_3 vektorlarning vektor ko'paytmasi, V_*- e_1, e_2, e_3 vektorlarning aralash ko'paytmasi.

d) Metrik tenzor komponentlari ma'lum bo'lsa, e_i va e^j vektorlar uzunligi qancha?

4. a) O'zaro ortonormallangan bazisni toping;

b) Har ikkitasining orasidagi burchagi $\frac{\pi}{3}$ ga teng bo'lgan e_i birlik vektorlar bazisi berilgan bo'lsin. Ularning o'zaro bazislarini toping. e^i vektor uzunligi qancha? $v = ae^1 + be^2 + ce^3$ vektorning kontravariant komponentalarini toping.

5. Ekvivalent tenzor komponentlari uchun quyidagi munosabatlarni to'g'riligini ko'rsating:

a) $s_{ij} = s_{ji} \Leftrightarrow s^{kl} = s^{lk} \Leftrightarrow s^{m}_{\cdot n} = s_{n}^{\cdot m}$;

b) $a_{ij} = -a_{ji} \Leftrightarrow a^{kl} = -a^{lk} \Leftrightarrow a^{m}_{\cdot n} = -a_{n}^{\cdot m}$.

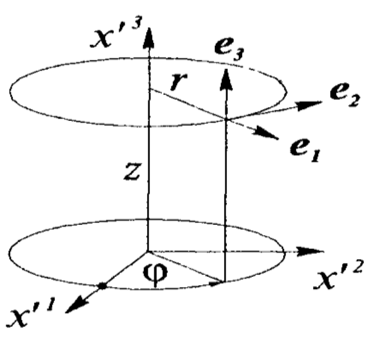
6. Ikkinchi rang tenzor uchun quyidagi munosabatlar to'g'rimi:

a) $t^{i}_{\cdot j} = t_{j}^{\cdot i}$; b) $t^{i}_{\cdot i} = t_{k}^{\cdot k}$?

7. Silindr koordinatalar sistemasi

$$x^1 = r, \qquad x^2 = \varphi, \qquad x^3 = z,$$

dekart kordinatalari bilan quyidagicha bog'langan (3-rasmga qarang):



3-rasm.

a) quyidagi nuqtalarda silindrik kordinatalar sistemasi bazis vektorlarini toping va ularni dekart koordinatalar sistemasi bazislari bilan ifodalang:

$$M_1(r=5;\ \varphi=0; z=0) \text{ va } M_2(r=10;\ \varphi=\frac{\pi}{6}; z=1).$$

b) silindr kordinatalar sistemasida metrik tenzorning kovariant, kontvariant va aralash komponentlarini toping.

c) $M_1\ va\ M_2$ nuqtalarda o'zaro bazis vektorlarni toping.

8. Tekislikdagi Dekart koordinatalar sistemasi bilan (x'^1, x'^2) $x'^1 = rcos\varphi,\ x'^2 = rsin\varphi$ ko'rinishida bog'langan $e'^1\ va\ e'^2$ bazis vektorlarni $x^1 = r, x^2 = \varphi$ polyar koordinatalar sistemasi bo'yicha yoying.

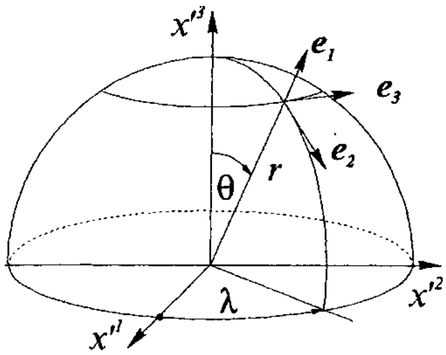
9. (7) masaladagi silindr kordinatalar sistemasida p ikkinchi rang tenzor $p^{11} = a, p^{22} = {}^{b}/_{r^2}$ komponentalarga ega, qolgan komponentalari esa nolga teng. Shu tenzorning dekart kordinatalar sistemasidagi komponentalarini toping.

10. $x^1 = r, x^2 = \theta, x^3 = \lambda$ sferik koordinatalarni dekart koordinatalar bilan quyidagicha bog'langan (4- rasmga qarang),

$$x'^1 = rsin\theta cos\lambda,$$

$$x'^2 = rsin\theta sin\lambda,$$

$$x'^3 = rcos\theta.$$



4-rasm.

Sferik kordinatalar sistemasi bazisini toping va ularni dekart sistema bazisi bilan ifodalang. Sferik kordinatalar sistemasida metrik tenzorning kovariant, kontvariant va aralash komponentlarini toping.

11. Dekart kordinatalar sistemasida v vektor quyidagi tenglik orqali aniqlangan bolsa, sferik kordinatalar sistemasida uni komponentlarini toping:

$$v = \frac{x'^1 e'_1 + x'^2 e'_2 + x'^3 e'_3}{\sqrt{(x'^1)^2 + (x'^2)^2 + (x'^3)^2}}.$$

12. $(x, \ y, \ z)$ dekart koordinatalari bilan $x^1 = r, \ x^2 = \varphi, \ x^3 = z$ elliptik kordinatalar sistemasi quyidagicha bog'langan bo'lsa,

$$x = \sqrt{r^2 + a^2} cos\varphi, \qquad y = rsin\varphi, \qquad r \geq 0 .$$

u holda $dx^1 e_1 + dx^2 e_2 + dx^3 e_3$ element uzunligi kvadratini toping.

13. Uchta a_1, a_2, a_3 vektorlardan hosil qilingan $a_1(a_2 a_3) - a_2(a_3 a_1)$ vektorning a_3 ga perpendikulyarligini isbotlang.

14. Agar a_1 va $a_2 + a_3$ vektorlar bir-biriga perpendikulyar bo'lsa, a_1+a_2+a_3 va a_1-a_2-a_3 vektorlarning modullarini teng bo'lishini isbotlang.

15. a=$i + j - k$ va $b = i - j + k$ vektorlarning modullari orasidagi burchagini va a_b, b_a proeksiyalarini toping.

16. Komplanar bo'lmagan a_1, a_2, a_3 vektorlarlar bilan komplanar bo'lmagan , b_1, b_2, b_3 vektorlar quyidagicha bog'langan bo'lsin:

$$a_1 = \alpha_{11} b_1 + \alpha_{12} b_2 + \alpha_{13} b_3,$$
$$a_2 = \alpha_{21} b_1 + \alpha_{22} b_2 + \alpha_{23} b_3,$$
$$a_3 = \alpha_{31} b_1 + \alpha_{32} b_2 + \alpha_{33} b_3.$$

a_1, a_2, a_3 va b_1, b_2, b_3 vektorlar uchtaliklari bir orientatsiyali bo'lsa,

$\begin{vmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{vmatrix}$>0 bo'ladi, turli orientatsiyali bo'lsa,

$\begin{vmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{vmatrix}$<0 bo'ladi. Shularni isbotlang.

17-23 masalalarda quyidagilarni aniqlang:

a) ortoganallik sharti bajarilishini;

b) A(1,2,4) nuqtaning shtrixli koordinatalar sistemasidagi koordinatalarini;

c) $a(a_1, \ a_2, \ a_3)$ vektorni shtrixli koordinatalar sistemasida ifodalang;

d) $Ax + By + Cz + D = 0$ tekislik tenglamasini shtrixli koordinatalar sistemasida ifodalang.

17.

	x_1	x_2	x_3

x_1'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$

18.

	x_1	x_2	x_3
x_1'	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
x_2'	$-\frac{1}{3}$	$-\frac{2}{3}$	0
x_3'	0	0	1

19.

	x_1	x_2	x_3
x_1'	0	$-\frac{4}{5}$	$\frac{3}{5}$
x_2'	1	0	0
x_3'	0	$\frac{3}{5}$	$\frac{4}{5}$

20.

	x_1	x_2	x_3
x_1'	0	$-\frac{2}{3}$	$\frac{1}{3}$
x_2'	1	0	1
x_3'	0	$\frac{1}{3}$	$\frac{2}{3}$

21.

	x_1	x_2	x_3
x_1'	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
x_2'	0	0	1
x_3'	$-\frac{1}{3}$	$-\frac{2}{3}$	0

22.

	x_1	x_2	x_3
x_1'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$

23.

	x_1	x_2	x_3
x_1'	$-\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$
x_2'	$\frac{4}{5}$	0	$-\frac{3}{5}$
x_3'	$\frac{3}{5\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5\sqrt{2}}$

II BOB. TENZORLAR USTIDA AMALLAR

1 §. Tenzor.

Ta’rif: Koordinata sistemasini tanlanishiga bog’liq bo’lmagan va diadalarning ushbu

$$T = T^{\alpha\beta}\overline{\Im}_{\alpha}\overline{\Im}_{\beta} = T_{\alpha\beta}\overline{\Im}^{\alpha}\overline{\Im}^{\beta} = T^{\alpha\cdot}_{\cdot\beta}\,\overline{\Im}_{\alpha}\Im^{\beta} = T^{\cdot\beta}_{\alpha\cdot}\,\overline{\Im}^{\alpha}\overline{\Im}_{\beta} \qquad (2.1.1)$$

chiziqli ifodasi ko’rinishida beriladigan miqdor ikkinchi rang tenzor deb ataladi.

Ta’rif: Ikkita $\vec{a}$ va $\vec{b}$ lar orqali tuzilgan ushbu ifoda D= $\vec{a}\otimes\vec{b}$ diada deyiladi. $\vec{a}$ − diadaning chap vektori, $\vec{b}$ −.diadaning o’ng vektori deyiladi. $\otimes$ − diadik ko’paytma belgisi, $a^i b^j$ − sonlar to’plami esa Dning komponentalari deyiladi.

Ta’rif: $\vec{\Im_i}$ va $\vec{\Im^j}$ lardan tuzigan E diada birlik diada deyiladi.

(2.1.1) ifodadagi diadalar oldidagi koeffitsientlar - tenzorning tashkil etuvchilari 4 xil bo’lib, $T^{\alpha\beta}$ - kontravariant, $T_{\alpha\beta}$ - kovariant, $T^{\alpha\cdot}_{\cdot\beta}$ va $T^{\cdot\beta}_{\alpha\cdot}$ lar esa aralash komponentalar deb ataladi.

Ikkinchi rang tenzor komponentalarining indeksi 1, 2, 3 qiymatlarni qabul qilgani tufayli, unga tegishli komponentalarning soni $3^2 = 9$ ta bo’ladi.

Ta’rifga ko’ra, tenzor invariant miqdordir. Shu sababli uning tashkil etuvchilari va diadalar koordinata sistemasi almashtirilganda o’zaro teskari matritsalar yordamida almashtiriladi. Masalan, tenzorning ta’rifiga va (1.11.5)ga ko’ra o’rinli bo’lgan

$$T^{\alpha\beta}\overline{\Im}_{\alpha}\overline{\Im}_{\beta} = \hat{T}^{ij}\hat{\Im}_i\hat{\Im}_j,\ \overline{\Im}_{\alpha}\overline{\Im}_{\beta} = \hat{\Im}_i\frac{\partial\eta^i}{\partial\xi^{\alpha}}\hat{\Im}_j\frac{\partial\eta^j}{\partial\xi^{\beta}}$$

munosabatlardan quyidagi almashtirish qoidasi kelib chiqadi.

$$\hat{T}^{ij} = T^{\alpha\beta}\frac{\partial\eta^i}{\partial\xi^{\alpha}}\frac{\partial\eta^j}{\partial\xi^{\beta}} \qquad (2.1.2)$$

Ikkinchi tomondan, (1.11.5)ning birinchi formulasidan esa

$$\hat{\Im}_i\hat{\Im}_j = \overline{\Im}_{\alpha}\frac{\partial\xi^{\alpha}}{\partial\eta^i}\overline{\Im}_{\beta}\frac{\partial\xi^{\beta}}{\partial\eta^j} = \frac{\partial\xi^{\alpha}}{\partial\eta^i}\frac{\partial\xi^{\beta}}{\partial\eta^j}\overline{\Im}_{\alpha}\overline{\Im}_{\beta}$$

almashtirish formulasi o'rinli ekanligini ko'ramiz. Ushbu diadaning almashtirish matrisalari va (2.1.2) formuladagi matrisalar o'zaro teskari matrisalardir:

$$\left\| \frac{\partial \xi^{\alpha}}{\partial \eta^{i}} \right\| \cdot \left\| \frac{\partial \eta^{i}}{\partial \xi^{\alpha}} \right\| = 1, \qquad \left\| \frac{\partial \xi^{\beta}}{\partial \eta^{j}} \right\| \cdot \left\| \frac{\partial \eta^{j}}{\partial \xi^{\beta}} \right\| = 1.$$

Agar (1.11.5) va (1.11.6) formulalarga ko'ra

$$\hat{\ni}_{i} = \overline{\ni}_{\alpha} \frac{\partial \xi^{\alpha}}{\partial \eta^{i}}, \quad \hat{\ni}^{j} = \frac{\partial \eta^{j}}{\partial \xi^{\beta}} \overline{\ni}^{\beta}$$

ekanligini nazarda tutsak, quyidagi munosabatlar hosil qilinadi.

$$T^{\alpha\beta} = \hat{T}^{ij} \frac{\partial \xi^{\alpha}}{\partial \eta^{i}} \frac{\partial \xi^{\beta}}{\partial \eta^{j}}, \quad T^{\alpha \cdot}_{\cdot \beta} = \hat{T}^{i \cdot}_{\cdot j} \frac{\partial \xi^{\alpha}}{\partial \eta^{i}} \frac{\partial \eta^{j}}{\partial \xi^{\beta}} \tag{2.1.3}$$

Tenzor komponentalari uchun indekslarni ko'tarish (tushirish) qoidasi o'rinlidir. Masalan, (2.1.1) va (1.3.2) dan quyidagi munosabatlar kelib chiqadi;

$$T^{\alpha\beta} \overline{\ni}_{\alpha} \overline{\ni}_{\beta} = T^{\alpha \cdot}_{\cdot \beta} \overline{\ni}_{\alpha} \overline{\ni}^{\beta} = T^{\alpha \cdot}_{\cdot \beta} \overline{\ni}_{\alpha} g^{\beta\gamma} \overline{\ni}_{\gamma} = T^{\alpha \cdot}_{\cdot \beta} g^{\beta\gamma} \overline{\ni}_{\alpha} \overline{\ni}_{\gamma}$$

Bu yerda β va γ befarq indekslardir. Agar ularning o'rnini almashtirsak, quyidagi tenglik hosil bo'ladi.

$$T^{\alpha\beta} = T^{\alpha \cdot}_{\cdot \gamma} g^{\beta\gamma} \tag{2.1.4}$$

Kovariant va aralash komponentalarda ham indekslarni ko'tarish yoki tushirish shunga o'xshash bajariladi.

To'g'ri burchakli dekart koordinatalar sistemasida $g^{\alpha\beta} = \delta^{\alpha\beta}$ tengligidan muayyan indeksni yuqorida yoki quyida yozishning farqi yo'q, yani

$$T^{\alpha\beta} = T^{\alpha \cdot}_{\cdot \beta} = T_{\alpha\beta} = T^{\cdot \beta}_{\alpha \cdot}$$

tengliklar o'rinli bo'ladi.

Yuqorida olingan munosabatlar tenzorni boshqacha ta'riflash mumkinligini anglatadi. Buni quyida n - rang tenzor uchun ko'ramiz.

2 - rang tenzorning ta'rifidan va poliadalarning xususiyatidan ko'rinib turibdiki, n -rang poliada yordamida ham tenzor tushunchasini kiritish mumkin.

Ta'rif: Koordinata sistemasini tanlanishiga bog'liq bo'lmagan va muayyan koordinata sistemasida n - rang poliadalarning ushbu

$$T = T^{\alpha_1\alpha_2\ldots\alpha_n}\,\overline{Э}_{\alpha_1}\overline{Э}_{\alpha_2}\ldots\overline{Э}_{\alpha_n} = T_{\alpha_1}^{\cdot\alpha_2\ldots\alpha_n}\,\overline{Э}^{\alpha_1}\overline{Э}_{\alpha_2}\ldots\overline{Э}_{\alpha_n} =$$
$$= T_{\alpha_1\ldots\alpha_{n-1}}^{\cdot\ \ldots\ \alpha_n}\,\overline{Э}^{\alpha_1}\ldots\overline{Э}^{\alpha_{n-1}}\overline{Э}_{\alpha_n} = T_{\alpha_1\alpha_2\ldots\alpha_n}\overline{Э}^{\alpha_1}\overline{Э}^{\alpha_2}\ldots\overline{Э}^{\alpha_n} \tag{2.1.5}$$

chiziqli ifodalar shaklida beriladigan miqdor *n* - rang tenzor deb ataladi.

Ta'rifga ko'ra, (2.1.5) formulada poliadalarning $\overline{Э}^{\alpha_\kappa}$ va $\overline{Э}_{\alpha_\kappa}$ $(\kappa = \overline{1,n})$ bazis vektorlar yordamida hosil qilinish mumkin bo'lgan barcha chiziqli ifodalari nazarda tutiladi. Mazkur (1.1.5) chiziqli ifodalarning barcha indekslari yuqorida joylashgan koeffitsientlari tenzorning kontravariant, barcha indekslari quyida joylashgani - kovariant va indekslari yuqorida ham, quyida ham joylashganlari - aralash komponentalari deb ataladi. Demak, n - rang tenzor o'zining tashkil etuvchilari - komponentalari bilan aniqlanadi. Muayyan xildagi komponentalarning soni $N = 3^n$ ga teng bo'ladi.

Ushbu ta'rif $n = 1$ $n = 0$ bo'lsa ham o'rinli. Agar $n = 1$ $(N = 3)$ bo'lsa, tenzor birinchi rang ob'yekt, yani vektor bo'ladi:

$$T = T^{\alpha}\overline{Э}_{\alpha} = T_{\beta}\overline{Э}^{\beta}$$

Agar $n = 0$ $(N = 1)$ bo'lsa, tenzor nolinchi rang ob'yekt - bir son bilan aniqlanuvchi invariant miqdor, yani skalyar bo'ladi.

Ikkinchi rang tenzor uchun yuqorida keltirilgan barcha xossalar n - rang tenzor uchun ham o'rinli. Masalan, tenzor komponentalarini almashtirish qoidalari (2.1.2) va (2.1.3) kabi ushbu

$$\hat{T}_{\beta_1}^{\cdot\beta_2\ldots\beta_n} = T_{\alpha_1}^{\cdot\alpha_2\ldots\alpha_n}\frac{\partial\xi^{\alpha_1}}{\partial\eta^{\beta_1}}\frac{\partial\eta^{\beta_2}}{\partial\xi^{\alpha_2}}\cdots\frac{\partial\eta^{\beta_n}}{\partial\xi^{\alpha_n}} \tag{2.1.6}$$

$$T_{\alpha_1}^{\cdot\alpha_2\ldots\alpha_n} = \hat{T}_{\beta_1}^{\cdot\beta_2\ldots\beta_n}\frac{\partial\eta^{\beta_1}}{\partial\xi^{\alpha_1}}\frac{\partial\xi^{\alpha_2}}{\partial\eta^{\beta_2}}\cdots\frac{\partial\xi^{\alpha_n}}{\partial\eta^{\beta_n}} \tag{2.1.7}$$

formulalar bilan aniqlanadi. Indekslarni ko'tarish (tushirish) esa, (2.1.4) formula kabi ushbu

$$T_{\alpha\ \ldots\alpha_{\kappa-1}\alpha_\kappa\alpha_{\kappa+1}\ldots\alpha_n} = g_{\alpha_\kappa\sigma}T_{\alpha_1\ldots\alpha_{\kappa-1}.\alpha_{\kappa+1}\ldots\alpha_n}^{\ldots\ldots\ldots\sigma\ldots\ldots\ldots}$$
$$T^{\alpha_1\ldots\alpha_{\kappa-1}\alpha_\kappa\alpha_{\kappa+1}\ldots\alpha_n} = g^{\sigma\alpha_\kappa}T^{\alpha_1\ldots\alpha_{\kappa-1}.\alpha_{\kappa+1}\ldots\alpha_n}_{\ldots\ldots\ldots\sigma\ldots\ldots\ldots} \tag{2.1.8}$$

qoidaga binoan bajariladi.

Tenzorning ikkinchi ta’rifi (2.1.6) va (2.1.7) almashtirish qoidalaridan foydalanib beriladi.

Ta’rif: Muayyan koordinatalar sistemasida 3^n ta son- tashkil etuvchilari berilgan va koordinatalar sistemasi almashtirilganda mazkur komponentalar (2.1.6) va (2.1.7) formulalarga binoan o’zgaradigan miqdor n - rang tenzor deb ataladi.

Ushbu ta’rifga ko’ra, tenzor komponentalari va tenzorning invariantligini taminlovchi almashtirish qoidalari beriladi. Birinchi ta’rifda esa, uning invariant bo’lishi talab etiladi. Invariantlik esa chiziqli ifodani hosil qiluvchi poliada va koeffitsientlarni almashtirish qoidalari, yuqorida ko’rganimizdek, o’zaro teskari matrisalar bilan aniqlanishi evaziga bajariladi.

Tenzorlarning yoyilmasini normalangan bazislar

$$\bar{e}_\alpha = \frac{\bar{Э}_\alpha}{Э_\alpha}\ , \qquad \bar{e}^\alpha = \frac{\bar{Э}^\alpha}{Э^\alpha}$$

orqali ham yozish mumkin.

Masalan,

$$T = T^{\alpha_1\alpha_2..\alpha_n}\,\bar{Э}_{\alpha_1}\bar{Э}_{\alpha_2}...\bar{Э}_{\alpha_n} =$$

$$= T^{\alpha_1\alpha_2..\alpha_n}\,Э_{\alpha_1}Э_{\alpha_2}...Э_{\alpha_n}\,\bar{e}_{\alpha_1}\bar{e}_{\alpha_2}...\bar{e}_{\alpha_n} = T*^{\alpha_1\alpha_2..\alpha_n}\,\bar{e}_{\alpha_1}\bar{e}_{\alpha_2}...\bar{e}_{\alpha_n}$$

Ta’rif: Tenzorning so’nggi ko’rinishida normalangan bazislar oldidagi koeffitsientlar

$$T*^{\alpha_1\alpha_2..\alpha_n} = T^{\alpha_1\alpha_2..\alpha_n}\,Э_{\alpha_1}Э_{\alpha_2}...Э_{\alpha_n}$$

tenzorning fizik komponentalari deb ataladi.

Tenzorning boshqa xil fizik komponentalari ham shunga o’xshash yoziladi:

$$T*^{\alpha_1\alpha_2..\alpha_n} = T_{\alpha_1}^{\cdot\alpha_2..\alpha_n}\,Э^{\alpha_1}Э_{\alpha_2}...Э_{\alpha_n}$$

$$T*^{\alpha_1\alpha_2..\alpha_n} = T_{\alpha_1\alpha_2..\alpha_n}\,Э^{\alpha_1}Э^{\alpha_2}...Э^{\alpha_n}$$

2 §. Tenzorlar ustida amallar.

1. Tenzorlarni qo'shish amali.

Tenzorlarni qo'shish uchun ularning ranglari bir xil bo'lishi kerak. n - rang P va Q tenzorlarning bir xil komponentalarini almashtirish qoidasi (2.1.6) ga ko'ra

$$\hat{P}^{\alpha_1..\alpha_n} = P^{\beta_1...\beta_n}\frac{\partial\eta^{\alpha_1}}{\partial\xi^{\beta_1}}\frac{\partial\eta^{\alpha_2}}{\partial\xi^{\beta_2}}\cdots\frac{\partial\eta^{\alpha_n}}{\partial\xi^{\beta_n}}$$
$$\hat{Q}^{\alpha_1..\alpha_n} = Q^{\beta_1...\beta_n}\frac{\partial\eta^{\alpha_1}}{\partial\xi^{\beta_1}}\frac{\partial\eta^{\alpha_2}}{\partial\xi^{\beta_2}}\cdots\frac{\partial\eta^{\alpha_n}}{\partial\xi^{\beta_n}} \qquad (2.2.1)$$

formulalar bilan beriladi. Bu tengliklarning ikkala tomonini hadma-had qo'shsak, quyidagi munosabatlar hosil bo'ladi:

$$\hat{P}^{\alpha_1..\alpha_n} + \hat{Q}^{\alpha_1..\alpha_n} = \left(P^{\beta_1...\beta_n} + Q^{\beta_1...\beta_n}\right)\frac{\partial\eta^{\alpha_1}}{\partial\xi^{\beta_1}}\cdots\frac{\partial\eta^{\alpha_n}}{\partial\xi^{\beta_n}} \qquad (2.2.2)$$

Bu yerdan (2.1.6) qoidani e'tiborga olsak, tenzorning ikkinchi ta'rifiga ko'ra

$$P^{\beta_1\cdots\beta_n} + Q^{\beta_1\cdots\beta_n} = T^{\beta_1\cdots\beta_n}$$

miqdorlar n - rang tenzor komponentalari ekanligi kelib chiqadi. P va Q tenzorlarning yig'indisi

$$T = P + Q \qquad (2.2.3)$$

tenglik bilan belgilanadi.

Yig'indisi hisoblanishi kerak bo'lgan tenzorlarning turli xil komponentalari berilgan bo'lsa, ular indekslarni ko'tarish va tushirish qoidasidan foydalanib, bir xil komponentalar ko'rinishida yoziladi, so'ngra qo'shish amali bajariladi. Masalan, ikkinchi rang P va Q tenzorlarning $P^{\alpha\beta}$ va $Q^{\alpha\cdot}_{\cdot\beta}$ komponentalari berilgan bo'lsin. $P+Q$ yig'indini topish uchun qo'shish amali ushbu $P^{\alpha\beta}$ va $Q^{\alpha\beta} = g^{i\beta}Q^{\alpha\cdot}_{\cdot i}$ kontravariant yoki $P^{\alpha\cdot}_{\cdot\beta} = g_{\beta i}P^{\alpha i}$ va Q^{α}_{β} aralash komponentalar bilan bajariladi.

2. Tenzorlarni ko'paytirish amali.

Bu amal har biri ixtiyoriy rangli ikkita tenzor uchun aniqlangan bo'lib, ko'paytmaning rangi ko'paytiruvchilar ranglarining yig'indisiga teng.

Ranglari n va m bo'lgan P va Q tenzorlar berilgan bo'lsin. Ularning ko'paytmasi $T = PQ$ $(n+m)$ - rang tenzor bo'ladi. Buni isbotlash uchun, masalan, P tenzorning kontravariant va Q tenzorning kovariant komponentalarining ushbu almashtirish formulalarini

$$
\begin{gathered}
\hat{P}^{\alpha_1 .. \alpha_n} = P^{\beta_1 .. \beta_n} \frac{\partial \eta^{\alpha_1}}{\partial \xi^{\beta_1}} \cdots \frac{\partial \eta^{\alpha_n}}{\partial \xi^{\beta_n}} \\
\hat{Q}_{i_1 .. i_m} = Q_{j_1 \dots j_m} \frac{\partial \xi^{j_1}}{\partial \eta^{i_1}} \cdots \frac{\partial \xi^{j_m}}{\partial \eta^{i_m}}
\end{gathered} \tag{2.2.4}
$$

hadma-had ko'paytiramiz:

$$
\hat{P}^{\alpha_1 .. \alpha_n} \hat{Q}_{i_1 .. i_m} = P^{\beta_1 .. \beta_n} Q_{j_1 ... j_m} \frac{\partial \eta^{\alpha_1}}{\partial \xi^{\beta_1}} \cdots \frac{\partial \eta^{\alpha_n}}{\partial \xi^{\beta_n}} \frac{\partial \xi^{j_1}}{\partial \eta^{i_1}} \cdots \frac{\partial \xi^{j_m}}{\partial \eta^{i_m}}
$$

Bu tenglikdan

$$
T^{\beta_1 \dots \beta_n}{}_{j_1 \dots j_m} = P^{\beta_1 \dots \beta_n} Q_{j_1 \dots j_m} \tag{2.2.5}
$$

miqdorlar $(n+m)$ - rang tenzorning komponentalari ekanligi kelib chiqadi. Boshqa xil komponentalari bilan berilgan tenzorlarni ko'paytirish ham shunga o'xshash bajariladi.

3. Tenzorni skalyar miqdorga ko'paytirish amali.

Agar ρ skalyar miqdor bo'lsa, ushbu

$$
T = \rho P \tag{2.2.6}
$$

ko'paytma komponentalari

$$
T^{\alpha_1 \dots \alpha_n} = \rho P^{\alpha_1 \dots \alpha_n}
$$

tenglik bilan aniqlangan n - rang tenzorni hosil qiladi.

4. Tenzorlarni yig'ishtirish amali.

Bu amal aralash indeksli komponentalar bilan bajariladi. Bu amal yordamida berilgan n- rang $(n \geq 2)$ tenzorga $(n-2)$ - rang tenzor mos keltiriladi;

$$
\begin{gathered}
T_{\alpha_1}^{\cdot \alpha_2 \dots \alpha_n} \div T_{\alpha_1}^{\cdot \alpha_1 \alpha_3 \dots \alpha_n} \\
T^{\alpha_1 \dots \alpha_\kappa \cdot \ \alpha_{\kappa+2} \dots \alpha_n}_{\qquad \alpha_{\kappa+1}} \div T^{\alpha_1 \dots \alpha_\kappa \cdot \ \alpha_{\kappa+2} \dots \alpha_n}_{\qquad \alpha_\kappa}
\end{gathered} \tag{2.2.7}
$$

va hokazo. 2 - rang tenzorga esa nolinchi rang tenzor - skalyar mos keltiriladi:

$$T_{\alpha_1 \cdot}^{\cdot \alpha_2} \div T_{\alpha_1 \cdot}^{\cdot \alpha_1} \qquad (2.2.8)$$

Hosil qilingan $(n-2)$- rang ob'yektlar tenzor ekanligi va ular uchun tenzor komponentalarini almashtirish qoidasi o'rinli ekanligidan kelib chiqadi. Masalan,

$$\hat{T}_{\beta_1}^{\cdot \beta_1 \ldots \beta_n} = T_{\alpha_1}^{\cdot \alpha_1 \alpha_3 \ldots \alpha_n} \frac{\partial \eta^{\beta_3}}{\partial \xi^{\alpha_3}} \ldots \frac{\partial \eta^{\beta_n}}{\partial \xi^{\alpha_n}}$$

almashtirish qoidasi o'rinli bo'ladi. Buni ko'rish uchun (2.1.6) almashtirishda $\beta_1 = \beta_2$, $\alpha_1 = \alpha_2$ deb qabul qilish yetarli.

Agar tenzor rangi *n ye*tarlicha katta bo'lsa, yig'ishtirish amalini turli juft indekslar (masalan, birinchi va ikkinchi, birinchi va uchinchi va hokazo) bilan bajarish mumkin.

Yig'ishtirish amalini bir necha marta qaytarish ham mumkin. Bunda tenzor juft rangli bo'lganda u nolinchi rang tenzor-skalyarga mos keladi. Agar tenzor toq rangli bo'lsa - birinchi rang tenzor-vektorga mos keladi. Demak, yig'ishtirish amali yordamida invariantlar hosil qilish mumkin. Masalan:

$T^{ij} g_{ij} = T_{\cdot i}^{i \cdot} = T_{\cdot 1}^{1 \cdot} + T_{\cdot 2}^{2 \cdot} + T_{\cdot 3}^{3 \cdot}$ - son koordinata sistemasiga bog'liq emas. Demak bu invariant.

Ta'rif: *n* - rang tenzorni yig'ishtirish natijasida hosil qilingan (*n*-*2*) - rang tenzorga tenzor o'rami deb ataladi.

O'ramlarda indekslarni ko'tarish va tushirish amali odatdagidek bajariladi. Masalan,

$$T_{\alpha_1}^{\cdot \alpha_1 \alpha_3 \ldots \alpha_n} = g^{\alpha_1 \gamma} g_{\alpha_1 \beta} \, T_{\cdot \gamma}^{\beta \cdot \alpha_3 \ldots \alpha_n} \qquad (2.2.9)$$

5. Tenzorlarni skalyar ko'paytirish amali.

Bu amal har biri ixtiyoriy rangli ikkita tenzor bilan bajariladi. Buning uchun avval ko'paytirish amalidan foydalanib, ko'paytma hosil qilinadi, keyin birinchi ko'paytuvchining oxirgi indeksi va ikkinchi ko'paytuvchining birinchi indeksi bo'yicha yig'ishtirish amali bajariladi.

Masalan, *n* - rang *P* tenzori va *m* - rang *Q* tenzorining skalyar ko'paytmasi $(m+n-2)$-rang *T* tenzor - o'ram bo'ladi:

$$T = P \cdot Q, T^{\alpha_2 .. \alpha_{n-1}}_{\beta_2 ... \beta_n} = P^{\alpha_1 .. \alpha_{n-1} \alpha} Q_{\alpha\beta_2 ... \beta_n} \tag{2.2.10}$$

2-rang tenzorni o'zini-o'ziga skalyar ko'paytirish natijasida yana 2-rang tenzor hosil bo'ladi:

$$\begin{aligned} &T = T^{\alpha\cdot}_{\cdot\beta}\, \overline{Э}_\alpha \overline{Э}^\beta, \qquad T \cdot T = T^{\alpha\cdot}_{\cdot\sigma}\, T^{\sigma\cdot}_{\cdot\beta}\, \overline{Э}_\alpha \overline{Э}^\beta = T^2, \\ &T^2 \cdot T = T^{\alpha\cdot}_{\cdot\sigma}\, T^{\sigma}_{\cdot\gamma}\, T^{\gamma}_{\cdot\beta}\, \overline{Э}_\alpha \overline{Э}^\beta = T^3, \ \ldots \end{aligned} \tag{2.2.11}$$

Bu yerda T^2 tenzorning kvadrati, T^3- kubi va hokazo deb ataladi. Ushbu ketma-ketlik to'la bo'lishi uchun

$$G = g_{\alpha\beta} \overline{Э}^\alpha \overline{Э}^\beta \tag{2.2.12}$$

ko'rinishidagi metrik tenzor deb ataluvchi tenzorni T^0-nolinchi darajali tenzor deb qabul qilinadi.

Yig'ishtirish amali yordamida (8.11) ketma-ketlikka quyidagi o'ramlar ketma-ketligi mos keltiriladi:

$$J_1 = T^{\alpha\cdot}_{\cdot\alpha}, \quad J_2 = T^{\alpha\cdot}_{\cdot\sigma}\, T^{\sigma\cdot}_{\cdot\alpha}, \quad J_3 = T^{\alpha}_{\cdot\sigma}\, T^{\sigma\cdot}_{\cdot\beta}\, T^{\beta}_{\alpha} \tag{2.2.13}$$

Tenzorlarni skalyar ko'paytirish amalini tenzorning birinchi ta'rifi va poliadalarni skalyar ko'paytirish qoidasidan foydalanib ham bajarish mumkin. Bunda ortiqcha (indekslarni tushirish yoki ko'tarish) amallarni bajarmaslik uchun birinchi tenzorning oxirgi va ikkinchi tenzorning birinchi bazis vektorlarining o'zaro teskari ko'rinishdagi ifodalarini olish qulay bo'ladi.

Masalan, 2-rang tenzor T va $\bar{a}$ vektorning skalyar ko'paytmasi quyidagicha bajariladi;

$$\begin{aligned} &T \cdot \bar{a} = \left(T^{\alpha\cdot}_{\cdot\beta} \overline{Э}_\alpha \overline{Э}^\beta \right) \cdot \left(a^\sigma \overline{Э}_\sigma \right) = T^{\alpha\cdot}_{\cdot\beta} a^\sigma \overline{Э}_\alpha \left(\overline{Э}^\beta \cdot \overline{Э}_\sigma \right) = \\ &= T^{\alpha\cdot}_{\cdot\beta}\, a^\sigma\, \delta^{\beta\cdot}_{\cdot\sigma}\, \overline{Э}_\alpha = T^{\alpha\cdot}_{\cdot\beta}\, a^\beta \overline{Э}_\alpha \end{aligned} \tag{2.2.14}$$

Shunga o'xshash

$$\begin{aligned} &T^2 = T \cdot T = \left(T^{\alpha\cdot}_{\cdot\beta} \overline{Э}_\alpha \overline{Э}^\beta \right) \cdot \left(T^{\sigma\cdot}_{\cdot\gamma} \overline{Э}_\sigma \overline{Э}^\gamma \right) = T^{\alpha\cdot}_{\cdot\beta} T^{\sigma\cdot}_{\cdot\gamma} \overline{Э}_\alpha \left(\overline{Э}^\beta \overline{Э}_\sigma \right) \overline{Э}^\gamma = \\ &= T^{\alpha\cdot}_{\cdot\beta} T^{\sigma\cdot}_{\cdot\gamma} \delta^{\beta\cdot}_{\cdot\sigma} \overline{Э}_\alpha \overline{Э}^\gamma = T^{\alpha\cdot}_{\cdot\beta} T^{\beta\cdot}_{\cdot\gamma} \overline{Э}_\alpha \overline{Э}^\gamma \end{aligned} \tag{2.2.15}$$

bajariladi.

6. Tenzorlarni vektor ko'paytirish amali.

Bu amal ham har biri ixtiyoriy rangli ikkita tenzor bilan ularning birinchi ta'rifi va poliadalarni vektor ko'paytirish qoidasi (1.7.1) va (1.7.2) yordamida bajariladi.

Masalan,

$$\begin{aligned} T \times \bar{a} &= \left(T^{\alpha\beta} \overline{Э}_\alpha \overline{Э}_\beta\right) \times \left(a^\sigma \overline{Э}_\sigma\right) = T^{\alpha\beta} a^\sigma \overline{Э}_\alpha e_{\beta\sigma\gamma} \overline{Э}^\gamma = \\ &= T^{\alpha\beta} e_{\beta\sigma\gamma}\, a^\sigma\, \overline{Э}_\alpha\, \overline{Э}^\gamma \end{aligned} \tag{2.2.16}$$

Shunga o'xshash

$$\begin{aligned} T \times T &= \left(T^{\alpha\beta} \overline{Э}_\alpha \overline{Э}_\beta\right) \times \left(T^{\sigma\gamma} \overline{Э}_\sigma \overline{Э}_\gamma\right) = \\ &= T^{\alpha\beta} T^{\sigma\gamma} \overline{Э}_\alpha \left(\overline{Э}_\beta \times \overline{Э}_\sigma\right) \overline{Э}_\gamma = T^{\alpha\beta} T^{\sigma\gamma} e_{\beta\sigma q} \overline{Э}_\alpha \overline{Э}^q \overline{Э}_\gamma \end{aligned} \tag{2.2.17}$$

Tenzorlarni vektor ko'paytirilganda ularning qaysi xil ifodasini olishning ahamiyati yo'q.

3 §. Simmetrik va antisimmetrik tenzorlar.

Ta'rif: Ikkita indeksining o'rni almashtirilganda o'zgarmaydigan tenzor shu indekslarga nisbatan simmetrik tenzor deb ataladi.

Ta'rrif: Ikkita indeksining o'rni almashtirilganda faqat ishorasi o'zgaradigan tenzor esa shu indekslarga nisbatan antisimmetrik tenzor deb ataladi.

Masalan, birinchi va uchinchi indekslari bo'yicha simmetrik bo'lgan n - rang tenzor uchun

$$T^{\alpha_1\alpha_2\alpha_3\alpha_4\ldots\alpha_n} = T^{\alpha_3\alpha_2\alpha_1\alpha_4\ldots\alpha_n} \tag{2.3.1}$$

tenglik o'rinli bo'ladi.

Agar tenzor mazkur indekslari bo'yicha antisimmetrik bo'lsa,

$$T^{\alpha_1\alpha_2\alpha_3\alpha_4\ldots\alpha_n} = -T^{\alpha_3\alpha_2\alpha_1\alpha_4\ldots\alpha_n} \tag{2.3.2}$$

tenglik o'rinli bo'ladi.

Endi qo'sh o'ramlar uchun o'rinli bo'lgan quyidagi to'g'ri va teskari teoremalarni isbotsiz keltiramiz.

Teorema. Ikkita har birining rangi ixtiyoriy va yig'ishtirilayotgan indekslari bo'yicha biri simmetrik, ikkinchisi esa antisimmetrik bo'lgan tenzorlarning qo'sh o'rami nolga teng.

Teskari teorema. Agar rangi ixtiyoriy ikkita tenzorning qo'sh o'rami nolga teng bo'lsa va ushbu tenzorlardan biri yig'ishtiralayotgan indekslari bo'yicha simmetrik (antisimmetrik) bo'lsa, ikkinchi tenzor esa shu indekslari bo'yicha antisimmetrik (simmetrik) bo'ladi.

Komponentasi $T^{\alpha_1 \dots \alpha_n}$ bo'lgan ixtiyoriy tenzor berilgan bo'lsin. Unga ixtiyoriy nomerli ikkita indeksi bo'yicha simmetrik (antisimmetrik) tenzorni mos keltirish mumkin. Masalan, α_1 va α_2 indekslari bo'yicha simmetrik va antisimmetrik tenzorlar quyidagicha hosil qilinadi:

$$T^{(\alpha_1\alpha_2)\alpha_3..\alpha_n} = \frac{1}{2!}\left(T^{\alpha_1\alpha_2..\alpha_n} + T^{\alpha_2\alpha_1\alpha_3..\alpha_n}\right) \tag{2.3.3}$$

$$T^{[\alpha_1\alpha_2]\alpha_3..\alpha_n} = \frac{1}{2!}\left(T^{\alpha_1\alpha_2..\alpha_n} - T^{\alpha_2\alpha_1\alpha_3..\alpha_n}\right) \tag{2.3.4}$$

Bu yerda $(\alpha_1\alpha_2)$ va $[\alpha_1\alpha_2]$ belgilar ushbu indekslar bo'yicha simmetrik va antisimmetrik xususiyatlarga ega ekanligini bildiradi.

Tenzor bir xil nomerlarda joylashgan guruh indekslarining har bir jufti bo'yicha simmetrik (antisimmetrik) bo'lsa, u holda ushbu tenzor mazkur guruh indekslari bo'yicha simmetrik (antisimmetrik) tenzor deb ataladi.

Ixtiyoriy tenzor yordamida simmetrik tenzor hosil qilish mumkin. Buning uchun indekslari barcha mumkin bo'lgan o'rin almashtirishlar natijasida hosil qilingan tenzorlarning yig'indisini olish kifoya. Ushbu yig'indining o'rta arifmetik qiymati berilgan tenzorga mos keltirilgan simmetrik tenzor sifatida qabul qilinadi. Masalan,

$$T_{(\alpha\beta\gamma)} = \frac{1}{3!}\left(T_{\alpha\beta\gamma} + T_{\beta\gamma\alpha} + T_{\gamma\alpha\beta} + T_{\beta\alpha\gamma} + T_{\gamma\beta\alpha} + T_{\alpha\gamma\beta}\right)$$

Indekslarni juft o'rniga qo'yish natijasida hosil qilingan tenzorlarni musbat ishora bilan, toq o'rniga qo'yish natijasida hosil qilingan tenzorlarni manfiy ishora bilan, olingan yig'indi esa antisimmetrik tenzorni beradi. Ushbu yig'indining o'rta arifmetik qiymati berilgan tenzorga mos keltirilgan antisimmetrik tenzor sifatida qabul qilinadi.

Masalan,

$$T_{(\alpha\beta\gamma)} = \frac{1}{3!}\left(T_{\alpha\beta\gamma} + T_{\beta\gamma\alpha} + T_{\gamma\alpha\beta} - T_{\beta\alpha\gamma} - T_{\alpha\gamma\beta} - T_{\gamma\beta\alpha}\right)$$

mazkur mos keltirish amallari simmetriklash va alternirlash deb ataladi.

Simmetrik tenzorning xossalari

a) Simmetrik *T* tenzor uchun quidagi munosabatlar o'rinli ekanligidan

$$T_{\beta\alpha} = T_{\alpha\beta}\,, \qquad T^{\beta\alpha} = T^{\alpha\beta}$$

$A_1 = \|T_{\alpha\beta}\|$ va $A_4 = \|T^{\alpha\beta}\|$ matritsalar ham simmetrik ekanligi kelib chiqadi;

b) Indekslarni ko'tarish (tushirish) qoidasiga ko'ra quyidagi tengliklar o'rinli bo'ladi;

$$T_{\beta\sigma}g^{\sigma\alpha} = T_{\sigma\beta}g^{\sigma\alpha}\,, \qquad T_{\beta\cdot}^{\cdot\alpha} = T_{\cdot\beta}^{\alpha\cdot}.$$

Demak, $A_2 = \|T_{\cdot\beta}^{\alpha\cdot}\|$ va $A_3 = \|T_{\cdot\beta}^{\alpha\cdot}\|$ matritsalar bir-biriga teng bo'ladi: $A_2 = A_3$

c) *T* tenzor simmetrik bo'lganligi uchun quyidagi munosabatlar o'rinli bo'ladi;

$$T_{\cdot\beta}^{\alpha\cdot}\, a^{\beta} = T_{\beta\cdot}^{\cdot\alpha}\, a^{\beta}\,, \qquad T\cdot\bar{a} = \bar{a}\cdot T$$

Tenzorning vektorga skalyar ko'paytmasi kommutativlik xossasiga egadir. Shuning uchun simmetrik tenzorga yagona chiziqli vektor funksiya mos keladi:

$$\bar{b} = T\cdot\bar{a}$$

Demak, simmetrik tenzorning ikkala uchlik xos vektorlari, yani $\bar{a}_a$ va $\bar{a}'_{\alpha}$ bosh yo'nalishlari ustma-ust tushadi.

Tenzorning ushbu xos vektorlari

$$\lambda_{\rho}\bar{a}_{\rho} = T\bar{a}_{\rho}\,, \qquad \left(\lambda_{\rho}\,\delta_{\beta}^{\alpha} - T_{\beta}^{\alpha}\right)a_{\rho}^{\beta} = 0$$

tenglamalardan aniqlanadi:

$$\left(\lambda\delta_{\cdot\beta}^{\alpha\cdot} - T_{\cdot\beta}^{\alpha\cdot}\right)a^{\beta} = 0. \qquad (2.3.5)$$

Simmetrik tenzor bosh yo'nalishlarining quyidagi xossalari mavjud:

1) Xarakteristik tenglamaning turli ildizlariga mos kelgan xos vektorlar o'zaro ortogonal:

$$\lambda_{\mu} \neq \lambda_{\nu}\,, \quad \bar{a}_{\mu}\cdot\bar{a}_{\nu} = 0\,, \qquad a_{(\mu)}^{\alpha}\cdot a_{\alpha}^{(\nu)} = 0$$

2) Agar xarakteristik tenglamaning ildizlarining ixtiyoriy ikkitasi bir-biriga teng bo'lsa, bosh yo'nalishlar tekisligi va shu tekislikka ortogonal uchinchi bosh yo'nalish mavjud bo'ladi; ildizlarning uchalasi ham bir-biriga teng bo'lsa, hamma yo'nalishlar bosh bo'ladi.

Xarakteristik tenglama ildizlari quyidagi xossalarga ega:

a) 2-rang simmetrik tenzor xarakteristik tenglamasining hamma ildizlari haqiqiy bo'ladi;

b) 2-rang simmetrik tenzorning kvadratik formasi musbat aniqlangan bo'lsa, xarakteristik tenglamaning hamma ildizlari musbat va haqiqiy bo'ladi.

Ta'rif: Agar kvadratik forma faqat $x^{\alpha}=0 \quad (\alpha=1,2,3)$ bo'lgandagina nolga teng bo'lib, x^{α} ning barcha haqiqiy qiymatlarida musbat bo'lsa, u musbat aniqlangan kvadratik forma deb ataladi.

4 §. Tenzorlarni bo'lish teoremasi.

Tenzorning asosiy xususiyati - invariantlik uning komponentalarini almashtirish qoidalari bilan aniqlanadi. Ammo biror berilgan miqdorning tenzor ekanligini aniqlash uchun uning komponentalarini almashtirish qoidalarini bajarilishini tekshirish shart emas. Buning uchun quyidagi teorema bilan beriladigan shartni tekshirish kifoya.

Teorema (tenzorlarni bo'lish teoremasi**):** Biror ξ^{α} koordinata sistemasida 3^n ta son-komponentalar $T^{\alpha_1 \dots \alpha_n}$ yordamida T ob'yekt berilgan bo'lsin. Uni ixtiyoriy m - rang $(m \le n)$ tenzorga ko'paytirib, so'ngra $2m$ indeksi bo'yicha yig'ishtirish amali bajarilganda $(n\text{-}m)$ -rangli P tenzor hosil bo'lsa, mazkur T ob'yekt tenzor bo'ladi.

Teoremaga ko'ra T ob'yekti tenzor bo'lishi uchun ushbu (2.2.5) va (2.2.7) formulalar

$$P = TQ = T^{\alpha_1 .. \alpha_m \alpha_{m+1} .. \alpha_n} Q_{\beta_1 .. \beta_m},$$

$$T^{\alpha_1 .. \alpha_m \alpha_{m+1} .. \alpha_n} Q_{\beta_1 \dots \beta_n} \div T^{\alpha_1 .. \alpha_m \alpha_{m+1} .. \alpha_n} Q_{\alpha_1 .. \alpha_m} = P^{\alpha_{m+1} .. \alpha_n}$$

yordamida hosil qilingan

$$P^{\alpha_{m+1}\dots\alpha_n} = T^{\alpha_1\dots\alpha_m\alpha_{m+1}\dots\alpha_n} Q_{\alpha_1\dots\alpha_m} \qquad (2.4.1)$$

sonlar *(n-m)* - rang tenzorning komponentalari bo'lishi kerak.

Isbot. Teoremaning shartiga ko'ra P - tenzor. Shuning uchun so'nggi tenglikni «yangi» η^α koordinata sistemasida yozish mumkin;

$$\hat{T}^{\beta_1\dots\beta_m\beta_{m+1}\dots\beta_n}\hat{Q}_{\beta_1\dots\beta_n} = \hat{P}^{\beta_{m+1}\dots\beta_n} = P^{\alpha_{m+1}\dots\alpha_n}\frac{\partial\eta^{\beta_{m+1}}}{\partial\xi^{\alpha_{m+1}}}\dots\frac{\partial\eta^{\beta_n}}{\partial\xi^{\alpha_n}}$$

Bu munosabatning o'ng tomonida $P^{\alpha_{m+1}\dots\alpha_n}$ ni (2.4.1) dan foydalanib almashtiriksak va Q tenzor komponentalarini yangi koordinata sistemasida yozsak, ushbu

$$\hat{T}^{\beta_1\dots\beta_m\beta_{m+1}\dots\beta_n}\hat{Q}_{\beta_1\dots\beta_m} = T^{\alpha_1\dots\alpha_m\alpha_{m+1}\dots\alpha_n}\hat{Q}_{\beta_1\dots\beta_m}\frac{\partial\eta^{\beta_1}}{\partial\xi^{\alpha_1}}\dots\frac{\partial\eta^{\beta_{n-1}}}{\partial\xi^{\alpha_{n-1}}}\frac{\partial\eta^{\beta_n}}{\partial\xi^{\alpha_n}}$$

munosabatni olamiz. Bu yerdan Q ixtiyoriy tenzor bo'lgani uchun ushbu

$$\hat{T}^{\beta_1\dots\beta_m\beta_{m+1}\dots\beta_n} = T^{\alpha_1\dots\alpha_m\alpha_{m+1}\dots\alpha_n}\frac{\partial\eta^{\beta_1}}{\partial\xi^{\alpha_1}}\dots\frac{\partial\eta^{\beta_n}}{\partial\xi^{\alpha_n}}$$

tenzor komponentalarini almashtirish qoidasi kelib chiqadi. Demak, teorema isbotlandi.

Ta'rif: Berilgan ikki P va Q tenzorlarning ko'paytmasi $T = PQ$ ni ixtiyoriy ikkita indeksi bo'yicha yig'ishtirish amali natijasida hosil bo'lgan tenzor qo'sh o'ram deb ataladi. Masalan,

$$T^{\alpha_1\alpha_2\dots\alpha_n}{}_{\beta_1\beta_2\dots\beta_n} = P^{\alpha_1\alpha_2\dots\alpha_n}Q_{\beta_1\beta_2\dots\beta_n}$$

ko'paytma uchun $T^{\alpha_1\alpha_2\dots\alpha_{n-2}\beta_1\beta_2}{}_{\beta_1\beta_2\dots\beta_n}$ tenzor qo'sh o'ram bo'ladi.

5 §. Metrik va diskriminant tenzorlar.

Metrik tenzor. Nolinchi darajali tenzor sifatida (2.2.12) formula bilan berilgan

$$G = g_{\alpha\beta}\overline{\ni}^{\alpha}\overline{\ni}^{\beta}$$

ob'yektni qabul qilib, uni metrik tenzor deb atagan edik.

G haqiqatan tenzor ekanligini aniqlash uchun

$$G = g_{\alpha\beta}\overline{\ni}^{\alpha}\overline{\ni}^{\beta} = \hat{g}_{ij}\hat{\ni}^{i}\hat{\ni}^{j} = \hat{G}$$

munosabat yoki (1.8.5) almashtirish formulalari o'rinli ekanligini ko'rsatish kerak. Buni (1.4.7) formula bilan aniqlangan $g_{\alpha\beta}$ ning ξ^{α} va η^{α} koordinata sistemalaridagi quyidagi ko'rinishlaridan foydalanib, osonlik bilan ko'rsatish mumkin.

$$g_{\alpha\beta} = \overline{\ni}_{\alpha}\cdot\overline{\ni}_{\beta};\ \hat{g}_{ij} = \hat{\ni}_{i}\cdot\hat{\ni}_{j} \tag{2.5.1}$$

$$G = g_{\alpha\beta}\overline{\ni}^{\alpha}\overline{\ni}^{\beta}$$

Agar $g_{\alpha\beta}$ ni asosiy kvadratik forma

$$dS^2 = g_{\alpha\beta}\, dx^{\alpha} dx^{\beta} \tag{2.5.2}$$

koeffitsientlari sifatida qarab, ushbu 9 ta son x^{α} koordinata sistemasida biror G obektni aniqlaydi desak, mazkur obekt ikkinchi rang tenzor ekanligini isbot qilingan teorema yordamida ko'rsatish mumkin.

Haqiqatan, dx^{α} va dx^{β} differensiallar ixtiyoriy bo'lgani uchun ularning ko'paytmasi ixtiyoriy 2-rang T tenzorning komponentalari bo'ladi. Shuning uchun tenglikning o'ng tomonini TG ko'paytmaning ikkala indeksi bo'yicha yig'ishtirish natijasi deb qarash mumkin. Ikkinchi tomonidan bu yerda $n = m = 2$ hamda tenglikning chap tomoni dS^2 skalyar miqdor. Demak, mazkur teoremaning shartlari bajariladi va G 2-rang tenzor ekanligi kelib chiqadi. Uni (1.6.1) va (1.6.2) indekslarni ko'tarish va tushirish amallari yordamida olinadigan ushbu

$$G = g_{\alpha\beta}\overline{\ni}^{\alpha}\overline{\ni}^{\beta} = g^{\alpha\beta}\overline{\ni}_{\alpha}\overline{\ni}_{\beta} = \delta^{\alpha\cdot}_{\cdot\beta}\,\overline{\ni}_{\alpha}\overline{\ni}^{\beta} = \delta^{\cdot\beta}_{\alpha\cdot}\,\overline{\ni}^{\alpha}\overline{\ni}_{\beta} \tag{2.5.3}$$

ko'rinishlarda yozish mumkin.

Ta'rif: (2.5.2) ga muvofiq koordinatalarning differensiallari orqali masofa differensialining kvadratini aniqlovchi simmetrik $g_{\alpha\beta}$ tenzor kovariant metrik tenzor deb ataladi.

Kovariant metrik $g_{\alpha\beta}$ tenzorning o'zaro bog'lanmagan komponentalarining umumiy soni $\frac{n(n+1)}{2}$ ga teng bo'ladi.

Kovariant metrik $g_{\alpha\beta}$ tenzor komponentalaridan tuzilgan determinantni g orqali belgilaylik:

$$g = |g_{\alpha\beta}| = \begin{vmatrix} g_{11} & g_{12} & \cdots \\ g_{21} & g_{22} & \cdots \\ \cdots & \cdots & \cdots \end{vmatrix}$$

Ta'rif: Kovariant metrik $g_{\alpha\beta}$ tenzorga teskari bo'lgan simmetrik $g^{\alpha\beta}$ tenzor kontravariant metric tenzor deb ataladi.

Kovaraint va kontravariant metrik tenzorlarning determinantlarining ko'paytmasi birga teng bo'ladi.

$$|g_{\alpha\beta}||g^{\alpha\beta}| = 1$$

Shunday qilib, dekart koordinatalar sistemasida metrik tenzorning bir xil indeksli komponentalari birga teng, har xil indeksli komponentalari nolga teng bo'ladi.

Metrik tenzorning xossalari

1. G - simmetrik tenzor. Bu xususiyat (2.5.1)dan kelib chiqadi.

2. Metrik tenzorning aralash komponentlari ixtiyoriy koordinatalar sistemasida Kroneker deltalarini beradi.

$$\hat{G}^{\alpha\cdot}_{\cdot\beta} = G^{i\cdot}_{\cdot j}\frac{\partial\eta^{\alpha}}{\partial\xi^{i}}\frac{\partial\xi^{j}}{\partial\eta^{\beta}} = \delta^{i\cdot}_{\cdot j}\frac{\partial\eta^{\alpha}}{\partial\xi^{i}}\frac{\partial\xi^{j}}{\partial\eta^{\beta}} = \frac{\partial\eta^{\alpha}}{\partial\eta^{\beta}} = \hat{\delta}^{\alpha\cdot}_{\cdot\beta} \qquad (2.5.4)$$

3. Ixtiyoriy P tenzor bilan G ning skalyar ko'paytmasi yana P tenzorni beradi:

$$P \cdot G = P \qquad (2.5.5)$$

ya'ni, metrik tenzor skalyar ko'paytirish amalida birlik tenzor rolini bajaradi. Masalan, P 3-rang tenzor bo'lsa, skalyar ko'paytirish formulasi (2.2.10)ga binoan olingan

$$T = P \cdot G \ \ (Q = G),$$

$$T^{\alpha_1\alpha_2\beta_2} = P^{\alpha_1\alpha_2\alpha}\delta^{\cdot\beta_2}_{\alpha\cdot} = P^{\alpha_1\alpha_2\beta_2}$$

munosabat (2.5.5) o'rinli ekanligini ko'rsatadi. Bu xususiyatdan G ning ixtiyoriy darajasi G ga teng ekanligi kelib chiqadi.

Diskriminant tenzor.

Biz avvalgi mavzularda bazis vektorlarning vektor ko’paytmasi kontravariant yoki kovariant bazis vektorlarning yoyilmasi (1.7.1) ko’rinishda ifodalanishini ko’rgan edik:

$$\begin{aligned} &\overline{Э}_{\alpha}\times\overline{Э}_{\beta}=e_{\alpha\beta\gamma}\,\overline{Э}^{\gamma}\,, \qquad \overline{Э}^{\alpha}\times\overline{Э}^{\beta}=e^{\alpha\beta\gamma}\,\overline{Э}_{\gamma} \\ &\overline{Э}_{\alpha}\times\overline{Э}^{\beta}=e_{\alpha}^{\cdot\beta\gamma}\,\overline{Э}_{\gamma}=e_{\alpha\cdot\gamma}^{\cdot\beta\cdot}\,\overline{Э}^{\gamma} \end{aligned} \tag{2.5.6}$$

Bu tengliklardan birinchisining ikkala tomonini $\overline{Э}_{\sigma}$, ikkinchisini $\overline{Э}^{\sigma}$, uchinchisini esa $\overline{Э}^{\sigma}$ va $\overline{Э}_{\sigma}$ vektorlarga skalyar ko’paytirsak, ushbu

$$\begin{aligned} &\overline{Э}_{\sigma}\cdot\left(\overline{Э}_{\alpha}\times\overline{Э}_{\beta}\right)=\left(\overline{Э}_{\sigma}\overline{Э}_{\alpha}\overline{Э}_{\beta}\right)=e_{\alpha\beta\gamma}\,\overline{Э}_{\sigma}\cdot\overline{Э}^{\gamma}=e_{\alpha\beta\gamma}\,\delta_{\sigma\cdot}^{\cdot\beta}=e_{\alpha\beta\sigma} \\ &\overline{Э}^{\sigma}\cdot\left(\overline{Э}^{\alpha}\times\overline{Э}^{\beta}\right)=\left(\overline{Э}^{\gamma}\overline{Э}^{\alpha}\overline{Э}^{\beta}\right)=e^{\alpha\beta\gamma}\,\overline{Э}_{\sigma}\cdot\overline{Э}_{\gamma}=e^{\alpha\beta\gamma}\,\delta_{\cdot\gamma}^{\alpha\cdot}=e^{\alpha\beta\sigma} \\ &\overline{Э}^{\sigma}\cdot\left(\overline{Э}_{\alpha}\times\overline{Э}^{\beta}\right)=\left(\overline{Э}^{\sigma}\overline{Э}_{\alpha}\overline{Э}^{\beta}\right)=e_{\alpha}^{\cdot\beta\gamma}\,\delta_{\cdot\gamma}^{\sigma\cdot}=e_{\alpha}^{\beta\sigma} \\ &\overline{Э}_{\sigma}\cdot\left(\overline{Э}_{\alpha}\times\overline{Э}^{\beta}\right)=\left(\overline{Э}_{\sigma}\overline{Э}_{\alpha}\overline{Э}^{\beta}\right)=e_{\alpha\cdot\gamma}^{\cdot\beta}\,\delta_{\sigma\cdot}^{\cdot\gamma}=e_{\alpha\cdot\sigma}^{\cdot\beta\cdot} \end{aligned} \tag{2.5.7}$$

munosabatlarga ega bo’lamiz. Demak, (2.5.6) yoyilmalarning koeffitsientlarini almashtirish qoidasini aniqlash uchun bazis vektorlarning aralash ko’paytmasini tekshirish kifoya qilar ekan. Haqiqatan (1.11.5) formulalardan foydalanib, η^{α} koordinatalar sistemasida aralash ko’paytmalarning, masalan, birinchisini yozsak, quyidagi tengliklarga ega bo’lamiz:

$$\begin{aligned} \hat{e}_{ij\kappa}&=\left(\hat{Э}_{i}\hat{Э}_{j}\hat{Э}_{\kappa}\right)=\left(\overline{Э}_{\alpha}\frac{\partial\xi^{\alpha}}{\partial\eta^{i}}\,\overline{Э}_{\beta}\frac{\partial\xi^{\beta}}{\partial\eta^{j}}\,\overline{Э}_{\gamma}\frac{\partial\xi^{\gamma}}{\partial\eta^{\kappa}}\right)= \\ &=\left(\overline{Э}_{\alpha}\overline{Э}_{\beta}\overline{Э}_{\gamma}\right)\frac{\partial\xi^{\alpha}}{\partial\eta^{i}}\frac{\partial\xi^{\beta}}{\partial\eta^{j}}\frac{\partial\xi^{\gamma}}{\partial\eta^{\kappa}}=e_{\alpha\beta\gamma}\frac{\partial\xi^{\alpha}}{\partial\eta^{i}}\frac{\partial\xi^{\beta}}{\partial\eta^{j}}\frac{\partial\xi^{\gamma}}{\partial\eta^{\kappa}} \end{aligned}$$

Demak, ξ^{α} koordinata sistemasida berilgan $3^{3}=27$ ta $e_{\alpha\beta\gamma}$ sonlarning almashtirish qoidasi quyidagi formula bilan aniqlanadi:

$$\hat{e}_{ij\kappa}=e_{\alpha\beta\gamma}\frac{\partial\xi^{\alpha}}{\partial\eta^{i}}\frac{\partial\xi^{\beta}}{\partial\eta^{j}}\frac{\partial\xi^{\gamma}}{\partial\eta^{\kappa}} \tag{2.5.8}$$

Bundan tenzorning ikkinchi ta’rifiga ko’ra, $e_{\alpha\beta\gamma}$ 3-rang tenzorning kovariant komponentalari ekanligi kelib chiqadi. Indekslari boshqacha joylashgan $e^{\alpha\beta\gamma}$, $e_{\alpha}^{\cdot\beta\gamma}$, $e_{\alpha\cdot\gamma}^{\cdot\beta\cdot}$ va hokazo sonlar ham 3-rang tenzorning komponentalari ekanligi shunga o'xshash ko'rsatiladi. Indekslarni ko'tarish va tushirish amali yordamida esa bazis vektorlarning turli xil aralash ko'paytmalari hosil qiladigan barcha miqdorlar bir tenzorning turli xil komponentalari ekanligi oson isbot qilinadi.

Ta'rif: Quyidagi shaklda ifodalangan tenzor diskriminant tenzor deb ataladi.

$$E = e_{\alpha\beta\gamma}\overline{Э}^{\alpha}\overline{Э}^{\beta}\overline{Э}^{\gamma} = e^{\alpha}_{\cdot\beta\gamma}\overline{Э}_{\alpha}\overline{Э}^{\beta}\overline{Э}^{\gamma} = ...e^{\alpha\beta\gamma}\overline{Э}_{\alpha}\overline{Э}_{\beta}\overline{Э}_{\gamma} \qquad (2.5.9)$$

Bazis vektorlarning aralash ko'paytmasini aniqlovchi (1.4.5), (1.5.1), (1.5.2) formulalardan va diskriminant tenzor komponentalarining ular orqali ifodasi (2.5.7)dan

$$e_{\alpha\beta\gamma} = -e_{\beta\alpha\gamma}\,, \quad e_{\alpha\beta\gamma} = -e_{\alpha\gamma\beta}\,, \quad e_{\alpha\beta\gamma} = -e_{\gamma\beta\alpha} \qquad (2.5.10)$$

munosabatlar o'rinli ekanligi kelib chiqadi. Demak, diskriminant tenzor uchchala indeksi bo'yicha antisimmetrik bo'lib, uning ikkita indeksi bir xil bo'lgan komponentalari nolga teng, yani faqat barcha indekslari turli bo'lgan komponentalarigina noldan farqli bo'ladi.

Ikkinchi tomondan, (1.4.5) va (1.4.9)ga ko'ra quyidagi tenglik o'rinli bo'ladi:

$$\left(\overline{Э}_1\overline{Э}_2\overline{Э}_3\right)^2 = |g_{\alpha\alpha}| = g$$

Agar

$$e_{123} = \left(\overline{Э}_1\overline{Э}_2\overline{Э}_3\right) = J^{-1} = \sqrt{g} \qquad (2.5.11)$$

deb qabul qilinsa va E ning (2.5.10) antisimmetriklik xususiyatlari e'tiborga olinsa, diskriminant tenzor komponentalari ushbu qoida bilan aniqlanadi:

$e_{\alpha\beta\gamma} = \sqrt{g}$, agar $\alpha\beta\gamma$ sonli indekslar 1, 2, 3 dan juft o'rin almashtirish natijasida hosil qilingan bo'lsa;

$e_{\alpha\beta\gamma} = -\sqrt{g}$, agar $\alpha\beta\gamma$ sonli indekslar 1, 2, 3 dan toq o'rin almashtirish natijasida hosil qilingan bo'lsa:

$e_{\alpha\beta\gamma} = 0$, boshqa hollarda, yani ikki yoki uchchala indeksi bir xil qiymat qabul qilsa.

Shunga o'xshash, (1.5.6)ga binoan,

$$(\overline{Э}^1\overline{Э}^2\overline{Э}^3) = \left|g^{\alpha\beta}\right|^{\frac{1}{2}} = \sqrt{g_1}$$

deb qabul qilsak va E ning (2.5.10) antisimmetrik xususiyatlarini e'tiborga olsak, diskriminant tenzorning kontravariant komponentalari

$$e^{\sigma\kappa r} = g^{\sigma\alpha} g^{\kappa\beta} g^{r\gamma} e_{\alpha\beta\gamma}$$

qoida bilan aniqlanishi kelib chiqadi:

$e^{\sigma kr} = \sqrt{g}$, agar σkr sonli indekslar 1, 2, 3 dan juft o'rin almashtirish natijasida hosil qilingan bo'lsa;

$e^{\sigma kr} = -\sqrt{g}$, agar σkr sonli indekslar 1, 2, 3 dan toq o'rin almashtirish natijasida hosil qilingan bo'lsa;

$e^{\sigma kr} = 0$, boshqa hollarda, yani ikki yoki uhchala indeksi bir xil qiymat qabul qilsa.

Diskriminant tenzorlar vektor ko'paytmani va aralash ko'paytmani (uchinchi tartibli determinantni) indeksli yozishda qo'llaniladi. Masalan,

$$\overline{a} \times \overline{b} = \overline{c} = c_\kappa \overline{Э}^\kappa = c^\kappa \overline{Э}_\kappa$$

vektor ko'paytmaning kovariant komponentalarini

$$c_\kappa = e_{ij\kappa}\, a^i b^j \tag{2.5.12}$$

ko'rinishda yozilishi quyidagi munosabatlardan kelib chiqadi:

$$\left(\overline{a} \times \overline{b}\right) = \left(a^i \overline{э}_i \times b^j \overline{э}_j\right) = a^i b^j \left(\overline{э}_i \times \overline{э}_j\right) = a^i b^j e_{ij\kappa} \overline{э}^\kappa$$

$\overline{c}$ vektorning kontravariant komponentasi esa

$$c^\kappa = e^{ij\kappa}\, a_i b_j \tag{2.5.13}$$

shaklda yozilishini ham ko'rsatish mumkin. Shunga o'xshash $\left(\overline{a}\,\overline{b}\,\overline{c}\right)$ aralash ko'paytmani ham indeksli ko'rinishda, yani

$$\left(\overline{a}\overline{b}\overline{c}\right) = a^i b^j c^\kappa \left(\overline{э}_i \overline{э}_j \overline{э}_\kappa\right) = a^i b^j c^\kappa\, e_{ij\kappa} \tag{2.5.14}$$

tenglikning o'ng tomonidagi i, j, k indekslar bo'yicha yig'indini ushbu

$$e_{ij\kappa} a^i b^j c^\kappa = \sqrt{g}\left(a^1b^2c^3 - a^1b^3c^2 + a^2b^3c^1 - a^2b^1c^3 + a^3b^1c^2 - a^3b^2c^1\right)$$

ko'rinishida ochib yozish mumkin. So'nggi tenglikda qavs ichidagi ifoda $\overline{a},\overline{b},\overline{c}$ vektorlarning komponentalaridan hosil qilingan determinantga teng. Shuning uchun uchinchi tartibli determinantni indeksli belgilash qoidasini beruvchi quyidagi munosabat o'rinli bo'ladi:

$$e_{ij\kappa}a^i b^j c^\kappa = \sqrt{g}\begin{vmatrix} a^1 & a^2 & a^3 \\ b^1 & b^2 & b^3 \\ c^1 & c^2 & c^3 \end{vmatrix} \tag{2.5.15}$$

6 §. Levi-Chivita tenzori.

Ta'rif: Ushbu $\varepsilon_{ij\kappa} = \dfrac{e_{ij\kappa}}{\left(\bar{Э}_1\bar{Э}_2\bar{Э}_3\right)}$, $\varepsilon^{ij\kappa} = \dfrac{e^{ij\kappa}}{\left(\bar{Э}^1\bar{Э}^2\bar{Э}^3\right)}$ (2.6.1)

formulalar bilan aniqlanadigan tenzorlar Levi-Chivita tenzori deb ataladi.

Levi-Chivita tenzori yordamida (2.5.15) munosabat

$$\varepsilon_{ij\kappa}a^i b^j c^\kappa = \begin{vmatrix} a^1 & a^2 & a^3 \\ b^1 & b^2 & b^3 \\ c^1 & c^2 & c^3 \end{vmatrix} \tag{2.6.2}$$

ko'rinishida yoziladi. Shunga o'xshash kovariant komponentalar hosil qilgan determinantni quyidagi indeksli ifoda shaklida yozish mumkin:

$$\varepsilon^{ij\kappa}a_i b_j c_\kappa = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \tag{2.6.3}$$

Yuqorida ko'rganimizdek, to'g'ri burchakli dekart koordinatalar sistemasida $g = g_1 = 1$ bo'lgani uchun

$$e_{ij\kappa} = e^{ij\kappa} = \varepsilon_{ij\kappa}$$

tenglik o'rinli, yani indekslarning yuqorida yoki quyida joylashtirishning farqi bo'lmaydi.

Diskriminant va Levi-Chivita tenzorlari (2.5.11)ga ko'ra yakobiani musbat bo'lgan almashtirishlar uchun kiritildi. Quyida $J < 0$ bo'lgan hollarni ham qarash mumkinligi ko’rsatiladi. Ortogonal dekart koordinatalar sistemasi y^α ni ξ^q sistemasiga va y^α ni η^r sistemasiga almashtirish matrisalarining determinantlari (1.4.5)ga binoan quyidagi ko’rinishga ega:

$$(\bar{Э}_1\bar{Э}_2\bar{Э}_3)=\begin{vmatrix}\frac{\partial y^1}{\partial\xi^1} & \frac{\partial y^2}{\partial\xi^1} & \frac{\partial y^3}{\partial\xi^1}\\ \frac{\partial y^1}{\partial\xi^2} & \frac{\partial y^2}{\partial\xi^2} & \frac{\partial y^3}{\partial\xi^2}\\ \frac{\partial y^1}{\partial\xi^3} & \frac{\partial y^2}{\partial\xi^3} & \frac{\partial y^3}{\partial\xi^3}\end{vmatrix},\ (\hat{Э}_1\hat{Э}_2\hat{Э}_3)=\begin{vmatrix}\frac{\partial y^1}{\partial\eta^1} & \frac{\partial y^2}{\partial\eta^1} & \frac{\partial y^3}{\partial\eta^1}\\ \frac{\partial y^1}{\partial\eta^2} & \frac{\partial y^2}{\partial\eta^2} & \frac{\partial y^3}{\partial\eta^2}\\ \frac{\partial y^1}{\partial\eta^3} & \frac{\partial y^2}{\partial\eta^3} & \frac{\partial y^3}{\partial\eta^3}\end{vmatrix}$$

Determinantlarning hossasiga ko’ra ularning nisbati ξ^q sistemani η^r sistemaga almashtirish matrisasining determinanti - yakobianga teng bo’ladi:

$$\frac{\left(\bar{Э}_1\bar{Э}_2\bar{Э}_3\right)}{\left(\hat{Э}_1\hat{Э}_2\hat{Э}_3\right)}=\left|\frac{\partial\eta^r}{\partial\xi^q}\right|=J \tag{2.6.4}$$

Levi-Chivita tenzorining komponentalarini almashtirish qoidasi (2.5.8) va (2.6.1) formulalarga ko’ra quyidagi ko’rinishga ega.

$$\bar{\varepsilon}_{ij\kappa}=\frac{\left(\bar{Э}_1\bar{Э}_2\bar{Э}_3\right)}{\left(\hat{Э}_1\hat{Э}_2\hat{Э}_3\right)}\varepsilon_{\alpha\beta\gamma}\frac{\partial\xi^\alpha}{\partial\eta^i}\frac{\partial\xi^\beta}{\partial\eta^j}\frac{\partial\xi^\gamma}{\partial\eta^\kappa}=J\varepsilon_{\alpha\beta\gamma}\frac{\partial\xi^\alpha}{\partial\eta^i}\frac{\partial\xi^\beta}{\partial\eta^j}\frac{\partial\xi^\gamma}{\partial\eta^\kappa} \tag{2.6.5}$$

Xususan, ξ^q va η^r ortogonal dekart koordinata sistemalari bo’lsa, $J=\pm 1$ bo’ladi.

Demak, Levi-Chivita tenzori komponentalarini almashtirish formulasi (10.21) yuqorida tenzorning komponentalari uchun berilgan (7.1) formuladan J vazni bilan farq qiladi.

Ta’rif: Almashtirish formulalari vaznli bo’lgan, yani komponentalari

$$\hat{T}_{\beta_1}^{\cdot\beta_2\ldots\beta_n}=J^m T_{\alpha_1}^{\cdot\alpha_2\ldots\alpha_n}\frac{\partial\xi^{\alpha_1}}{\partial\eta^{\beta_1}}\frac{\partial\eta^{\beta_2}}{\partial\xi^{\alpha_2}}\cdots\frac{\partial\eta^{\beta_n}}{\partial\xi^{\alpha_n}} \tag{10.22}$$

qoida bilan almashadigan ob’yektlar psevdotenzor deb ataladi.

Bu munosabat, xususan $m=0$ bo’lganda, tenzor komponentalarini almashtirish formulasi (2.1.6) ko’rinishini oladi. Demak, psevdotenzorlar tenzor tushunchasini umumlashtirishdir.

7 §. Ikkinchi rang tenzorlar va matrisalar.

2-rang T tenzor uchun quyidagi formulalar o'rinli:

$$T = T_{\alpha\beta}\overline{Э}^{\alpha}\overline{Э}^{\beta} = T^{\alpha\cdot}_{\cdot\beta}\overline{Э}_{\alpha}\overline{Э}^{\beta} = T^{\cdot\beta}_{\alpha\cdot}\overline{Э}^{\alpha}\overline{Э}_{\beta} = T^{\alpha\beta}\overline{Э}_{\alpha}\overline{Э}_{\beta} \qquad (2.7.1)$$

Muayyan koordinata sistemasida 2-rang tenzorga to'rtta turli uchinchi tartibli kvadrat matrisalar mos keladi:

$$A_1 = \left\| T_{\alpha\beta} \right\|, \quad A_2 = \left\| T^{\alpha\cdot}_{\cdot\beta} \right\|, \quad A_3 = \left\| T^{\cdot\beta}_{\alpha\cdot} \right\|, \quad A_4 = \left\| T^{\alpha\beta} \right\| \qquad (2.7.2)$$

Bu tengliklarda hamma birinchi indekslar matrisa elementi joylashgan satrning, ikkinchilari esa - ustunning nomerini ko'rsatadi.

Agar

$$T_{\alpha\beta} = g_{\alpha\sigma}T^{\sigma\cdot}_{\cdot\beta} = T^{\cdot\sigma}_{\alpha\cdot}g_{\sigma\beta} = g_{\alpha\sigma}T^{\sigma\tau}g_{\tau\beta}$$

munosabatlarni hisobga olib, matritsalarni ko'paytirish qoidasini eslasak,

$$A_1 = g_* A_2 = A_3 g_* = g_* A_4 g_*, \; g_* = \left\| g_{\alpha\gamma} \right\| \qquad (2.7.3)$$

tenglik hosil bo'ladi. Shunday qilib, $A_i \left(i = \overline{1,4} \right)$ matritsalarning birontasi ma'lum bo'lsa, qolganlarini (2.7.3) dan foydalanib aniqlash qiyin emas.

2-rang tenzorlar uchun kiritilgan qo'shish, ko'paytirish, skalyar ko'paytirish amallariga mos kelgan amallar matrisalar uchun ham o'rinli. Bu esa matrisani hisoblash usullari va unda olingan natijalarni tenzor hisob nazariyasida ham qo'llash mumkinligini bildiradi.

Ma'lum bir koordinatalar sistemasida A_i larning birortasi berilgan bo'lsa, (2.7.2) va (2.7.3) tengliklarga binoan shu sistemada T tenzor ham aniqlangan bo'ladi. Shunday qilib, 2-rang tenzorlar va uchinchi tartibli matrisalar orasida bir qiymatli moslik mavjud.

2-rang tenzorlarni simmetrik va antisimmetrik tenzorlar orqali ifodalash ham mumkin.

Ixtiyoriy 2-rang tenzorning kovariant komponentalarini simmetrik va antisimmetrik tenzor komponentalarining yig'indisi shaklida yozish mumkin:

$$T_{\alpha\beta} = \frac{1}{2}\left(T_{\alpha\beta} + T_{\beta\alpha}\right) + \frac{1}{2}\left(T_{\alpha\beta} - T_{\beta\alpha}\right) = T_{(\alpha\beta)} + T_{[\alpha\beta]} \qquad (2.7.4)$$

Teorema: Har bir 2-rang tenzor uchun (2.7.4) ko'rinish yagonadir.

Isbot. Teskarisini faraz qilaylik, yani (2.7.4) bilan birga

$$T_{\alpha\beta} = S_{\alpha\beta} + A_{\alpha\beta} \tag{2.7.5}$$

tenglik ham o'rinli bo'lsin. Bu yerda S - simmetrik $\left(S_{\alpha\beta} = S_{\beta\alpha}\right)$, A esa antisimmetrik $\left(A_{\beta\alpha} = -A_{\alpha\beta}\right)$ tenzorlar bo'lgani uchun (2.7.4) va (2.7.5) lardan quyidagilar hosil bo'ladi:

$$T_{(\alpha\beta)} + T_{[\alpha\beta]} = S_{\alpha\beta} + A_{\alpha\beta} \tag{2.7.6}$$

$$T_{(\beta\alpha)} + T_{[\beta\alpha]} = S_{\beta\alpha} + A_{\beta\alpha} \tag{2.7.7}$$

Bu tengliklarni chap va o'ng tomonlarini mos ravishda qo'shsak,

$$T_{(\alpha\beta)} + T_{(\beta\alpha)} + T_{[\alpha\beta]} + T_{[\beta\alpha]} = S_{\alpha\beta} + S_{\beta\alpha} + A_{\alpha\beta} + A_{\beta\alpha}$$

$$2\,T_{(\alpha\beta)} = 2\,S_{\alpha\beta} \;\Rightarrow\; T_{(\alpha\beta)} = S_{\alpha\beta} \tag{2.7.8}$$

munosabat kelib chiqadi.

Endi (2.7.6) va (2.7.7) tengliklarni hadma-had ayirsak,

$$T_{(\alpha\beta)} - T_{(\beta\alpha)} + T_{[\alpha\beta]} - T_{[\beta\alpha]} = S_{\alpha\beta} - S_{\beta\alpha} + A_{\alpha\beta} - A_{\beta\alpha}$$

munosabat kelib chiqadi. Bu yerda

$$2\,T_{[\alpha\beta]} = 2\,A_{\alpha\beta} \quad\Rightarrow\quad T_{[\alpha\beta]} = A_{\alpha\beta} \tag{2.7.9}$$

ekanligini ko'ramiz. (2.7.8) va (2.7.9) lardan esa (2.7.4) ko'rinishning yagonaligi kelib chiqadi. Shunday qilib, 2-rang T tenzor uchun yagona quyidagi ko'rinish o'rinli;

$$T = S + A \tag{2.7.10}$$

S va A lar (2.7.8) va (2.7.9) formulalar bilan aniqlanadi. T, S, A tenzorlarning birinchi invariantalarini

$$J_1^T = T^{\alpha\cdot}_{\cdot\alpha}\,, \qquad J_1^S = S^{\alpha\cdot}_{\cdot\alpha}\,, \qquad J_1^A = A^{\alpha\cdot}_{\cdot\alpha}$$

deb belgilaylik. Qo'sh o'ram haqidagi teoremaga ko'ra antisimmetrik tenzorning birinchi invarianti

$$J_1^A = A^{\alpha\cdot}_{\cdot\alpha} = g^{\alpha\beta} A_{\beta\alpha} = 0$$

ekanligini topamiz. Bu yerdan va (2.7.10 formuladan

$$J_1^T = J_1^S + J_1^A = J_1^S \tag{2.7.11}$$

tenglik, yani tenzor va uning simmetrik qismining birinchi invariantlari teng ekanligi kelib chiqadi.

8 §. Shar va deviator tenzori.

Ta'rif: 2-rang P tenzor metrik tenzordan skalyar ko'paytma bilangina farq qilsa, u tenzor shar tenzori deb ataladi;

$$P = f\, G, \qquad P_{\cdot\beta}^{\alpha\cdot} = f\, \delta_{\cdot\beta}^{\alpha\cdot} \tag{2.8.1}$$

Demak, shar tenzorning birinchi invarianti

$$J_1^P = P_{\cdot\alpha}^{\alpha\cdot} = f\, \delta_{\cdot\alpha}^{\alpha\cdot} = 3f \tag{2.8.2}$$

bo'ladi.

Ta'rif: 2-rang simmetrik D tenzorning birinchi invarianti J_1^D nolga teng bo'lsa, D tenzor deviator tenzori deb ataladi.

Ixtiyoriy 2-rang simmetrik tenzor S ga quyidagi formula asosida deviator tenzorni mos keltirish mumkin:

$$D = S - P\ , \qquad P = \frac{1}{3} J_1^S G \tag{2.8.3}$$

Bu yerda D deviator ekanligini isbotlash uchun $J_1^D = 0$ ekanligini ko'rsatish kerak. Haqiqatan,

$$J_1^D = D_{\cdot\alpha}^{\alpha\cdot} = S_{\cdot\alpha}^{\alpha\cdot} - P_{\cdot\alpha}^{\alpha\cdot} = S_{\cdot\alpha}^{\alpha\cdot} - \frac{1}{3} J_1^S \delta_{\cdot\alpha}^{\alpha\cdot} = J_1^S - \frac{1}{3} J_1^S \cdot 3 = 0$$

Demak, 2-rang simmetrik tenzor S ni

$$S = P + D \tag{2.8.4}$$

ko'rinishida, yani shar va deviator tenzorning yig'indisi ko'rinishida yozish mumkin. Ixtiyoriy T tenzorni (2.7.10) va (2.8.4) ga asosan ushbu

$$T = P + D + A$$

yig'indi ko'rinishida yozish mumkin. Bu yerda (2.7.11) va (2.8.3)ga asosan shar tenzorini $P = \frac{1}{3} J_1^T G$ formula bilan ifodalash mumkin.

Tarif: 2-rang T va T^{-1} tenzorlarning skalyar ko'paytmasi metrik tenzorga teng bo'lsa,

$$T \cdot T^{-1} = G$$

ular o'zaro teskari tenzorlar deb ataladi.

2-rang T tenzor uchun, umuman olganda, ikkita teskari tenzor mavjud: T_1^{-1} - chap teskari tenzor, T_2^{-1} - o'ng teskari tenzor

$$T_1^{-1} \cdot T = G \ , \qquad T \cdot T_2^{-1} = G$$

Lekin $T_1^{-1} = T_2^{-1}$ ekanligini ko'rsatish qiyin emas. Buning uchun birinchi tenglikni T_2^{-1} ga skalyar ko'paytirib, ikkinchi tenglikdan va assotsiativlik qonunidan foydalanish yetarli;

$$T_1^{-1} \cdot \left(T \cdot T_2^{-1}\right) = G \cdot T_2^{-1} = T_2^{-1} \Rightarrow T_1^{-1} \cdot G = T_2^{-1} \Rightarrow T_1^{-1} = T_2^{-1}$$

Endi teskari tenzor yagona ekanligini ko'rsatamiz. Ikkita T^{-1} va Q teskari tenzorlar mavjud deb faraz qilsak,

$$T \cdot T^{-1} = TQ$$

tenglik o'rinli bo'lar edi.

Endi bu tenglikni chapdan T^{-1} ga ko'paytirib assotsiativlik qonunidan foydalanish kifoya:

$$\left(T^{-1} \cdot T\right) \cdot T^{-1} = \left(T^{-1} \cdot T\right) \cdot Q \Rightarrow G \cdot T^{-1} = G \cdot Q \Rightarrow T^{-1} = Q$$

Ta'rif:_Agar T tenzorning determinanti $\left|T^{l\cdot}_{\cdot m}\right|$ nolga teng bo'lsa, u xos tenzor deb ataladi.

9 §. Ikinchi rang tenzorning bosh yo'nalishlari. Xarakteristik tenglama.

Malumki, 2-rang tenzorga ikkita

$$\bar{b} = T \cdot \bar{a} \ , \qquad \bar{b}\,' = \bar{a} \cdot T \qquad (2.9.1)$$

chiziqli vektor-funktsiyalarni mos keltirish mumkin. Bu formulalardagi T tenzorga $\bar{a}$ vektorning uzunligini va yo'nalishini o'zgartiruvchi operator deb qarash mumkin.

Ta’rif: Agar T tenzorning ta’siri natijasida $\bar{a}$ vektorning faqat uzunligi o’zgarsa, yani shunday λ va μ sonlar mavjud bo’lsaki,

$$\bar{b} = \lambda\,\bar{a} \qquad \text{yoki} \qquad \bar{b}\,' = \mu\,\bar{a} \tag{2.9.2}$$

tenglik o’rinli bo’lsa, $\bar{a}$ vektorning yo’nalishi T tenzorning bosh yo’nalishi deb ataladi.

Bu holda (2.9.1) tengliklarni quyidagi ko’rinishda yozish mumkin;

$$\lambda\,\bar{a} = T\cdot\bar{a} \Rightarrow \left(\lambda\,G - T\right)\cdot\bar{a} = 0 \Rightarrow \left(\lambda\,\delta^{\alpha\cdot}_{\cdot\beta} - T^{\alpha\cdot}_{\cdot\beta}\right)a^{\beta} = 0 \tag{2.9.3}$$

$$\mu\,\bar{a} = \bar{a}\cdot T \Rightarrow \bar{a}\left(\mu\,G - T\right) = 0 \Rightarrow \left(\mu\,\delta^{\cdot\alpha}_{\beta\cdot} - T^{\cdot\alpha}_{\beta\cdot}\right)a^{\beta} = 0 \tag{2.9.4}$$

Shunday qilib, tenzorning bosh yo’nalishini aniqlash masalasi (2.9.3), (2.9.4) sistemalarning noldan farqli yechimlarini topish masalasiga keltirildi. Bunday yechimlar mavjud bo’lishi uchun esa (2.9.3), (2.9.4) sistemaning determinanti nolga teng bo’lishi kerak:

$$\left|\lambda\,\delta^{\alpha\cdot}_{\cdot\beta} - T^{\alpha\cdot}_{\cdot\beta}\right| = 0\,, \qquad \left|\mu\,\delta_{\beta}{}^{\alpha} - T_{\beta}{}^{\alpha}\right| = 0 \tag{2.9.5}$$

Ikkinchi tomondan, bu yerda indekslarni ko’tarish (tushirish) qoidasiga ko’ra o’rinli bo’lgan

$$\lambda\,\delta^{\cdot\alpha}_{\beta\cdot} - T^{\cdot\alpha}_{\beta\cdot} = g_{\beta\sigma}\left(\lambda\,\delta^{\sigma\cdot}_{\cdot\tau} - T^{\sigma\cdot}_{\cdot\tau}\right)g^{\tau\alpha}$$

tenglikdan va matrisalarni ko’paytirish qoidasidan ushbu

$$\left|\lambda\delta^{\cdot\alpha}_{\beta\cdot} - T^{\cdot\alpha}_{\beta\cdot}\right| = g\left|\lambda\delta^{\sigma\cdot}_{\cdot\tau} - T^{\sigma\cdot}_{\cdot\tau}\right|g_1 = \left|\lambda\delta^{\sigma\cdot}_{\cdot\tau} - T^{\sigma\cdot}_{\cdot\tau}\right|$$

munosabatlar kelib chiqadi. Bundan (2.9.3), (2.9.4) tenglamalar bitta

$$\left|\lambda\cdot\delta^{\alpha\cdot}_{\cdot\beta} - T^{\alpha\cdot}_{\cdot\beta}\right| = 0 \tag{2.9.5}$$

tenglama ekanligini ko’ramiz.

Ta’rif: (2.9.5) tenglama T tenzorning xarakteristik tenglamasi deb ataladi.

Xarakteristik tenglama invariant ekanligini, yani turli koordinatalar sistemalarida determinantning qiymati o’zgarmasligini ko’rsatamiz. Buning uchun $\lambda G - T$ tenzorning aralash komponentalarining ushbu

$$\lambda\hat{\delta}^{\mu\cdot}_{\cdot\nu} - \hat{T}^{\mu\cdot}_{\cdot\nu} = \left(\lambda\delta^{\alpha\cdot}_{\cdot\beta} - T^{\alpha\cdot}_{\cdot\beta}\right)\frac{\partial\eta^{\mu}}{\partial\xi^{\alpha}}\frac{\partial\xi^{\beta}}{\partial\eta^{\nu}}$$

almashtirish formulalarini determinantlar orqali yozish kifoya:

$$\left|\hat{\delta}_{\nu}^{\mu}-\hat{T}_{\nu}^{\mu}\right|=\left|\lambda\delta_{\beta}^{\alpha}-T_{\beta}^{\alpha}\right|J\cdot J^{-1}=\left|\lambda\delta_{\beta}^{\alpha}-T_{\beta}^{\alpha}\right|$$

demak, xarakteristik tenglamaning invariantligi isbotlandi.

10 §. Tenzorning xos vektorlari

Xarakteristik tenglamaning hamma yechimlari haqiqiy va turli bo'lgan holni ko'raylik. Mazkur λ_{ρ} $(\rho=1,2,3)$ ildizlarni (2.9.3) va (2.9.4) tenglikka keltirib qo'ysak,

$$\lambda_{\rho}\overline{a}_{\rho}=T\overline{a}_{\rho}\,,\qquad \left(\lambda_{\rho}\,\delta_{\beta}^{\alpha}-T_{\beta}^{\alpha}\right)a_{\rho}^{\beta}=0 \tag{2.10.1}$$

$$\lambda_{\rho}\overline{a}'_{\rho}=\overline{a}'_{\rho}\cdot T\,,\qquad \left(\lambda_{\rho}\delta_{\beta\cdot}^{\cdot\alpha}-T_{\beta\cdot}^{\cdot\alpha}\right)a'^{\beta}_{\rho}=0 \tag{2.10.2}$$

tenglamalarni olamiz. Ular, umuman olganda, turli $\overline{a}_{\rho}$ va $\overline{a}'_{\rho}$ vektorlar uchligini aniqlaydi. Bu vektorlar T tenzorning bosh yo'nalishi bo'ylab yo'nalgan bo'lib, tenzorning xos vektorlari deb ataladi. Xarakteristik tenglamaning turli ildizlariga mos kelgan xos vektorlar o'zaro chiziqli erkli ekanligini ko'rsatamiz. Teskarisini, yani $\overline{a}_{\rho}$ vektorlar o'zaro chiziqli bog'liq deb faraz qilamiz:

$$c^{1}\overline{a}_{1}+c^{2}\overline{a}_{2}+c^{3}\overline{a}_{3}=0$$

Bu tenglikni chap tomondan T tenzorga skalyar ko'paytirsak va (2.10.1) munosabatlardan foydalansak,

$$\lambda_{1}c^{1}\overline{a}_{1}+\lambda_{2}c^{2}\overline{a}_{2}+\lambda_{3}c^{3}\overline{a}_{3}=0 \tag{2.10.3}$$

tenglikni olamiz. Shu amalni (2.10.3) ga qo'llab esa

$$\lambda_{1}^{2}c^{1}\overline{a}_{1}+\lambda_{2}^{2}c^{2}\overline{a}_{2}+\lambda_{3}^{2}c^{3}\overline{a}_{3}=0 \tag{2.10.4}$$

tenglikni olamiz. Ushbu bir jinsli chiziqli tenglamalar sistemasi noldan farqli $c^{1}\overline{a}_{1},c^{2}\overline{a}_{2},c^{3}\overline{a}_{3}$ yechimlarga ega bo'lgani uchun uning determinanti nolga teng bo'lishi kerak. Lekin bu determinant

$$\begin{vmatrix}1&1&1\\ \lambda_{1}&\lambda_{2}&\lambda_{3}\\ \lambda_{1}^{2}&\lambda_{2}^{2}&\lambda_{3}^{2}\end{vmatrix}=\left(\lambda_{2}-\lambda_{1}\right)\left(\lambda_{3}-\lambda_{1}\right)\left(\lambda_{3}+\lambda_{1}-\lambda_{2}-\lambda_{1}\right)\neq 0$$

bu yerdan $c^1 = c^2 = c^3 = 0$. Demak, $\bar{a}_\rho$ vektorlar chiziqli erkli ekanligi kelib chiqadi. Shunga o'xshash a'_ρ vektorlar ham chiziqli erkli ekanligi ko'rsatiladi.

Endi $\bar{a}_\rho$ va a'_ρ vektorlarning o'zaro munosabatlarini aniqlaymiz. Buning uchun (2.10.2) ni $\bar{a}_\mu$ ga skalyar ko'paytirib (2.10.1) larni hisobga olsak, quyidagilarga ega bo'lamiz:

$$\lambda_\rho \bar{a}'_\rho \cdot \bar{a}_\mu = \left(\bar{a}'_\rho \cdot T\right) \cdot \bar{a}_\mu = \bar{a}'_\rho \cdot \left(T \cdot \bar{a}_\mu\right) =$$
$$= \lambda_\mu \bar{a}'_\rho \cdot \bar{a}_\mu \quad \Rightarrow \quad \left(\lambda_\rho - \lambda_\mu\right) \bar{a}'_\rho \cdot \bar{a}_\mu = 0$$

bu yerdan

$$\bar{a}'_\rho \cdot \bar{a}_\mu = 0, \quad \rho \neq \mu; \qquad \bar{a}'_\rho \cdot \bar{a}_\mu = f, \quad \rho = \mu \tag{2.10.5}$$

ekanligi kelib chiqadi.

Bu yerda f - ixtiyoriy chegaralangan miqdor. Shu tufayli $f = 1$ deb olsa bo'ladi. Unda (2.10.5) ga o'zarolik munosabatlari deb qaralsa, $\bar{a}_\rho$ va $\bar{a}'_\rho$ vektorlar o'zaro uchlik vektorlar bo'ladi.

11 §. Tenzorning bosh qiymatlari va kanonik ko'rinishi

Koordinata bazisi sifatida xos vektorlar uchligi $\bar{a}_\rho$ larni $\left(Э_\rho = \bar{a}_\rho\right)$ olinsa, (2.9.1) vektor funksiyalarni ushbu ko'rinishda yozish mumkin;

$$T \cdot \bar{Э}_\beta = T^{i\cdot}_{\cdot j} \bar{Э}_i \bar{Э}^j_{\cdot} \cdot \bar{Э}_\beta = T^{i\cdot}_{\cdot\beta} \bar{Э}_i \Rightarrow \bar{Э}^\alpha \cdot \left(T \cdot \bar{Э}_\beta\right) = T^{\alpha\cdot}_{\cdot\beta}$$
$$\bar{Э}_\alpha \cdot T = T^j_i \bar{Э}_\alpha \cdot \bar{Э}^i_{\cdot} \bar{Э}_j = T^j_\alpha \bar{Э}_j \Rightarrow \left(\bar{Э}_\alpha \cdot T\right) \cdot \bar{Э}^\beta = T^\beta_\alpha$$

Bu yerda (2.10.1) ga ko'ra

$$\bar{Э}^\alpha \cdot \left(T \cdot \bar{Э}_\beta\right) = \bar{Э}^\alpha \cdot \lambda_\beta \bar{Э}_\beta = \lambda_\beta \delta^{\alpha\cdot}_{\cdot\beta}$$

tenglik o'rinli.

Demak, T tenzorning aralash komponentalari

$$T^\alpha_\beta = \lambda_\beta \delta^\alpha_\beta \tag{2.11.1}$$

qiymatlarni qabul qiladi. Ushbu tenzorning matrisasi esa diagonal ko'rinishga ega bo'ladi:

$$A_2 = \left\| T^{\alpha}_{\beta} \right\| = \begin{Vmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{Vmatrix} \qquad (2.11.2)$$

Shunga o'xshash, $\bar{э}_\alpha = \bar{a}'_\alpha$ deb qabul qilsak, $T^{\beta}_{\alpha} = \lambda_\beta \delta_\alpha^{\ \beta}$ tenglik o'rinli bo'ladi va A_3 matrisa diagonal ko'rinishni oladi:

$$A_3 = \left\| T^{\beta}_{\alpha} \right\| = \begin{Vmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{Vmatrix} \qquad (2.11.3)$$

Mazkur (2.11.2) va (2.11.3) matrisalar tenzorning kanonik ko'rinishi deb ataladi. Shu matrisalarning noldan farqli λ_β komponentalari esa, tenzorning bosh qiymatlari (komponentalari) deb ataladi.

12 §. Tenzorning asosiy invariantlari.

2-rang tenzorning komponentalaridan tuzilgan determinantni yoyib yozilsa, (2.9.6) tenglama ushbu

$$\left| \lambda \delta^{\alpha\cdot}_{\cdot\beta} - T^{\alpha\cdot}_{\cdot\beta} \right| = I_0 \lambda^3 - I_1 \lambda^2 + I_2 \lambda - I_3 = 0 \qquad (2.12.1)$$

ko'rinishni oladi. Bu tenglamaning koeffitsientlari quyidagi tengliklar bilan aniqlanadi:

$$I_0 = 1\ ,$$

$$I_1 = \frac{1}{3}\left(\delta^{\gamma\cdot}_{\cdot\omega} T^{\omega\cdot}_{\cdot\gamma} + \delta^{\beta\cdot}_{\cdot\tau} T^{\tau\cdot}_{\cdot\beta} + \delta^{\alpha\cdot}_{\cdot\sigma} T^{\sigma\cdot}_{\cdot\alpha} \right) = \frac{1}{3}\left(T^{\omega}_{\omega} + T^{\tau}_{\tau} + T^{\sigma}_{\sigma} \right) = T^{\sigma}_{\sigma} = J_1 \qquad (2.12.2)$$

$$I_2 = \frac{1}{2}\left(J_1^2 - J_2 \right)$$

$$I_3 = \left| T^{\sigma}_{m} \right| = \frac{1}{6}\left(J_1^3 - 3 J_1 J_2 + 2 J_3 \right)$$

bu yerda $J_i \quad \left(i = \overline{1,3} \right)$ - tenzorning o'ramlari

$$J_0 = 1, \quad J_1 = T^{\sigma}_{\sigma}\,, \quad J_2 = T^{\alpha}_{\sigma} T^{\sigma}_{\alpha}\,, \quad J_3 = T^{\alpha}_{\sigma} T^{\sigma}_{\tau} T^{\tau}_{\alpha}$$

Demak, I_a koeffitsientlari $\left(\alpha = \overline{1,3} \right)$ ham invariant miqdorlar bo'lib, ular bosh invariantlar deb ataladi. Endi (2.12.1) tenglamaning koeffisientlari va ildizlari orasidagi munosabatlarni hisobga olinsa, bosh invariantlarning bosh komponentalari orqali ifodasi kelib chiqadi:

$$I_1 = \lambda_1 + \lambda_2 + \lambda_3, \quad I_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3, \quad I_3 = \lambda_1\lambda_2\lambda_3 \qquad (2.12.3)$$

13 §. Bazis vektorni koordinatalar bo'yicha differensiallash. Kristofell belgilari va ularning xossalari.

Bir nuqtadan ikkinchi nuqtaga o'tganda $\bar{\ni}_\alpha = \bar{\ni}_\alpha\left(x^1, x^2, x^3\right)$ va $\bar{\ni}^\alpha = \bar{\ni}^\alpha\left(x^1, x^2, x^3\right)$ $(\alpha = 1,\ 2,\ 3)$ bazis vektorlarning o'zgarish tezligi koordinata bo'yicha olingan hosila bilan ta'riflanadi. Hosila olish natijasi yana vektor miqdorni beradi:

$$\bar{\ni}_\alpha = \frac{\partial \bar{r}}{\partial x^\alpha} = \frac{\partial y^\gamma}{\partial x^\alpha}\bar{k}_\gamma \;\Rightarrow\; \frac{\partial \bar{\ni}_\alpha}{\partial x^\beta} = \frac{\partial^2 \bar{r}}{\partial x^\beta \partial x^\alpha} = \frac{\partial^2 y^\gamma}{\partial x^\alpha \partial x^\beta}\bar{k}_\gamma \, .$$

Demak, bazis vektorlarning hosilasini kovariant va kontravariant bazislarga yoyish mumkin:

$$\frac{\partial \bar{\ni}_\alpha}{\partial x^\beta} = \Gamma^{\sigma}_{\cdot\alpha\beta}\,\bar{\ni}_\sigma = \Gamma_{\tau\alpha\beta}\bar{\ni}^\tau \qquad (2.13.1)$$

Ta'rif: (2.13.1) yoyilmaning $\Gamma_{\tau\alpha\beta}$ va $\Gamma^{\sigma}_{\cdot\alpha\beta}$ koeffitsientlari birinchi xil va ikkinchi xil Kristoffel belgilari deb ataladi.

Kristoffel belgilari quyidagi xossalarga ega:

1) (2.13.1) tenglikni $\bar{\ni}_\gamma\left(\bar{\ni}^\gamma\right)$ ga hadma-had skalyar ko'paytiramiz:

$$\bar{\ni}_\gamma \cdot \frac{\partial \bar{\ni}_\alpha}{\partial x^\beta} = \Gamma_{\tau\alpha\beta}\bar{\ni}^\tau \cdot \bar{\ni}_\gamma$$

$$\bar{\ni}^\gamma \cdot \frac{\partial \bar{\ni}_\alpha}{\partial x^\beta} = \Gamma^{\sigma}_{\cdot\alpha\beta}\bar{\ni}_\sigma \cdot \bar{\ni}^\gamma$$

Endi o'zarolik munosabatlarini $\left(\left(\bar{\ni}^\tau \cdot \bar{\ni}_\gamma\right) = \delta^\tau_\gamma, \quad \left(\bar{\ni}_\sigma \cdot \bar{\ni}^\gamma\right) = \delta^\gamma_\sigma\right)$

eslasak, Kristoffel belgilari

$$\Gamma_{\gamma\alpha\beta} = \bar{\ni}_\gamma \cdot \frac{\partial \bar{\ni}_\alpha}{\partial x^\beta}, \; \Gamma^{\sigma}_{\cdot\alpha\beta} = \bar{\ni}^\sigma_{\cdot} \cdot \frac{\partial \bar{\ni}^\alpha}{\partial x^\beta} \qquad (2.13.2)$$

tengliklar bilan aniqlanadi.

2) Bazislarning vektorlarning ta’rifiga ko’ra:

$$\mathit{\Gamma}_{\gamma\alpha\beta} = \overline{Э}_{\gamma} \cdot \frac{\partial^2 \bar{r}}{\partial x^{\alpha} \partial x^{\beta}} \quad , \qquad \mathit{\Gamma}^{\gamma}_{\cdot\alpha\beta} = \frac{\partial^2 \bar{r}}{\partial x^{\alpha} \partial x^{\beta}} \cdot \overline{Э}^{\gamma}$$

tengliklar o’rinlidir.

Bu yerdan

$$\Gamma_{\gamma\alpha\beta} = \Gamma_{\gamma\beta\alpha} \quad , \qquad \Gamma^{\gamma}_{\cdot\alpha\beta} = \Gamma^{\gamma}_{\cdot\beta\alpha} \tag{2.13.3}$$

ekanligi, ya’ni Kristoffel belgilari oxirgi ikkita erkin indekslari bo’yicha simmetrik ekanligi kelib chiqadi.

Bazislarning yoyilmasidan foydalanib, ushbu:

$$\begin{aligned} \mathit{\Gamma}_{\gamma\alpha\beta} &= g_{\gamma\omega} \overline{Э}^{\omega} \cdot \frac{\partial \overline{Э}_{\alpha}}{\partial x^{\beta}} = g_{\gamma\omega} \mathit{\Gamma}^{\omega}_{\cdot\alpha\beta} \\ \mathit{\Gamma}^{\gamma}_{\cdot\alpha\beta} &= g^{\gamma\omega} \overline{Э}_{\omega} \cdot \frac{\partial \overline{Э}_{\alpha}}{\partial x^{\beta}} = g^{\gamma\omega} \mathit{\Gamma}_{\omega\alpha\beta} \end{aligned} \tag{2.13.4}$$

munosabatlarni olamiz. Bu tengliklardan Kristoffel belgilari uchun ham indekslarni ko’tarish va tushirish umumiy qoidalari o’rinli ekanligini kelib chiqadi.

3) Kristoffel belgilarini metrik tenzor komponentalari orqali ifodalash mumkin. Buning uchun

$$g_{\gamma\alpha} = \overline{Э}_{\gamma} \cdot \overline{Э}_{\alpha}$$

tenglikdan x^{β} bo’yicha hosila olamiz:

$$\overline{Э}_{\gamma} \cdot \frac{\partial \overline{Э}_{\alpha}}{\partial x^{\beta}} + \overline{Э}_{\alpha} \cdot \frac{\partial \overline{Э}_{\gamma}}{\partial x^{\beta}} = \frac{\partial g_{\gamma\alpha}}{\partial x^{\beta}}$$

Bu tenglik, (14.2) ni hisobga olib, quyidagicha yoziladi:

$$\Gamma_{\gamma\alpha\beta} + \Gamma_{\alpha\gamma\beta} = \frac{\partial g_{\gamma\alpha}}{\partial x^{\beta}} \tag{2.13.5}$$

Bu yerda α, β, γ indekslarni siklik almashtirsak, yani $g_{\alpha\beta}$ dan x^{γ} bo’yicha hosila olsak, (14.5) ga o’xshash

$$\Gamma_{\alpha\beta\gamma} + \Gamma_{\beta\alpha\gamma} = \frac{\partial g_{\alpha\beta}}{\partial x^{\gamma}} \quad (\gamma \rightleftarrows \beta) \tag{2.13.6}$$

$$\Gamma_{\beta\gamma\alpha} + \Gamma_{\gamma\beta\alpha} = \frac{\partial g_{\beta\gamma}}{\partial x^{\alpha}} \qquad (\beta \rightleftarrows \alpha) \tag{2.13.7}$$

munosabatlarni olamiz.

Endi (2.13.6) ga (2.13.7) ni qo'shib, natijadan (2.13.5) ni ayirsak va (2.13.3) ni hisobga olsak, birinchi tur Kristoffel belgilari uchun

$$\Gamma_{\beta\gamma\alpha} = \frac{1}{2}\left(\frac{\partial g_{\beta\gamma}}{\partial x^{\alpha}} + \frac{\partial g_{\beta\alpha}}{\partial x^{\gamma}} - \frac{\partial g_{\gamma\alpha}}{\partial x^{\beta}} \right) \tag{2.13.8}$$

formulani topamiz. Bu yerdan (14.7) yordamida ikkinchi tur Kristoffel belgilari uchun

$$\Gamma^{\beta}_{\cdot\gamma\alpha} = \frac{1}{2} g^{\beta\omega} \cdot \left(\frac{\partial g_{\omega\gamma}}{\partial x^{\alpha}} + \frac{\partial g_{\omega\alpha}}{\partial x^{\gamma}} + \frac{\partial g_{\gamma\alpha}}{\partial x^{\omega}} \right) \tag{2.13.9}$$

formula kelib chiqadi.

5)Kontravariant koordinata bazisi vektorlaridan koordinatalar bo'yicha hosila ushbu yoyilmalar ko'rinishida yoziladi:

$$\frac{\partial \overline{Э}^{\alpha}}{\partial x^{\beta}} = G^{\alpha}_{\cdot\beta\sigma} \overline{Э}^{\sigma} = G^{\alpha\cdot\tau}_{\beta} \overline{Э}_{\tau} \tag{2.13.10}$$

Bu yerdan yoyilmalarning koeffitsientlari

$$G^{\alpha}_{\cdot\beta\gamma} = \frac{\partial \overline{Э}^{\alpha}}{\partial x^{\beta}} \cdot \overline{Э}_{\gamma} \ , \qquad G^{\alpha\cdot\gamma}_{\beta} = \overline{Э}^{\gamma} \cdot \frac{\partial \overline{Э}^{\alpha}}{\partial x^{\beta}} \tag{2.13.11}$$

formulalar bilan aniqlanishi osonlik bilan topiladi. Bu koeffitsientlarni Kristoffel belgilari orqali ham ifodalash mumkin:

$$G^{\alpha}_{\cdot\beta\gamma} = \frac{\partial \overline{Э}^{\alpha}}{\partial x^{\beta}} \cdot \overline{Э}_{\gamma} = \frac{\partial}{\partial x^{\beta}}\left(\overline{Э}^{\alpha} \cdot \overline{Э}_{\gamma} \right) - \overline{Э}^{\alpha} \cdot \frac{\partial \overline{Э}_{\gamma}}{\partial x^{\beta}} = -\Gamma^{\alpha}_{\cdot\gamma\beta} = -\Gamma^{\alpha}_{\cdot\beta\gamma} \tag{2.13.12}$$

$$G^{\alpha\cdot\gamma}_{\beta} = \frac{\partial \overline{Э}^{\alpha}}{\partial x^{\beta}} \cdot \overline{Э}^{\gamma} = \frac{\partial \overline{Э}^{\alpha}}{\partial x^{\beta}} \cdot \overline{Э}_{\sigma} g^{\sigma\gamma} = -\Gamma^{\alpha}_{\cdot\beta\sigma} g^{\sigma\gamma} \tag{2.13.13}$$

Demak, (2.13.10) ning o'rniga

$$\frac{\partial \overline{Э}^{\alpha}}{\partial x^{\beta}} = -\Gamma^{\alpha}_{\cdot\beta\gamma} \overline{Э}^{\gamma} = -\Gamma^{\alpha}_{\cdot\beta\sigma} g^{\sigma\gamma} \overline{Э}_{\gamma} \tag{2.13.14}$$

formulalarni yozsa bo'ladi.

Shunday qilib, kovariant va kontravariant bazis vektorlarning fazoning ixtiyoriy nuqtasi atrofidagi o'zgarishi (2.13.1) va (2.13.14) formulalar bilan beriladi.

6)Aralash ko'paytmaning koordinatalar bo'yicha hosilasini ko'paytmadan hosila olish qoidasiga ko'ra (2.13.1) yordamida hisoblaymiz:

$$\frac{\partial\sqrt{g}}{\partial x^{\beta}}=\left(\frac{\partial\overline{Э}_1}{\partial x^{\beta}}\,\overline{Э}_2\overline{Э}_3\right)+\left(\overline{Э}_1\,\frac{\partial\overline{Э}_2}{\partial x^{\beta}}\,\overline{Э}_3\right)+\left(\overline{Э}_1\overline{Э}_2\,\frac{\partial\overline{Э}_3}{\partial x^{\beta}}\right)$$

Aralash ko'paytma uchun $(\overline{Э}_i\overline{Э}_i\overline{Э}_j=0)$ tenglik o'rinli ekanligini va (2.13.1) ni e'tiborga olsak, bu formula

$$\begin{aligned}\frac{\partial\sqrt{g}}{\partial x^{\alpha}}&=\Gamma^{\sigma}_{\cdot 1\alpha}\left(\overline{Э}_{\sigma}\overline{Э}_2\overline{Э}_3\right)+\Gamma^{\sigma}_{\cdot 2\alpha}\left(\overline{Э}_1\overline{Э}_{\sigma}\overline{Э}_3\right)+\Gamma^{\sigma}_{\cdot 3\alpha}\left(\overline{Э}_1\overline{Э}_2\overline{Э}_{\sigma}\right)=\\&=\left(\Gamma^{1}_{\cdot 1\alpha}+\Gamma^{2}_{\cdot 2\alpha}+\Gamma^{3}_{\cdot 3\alpha}\right)\left(\overline{Э}_1\overline{Э}_2\overline{Э}_3\right)=\Gamma^{\sigma}_{\cdot\sigma\alpha}\sqrt{g}\end{aligned}$$

ko'rinishini oladi, yani

$$\Gamma^{\sigma}_{\cdot\sigma\alpha}=\frac{1}{\sqrt{g}}\;\frac{\partial\sqrt{g}}{\partial x^{\alpha}}\qquad(2.13.15)$$

formula hosil bo'ladi.

Kristoffel belgilari uchun quyidagi almashtirish formulalari o'rinli.

Bazis vektorlarini almashtirish formulalariga va (2.13.2) tengliklarga asoslanib, quyidagilarni olamiz:

$$\begin{aligned}\hat{\Gamma}^{\lambda}_{\cdot\mu\nu}&\equiv\hat{\overline{Э}}^{\lambda}\cdot\frac{\partial\hat{\overline{Э}}_{\mu}}{\partial\xi^{\nu}}=\left(\overline{Э}^{\alpha}\,\frac{\partial\xi^{\lambda}}{\partial x^{\alpha}}\right)\cdot\frac{\partial}{\partial\xi^{\nu}}\left(\overline{Э}_{\beta}\,\frac{\partial x^{\beta}}{\partial\xi^{\mu}}\right)=\\&=\frac{\partial\xi^{\lambda}}{\partial x^{\alpha}}\left(\overline{Э}^{\alpha}\cdot\overline{Э}_{\beta}\,\frac{\partial^2x^{\beta}}{\partial\xi^{\mu}\partial\xi^{\nu}}+\overline{Э}^{\alpha}\cdot\frac{\partial\overline{Э}_{\beta}}{\partial x^{\gamma}}\frac{\partial x^{\gamma}}{\partial\xi^{\nu}}\frac{\partial x^{\beta}}{\partial\xi^{\mu}}\right)\end{aligned}$$

$$\begin{aligned}\hat{\Gamma}_{\lambda\mu\nu}&\equiv\hat{\overline{Э}}_{\lambda}\cdot\frac{\partial\hat{\overline{Э}}_{\mu}}{\partial\xi^{\nu}}=\left(\overline{Э}_{\alpha}\,\frac{\partial x^{\alpha}}{\partial\xi^{\lambda}}\right)\cdot\frac{\partial}{\partial\xi^{\nu}}\left(\overline{Э}_{\beta}\,\frac{\partial x^{\beta}}{\partial\xi^{\mu}}\right)=\\&=\frac{\partial x^{\alpha}}{\partial\xi^{\lambda}}\left(\overline{Э}_{\alpha}\cdot\overline{Э}_{\beta}\frac{\partial^2x^{\beta}}{\partial\xi^{\mu}\partial\xi^{\nu}}+\overline{Э}_{\alpha}\cdot\frac{\partial\overline{Э}_{\beta}}{\partial x^{\gamma}}\frac{\partial x^{\gamma}}{\partial\xi^{\nu}}\frac{\partial x^{\beta}}{\partial\xi^{\mu}}\right)\end{aligned}$$

yoki

$$\hat{\Gamma}^{\lambda}_{\cdot\mu\nu} = \frac{\partial\xi^{\lambda}}{\partial x^{\alpha}}\left(\frac{\partial^2 x^{\alpha}}{\partial\xi^{\mu}\partial\xi^{\nu}} + \Gamma^{\alpha}_{\cdot\beta\gamma}\frac{\partial x^{\beta}}{\partial\xi^{\mu}}\frac{\partial x^{\gamma}}{\partial\xi^{\nu}}\right) \tag{2.13.16}$$

$$\hat{\Gamma}_{\lambda\mu\nu} = \frac{\partial x^{\alpha}}{\partial\xi^{\lambda}}\left(g_{\alpha\beta}\frac{\partial^2 x^{\beta}}{\partial\xi^{\mu}\partial\xi^{\nu}} + \Gamma_{\alpha\beta\gamma}\frac{\partial x^{\beta}}{\partial\xi^{\mu}}\frac{\partial x^{\gamma}}{\partial\xi^{\nu}}\right) \tag{2.13.17}$$

Bu formulalar Kristoffel belgilari uchun almashtirish qonunlarini beradi. Ular tenzor komponentalarini almashtirish qoidasidan farq qiladi va Kristoffel belgilari, umuman olganda, tenzor komponentalari bo’lmaydi. Lekin $x^{\alpha} = x^{\alpha}\left(\xi^1,\, \xi^2,\, \xi^3\right)$ almashtirish formulalari chiziqli bo’lsa, (2.13.16) va (2.13.17) lardagi ikkinchi tartibli hosilalar nolga teng bo’ladi va ular tenzor komponentalarini almashtirish qonunlariga aylanadi. Demak, koordinatalarni affin almashtirishda Kristoffel belgilari tenzor komponentalari bo’ladi.

Kristoffel belgilarining ortogonal koordinatalar sistemasidagi ko’rinishi.

Agar x^{α} koordinatalar sistemasi ortogonal bo’lsa, metrik tenzor komponentalari uchun

$$g_{\alpha\beta} = g^{\alpha\beta} = 0 \quad \left(\alpha \neq \beta\right), \qquad g^{\alpha\alpha} = g^{-1}_{\alpha\alpha}$$

tengliklar o’rinli bo’adi. Bundan foydalanib, Kristoffel belgilarining indekslari turli bo’lsa quyidagicha;

$$\Gamma_{\beta\gamma\alpha} = \frac{1}{2}\left(\frac{\partial g_{\beta\gamma}}{\partial x^{\alpha}} + \frac{\partial g_{\beta\alpha}}{\partial x^{\gamma}} - \frac{\partial g_{\gamma\alpha}}{\partial x^{\beta}}\right) = 0 \qquad \left(\alpha \neq \beta \neq \gamma \neq \alpha\right),$$

ikkita oxirgi indekslari bir-biriga teng bo’lsa esa;

$$\Gamma_{\beta\alpha\alpha} = \frac{1}{2}\left(\frac{\partial g_{\beta\alpha}}{\partial x^{\alpha}} + \frac{\partial g_{\beta\alpha}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\alpha}}{\partial x^{\beta}}\right) = -\frac{1}{2}\,\frac{\partial g_{\alpha\alpha}}{\partial x^{\beta}} \qquad \left(\alpha \neq \beta\right),$$

birinchi indeksi ikkinchisiga yoki uchinchisiga teng bo’lsa

$$\Gamma_{\beta\beta\alpha} = \Gamma_{\beta\alpha\beta} = \frac{1}{2}\left(\frac{\partial g_{\beta\beta}}{\partial x^{\alpha}} + \frac{\partial g_{\beta\alpha}}{\partial x^{\beta}} - \frac{\partial g_{\beta\alpha}}{\partial x^{\beta}}\right) = \frac{1}{2}\,\frac{\partial g_{\beta\beta}}{\partial x^{\alpha}}\,,$$

va nihoyat indekslarning hammasi bir-biriga teng bo’lsa

$$\Gamma_{\beta\beta\beta} = \frac{1}{2}\left(\frac{\partial g_{\beta\beta}}{\partial x^{\beta}} + \frac{\partial g_{\beta\beta}}{\partial x^{\beta}} - \frac{\partial g_{\beta\beta}}{\partial x^{\beta}}\right) = \frac{1}{2}\,\frac{\partial g_{\beta\beta}}{\partial x^{\beta}}$$

formulalarga ega bo’lamiz.

Demak, fazoning metrikasi berilgan bo'lsa, Kristoffel belgilari quyidagi formulalar yordamida hisoblanadi:

$$\Gamma_{\beta\gamma\alpha} = 0 \qquad (\beta \neq \gamma \neq \alpha \neq \beta),$$
$$\Gamma_{\beta\alpha\alpha} = -\frac{1}{2}\frac{\partial g_{\alpha\alpha}}{\partial x^{\beta}}, \tag{2.14.18}$$
$$\Gamma_{\beta\beta\alpha} = \Gamma_{\beta\alpha\beta} = \frac{1}{2}\frac{\partial g_{\beta\beta}}{\partial x^{\alpha}}, \qquad \Gamma_{\beta\beta\beta} = \frac{1}{2}\frac{\partial g_{\beta\beta}}{\partial x^{\beta}}$$

Turli xildagi Kristoffel belgilari orasida quyidagi munosabatlar o'rinli bo'lganligi tufayli

$$\Gamma^{\beta}_{\cdot\gamma\alpha} = g^{\beta\omega}\,\Gamma_{\omega\gamma\alpha} = g^{\beta\beta}\,\Gamma_{\beta\gamma\alpha} = \frac{1}{g_{\beta\beta}}\,\Gamma_{\beta\gamma\alpha}$$

ikkinchi xil Kristoffel belgilari uchun (2.14.18) ga ko'ra ushbu

$$\Gamma^{\beta}_{\cdot\gamma\alpha} = 0 \qquad (\alpha \neq \beta \neq \gamma \neq \alpha)$$
$$\Gamma^{\beta}_{\cdot\alpha\alpha} = -\frac{1}{2g_{\beta\beta}}\frac{\partial g_{\alpha\alpha}}{\partial x^{\beta}} = -\frac{Э_{\alpha}}{Э^{2}_{\beta}}\frac{\partial Э_{\alpha}}{\partial x^{\beta}} \qquad (\beta \neq \alpha) \tag{2.14.19}$$
$$\Gamma^{\beta}_{\cdot\alpha\beta} = \frac{1}{2g_{\beta\beta}}\frac{\partial g_{\beta\beta}}{\partial x^{\alpha}} = \frac{1}{Э^{\beta}}\frac{\partial Э_{\beta}}{\partial x^{\alpha}}$$

formulalarni olamiz. (2.14.18) va (2.14.19) larda yig'indi olish amali bajarilmaydi.

14 §. Skalyar, vektor va ikkinchi rang tenzorni koordinatalar bo'yicha differensiallash.

Ta'rif. *Har bir nuqtasida vektor miqdor aniqlangan soha vektor maydon* deb ataladi.

Demak, bu maydonda $\bar{a} = \bar{a}\left(x^{1},\, x^{2},\, x^{3}\right)$ bo'ladi.

Vektor maydonda o'tkazilgan biror chiziqning har bir nuqtasida maydon vektori va chiziq urinmasi bir xil yo'nalishga ega bo'lsa, ushbu chiziq vektor chizig'i deb ataladi. Demak, vektor chizig'i bo'ylab elementar ko'chish vektori $d\,\bar{r}$ maydon vektori $\bar{a}$ ga parallel bo'ladi:

$$d\,\bar{r} = \bar{a}\,d\lambda \tag{2.14.1}$$

Bu yerda $d\lambda$ - proporsionallik koeffitsienti. Ushbu (2,14.1) vektor tenglama quyidagi skalyar tenglamalarga ekvivalent:

$$dx^1 = a^1 d\lambda, \quad dx^2 = a^2 d\lambda, \quad dx^3 = a^3 d\lambda$$

yoki

$$\frac{dx^1}{a^1\left(x^1, x^2, x^3\right)} = \frac{dx^2}{a^2\left(x^1, x^2, x^3\right)} = \frac{dx^3}{a^3\left(x^1, x^2, x^3\right)} \qquad (2.14.2)$$

Differensial tenglamalar nazariyasidan ma'lumki, $a^\alpha = a^\alpha\left(x^1, x^2, x^3\right)$ funksiyalar uzluksiz bo'lib, uzluksiz xususiy hosilalarga ega bo'lsa, fazoning har bir nuqtasidan yagona vektor chizig'i o'tkazish mumkin. Bu funksiyalarning barchasi nolga yoki cheksizga teng bo'lgan nuqtalar (2.14.2) sistemaning maxsus nuqtalari deb ataladi. Maxsus nuqtalarda sistema yechimining mavjudlik va yagonalik teoremasining shartlari bajarilmaydi va bunday nuqtalar orqali bir nechta vektor chiziqlari o'tishi mumkin.

Vektor maydonida biror vektor chizig'i bo'lmagan L chiziqning har bir nuqtasidan vektor chiziqlari o'tkazish natijasida hosil qilingan sirt $\sum$ vektor sirti deb ataladi. Bu sirtning har bir nuqtasida maydon vektori urinma tekislikda yotadi. L konturi yopiq bo'lsa $\sum$ naysimon sirt deb ataladi va maydonning $\sum$ bilan chegaralangan qismi vektor naychasi deb ataladi.

Vektor maydonining $M\left(x^\alpha\right)$ nuqtasidan $M'\left(x^\alpha + dx^\alpha\right)$ nuqtasiga o'tganda $\bar{a}\left(x^1, x^2, x^3\right)$ vektorning orttirmasi

$$d\bar{a} = dx^\alpha \frac{\partial \bar{a}}{\partial x^\alpha} = dx_\alpha \frac{\partial \bar{a}}{\partial x_\alpha}$$

ga teng bo'ladi. Bu yerda $dx^\alpha\left(dx_\alpha\right)$ ixtiyoriy elementar ko'chish vektori dr ning komponentalari, $d\bar{a}$ esa vektor. Shu tufayli tenzorlarni bo'lish teoremasiga asosan $\ni^\alpha \dfrac{\partial \bar{a}}{\partial x^\alpha}$ obekt 2-rang tenzor ekanligi kelib chiqadi. Shu tenzor vektor gradienti deb ataladi va quyidagicha belgilanadi:

$$\nabla\bar{a} \equiv \frac{\nabla\bar{a}}{\partial\bar{r}} = \bar{Э}^{\alpha}\,\frac{\partial\bar{a}}{\partial x^{\alpha}} = \bar{Э}_{\alpha}\,\frac{\partial\bar{a}}{\partial x_{\alpha}} \tag{2.14.3}$$

Ko'paytmaning hosilasini hisoblash qoidasiga binoan

$$\begin{gathered}\frac{\partial\bar{a}}{\partial x^{\alpha}} = \frac{\partial}{\partial x^{\alpha}}\left(a^{\sigma}\bar{Э}_{\sigma}\right) = \frac{\partial a^{\sigma}}{\partial x^{\alpha}}\bar{Э}_{\sigma} + a^{\sigma}\frac{\partial\bar{Э}_{\sigma}}{\partial x^{\alpha}} = \\ = \frac{\partial a^{\sigma}}{\partial x^{\alpha}}\bar{Э}_{\sigma} + a^{\sigma}\Gamma^{\beta}_{\sigma\alpha}\bar{Э}_{\beta} = \left(\frac{\partial a^{\beta}}{\partial x^{\alpha}} + a^{\sigma}\Gamma^{\beta}_{\sigma\alpha}\right)\bar{Э}_{\beta} \equiv \nabla_{\alpha}a^{\beta}\bar{Э}_{\beta}\end{gathered} \tag{2.14.4}$$

munosabatlar o'rinli. Bu yerda

$$\nabla_{\alpha}a^{\beta} = \frac{\partial a^{\beta}}{\partial x^{\alpha}} + a^{\sigma}\Gamma^{\beta}_{\cdot\sigma\alpha} \tag{2.14.5}$$

ifodalar, 2-rang tenzorning komponentalari bo'lib, $\bar{a}$ vektorning kontravariant komponentalarining kovariant hosilasi deb ataladi.

Demak, vektor gradienti ushbu tenglik bilan aniqlanadi:

$$\nabla\bar{a} = \nabla_{\alpha}a^{\beta}\bar{Э}_{\beta}\bar{Э}^{\alpha} \tag{2.14.6}$$

Kristoffel belgilari tenzor komponentalari bo'lmagani tufayli, vektor komponentalarining koordinata bo'yicha odatdagi hosilasi, umuman olganda, tenzor komponentalari bo'lmaydi. Dekart koordinata sistemasida esa Kristoffel belgilarining barchasi nolga tengligidan kovariant hosila odatdagi hosilaga teng ekanligi kelib chiqadi:

$$\nabla_{\alpha}a^{\beta} = \frac{\partial a^{\beta}}{\partial x^{\alpha}} \tag{2.14.7}$$

Agar vektorning kovariant komponentalari orqali yoyilmasini olsak,

$$\frac{\partial\bar{a}}{\partial x^{\alpha}} = \frac{\partial}{\partial x^{\alpha}}\left(a_{\sigma}\bar{Э}^{\sigma}\right) = \nabla_{\alpha}a_{\beta}\bar{Э}^{\beta}\ , \qquad \nabla\bar{a} = \nabla_{\alpha}a_{\beta}\bar{Э}^{\beta}\bar{Э}^{\alpha} \tag{2.14.8}$$

munosabatlarni topish mumkin. Bu yerda

$$\nabla_{\alpha}a_{\beta} = \frac{\partial a_{\beta}}{\partial x^{\alpha}} - a_{\sigma}\Gamma^{\sigma}_{\cdot\beta\alpha} \tag{2.14.9}$$

ifodalar vektorning kovariant komponentalarining kovariant hosilalari deb ataladi.

Shunga o'xshash, (2.14.7) va (2.14.8) dan foydalanib quyidagi tenglikni

$$\left(\frac{\partial\varphi}{\partial x_\beta}-\frac{\partial\varphi}{\partial x^\alpha}g^{\alpha\beta}\right)\overline{Э}_\beta=0\Rightarrow\frac{\partial\varphi}{\partial x_\beta}=\frac{\partial\varphi}{\partial x^\alpha}g^{\alpha\beta}\Leftrightarrow\nabla^\beta\varphi=g^{\alpha\beta}\nabla_\alpha\varphi \tag{2.14.9}$$

etiborga olib, vektor komponentalaridan olingan kontravariant hosila tushunchasini ham kiritish mumkin:

$$\frac{\partial\overline{a}}{\partial x_\beta}=\frac{\partial\overline{a}}{\partial x^\alpha}\frac{\partial x^\alpha}{\partial x_\beta}=g^{\alpha\beta}\frac{\partial\overline{a}}{\partial x^\alpha}=\nabla^\beta a^\sigma\overline{Э}_\sigma=\nabla^\beta a_\tau\overline{Э}^\tau \tag{2.14.10}$$

Bu yerda

$$\begin{aligned}\nabla^\beta a^\sigma&=g^{\alpha\beta}\nabla_\alpha a^\sigma\\ \nabla^\beta a_\sigma&=g^{\alpha\beta}\nabla_\alpha a_\sigma\end{aligned} \tag{2.14.11}$$

ifodalar $\overline{a}$ vektorning kontravariant va kovariant komponentalarining kontravariant hosilasi deb ataladi.

Vektor gradienti deb atalgan 2-rang tenzor quyidagi ko'rinishlarga ega:

$$\begin{aligned}\nabla\overline{a}&=\nabla_\alpha a_\beta\overline{Э}^\alpha\overline{Э}^\beta=\nabla_\alpha a^\beta\overline{Э}^\alpha\overline{Э}_\beta=\\ &=\nabla^\alpha a_\beta\overline{Э}_\alpha\overline{Э}^\beta=\nabla^\alpha a^\beta\overline{Э}_\alpha\overline{Э}_\beta\end{aligned} \tag{2.14.12}$$

Maydonning barcha nuqtalarida vektorlar bir xil qiymatga ega bo'lsa, vektor maydoni bir jinsli deb ataladi. Bir jinsli vektor maydonida vektor gradienti nolga teng.

Demak, vektor maydonining bir jinsli bo'lish sharti

$$\nabla_\alpha a_\beta=0 \qquad (\alpha,\ \beta=1,\ 2,\ 3) \tag{2.14.13}$$

ko'rinishida yoziladi.

Vektor gradienti (2.14.12) ning birinchi invarianti vektor divergensiyasi deb ataladi va

$$div\,\overline{a}=\nabla_\alpha a^\alpha=\frac{\partial a^\alpha}{\partial x^\alpha}+a^\sigma\Gamma^{\alpha}_{\cdot\sigma\alpha} \tag{2.14.14}$$

ko'rinishida belgilanadi.

Misol. Dekart koordinatalar sistemasida

$$div\,\overline{a}=\frac{\partial a_1}{\partial y^1}+\frac{\partial a_2}{\partial y^2}+\frac{\partial a_3}{\partial y^3} \tag{2.15.15}$$

ekanligi ko'rsatilsin.

Har bir nuqtasida $div\,\overline{a}=0$ shart bajarilgan vektor maydoni solenoidal maydon deb ataladi.

Vektor maydonda komponentalari

$$\Omega^{\alpha}=e^{\alpha\beta\gamma}\nabla_{\beta}a_{\gamma} \tag{2.14.16}$$

tenglik bilan aniqlangan va vektor maydon uyurmasi deb ataluvchi vektor ham mavjud. Uyurma vektori odatda

$$\overline{\Omega}=rot\,\overline{a}$$

ko'rinishda belgilanadi. Uning kontravariant komponentalari (2.14.16) diskriminant tenzor komponentalarini hisoblash qoidasiga binoan ushbu

$$\Omega^{1}=\sqrt{g_1}\left(\nabla_2 a_3-\nabla_3 a_2\right)$$
$$\Omega^{2}=\sqrt{g_1}\left(\nabla_3 a_1-\nabla_1 a_3\right)$$
$$\Omega^{3}=\sqrt{g_1}\left(\nabla_1 a_2-\nabla_2 a_1\right)$$

ko'rinishda yoziladi. Bu yerda kovariant hosilala (2.14.9) formulaga ko'ra hisoblansa, Kristoffel belgilari qatnashgan hadlar qisqaradi va uyurma vektori komponentalari uchun ushbu

$$\begin{aligned}\Omega^{1}&=\sqrt{g_1}\left(\frac{\partial a_3}{\partial x^2}-\frac{\partial a_2}{\partial x^3}\right)\\ \Omega^{2}&=\sqrt{g_1}\left(\frac{\partial a_1}{\partial x^3}-\frac{\partial a_3}{\partial x^1}\right)\\ \Omega^{3}&=\sqrt{g_1}\left(\frac{\partial a_2}{\partial x^1}-\frac{\partial a_1}{\partial x^2}\right)\end{aligned} \tag{2.14.17}$$

formulalarga ega bolamiz.

Misol. Dekart koordinata sistemasida uyurma komponentalari ushbu ko'rinishda yozilishi ko'rsatilsin:

$$\Omega_1=\frac{\partial a_3}{\partial y^2}-\frac{\partial a_2}{\partial y^3}\,,\qquad \Omega_2=\frac{\partial a_1}{\partial y^3}-\frac{\partial a_3}{\partial y^1}\,,\qquad \Omega_3=\frac{\partial a_2}{\partial y^1}-\frac{\partial a_1}{\partial y^2} \tag{2.14.18}$$

Vektor maydonning har bir nuqtasida $rot\,\overline{a}=0$ shart bajarilsa, bu maydon uyurmasiz maydon deb ataladi. Maydon uyurmasiz bo'lishi uchun, (2.14.17) ga ko'ra

$$\frac{\partial a_3}{\partial x^2} = \frac{\partial a_2}{\partial x^3}\,, \qquad \frac{\partial a_1}{\partial x^3} = \frac{\partial a_3}{\partial x^1}\,, \qquad \frac{\partial a_2}{\partial x^1} = \frac{\partial a_1}{\partial x^2} \tag{2.14.19}$$

tenglik bajarilishi kerak.

Differensial tenglamalar nazariyasidan malumki, (2.14.19) chiziqli $a_\alpha dx^\alpha$ differensial formaning integrallanish shartini beradi. Shu tufayli $a_\alpha dx^\alpha = d\varphi$,yani $a_\alpha dx^\alpha$ to'la differensial bo'ladi.

Demak,

$$a_\alpha = \nabla_\alpha \varphi\,, \qquad \bar{a} = \nabla\varphi \tag{2.14.20}$$

deb yozish mumkin bo'ladi. Ikkinchi tomondan vektor maydoni potensialli bo'lsa, yani (2.14.20) o'rinli bo'lsa, (2.14.19) tengliklar aynan bajariladi. Bulardan uyurmasiz vektor maydoni potensialli bo'lishi va aksincha potensialli maydon uyurmasiz bo'lishi kelib chiqadi.

Potensialli vektor maydonida $\bar{a} = \nabla\varphi$ bo'lganligi tufayli $\bar{a}$ vektorning divergensiyasi

$$div\,\bar{a} = div\;grad\;\varphi = \nabla_\alpha \nabla^\alpha \varphi = \Delta\,\varphi \tag{2.14.21}$$

ko'rinishni oladi. Buyerda Δ - Laplas operatori. Demak, skalyardan olingan Laplas operatori invariantdir.

U dekart koordinatalar sistemasida

$$\nabla\varphi = \frac{\partial^2\varphi}{\left(\partial y^1\right)^2} + \frac{\partial^2\varphi}{\left(\partial y^2\right)^2} + \frac{\partial^2\varphi}{\left(\partial y^3\right)^2}$$

ko'rinishda yoziladi.

Skalyar maydonning biror $M\left(x^\alpha\right)$ nuqtasidan unga yaqin joylashgan $M'\left(x^\alpha + dx^\alpha\right)$ nuqtaga o'tilganda φ skalyar

$$d\varphi = dx^\alpha\;\frac{\partial\varphi}{\partial x^\alpha} \tag{2.14.22}$$

orttirma oladi. Tenglikning o'ng tomoni ikkita birinchi rang ob'yektlarning o'ramidir. Undan tashqari M' nuqta M nuqta atrofidagi ixtiyoriy nuqtalardan biri bo'lganligi tufayli, $dx^{\alpha}\overline{Э}_{\alpha}$ ixtiyoriy vektor bo'ladi. Shu sababli, tenzorlarni bo'lish teoremasidan $\overline{Э}^{\alpha}\dfrac{\partial\varphi}{\partial x^{\alpha}}$ ob'yektning vektor ekanligi kelib chiqadi. Bu ob'yekt skalyarning gradienti deb ataladi va

$$\nabla\varphi = grad\varphi \equiv \frac{\partial\varphi}{\partial\overline{r}} = \frac{\partial\varphi}{\partial x^{\alpha}}\overline{Э}^{\alpha} = \nabla_{\alpha}\varphi\,\overline{Э}^{\alpha} \qquad (2.14.23)$$

ko'rinishlarda belgilanadi.

Bu yerda $\dfrac{\partial\varphi}{\partial\overline{r}}$ ko'rinishdagi belgilashni quyidagicha izohlash mumkin:

$$\nabla\varphi = \frac{\partial\varphi}{\partial x^{\alpha}}\overline{Э}^{\alpha} = \frac{\partial\varphi}{\partial\overline{r}}\frac{\partial\overline{r}}{\partial x^{\alpha}}\overline{Э}^{\alpha} = \frac{\partial\varphi}{\partial\overline{r}}$$

Dekart koordinata sistemasida esa

$$\nabla\varphi = g\,rad\varphi = \frac{\partial\varphi}{\partial y^{\beta}}\overline{K}_{\beta} \qquad (2.14.24)$$

formula o'rinli. Buni (2.14.24) va (1.5.2) lardan foydalanib, oson isbot qilinadi:

$$\frac{\partial\varphi}{\partial x^{\alpha}}\overline{Э}^{\alpha} = \frac{\partial\varphi}{\partial y^{\beta}}\frac{\partial y^{\beta}}{\partial x^{\alpha}}\overline{Э}^{\alpha} = \frac{\partial\varphi}{\partial x^{\alpha}}\frac{\partial x^{\alpha}}{\partial y^{\beta}}\overline{K}_{\beta} = \frac{\partial\varphi}{\partial y^{\beta}}\overline{K}_{\beta}$$

Skalyar gradienti vektorining kovariant bazis vektorlari orqali yoyilmasi uchun

$$\nabla\varphi = \nabla^{\beta}\varphi\overline{Э}_{\beta} = \frac{\partial\varphi}{\partial x_{\beta}}\overline{Э}_{\beta} \qquad (2.14.25)$$

belgilashlar qabul qilingan. Ushbu formulalardan skalyar grandienti vektorining kovariant va kontravariant komponentalari orasidagi bog'lanishni olish mumkin:

$$\begin{gathered}\nabla\varphi = \frac{\partial\varphi}{\partial x^{\alpha}}\overline{Э}^{\alpha} = \frac{\partial\varphi}{\partial x_{\beta}}\overline{Э}_{\beta}\,, \qquad \frac{\partial\varphi}{\partial x_{\beta}}\overline{Э}_{\beta} = \frac{\partial\varphi}{\partial x_{\beta}}g_{\beta\gamma}\overline{Э}^{\gamma} \Rightarrow \\ \left(\frac{\partial\varphi}{\partial x^{\alpha}} - \frac{\partial\varphi}{\partial x_{\beta}}g_{\beta\alpha}\right)\overline{Э}^{\alpha} = 0 \;\Rightarrow\; \frac{\partial\varphi}{\partial x^{\alpha}} = \frac{\partial\varphi}{\partial x_{\beta}}g_{\alpha\beta} \Leftrightarrow \nabla_{\alpha}\varphi = g_{\alpha\beta}\nabla^{\beta}\varphi\end{gathered} \qquad (2.14.26)$$

Shunga o'xshash

$$\left(\frac{\partial\varphi}{\partial x_{\beta}} - \frac{\partial\varphi}{\partial x^{\alpha}}g^{\alpha\beta}\right)\overline{Э}_{\beta} = 0 \Rightarrow \frac{\partial\varphi}{\partial x_{\beta}} = \frac{\partial\varphi}{\partial x^{\alpha}}g^{\alpha\beta} \Leftrightarrow \nabla^{\beta}\varphi = g^{\alpha\beta}\nabla_{\alpha}\varphi \qquad (2.14.27)$$

munosabat olinadi. Ushbu (2.14.26) va (2.14.27) formulalar skalyar gradienti komponentalari uchun indekslarni ko'tarish (tushirish) qoidasini beradi.

Demak, skalyar gradienti komponentalari uchun ham indekslarni ko'tarish (tushirish) qoidalari o'rinli.

Invariant miqdor bo'lgan skalyar gradientining moduli uchun quyidagi munosabatlar o'rinli

$$\left|\frac{\partial\varphi}{\partial\bar{r}}\right|^2=\frac{\partial\varphi}{\partial x^\alpha}\,\frac{\partial\varphi}{\partial x_\alpha}=g^{\alpha\beta}\,\frac{\partial\varphi}{\partial x^\alpha}\,\frac{\partial\varphi}{\partial x^\beta}=g_{\sigma\tau}\,\frac{\partial\varphi}{\partial x_\sigma}\,\frac{\partial\varphi}{\partial x_\tau}$$

Skalyar gradientiga teng bo'lgan $\bar{a}=\nabla\varphi$ vektor potentsialli vektor, φ esa skalyar potentsial deb ataladi. Potentsialli vektor maydoni potentsial maydon deb ataladi. Mazkur maydonning har bir nuqtasida skalyar gradienti shu nuqtadan o'tuvchi $\varphi\left(x^1,\,x^2,\,x^3\right)=const$ sirtga o'tkazilgan normal bo'ylab yo'nalgan bo'ladi.

Haqiqatan, ushbu sirtning har bir nuqtasida

$$d\varphi=dx^\alpha\,\frac{\partial\varphi}{\partial x^\alpha}=d\,\bar{r}\cdot\nabla\varphi=0$$

tenglik o'rinli, yani qaralayotgan nuqtada $d\,\bar{r}\perp\nabla\varphi$ bo'ladi.

Potensial maydonida aniqlangan va tenglamasi $\bar{r}=\bar{r}(S)$ yoki $x^\alpha=x^\alpha(S)$ bo'lgan *L* chizig'i bo'ylab φ skalyar *S* o'zgaruvchining funksiyasi bo'ladi:

$$\varphi\left[x^1(S),\,x^2(S),\,x^3(S)\right]=\varphi(S)$$

Ushbu

$$\frac{\partial\varphi}{\partial S}=\frac{\partial\varphi}{\partial x^\alpha}\,\frac{dx^a}{dS}=\frac{\partial\varphi}{\partial\bar{r}}\cdot\frac{d\bar{r}}{dS}$$

hosila esa skalyarning *S* yo'nalishi bo'yicha hosilasi deb ataladi va $\varphi=\varphi(S)$ ning *L* chizig'i bo'ylab o'zgarish tezligini beradi.

Agar *S* deb *L* chiziqning yoy uzunligi belgilangan bo'lsa, $\dfrac{d\bar{r}}{dS}$ – *L* chizig'ining urinmasi bo'ylab *S* parametrning o'sish tomoniga yo'nalgan birlik vektordir. Demak, *S* yo'nalishi bo'ylab olingan hosila gradientning *L* chizig'i urinmasi yo'nalishi proeksiyasiga teng va maksimal tezlik $\frac{\partial \varphi}{\partial S}$ gradient yo'nalishida bo'ladi:

$$\frac{\partial \varphi}{\partial S} = \nabla \varphi \cdot \frac{d\bar{r}}{dS} = \left| \nabla \varphi \right| cos \left(\nabla \hat{\varphi, \; S} \right)$$

Har bir nuqtasida n -rang tenzor aniqlangan soha tenzor maydon deb ataladi. Demak, tenzor maydonda tenzor komponentalari va poliadalar qaralayotgan soha nuqtalarining funksiyasi bo'ladi:

$$\begin{aligned} &T\left(x^1, x^2, x^3\right) = T^{\alpha_1 \alpha_2 \ldots \alpha_n} \,\overline{Э}_{\alpha_1} \overline{Э}_{\alpha_2 \ldots} \overline{Э}_{\alpha_n} \\ &T^{\alpha_1 \alpha_2 \ldots \alpha_n} = T^{\alpha_1 \alpha_2 \ldots \alpha_n}\left(x^1, x^2, x^3\right), \overline{Э}_{\alpha_i} = \overline{Э}_{\alpha_i}\left(x^1, x^2, x^3\right) \end{aligned} \tag{2.14.28}$$

Avval 2-rang tenzor maydonini qaraymiz. Biror $M\left(x^\alpha\right)$ nuqtadan $M\left(x^\alpha + dx^\alpha\right)$ nuqtaga o'tganda T tenzor orttirmasi

$$dT = dx^\alpha \frac{\partial T}{\partial x^\alpha} = dx_\alpha \frac{\partial T}{\partial x_\alpha} \tag{2.14.29}$$

ifoda bilan aniqlanadi. Bu yerdan $\overline{Э}^\alpha \frac{\partial T}{\partial x^\alpha}$ ob'yekt 3-rang tenzor ekanligi kelib chiqadi. Uni tenzor gradienti deb ataladi va ushbu

$$\nabla T = \overline{Э}^\alpha \frac{\partial T}{\partial x^\alpha} = \overline{Э}_\alpha \frac{\partial T}{\partial x_\alpha} = \frac{\partial T}{\partial \bar{r}} \tag{2.14.30}$$

ko'rinishda belgilanadi.

Bu yerda tenzorning koordinata bo'yicha hosilasini, odatdagidek, ko'paytmadan hosila olish qoidasiga binoan hisoblasak va bazis vektorlarning koordinata bo'yicha hosilalarini aniqlovchi formulalarni eslasak, quyidagi munosabatlarni olamiz:

$$\begin{aligned} \frac{\partial T}{\partial x^\alpha} &= \frac{\partial}{\partial x^\alpha}\left(T^{\beta}_{\cdot\,\gamma} \overline{Э}_\beta \overline{Э}^\gamma\right) = \frac{\partial T^{\beta}_{\cdot\,\gamma}}{\partial x^\alpha} \overline{Э}_\beta \overline{Э}^\gamma + \\ &+ T^{\beta}_{\cdot\,\gamma} \Gamma^{\sigma}_{\cdot\,\beta\alpha} \overline{Э}_\sigma \overline{Э}^\gamma - T^{\beta}_{\cdot\,\gamma} \Gamma^{\gamma}_{\cdot\alpha\sigma} \overline{Э}_\beta \overline{Э}^\sigma = \\ &= \left(\frac{\partial T^{\beta}_{\cdot\,\gamma}}{\partial x^\alpha} + T^{\sigma\cdot}_{\cdot\,\gamma} \Gamma^{\beta}_{\cdot\,\sigma\alpha} - T^{\beta\cdot}_{\cdot\,\sigma} \Gamma^{\sigma}_{\cdot\,\gamma\alpha}\right) \overline{Э}_\beta \overline{Э}^\gamma \end{aligned} \tag{2.14.31}$$

Demak, tenzor gradienti

$$\nabla T = \nabla_\alpha T^{\beta}_{\cdot\,\gamma}\, \overline{\mathcal{Э}}^{\alpha} \overline{\mathcal{Э}}_{\beta} \overline{\mathcal{Э}}^{\gamma} \tag{2.14.32}$$

formula bilan, uning komponentalari esa *T* tenzor aralash komponentalarining kovariant hosilasi deb ataluvchi quyidagi munosabatlar bilan aniqlanuvchi 3-rang tenzor.

$$\nabla_\alpha T^{\beta\,\cdot}_{\cdot\,\gamma} = \frac{\partial T^{\beta\,\cdot}_{\cdot\,\gamma}}{\partial x^\alpha} + T^{\sigma\,\cdot}_{\cdot\,\gamma}\Gamma^{\beta}_{\cdot\sigma\alpha} - T^{\beta\,\cdot}_{\cdot\sigma}\Gamma^{\sigma}_{\cdot\gamma\alpha} \tag{2.14.33}$$

Bu yerdan Kristoffel belgilari tenzor komponentalari emasligi ko'rinib turibdi.

Demak, tenzor komponentasidan olingan odatdagi hosila $\frac{\partial T^\beta}{\partial x^\alpha}$ ham tenzor komponentasini hosil qilmaydi. Lekin dekart koordinata sistemasi bundan mustasno, chunki bu holda hosila

$$\nabla_\alpha T^{\beta\,\cdot}_{\cdot\,\gamma} = \frac{\partial T^{\beta\,\cdot}_{\cdot\,\gamma}}{\partial x^\alpha} \tag{2.14.34}$$

tenglik bilan aniqlanadi.

T tenzorning boshqa xil komponentalaridan ham kovariant hosila olish mumkin.

$$\nabla T = \nabla_\alpha T^{\beta\gamma} \overline{\mathcal{Э}}^{\alpha} \overline{\mathcal{Э}}_{\beta} \overline{\mathcal{Э}}_{\gamma}$$

Buyerda *T* tenzorning kovariant va kontravariant komponentalaridan olingan kovariant hosilalari

$$\begin{aligned} \nabla_\gamma T_{\alpha\beta} &= \frac{\partial T_{\alpha\beta}}{\partial x^\gamma} - T_{\sigma\beta}\Gamma^{\sigma}_{\cdot\,\alpha\gamma} - T_{\alpha\sigma}\Gamma^{\sigma}_{\cdot\beta\gamma} \\ \nabla_\alpha T^{\beta\gamma} &= \frac{\partial T^{\beta\gamma}}{\partial x^\alpha} + T^{\sigma\gamma}\Gamma^{\beta}_{\cdot\sigma\alpha} + T^{\beta\sigma}\Gamma^{\gamma}_{\cdot\,\sigma\alpha} \end{aligned} \tag{2.14.35}$$

formulalar bilan aniqlanishini ko'rsatish qiyin emas.

Endi (2.14.32) dan foydalanib,

$$\frac{\partial T}{\partial x_\alpha} = \frac{\partial T}{\partial x^\beta}\frac{\partial x^\beta}{\partial x_\alpha} = \mathrm{g}^{\alpha\beta}\nabla_\beta T^{\sigma\,\cdot}_{\cdot\tau}\overline{\mathcal{Э}}_{\sigma}\overline{\mathcal{Э}}^{\tau} = \nabla^\alpha T^{\sigma\,\cdot}_{\cdot\tau}\overline{\mathcal{Э}}_{\sigma}\overline{\mathcal{Э}}^{\tau}$$

munosabatni olamiz. Bu yerda

$$g^{\alpha\beta}\nabla_{\beta} T^{\sigma\cdot}_{\cdot\,\tau} = \nabla^{\alpha} T^{\sigma}_{\cdot\,\tau} \tag{2.14.36}$$

Demak, bu holda tenzor gradienti

$$\nabla T = \nabla^{\alpha} T^{\sigma}_{\cdot\,\tau} \overline{Э}_{\alpha} \overline{Э}_{\sigma} \overline{Э}^{\tau}$$

ko'rinishda yoziladi.

Ta'rif: $\nabla^{\alpha} T$ miqdor T tenzorning aralash komponentalaridan olingan kontravariant hosila deb ataladi.

(2.14.36) dan ko'rinib turibdiki, kontravariant hosila kovariant hosiladan indekslarni ko'tarish qoidasi yordamida olinadi.

2-rang tenzorlar uchun olingan koordinata bo'yicha dif-ferensiallash qoidalari *n*-rang tenzorlar uchun ham o'rinli bo'lib, differensiallash natijasida $(n+1)$ - rang tenzor hosil qilinadi.

Tenzor komponentalaridan kovariant hosila olish qoidasiga binoan

$$\nabla_{\gamma} g_{\alpha\beta} = \frac{\partial g_{\alpha\beta}}{\partial x^{\gamma}} - g_{\sigma\beta}\Gamma^{\sigma}_{\cdot\alpha\gamma} - g_{\alpha\sigma}\Gamma^{\sigma}_{\cdot\beta\gamma}$$

bo'ladi. Ikkinchi tomondan Kristoffel belgilarining (2.14.6) xossalari e'tiborga olinsa, ushbu hosila aynan nolga teng ekanligi kelib chiqadi:

$$\nabla_{\gamma} g_{\alpha\beta} = \left(\Gamma_{\alpha\beta\gamma} + \Gamma_{\beta\alpha\gamma}\right) - \Gamma_{\beta\alpha\gamma} - \Gamma_{\alpha\beta\gamma} = 0 \tag{2.14.37}$$

Shunga o'xshash

$$\nabla_{\gamma} g^{\alpha\beta} = 0, \;\; \nabla_{\gamma}\delta^{\gamma\cdot}_{\cdot\beta} = 0$$

ekanligini ham ko'rsatish mumkin.

Indekslarni ko'tarish qoidasi o'rinli bo'lgani tufayli olingan natijalar kontravariant hosilalar uchun ham o'rinli.

Demak, metrik tenzorning gradienti 3-rangli nol tenzor ekan. Bundan metrik tenzorning maydoni bir jinsli maydon ekanligi kelib chiqadi. Shunday qilib, metrik tenzorning komponentalari kovariant hosila olishda o'zini o'zgarmas kabi tutadi, yani ularni hosila belgisi ichiga kiritish va tashqarisiga chiqarish mumkin ekan.

Kovariant hosilaning ta'rifidan va (2.5.9) dan foydalanib,

$$\nabla_{\sigma} e_{\alpha\beta\gamma} = \frac{\partial e_{\alpha\beta\gamma}}{\partial x^{\sigma}} - e_{\tau\beta\gamma}\Gamma^{\tau}_{\alpha\sigma} - e_{\alpha\tau\gamma}\Gamma^{\tau}_{\beta\sigma} - e_{\alpha\beta\tau}\Gamma^{\tau}_{\cdot\gamma\sigma}$$

formulalarni olamiz. Bu yerda

$$\frac{\partial e_{\alpha\beta\gamma}}{\partial x^{\sigma}} = \frac{\partial}{\partial x^{\sigma}}\left(\overline{Э}_{\alpha}\,\overline{Э}_{\beta}\,\overline{Э}_{\gamma}\right) = \Gamma^{\tau}_{\cdot\alpha\sigma}\left(\overline{Э}_{\tau}\,\overline{Э}_{\beta}\,\overline{Э}_{\gamma}\right) + \Gamma^{\tau}_{\cdot\beta\sigma}\left(\overline{Э}_{\alpha}\,\overline{Э}_{\tau}\,\overline{Э}_{\gamma}\right) +$$
$$+\Gamma^{\tau}_{\cdot\gamma\sigma}\left(\overline{Э}_{\alpha}\,\overline{Э}_{\beta}\,\overline{Э}_{\tau}\right) = \Gamma^{\tau}_{\alpha\sigma}e_{\tau\beta\gamma} + \Gamma^{\tau}_{\beta\sigma}e_{\alpha\tau\gamma} + \Gamma^{\tau}_{\gamma\sigma}e_{\alpha\beta\tau}$$

ekanligini hisobga olsak, kovariant hosila

$$\nabla_{\alpha} e_{\alpha\beta\gamma} = 0 \tag{2.14.38}$$

ekanligi kelib chiqadi.

Indekslarning tuzilishi boshqacha bo’lgan hollarda ham bu xossa o’rinli ekanligini ko’rsatish qiyin emas. Demak, diskriminant tenzor maydoni ham bir jinsli maydon bo’lib, hosila olishda diskriminant tenzor komponentalarini hosila belgisining ichiga kiritish yoki tashqarisiga chiqarish mumkin.

Kovariant hosila olish amali oddiy hosila olish amalining ayrim xossalariga ega:

1) ikkita vektor yig’indisining hosilasi shu vektorlar hosilalarining yig’indisiga teng. Vektor kovariant komponentasining kovariant hosilasini hisoblash (2.15.9) formulaga ko’ra quyidagi tenglik o’rinli bo’ladi:

$$\begin{aligned}\nabla_{\sigma}\left(a_{\alpha} + b_{\alpha}\right) &= \frac{\partial}{\partial x^{\sigma}}\left(a_{\alpha} + b_{\alpha}\right) - \left(a_{\tau} + b_{\tau}\right)\Gamma^{\tau}_{\cdot\alpha\sigma} = \\ &= \frac{\partial a_{\alpha}}{\partial x^{\sigma}} - a_{\tau}\Gamma^{\tau}_{\cdot\alpha\sigma} + \frac{\partial b_{\alpha}}{\partial x^{\sigma}} - b_{\tau}\Gamma^{\tau}_{\cdot\alpha\sigma} = \nabla_{\sigma}a_{\alpha} + \nabla_{\sigma}b_{\alpha}\end{aligned} \tag{2.14.39}$$

2) indefenit ko’paytmaning kovariant hosilasi odatdagi ko’paytmadan hosila olish qoidasiga binoan hisoblanadi:

$$\nabla_{\sigma}\left(a^{\alpha}b^{\beta}\right) = \left(\nabla_{\sigma}a^{\alpha}\right)b^{\beta} + a^{\alpha}\left(\nabla_{\sigma}b^{\beta}\right) \tag{2.14.40}$$

Indefenit ko’paytma komponentalarining kovariant hosilasi uchun (2.15.5) ga ko’ra ushbu

$$\begin{aligned}\nabla_{\sigma}\left(a^{\alpha}b^{\beta}\right) &= \frac{\partial\left(a^{\alpha}b^{\beta}\right)}{\partial x^{\sigma}} + a^{\tau}b^{\beta}\Gamma^{\alpha}_{\cdot\tau\sigma} + a^{\alpha}b^{\tau}\Gamma^{\beta}_{\cdot\tau\sigma} = \\ &= \left(\frac{\partial a^{\alpha}}{\partial x^{\sigma}} + a^{\tau}\Gamma^{\alpha}_{\cdot\tau\sigma}\right)b^{\beta} + \left(\frac{\partial b^{\beta}}{\partial x^{\sigma}} + b^{\tau}\Gamma^{\beta}_{\cdot\tau\sigma}\right)a^{\alpha} = \\ &= \left(\nabla_{\sigma}a^{\alpha}\right)b^{\beta} + \left(\nabla_{\sigma}b^{\beta}\right)a^{\alpha}\end{aligned}$$

munosabatlar o’rinli bo’ladi.

3) kovariant hosila olish va yig’ishtirish amalining o’rinlarini almashtirish mumkin;

4) vektorning skalyar va vektor ko’paytmasidan kovariant hosila olish mumkin. Agar (2.14.40) ni $g_{\alpha\beta}$ ga ko’paytirib, α va β lar bo’yicha yig’indi hisoblash amalini bajarsak, ushbu

$$\bar{a} \cdot \bar{b} = a^{\alpha} b_{\alpha} = g_{\alpha\beta} a^{\alpha} b^{\beta}$$

skalyar ko’paytmaning kovariant hosilasi uchun

$$\nabla_{\sigma}\left(g_{\alpha\beta} a^{\alpha} b^{\beta}\right) = g_{\alpha\beta}\left(\nabla_{\alpha} a^{\alpha}\right) b^{\beta} + g_{\alpha\beta} a^{\alpha}\left(\nabla_{\sigma} b^{\beta}\right)$$

formula yoki

$$\nabla\left(\bar{a} \cdot \bar{b}\right) = \nabla\bar{a} \cdot b + \bar{a} \cdot \left(\nabla\bar{b}\right) \tag{2.14.41}$$

formula o’rinli.

Shunga o’xshash, (2.15.40) ni $e_{\alpha\beta\gamma}$ ga ko’paytirib, ushbu

$$\bar{a} \times \bar{b} = e_{\alpha\beta\gamma} a^{\alpha} b^{\beta} \bar{э}^{\gamma}$$

vektor ko’paytmaning kovariant hosilasi uchun quyidagi

$$\nabla_{\sigma}\left(e_{\alpha\beta\gamma} a^{\alpha} b^{\beta}\right) = e_{\alpha\beta\gamma}(\nabla_{\sigma} a^{\alpha}) b^{\beta} - e_{\beta\alpha\gamma} a^{\alpha}\left(\nabla_{\alpha} b^{\beta}\right)$$

yoki

$$\nabla\left(\bar{a} \times \bar{b}\right) = \nabla\,\bar{a} \times \bar{b} - \left(\nabla\,\bar{b}\right) \times \bar{a} \tag{2.14.42}$$

formulalarni olamiz. Shunday qilib, (2.14.41) va (2.14.42) formulalar vektorlarning skalyar va vektor ko’paytmasidan kovariant hosila olish qoidasini beradi.

Masalalar yechishga doir namunalar

1. $T = \begin{pmatrix} 2 & 5 & 1 \\ 2 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix}$

a) Tenzorni vektor shaklida yozing;

b) Tenzorni simmetrik va antisemmetrik tenzorlarga ajrating va ularning yig'indisidan iborat ekanligini ko'rsating;

c) Tenzorning sharsimon va deviatorlarini toping;

Yechish.

a) Tenzorni vektor ko'rinishiga o'tkazamiz

$$T = 2\vec{э}^1\vec{э}^1 + 5\vec{э}^1\vec{э}^2 + \vec{э}^1\vec{э}^3 + 2\vec{э}^2\vec{э}^1 + 4\vec{э}^2\vec{э}^2 + \vec{э}^3\vec{э}^1 + 4\vec{э}^3\vec{э}^3;$$

b) Simmetrik va antisimmetrik tenzorlarga ajratib, uni yig'indi ko'rinishida ifodalaymiz. Buning uchun quyidagi formulalardan foydalanamiz:

c) $T_{sim} = \frac{1}{2}(T_{ij} + T_{ji}); \quad T_{anti} = \frac{1}{2}(T_{ij} - T_{ji})$

$$T_{sim} = \begin{pmatrix} 2 & 3{,}5 & 1 \\ 3{,}5 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix}; \quad T_{anti} = \begin{pmatrix} 0 & 1{,}5 & 0 \\ -1{,}5 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$T = \begin{pmatrix} 2 & 5 & 1 \\ 2 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 3{,}5 & 1 \\ 3{,}5 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 1{,}5 & 0 \\ -1{,}5 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

d) Sharsimon va deviator tenzorni topish uchun esa quyidagi formulalardan foydalanamiz;

$$T_{shar} = \frac{1}{3}(T_{11} + T_{22} + T_{33})\delta_{ij};$$

$$T_{dev} = T - T_{shar};$$

$$T_{shar} = \begin{pmatrix} \frac{10}{3} & 0 & 0 \\ 0 & \frac{10}{3} & 0 \\ 0 & 0 & \frac{10}{3} \end{pmatrix}; \quad T_{dev} = \begin{pmatrix} \frac{-4}{3} & 5 & 1 \\ 2 & \frac{2}{3} & 0 \\ 1 & 0 & \frac{2}{3} \end{pmatrix};$$

2. $T = \begin{pmatrix} 4 & 6 & 3 \\ 3 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix}$ tenzor uchun quyidagilarni toping.

a) Tenzorning bosh komponentalarini toping;

b) Tenzorning bosh o'qlarini toping;

c) Tenzorni sharsimon va deviator qismlariga ajrating;

d) Tenzor sirti tenglamasini va shaklini toping.

Yechish.

a) Tenzorning bosh komponentalarini topish uchun

$$\begin{vmatrix} 4-\lambda & 6 & 3 \\ 3 & 4-\lambda & 0 \\ 1 & 0 & 4-\lambda \end{vmatrix} = 0$$

tenglamani yechamiz. Kubik tenglama hosil bo'ladi va natijada 3ta ildizga ega bo'lamiz

$$(4-\lambda)^3-3(4-\lambda)-18(4-\lambda)=0$$

$$(4-\lambda)((4-\lambda)^2-21)=0$$

Uning ildizlari $\lambda_1 = 4, \lambda_2 = 4 + \sqrt{21}, \lambda_3 = 4 - \sqrt{21}$.

b) Bosh qiymatlarga mos bosh yo'nalishlarini topish uchun esa quyidagi tenglamalar yechiladi:

$$\lambda = 4 \text{ uchun}$$

$$\begin{cases} 6n_2 + 3n_3 = 0 \\ 3n_1 = 0 \\ n_1 = 0 \\ n_1^2 + n_2^2 + n_3^2 = 1 \end{cases} \Rightarrow \begin{cases} n_1 = 0 \\ n_2 = -\dfrac{1}{\sqrt{5}} \\ n_3 = \dfrac{2}{\sqrt{5}} \end{cases}$$

$$\lambda = 4 + \sqrt{21} \text{ uchun}$$

$$\begin{cases} -\sqrt{21}n_1 + 6n_2 + 3n_3 = 0 \\ 3n_1 - \sqrt{21}n_2 = 0 \\ n_1 - \sqrt{21}n_3 = 0 \\ n_1^2 + n_2^2 + n_3^2 = 1 \end{cases} \Rightarrow \begin{cases} n_1 = \pm\sqrt{\dfrac{21}{31}} \\ n_2 = \pm\dfrac{3}{\sqrt{31}} \\ n_3 = \pm\dfrac{1}{\sqrt{31}} \end{cases}$$

$$\lambda = 4 - \sqrt{21} \text{ uchun}$$

$$\begin{cases} -\sqrt{21}n_1 + 6n_2 + 3n_3 = 0 \\ 3n_1 + \sqrt{21}n_2 = 0 \\ n_1 + \sqrt{21}n_3 = 0 \\ n_1^2 + n_2^2 + n_3^2 = 1 \end{cases} \Rightarrow \begin{cases} n_1 = \pm\sqrt{\dfrac{21}{31}} \\ n_2 = \pm\dfrac{3}{\sqrt{31}} \\ n_3 = \pm\dfrac{1}{\sqrt{31}} \end{cases}$$

c) Tenzor sirtini topamiz:

$$T_{ij}x_ix_j = 4x_1^2 + 6x_1x_2 + 3x_1x_3 + 3x_2x_1 + 4x_2^2 + 0x_1x_3 + x_3x_1 + 0x_3x_2 + 4x_3^2 = \pm c^2$$

yoki

$$4x_1^2 + 4x_2^2 + 4x_3^2 + 9x_1x_3 + 4x_1x_3 = \pm c^2.$$

Demak, bu tenzor yuqoridagi tenglama bilan ifodalanuvchi ellipsoiddan iborat ekan.

II bobga doir masalalar.

1. $T = \begin{pmatrix} 4 & 0 & 1 \\ 3 & 4 & 2 \\ 2 & 1 & 4 \end{pmatrix}$ tenzorni vektor shaklida yozing.

2. Ikkinchi rangli tenzorni ifodalovchi :

$$T'_{ij} = a_{im}\, a_{jn}\, T_{mn}$$

fo’rmula bilan ortogonallik shartidan foydalanib , T_{mn} ni T'_{ij} orqali ifodalang.

3. $T = \begin{pmatrix} 2 & 5 & 1 \\ 2 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix}$ tenzorni a)simmetrik va antisemmetrik tenzorlarga ajrating va tenzorni ularning yig’indisidan iborat ekanligini ko’rsating; b) tenzorning sharsimon va deviatorlarini toping.

4.Uchinchi rangli tenzorni ifodalovchi :

$$T'_{ijk} = a_{il}\, a_{jm}\, a_{kn}\, T_{lmn}$$

fo’rmula bilan ortogonallik shartidan foydalanib, T_{lmn} ni T'_{ijk} orqali ifodalang.

5. Ikki a, b vector komponentlaridan tuzilgan ikkinchi rangli tenzor $T_{ij} = a_i b_j$ ning chiziqli invariant T_{ij} aniqlang.

6. Birlik tenzor δ_{ij} (i, j = 1,2,3) ning chiziqli invarianti δ_{ij} ni toping.

7. Birlik tenzor δ_{ij} (i, j = 1,2,3) ning kvadratik invarianti $\delta_{ij}\ \delta_{ij}$ ni toping.

8. Birlik tenzor δ_{ij} (i, j = 1,2,3) ning kubik invariant, ya'ni diskriminanti $|\delta_{ij}|$ ni toping.

9. Birlik tenzor uchun $\delta_{ij}\ \delta_{jk}\ \delta_{ki}$ ni hisoblab toping.

10. $T = \begin{pmatrix} 4 & 6 & 3 \\ 3 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix}$ tenzorning bosh komponentalarini va bosh o'qlarini toping.

11. $\mathrm{T^{ij}} = \begin{pmatrix} 5 & 6 & 3 \\ 3 & 4 & 5 \\ 1 & 3 & 2 \end{pmatrix}$ tenzor sirti tenglamasini va shaklini toping.

12-14 masalalar uchun quyidagilarni aniqlang.

a) Tenzorni vektor shaklida yozing;

d) Tenzorni simmetrik va antisemmetrik tenzorlarga ajrating va ularning yig'indisidan iborat ekanligini ko'rsating;

e) Tenzorning sharsimon va deviatorlarini toping;

12. $\mathrm{T^{ij}} = \begin{pmatrix} 5 & 8 & 2 \\ 3 & 8 & 5 \\ 7 & 7 & 4 \end{pmatrix}$

13. $\mathrm{T^{ij}} = \begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{pmatrix}$

14. $\mathrm{T^{ij}} = \begin{pmatrix} 1 & 2 & 4 \\ 6 & 7 & 8 \\ 10 & 12 & 13 \end{pmatrix}$

15-17 masalalar uchun quyidagilarni aniqlang.

a) Tenzorning bosh komponentalarini toping;

b) Tenzorning bosh o'qlarini toping;

c) Tenzorni sharsimon va deviator qismlariga ajrating;

d) Tenzor sirti tenglamasini va shaklini toping.

15. $\mathrm{T^{ij}} = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 4 & 5 \\ 0 & 7 & 6 \end{pmatrix}$

16. $T^{ij} = \begin{pmatrix} 2 & -3 & 0 \\ 3 & 6 & 0 \\ 8 & 0 & 1 \end{pmatrix}$

17. $T^{ij} = \begin{pmatrix} 0 & 2 & 4 \\ 6 & 7 & 4 \\ 10 & 1 & 3 \end{pmatrix}$

18-21 masalalar uchun quyidagilarni aniqlang.

a)Tenzorni matritsa shaklida yozing;

b) Tenzorni simmetrik va antisemmetrik tenzorlarga ajrating va ularning yig'indisidan iborat ekanligini ko'rsating;

c) Tenzorning sharsimon va deviatorlarini toping;

d) Tenzorning bosh komponentalarini toping;

e) Tenzorning bosh o'qlarini toping;

f) Tenzorni sharsimon va deviator qismlariga ajrating;

g) Tenzor sirti tenglamasini va shaklini toping.

18. $A = 2\vec{\text{э}}^1\vec{\text{э}}^1 + 2\vec{\text{э}}^1\vec{\text{э}}^3 + 3\vec{\text{э}}^2\vec{\text{э}}^3 + 5\vec{\text{э}}^3\vec{\text{э}}^1 + 6\vec{\text{э}}^3\vec{\text{э}}^2 + 4\vec{\text{э}}^3\vec{\text{э}}^3$;

19. $A = 4\vec{\text{э}}^1\vec{\text{э}}^1 + 2\vec{\text{э}}^1\vec{\text{э}}^2 + +5\vec{\text{э}}^2\vec{\text{э}}^1 + 6\vec{\text{э}}^2\vec{\text{э}}^2 + 4\vec{\text{э}}^3\vec{\text{э}}^1 + 7\vec{\text{э}}^3\vec{\text{э}}^2$

20. $A = \vec{\text{э}}^1\vec{\text{э}}^1 + 2\vec{\text{э}}^1\vec{\text{э}}^2 + +5\vec{\text{э}}^1\vec{\text{э}}^3 + 6\vec{\text{э}}^2\vec{\text{э}}^1 + 4\vec{\text{э}}^2\vec{\text{э}}^2 + 3\vec{\text{э}}^3\vec{\text{э}}^2 + \vec{\text{э}}^3\vec{\text{э}}^3$

21. $A = 3\vec{\text{э}}^1\vec{\text{э}}^1 + 2\vec{\text{э}}^1\vec{\text{э}}^2 + +\vec{\text{э}}^1\vec{\text{э}}^3 + 4\vec{\text{э}}^2\vec{\text{э}}^2 + 6\vec{\text{э}}^2\vec{\text{э}}^3 + +3\vec{\text{э}}^3\vec{\text{э}}^1 + \vec{\text{э}}^3\vec{\text{э}}^2$

22. Berilgan T_{ij} tenzor bilan unga teskari T_{ij}^{-1} tenzordan skalyar argument σ bo'yicha olingan hosilalar o'zaro qanday bog'langan ?

23. *n*-o'lchovli fazoda birlik vektor uchun δ_{ii} hisoblab chiqing.

24. *n*-o'lchovli fazoda birlik psevdotenzor uchun $\varepsilon_{ij\ldots s}$ $\varepsilon_{ij\ldots s}$ hisoblab chiqing.

25. *n*-o'lchovli fazo birlik psevdotenzori $\varepsilon_{i_1 i_2 \ldots i_n}$ vositasida ***l*** rangli tenzordan *n-l* rangli psevdotenzor hosil qiling.

26. Metrik tenzor uchun:

$$g^{ij}g_{ij} = n$$

ekanligini ko'rsating.

27. To'rtinchi rangli $T^{\cdot\cdot i_3 i_4}_{i_1 i_2 \cdot\cdot}$ tenzorni mos indekslar bo'yicha yig'ishtirish yo'li bilan ikkinchi rangli to'rtta tenzorni hosil qilining.

28. Ikkinchi rangli kontvariant $T^{i_1 i_2}$ tenzorlar bilan kovariant A_{i_3}vektor ko'paytmasini mos indekslar bo'yicha yig'ishtirib, ikkita kontravariant vektor hosil qilining.

29. Kontravariant metrik g^{ij} tenzorni kovariant metrik g_{rs} tenzor orqali ifodalang.

30. Berilgan T^{ijk} tenzordan metrik tenzor vositasida T_{pqr} tenzor hosil qiling.

31. Ikkinchi rangli tenzor uchun :

$$T^{i.}_{.i} = T^{.i}_{i.}$$

ekanligini ko'rsating.

32. Ikkinchi rangli kontravariant antisimmetrik tenzor $A^{ij} = -A^{ji}$ uchun:

$$\Gamma^k_{ij} A^{ij} = 0$$

rangligini ko'rsating.

33. Berilgan $T^{..i_2 i_3}_{i_1..}$ tenzordan metrik tenzor vositasida $T^{j_1 i_2 i_3}$, $T^{..i_3}_{i_1 i_2 ..}$, $T^{..i_2}_{i_1 . i_3}$ tenzorlar hosil qiling.

34. Berilgan $T_{i_1 i_2 i_3}$ tenzordan metrik tenzor sifatida $T^{j_1 j_2}_{..i_3}$, $T^{.j_2 j_3}_{i_1}$, $T^{j_1 . j_3}_{.i_2 .}$ tenzorlar hosil qiling.

35. e_i $-$ ortonormallashgan bazisda t_{ij} $-$tenzorning komponentalari.

a) tenzorning komponentalari $\tau_{ij} = t_{ij}$ tenglikni qanoatlantirishini ko'rsating

b) Quyidagilarni tengligini ko'rsating:

1)$\tau_{ij} u_i u_j \; va \; t_{ij} u_i u_j$

2)$\tau_{ij} u_i v_j \; va \; t_{ij} u_i v_j$

bu yerda u_i, v_j $-$ vektor komponentalari.

36. Bir necha ortonormal bazislarda tenzor komponentlari quyidagi munosabatlarni qanoatlantirsa, ixtiyoriy ortonormal bazislar komponentlari uchun bu munosabatlar bajarilishini ko'rsating:

a) $t_{ij} = t_{ji}$; b) $t_{ij} = -t_{ji}$.

37. s_{ij} simmetrik va a_{kl} antisimmetrik tenzorlar bo'lsa, $s_{ij} a_{ij} = 0$ bo'lishligini ko'rsating.

38. Ixtiyoriy ortonormal bazisda a va b tenzor komponentalari bo'lsa, $a_{ijk} + b_{ijk}$ yigindi ham tenzor komponentasi bo'lishini ko'rsating.

39. Ixtiyoriy ortonormal bazisda B va ε $B_{ijkl}\varepsilon_{mn}$ kopaytmaning tenzor komponentalari bo'lsa, $B_{ijkl}\varepsilon_{kl}$ yigindining ham tenzor komponentalari bo'lishini ko'rsating.

40. Ixtiyoriy ikkinchi rang tenzorni simmetrik va antimmetrik tenzorlar yigindisi korinishida ifodalanishini ko'rsating. Bunday ifodalanish yagona bo'ladimi?

41. s ikkinchi rang $s_{ij}u_i v_j$simmetrik tenzorini $s_{ij}w_i w_j$ korinishda ifodalang, bu yerda- u_i, v_i, w_i vektor komponentalari.

42. Agar ikkinchi rang tenzorlar uchun ixtiyoriy v vektori quyidagi tenglikni qanoatlantirsa $t_{ij}v_i v_j = 0$, t tenzor antisimmetrik ekanligini ko'rsating.

43. *t* ikkinchi rang simmetrik tenzorning $t^{(sh)}$ shar va $t^{(d)}$deviator tenzorlari mos ravishda quyidagilarga teng:

$$t_{ij}^{(sh)} = \frac{1}{3}t_{kk}\delta_{ij} \text{ va } t_{ij}^{(d)} = t_{ij} - \frac{1}{3}t_{kk}\delta_{ij},$$

a) $(t^{(sh)})^{(d)}$ shar tenzorni deviator tenzorini toping;

b) $(t^{(d)})^{(sh)}$ deviator tenzorning shar tenzorini toping.

44. Agar barcha ortonormal bazislarda $t_{12} = 0$ bo'lsa, *t* ikinchi rang tenzorni umumiy korinishda ifodalang.

45. Quyidagi ko'rinishda berilgan tenzorlarning bosh komponentalarini va bosh o'qlarini toping:

a) $\begin{pmatrix} 1 & -\sqrt{3} & 0 \\ -\sqrt{3} & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$; b) $\begin{pmatrix} 1 & -\sqrt{3} & 0 \\ -\sqrt{3} & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

46. Quyidagi funktsiya komponenti ikkinchi darajali simmetrik tenzorni invarianti yekanligini ko'rsating.

47. Ikkinchi rang simmetrik tenzorlarning asosiy komponentalari uning invariantlari bo'ladimi?

III BOB. DEFORMATSIYALANUVCHI MUHIT KINEMATIKASI

1 §. Tutash muhitlar mehanikasining asosiy farazlari.

Tutash muhitlar mehanikasining asosiy farazlari

a) Haqiqiy jismlarning tuzilishi va doimiylik farazi.

Ma'lumki, barcha moddiy jismlar juda kichik zarralardan, ular molekula va atomlardan tashkil topgan. Atomlar esa elektronlarga va yadrolarga ega. Masalan, atom yadrosining radiusi 10^{-13} smga teng, vodorod molekulasi radiusi $1.36\cdot 10^{-8}$ sm. Lekin vodorodning asosiy massasi uning yadrosida joylashgan.

Ma’lumki, oddiy sharoitda (0^0C va dengiz yuzasidagi atmosfera bosimida) $1\ \text{sm}^3$ havo hajmida $N = 2.687\cdot 10^{19}$ ta molekula mavjud. 60 km yuqorida esa $N = 8\cdot 10^{15}$ ta, yulduzlar orasidagi muhitda esa $N = 1$ ta molekula bor deyish mumkin. Shuningdek, temir uchun $N = 8.622\cdot 10^{22}\ 1/\text{sm}^3$. Bundan ko’rinib turibdiki, jismlar massasi joylashgan hajmlar shu jism egallagan hajmlarning nihoyatda kichik qismini tashkil etadi. Ya’ni jismlar «bo’shliqlar»dan iborat. Atom va molekulalar doimiy xaotik harakatda bo’ladilar. Bundan ko’rinadiki, ularning individual harakatini o’rganish nihoyatda qiyin: chunki ularning soni juda ko’p. Masalan, kislorod molekulalari 1 sekundda $6.55\cdot 10^9$ marta to’qnashadi, ularning o’rtacha tezligi $v = 425$ m/sek atrofida, harakatlanuvchi bu zarralar o’rtasidagi ta’sir kuchlari ham ma’lum emas.

Tutash muhit mexanikasida zarralar o'rtasida ma'lum o'zaro ta'sirlar mavjud. Gazda ular faqat to'qnashuvlar bilan bog'liq. Suyuqlik va qattiq moddalarda zarralar bir-biriga yaqinlashadi va ularda o'zaro ta'sir kuchlari katta ahamiyatga ega. Zarralarni bir-biriga yaqinlashtirish uchun zarrachalarning millionlab darajadagi haroratlarda tartibsiz harakatlanishi natijasida vujudga keladigan ulkan energiya talab qilinadi.

Tutash muhit mexanikasida real jismlar o’rganilganda amaliyot uchun har bir zarraning trayektoriyasi, tezligi va boshqa xarakteristikalari emas, balki bu jism zarralari uchun o’rtacha bo’lgan xarakteristikalar kerak bo’ladi.

Real jismlarning harakati o’rganilganda statistik fizika usullari ham qo’llaniladi. Statistik usullar bilan ish ko’rilganda doimo ayrim qo’shimcha farazlarga murojaat qilinadi. Lekin bularning hammasini ham asosli deb bo’lavermaydi. Boshqa tomondan, statistik usul asosidagi tenglamalar murakkab ko’rinishga ega bo’lib, effektiv yechimlar topish qiyin masalaga aylanadi.

Moddiy jismlar harakatini o’rganishdagi ikkinchi yo’l bu - fenomenologik makroskopik nazariyadan iborat. Makroskopik nazariyalar amaliy muammolarni hal qilishning samarali vositasidir va ularning yordami bilan olingan ma'lumotlar tajribaga mos keladi.

b) Tutashlik farazi

Tutash muhit mexanikasining asosiy farazi *muhitning tutashligi* farazidir.

Jism fazosi yoki uning bo’lagini to’la ravishda qoplagan deb yuqorida aytilgan fikr real jismlar uchun taxminan mos keladi.

Deformatsiyalanuvchi jismlar tashqi kuchlar ta’sirida o’z o’lchamlari va shakllarini o’zgartiradilar, jismning fikran ajratilgan bo’laklari o’rtasida kuchlar, zo’riqishlar, bosimlar deb ataluvchi mexanik miqdorlar paydo bo’ladiki, bu kattaliklar faqat tashqi kuchlargagina emas, balki jism geometrik shakliga, muhitning tajribalar asosida olinishi mumkin bo’lgan makroskopik xossalariga ham bog’liq bo’ladi. Bu muammolar TMMda o’rganiladi.

Gaz, suyuq yoki qattiq holatdagi jismlar, ularning aralashmalari turli sabablar bilan fazoda harakatlanishi va harakatlanish jarayonida «moddiy zarralar» orasidagi masofalar o’zgarishi mumkin. Bu yerda shuni yaqqol ko’rish mumkinki, bir tomondan tutash muhit tushunchasini kiritmay turib yaxlit, alohida moddiy jism tasavvuriga ega bo’lish qiyin, ikkinchi tomondan, har qanday tutash muhit aslida mayda zarralardan tashkil topganligi va ularda «bo’shliqlar» borligini ta’kidlash kerak. Bu dialektika asosida yechilishi kerak bo’lgan ichki ziddiyat va uning tajribalar asosida ma’lum masalalarni hal etishga xizmat qila oladigan nazariyasi bo’lib, nazariyaning amalda ishlatilishidir.

d) Fazo va vaqt.

Fazo va vaqt tushunchalari haqida ham to’xtalib o’taylik. Bu tushunchalar nazariy mexanika kursida to’la bayon etilgan. Shuning uchun bu tushunchalarning ayrim mohiyatlari ustidagina to’xtalib o’tamiz.

Fazo deganda koordinatalar deb ataluvchi sonlar orqali berilgan nuqtalar to’plami tushuniladi. Agar fazoda ikki nuqta orasidagi masofa aniqlangan bo’lsa, bu fazo metrik fazo deyiladi.

Agar metrik fazoda ikki nuqta orasidagi masofa

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

bo’lsa, bu fazo Yevklid fazosidir. Yevklid fazosiga shu fazo uchun umumiy bo’lgan Dekart koordinatalar sistemasini kiritish mumkin. Bunday fazoda biz o’rganadigan mexanika Nyuton mexanikasi deb ataladi. Bu fazoda joylashgan tutash muhit uning ixtiyoriy geometrik nuqtasida mavjuddir.

Vaqt tushunchasi tajriba bilan bog'liq va mexanikada zarur. Har qanday mexanik xodisa har doim bazi kuzatuvchilar nuqtayi nazaridan tavsiflanadi Vaqt barcha kuzatuvchilar uchun bir xil yo'nalishda - poezdda, samolyotda, auditoriyada bir xil o’tadi deb taxmin qilamiz . TMMda absolut vaqt tushunchasi bilan ish ko’ramiz. Bizning vaqtimiz barcha koordinata sistemalarida bir xil o’zgaradi.

Shunday qilib, TMMda muhit harakati - kontinium harakati Yevklid fazosida absolyut vaqtdan foydalanib o’rganiladi.

TMMda o’rganilayotgan muhit harakat jarayoni davomida nuqtalar orasidagi masofalar o’zgarishi mumkin Harakat davomida muhit nuqtalari orasidagi masofa o’zgarmasa, bunday harakat nazariy mexanikada o’rganiladi.

Shunday qilib, yuqorida uchta fundamental farazlar kiritildi, ulardan foydalanib deformatsiyalanadigan jismlarning harakatlanish nazariyasi quriladi. Ushbu farazlarga asoslangan nazariyadan xulosalar ko'pincha, har doim ham tajribaga mos kelmaydi. Agar kerak bo'lsa, makon va vaqtning qabul qilingan modelini aniqlashtirish va umumlashtirish mumkin. Biroq, barcha keyingi umumlashmalar, yuqorida tavsiflangan fundamental farazlarga asoslanib, Nyuton mexanikasiga asoslangan.

2 §. Hamroh koordinata sistemasi.

Har qanday harakat kabi, kontinuum harakati har doim ba'zi koordinatalar sistemasi x^1, x^2, x^3 – kuzatuvchi sanoq sistemasiga nisbatan aniqlanadi. Bu koordinatalar sistemasi ihtiyoriy ravishda tanlanishi mumkin. U shart asosida kiritiladi, va uning tanlovi tadqiqotchiga bog'liq. Amalda u ko'pincha Yer bilan bog'liq, biroq shuningdek, Quyosh, yulduzlar, samolyot, vagon va boshqalar bilan ham bog'liq bo'lishi mumkin. Uni kiritish mazmuniga ko'ra, u harakatli yoki harakatsiz bo'lishi mumkin.

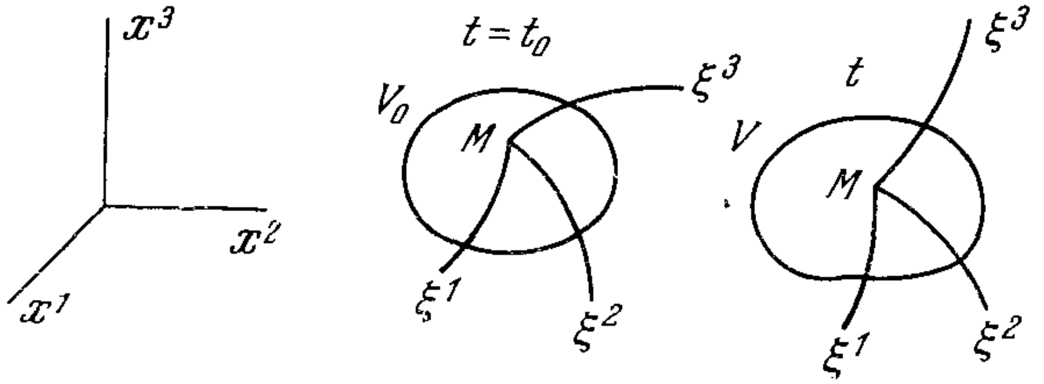
Nyuton mexanikasida, bir-biriga nisbatan parellel ravishda vaqt bo'yicha doimiy tezlik bilan harakatlanuvchi koordinatalarning inersial sistemalariga nisbatan harakatlanishi alohida fizik ahamiyatga ega. Bunday koordinata sistemalarining mavjudligi mexanikasining asosiy postulati, ya'ni isbot talab qilmaydigan qoidasi bo'lib hisoblanadi.

Amaliyotda, hayotda inersial koordinatalar sistemasi sifatida, uzoq yulduzlarni harakatsiz deb hisoblash mumkin bo'lgan Dekart koordinatalar sistemasini tanlash mumkin.

Bulardan tashqari tutash muhit harakatini ifodolovchi koordinatalar sistemasini kiritish lozim. x^1, x^2, x^3 koordinatalari bilan bir qatorda, ξ^1, ξ^2, ξ^3 nuqtalarning Lagranj koordinatalarini fazoning huddi shu nuqtalarining D sohasidagi boshqa koordinatalari tariqasida ko'rib chiqish mumkin.

Ta'rif: ξ^1, ξ^2, ξ^3 koordinatalar sistemasiga mos shu fazoda harakatclanuvchi, deformatsiyalanuvchi, egri chiziqli koordinatalar sistemasi hamroh koordinatalar sistemasi deb ataladi.

Agarda t_0 boshlang'ich vaqtda tutash muhitda, tutash muhit nuqtalaridan tashkil bo'lgan ξ^1, ξ^2, ξ^3 ayrim koordinata chiziqlari tanlansa, keyingi vaqtda ular kontinum nuqtalari bilan birgalikda qaytadan hamroh koordinata sistemasining chiziqlariga o'tadi. Lekin, boshlang'ich vaqtda ular to'g'ri chiziq bo'lib tanlangan bo'lsa, keyingi vaqtda ular egri chiziqqa aylanadi (5- rasm)



5-rasm. x^1, x^2, x^3- sanoq sistemasi, ξ^1, ξ^2, ξ^3-hamroh koordinata sistemasi.

Shunday qilib, koordinatalar sistemasi tutash muhit zarrachalari bilan bog'liq holda ko'rib chiqilsa, u vaqt o'tishi bilan o'zgarishi kuzatiladi. Bunday koordinatalar sistemasini tanlash istalgan vaqtda bizning qo'limizda, biroq vaqtning keyingi lahzalarida u bizning qo'limizda bo'lmaydi, chunki u muhitga "mustahkam kiritilgan" bo'lib, u bilan birgalikda o'zgaradi. Muhitga kiritilgan aynan bunday koordinatalar sistemasi yuqorida hamroh koordinatalar sistemasi deb belgilandi. Tutash muhitning barcha nuqtalari har doim harakatchan hamroh koordinatalar sistemasiga nisbatan joylashadi, chunki ularning ξ^1, ξ^2, ξ^3 koordinatalari hamroh sistemada o'zgarmaydi. Lekin sistemaning o'zi harakatlanadi, cho'ziladi, qisqaradi, buraladi va hokazo.

Biz tutash muhitning harakatlanishi haqida doimo gapirganimizda, nuqtalarni alohida ajratishimiz va lagranj koordinatalaridan foydalanishimiz zarur.

Shu sababli, doim tutash muhitning harakatlanishini ko'rib chiqqanimizda x^1, x^2, x^3 sanoq sistemasining mavjudligi nazarda tutilib, unga nisbatan hamroh koordinatalar sistemasining harakatlanishi ham ko'rib chiqiladi.

Erkin o'zgaruvchilar ξ^1, ξ^2, ξ^3 va t ning qo'llanilishi Lagranjning tutash muhit harakatlanishini o'rganishdagi nazariyasi bo'yicha, tutash muhitning har bir nuqtasining harakatlanishini alohida o'rganish kerak. Amaliyotda bunday ta'riflash odatda haddan tashqari batafsil va mukammal bo'ladi, biroq u doimo fizika qonunlarini shakllantirishda nazarda tutiladi. Tutash muhit harakatlanishiga ta'rif berish uchun, harakat qonunining tushunchasidan tashqari yana boshqa tushunchalarni, hususan, tezlik va tutash muhit nuqtalarining tezlashish tushunchalarini ham kiritish zarur.

Aytaylik, tutash muhitning ihtiyoriy nuqtasi t vaqtda fazoning M nuqtasida, $t+\Delta t$ vaqtda esa – M' *va* $M\,M'= \Delta r$ nuqtasida joylashgan bo'lsin. Δr Δt vaqtda tutash muhit nuqtasining yo'naltirilgan harakatidir. Fazoda r radius-vektorini kiritish mumkin bo'lsa, Δr qaralayotgan radius-vektorning orttirmasidir.

Ta'rif: Yevklid fazosida radius-vektordan vaqt bo'yicha olingan xususiy hosila $\frac{\partial \vec{r}}{\partial t}$ tutash muhit nuqtasining tezligi deb ataladi. Tezlik vektorini $\vec{v}$harf bilan belgilanadi.

$$\vec{v} = \frac{\partial \vec{r}}{\partial t}$$

Tezlik sanoq sistemasiga nisbatan hisoblanadi. Demak, hamroh koordinatalar sistemasiga nisbatan tezlik doimo nolga teng. Tutash muhitlar mexanikasida tezlikdan tashqari tezlanish ham o'rganiladi va u quyidagicha ifodalanadi:

$$\vec{a} = (\frac{\partial \vec{v}}{\partial t})_{\xi^i} = \vec{a}^i \ni_i$$

bu yerda $a^i = a^i(\xi^1, \xi^2, \xi^3, t)$ – tezlanish komponentalari. a tezlanish, ϑ tezlik kabilar, tutash muhitning individual nuqtasi uchun hisoblab chiqiladi. Tezlanishning aniqlanishi kuzatuvchining x^1, x^2, x^3 koordinatalar tizimini tanlash bilan bog'liq bo'lib, unda harakat qonuni o'rganiladi.

3 §. Muhitning harakat tenglamasi. Tutash muhit harakatini tavsiflashning Lagranj va Eyler usullari.

Tutash muhit uch o'lchovli Yevklid fazosida joylashgan va unda harakatlanadi deylik. Uning kinematikasini o'rganaylik. Shu maqsadda tutash muhit tushunchasi, asosiy farazlarini eslaylik.

Ushbu $|\vec{e}_i| = 1$ va $\vec{e}_i \cdot \vec{e}_j = \delta_i^j = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$

bazis vektorni kiritaylik.

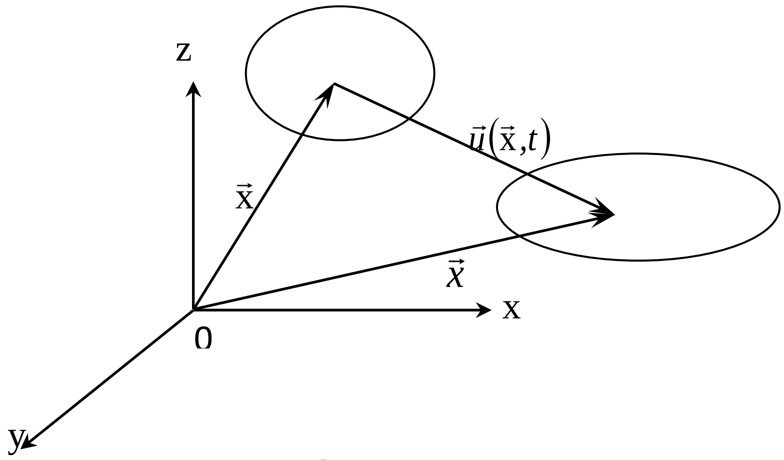
Tutash muhitning t_0 - dastlabki paytdagi holatini $\vec{X}(X^1, X^2, X^3)$ koordinatalar bilan va ixtiyoriy $t \geq t_0$ momentdagi koordinatalarini $\vec{x}\{x^1, x^2, x^3\}$ lar bilan belgilaylik. Bunda t_0 momentda tutash muhit egallagan τ_0 hajm $t \geq t_0$ momentda τ hajmni egallaydi (6-rasm).

Quyidagi ifodalarni keltirilgan shakl asosida yozaylik:

$$\vec{X} = X^k \cdot \vec{e}_k \tag{3.3.1}$$

$$\vec{x} = x^k \cdot \vec{e}_k \tag{3.3.2}$$

bu yerda X^k - tutash muhit zarrasi, fizik *M* nuqtaning dastlabki paytdagi ortogonal Dekart koordinatalaridir. x^k - shu fizik *M* nuqtaning bir oz vaqt o'tgach, ya'ni $t \geq t_0$ momentdagi ortogonal Dekart koordinatalaridir.



6-rasm

«Fizik *M* nuqta» deganimizda tutash muhit gipotezasiga ko'ra barcha geometrik nuqtalar va ularda «joylashgan» muhit zarralari tasavvur etiladi.

Shaklda tasvirlangani kabi, har bir $\vec{x}$ o'zining $t = t_0$ momentga mos kelgan dastlabki holatlari - fizik nuqtalariga ega, hamda ularga bog'liq bo'ladi. t_0 momentda ixtiyoriy olingan *M* nuqta τ_0 hajmdagi tutash muhitning ixtiyoriy fizik nuqtasi bo'la oladi va har bir *M* nuqtalar tutash muhit harakati tufayli biror t momentda τ hajmga tegishli nuqtani egallaydi.

Shunday qilib, quyidagini yoza olamiz:

$$x^\alpha = x^\alpha\{X^1, X^2, X^3, t\}, \ (\alpha = 1,2,3) \tag{3.3.3}$$

(3.3.3) o’rniga ushbu vektor tenglamani yozish mumkin:

$$\vec{x} = \vec{x}(\vec{X}, t) \tag{3.3.4}$$

(3.3.3) (yoki (3.3.4)) $\vec{X}$ nuqtaning harakat qonuni deyiladi. TMMning asosiy vazifasi shu harakat qonunini topish, uni o’rganishdir. Ikkinchi tomondan, (3.3.4) ga matematik nuqtayi nazardan qaralganda, undan $\vec{X}$ ni $\vec{x}$ ning funksiyasi sifatida topish masalasini qo’yish mumkin. $\vec{x}$ va $\vec{X}$ lar o’rtasida har bir $t \geq t_0$ uchun bir qiymatli akslantirish mavjudligi uchun ushbu yakobian noldan farqli bo’lishi kerak:

$$\begin{vmatrix} \frac{\partial x^1}{\partial X^1} & \frac{\partial x^1}{\partial X^2} & \frac{\partial x^1}{\partial X^3} \\ \frac{\partial x^2}{\partial X^1} & \frac{\partial x^2}{\partial X^2} & \frac{\partial x^2}{\partial X^3} \\ \frac{\partial x^3}{\partial X^1} & \frac{\partial x^3}{\partial X^2} & \frac{\partial x^3}{\partial X^3} \end{vmatrix} \neq 0$$

Bu shart bajarilsa, (3.3.4) dan yoza olamiz: $\vec{X} = \vec{X}(\vec{x}, t)$ ya’ni:

$$\begin{aligned} X^1 &= X^1\{x^1, x^2, x^3, t\} \\ X^2 &= X^2\{x^1, x^2, x^3, t\} \\ X^3 &= X^3\{x^1, x^2, x^3, t\} \end{aligned} \tag{3.3.5}$$

(3.3.5) dan (3.3.3) yoki (3.3.4) ga o’tish uchun o’tish ko’rilayotgan $t = t_0$ momentda:

$$\begin{vmatrix} \frac{\partial X^1}{\partial x^1} & \frac{\partial X^1}{\partial x^2} & \frac{\partial X^1}{\partial x^3} \\ \frac{\partial X^2}{\partial x^1} & \frac{\partial X^2}{\partial x^2} & \frac{\partial X^2}{\partial x^3} \\ \frac{\partial X^3}{\partial x^1} & \frac{\partial X^3}{\partial x^2} & \frac{\partial X^3}{\partial x^3} \end{vmatrix} \neq 0$$

bajarilishi kerak.

Tutash muhit harakati o’rganilayotganda ayrim $t \geq t_0$ onda yuqorida keltirilgan yakobianlar qiymati nolga teng bo’lishi mumkin. Yakobianlar qiymati ayrim nuqta yoki nuqtalarda, chiziqlarda, sirtlarda nolga teng bo’lib qolishi mumkin. Bunday nuqtalar kritik nuqtalar yoki kritik nuqtalar to'plami deyiladi. Biz tekshirishlarimizda yakobianlar noldan farqli deb faraz qilamiz.

6-rasmdan quyidagini yoza olamiz:

$$\vec{x} = \vec{X} + \vec{u} \tag{3.3.6}$$

$\vec{u}(X^1, X^2, X^3, t)$ vektor ko'chish vektori deyiladi. M nuqtaning harakatini o'rganganda, uning uchun (3.3.4) formula, $\vec{X}$ miqdor vaqt o'tishi bilan har bir individual fizik zarra uchun o'zgarmasligi sababli ular t ga bog'liq emas. Shuni e'tiborga olsak, ko'chish $\vec{u}$, tezlik $\vec{V}$ va tezlanish $\vec{W}$ ni istalgan t ondagi ifodasi mos ravishda quyidagicha bo'ladi:

$$\vec{u} = \vec{x} - \vec{X} \tag{3.3.7}$$

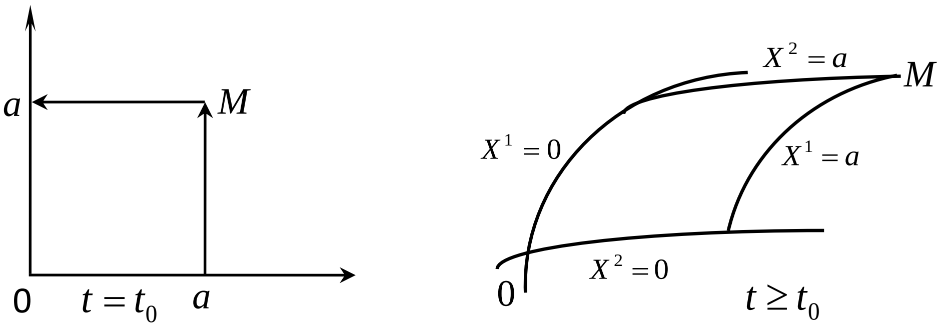
$$\vec{V} = \frac{\partial \vec{x}}{\partial t} \tag{3.3.8}$$

$$\vec{W} = \frac{\partial^2 \vec{x}}{\partial t^2} \tag{3.3.9}$$

$\vec{u}$, $\vec{V}$, $\vec{W}$ lar $\vec{X}$ va t ga bog'liq bo'lib, uning funksiyalaridir:

$$\begin{aligned} \vec{u} &= \vec{u}\{X^1, X^2, X^3, t\} \\ \vec{V} &= \vec{V}\{X^1, X^2, X^3, t\} \\ \vec{W} &= \vec{W}\{X^1, X^2, X^3, t\} \end{aligned} \tag{3.3.10}$$

Yuqorida foydalanilgan $\vec{X}$, t lar Lagranj koordinatalari deyiladi. Ular har bir individual fizik nuqta uchun $t = t_0$ momentda olgan qiymatlarini istalgan $t \geq t_0$ momentda saqlab qolgani sababli tutash muhitning $t \geq t_0$ momentdagi holati uning dastlabki holati bilan aniqlanadi. Lagranj koordinatalari $t = t_0$ da ortogonal Dekart koordinatasi sifatida qabul qilingan edi. Bu paytda metrik tenzor $\vec{e}_i \cdot \vec{e}_j = \delta_{ij}$ dan iborat bo'lsa ham, $t \geq t_0$ da Lagranj koordinatalari egri chiziqli koordinatalarga aylanadi va olinayotgan Yevklid fazosida metrik tenzor turlicha bo'lishi mumkin. Umuman aytganda, $t = t_0$ momentda olingan ortogonal koordinatalar sistemasi va u bilan bog'liq bo'lgan δ_{ij} metrik tenzor olinishi shart emas. Dastlabki $t = t_0$ momentda ixtiyoriy ortogonal bo'lmagan ma'lum metrik tenzorga ega koordinata sistemasini tanlab olish mumkin edi. Uni odatda $\dot{g}_{ij}$ bilan belgilash mumkin. Lekin umumiyatga zarar keltirmasdan TMM masalalarini o'rganganda dastlabki vaqtda $g_{ij}\big|_{t=t_0} = \delta_{ij}$ metrik tenzorga ega ortogonal Dekart koordinata sistemasini ishlatish qulay bo'ladi. Misol sifatida ushbu $0 \le X^1 \le a,\ 0 \le X^2 \le a$ Lagranj koordinatalarini tekislikda ko'raylik (7-rasm).



7-rasm

Lagranj koordinatalari tabiiy ravishda o'zgaradi:

$$(a, a, 0) \to (a, a, 0)$$
$$t = t_0 \ \to t > t_0$$

Biror bir F miqdor fizik zarra uchun Lagranj koordinatalarida aniqlangan bo'lsa (skalyar yoki vektor miqdor), uning o'zgarish tezligi

$$\frac{\partial F}{\partial t} = \frac{\partial F(\vec{X}, t)}{\partial t}$$

bo'ladi.

$F(\vec{X}, t)$ ning t momentda olingan ikki cheksiz kichik masofadagi X^k va $X^k + dX^k$ nuqtalardagi farqi quyidagicha bo'ladi:

$$\frac{\partial F}{\partial X^k} d\, X^k .$$

4 §. Lagranj va Eyler koordinatalariga o'zaro o'tish

Tutash muhit harakatini ixtiyoriy t momentda ortogonal Dekart koordinata sistemasida ko'raylik. Fazoning $\vec{x}$ - radius vektorini olaylik va uni harakat davomida o'zgartirmaylik. U holda radius-vektor belgilangan fazoning nuqtasi va ular to'plamiga turli parametrlar bilan xarakterlanuvchi turli tutash muhit zarralari o'tishini kuzatish mumkin. $\vec{x}$ nuqtada turli vaqtlarda turlicha fizik zarralar o'tadi. Demak, bu holda har bir individual fizik zarra harakati kuzatilmaydi, balki fazoning ayrim bo'lagiga kelayotgan va undan o'tib ketayotgan zarralar kuzatiladi, aniqrog'i, shu fazoning har bir nuqtasiga kelayotgan va undan o'tib ketayotgan fizik zarralar harakati kuzatiladi. Shunday qilib, $\vec{x}$ o'zgarmas fazosiga bog'liq maydon bilan ish ko'rishga to'g'ri keladi. Masalan, $\vec{V}(\vec{x},t)$- tezlik maydoni. Bu maydon Eyler maydoni deyiladi. $\vec{x}$ - esa Eyler koordinatasi deyiladi. Har bir $\vec{x}=const$ zarra bu maydonda o'z trayektoriyasini «chizib» o'tadi. Agar shu maydonga tegishli $\vec{V}(\vec{x},t)$ ma'lum bo'lsa, t momentda $\vec{x}$ nuqtadagi fizik zarra $\vec{V}(\vec{x},t)$ tezlikka va kichik dt vaqtda shu fizik nuqta $\vec{V}(\vec{x},t)dt$ ko'chishga ega bo'lishini ko'rish qiyin emas. Bu ko'chishni $d\vec{x}$ desak:

$$d\vec{x}=\vec{V}(\vec{x},t)dt$$

bo'ladi.

Bundan

$$\frac{dx^k}{dt}=V^k(x^1,x^2,x^3,t) \tag{3.4.1}$$

Faraz qilaylik, (2.11) differensial tenglamalar sistemasi integrallari topilgan deylik:

$$x^k=x^k(c_1,c_2,c_3,t) \tag{3.4.2}$$

dastlabki paytda esa fizik zarralar holati ma'lum bo'lgan va ular X^k lar bilan belgilangan deylik. U holda

$$x^k\Big|_{t=t_0}=X^k$$

c_1,c_2,c_3 lar X^k orqali ifodalanadi.

U holda

$$X^k=X^k(\vec{X},t) \tag{3.4.3}$$

Demak, x^1, x^2, x^3, t Eyler koordinatadan X^1, X^2, X^3, t Lagranj koordinatalariga, agar Eyler maydoni va undagi $V^k(x^1, x^2, x^3, t)$ ma'lum bo'lsa, o'tish formulalarini ko'rdik.

Shuningdek, Lagranj koordinatalaridan Eyler koordinatalariga ham o'tish mumkin. Masalan, $\vec{u} = \vec{u}(\vec{X}, t)$ ma'lum deylik (ya'ni ko'chish Lagranj koordinatalarida berilgan) va $\vec{V} = \vec{V}(\vec{x}, t)$ topilishi talab qilingan deylik.

Quyidagilarni yoza olamiz:

$$\vec{x} = \vec{X} + \vec{u}(\vec{X}, t) = \vec{x}(\vec{X}, t)$$

$$\vec{V} = \frac{\partial \vec{u}}{\partial t} = \vec{V}(\vec{X}, t)$$

Bundan

$$\vec{V} = \vec{V}\left\lfloor \vec{X}(\vec{x}, t), t \right\rfloor = \vec{V}(\vec{x}, t)$$

kelib chiqadi.

Eyler koordinata sistemasida tezlik va tezlanish. Turli nuqtalar tezligi $\vec{V} = \vec{V}(x^1, x^2, x^3, t)$. Agar $\vec{x} = \vec{x}(\vec{X}, t)$ ma'lum bo'lsa va uni yuqoriga qo'yilsa, $\vec{V} = \vec{V}(x^1(\vec{X}, t), x^2(\vec{X}, t), x^3(\vec{X}, t), t)$ $\vec{X}$ nuqta uchun topiladi. Agar

$$V^k = V^k(x^i, t)$$

bo'lsa,

$$\frac{\partial V^k}{\partial t}$$

tezlanish bo'la olmaydi.

Tutash muhit fizik zarrasi tezlanishi $\vec{W} = W^i \cdot \vec{e}_i$ bo'lib,

$$W^i = \frac{dV^i}{dt} = \frac{\partial V^i}{\partial t} + \frac{\partial V^i}{\partial x^j} \cdot \frac{\partial x^j}{\partial t} = \frac{\partial V^i}{\partial t} + V^j \cdot \frac{\partial V^i}{\partial x^j}$$

bo'ladi.

Har ikki usul o'zaro ekvivalent va ulardan foydalaniladi. Yuqorida ko'rganimizdek, bir koordinata sistemasidan ikkinchisiga yoki aksincha o'tish mumkin. TMMda masalaning qo'yilishi va tutash muhit modellariga qarab u yoki bu koordinatalardan foydalaniladi. Odatda, suyuqlik harakatini o'rganishda Eyler koordinatalari ko'p ishlatilsa, deformatsiyalanuvchi qattiq jism uchun esa Lagranj koordinatalari qo'l keladi. Ayrim masalalarni yechishda bu an'ana buzilishi ham mumkin.

5 §. Skalyar va vektor maydonlar va ularning ayrim xossalari.

Ta'rif. Har bir nuqtasida skalyar miqdor aniqlangan soha skalyar maydon deb ataladi.

Ta ‘rif. Fazodagi biror D sohaning har bir M nuqtasiga aniq qonun bo‘yicha biror u(M) son mos qo ‘yilgan bo'lsa, bu sohada u=u(M) skalyar maydon berilgan deyiladi. D soha sifatida fazoning biror bo‘lagi, sirti yoki chizig'i bo'lishi mumkin.

Agar skalyar maydon sohaning barcha nuqtalarida bir xil bo‘lsa, bunday maydonni bir jinsli maydon deyiladi. Agar skalyar maydonning qiymati bir nuqtadan boshqa nuqtaga ko‘chganda o‘zgarsa bunday maydonga bir jinssiz maydon deymiz.

Ba'zan skalyar maydonning qiymati vaqtga qarab ham o'zgarib borishi mumkin. Masalan, qizdirilgan jism temperaturasi tashqi muhit temperaturasiga qarab o‘zgaradi. Bunday maydonlar nostasionar skalyar maydonlarni tashkil qiladi. Agar skalyar maydon vaqtga bog‘liq bo'lmasa bunday maydonlarni stasionar (barqaror) maydonlar deyiladi.

Skalyar - har bir nuqtada yagona qiymatga ega bo'ladi, ya'ni u koordinatalarning bir qiymatli funktsiyasidir:

$$\varphi = \varphi\left(x^1,\, x^2,\, x^3\right)$$

skalyar miqdor bir xil qiymatga ega bo'lgan nuqtalar ushbu

$$\varphi\left(x^1,\, x^2,\, x^3\right) = const \qquad (3.5.1)$$

shartdan topiladi. Bu shart fazoda sirt tenglamasini beradi. Ushbu sirt ekvipotensial sirt yoki saviya (daraja) sirti deb ataladi. Konstantaga har xil qiymatlar berilsa mazkur sirtlarning oilasi hosil bo'ladi. Fazoning har bir nuqtasidan yagona ekvipotensial sirti o'tadi.

Ta’rif. Har bir nuqtasida vektor miqdor aniqlangan soha vektor maydon deb ataladi.

Demak, bu maydonda $\overline{a} = \overline{a}\left(x^1,\ x^2,\ x^3\right)$ bo’ladi.

Demak, maydon skalyar yoki vektor maydondan iborat bo’lishi mumkin ekan. Bunga ko’pgina misollar keltirish mumkin. Tutash muhit mexanikasida tezlik vektori va fazoning bo’lagini uzluksiz egallagan tutash muhit zarralari uchun tekshirilganda tezlik maydoni tushunchasiga kelamiz: har bir tutash muhit zarrasi har bir onda o’z tezligiga ega va bu tezlik vektori fazoning (masalan, Eyler koordinatalarida) va vaqtning funksiyasi sifatida tezlik maydonini tashkil etadi.

Endi vektorning sirkulyatsiyasini ko’raylik. Biror vaqt uchun harakat jarayoni davomida tutash muhit egallagan fazoga tegishli *A* va *B* nuqtalar bilan birga shu nuqtalarni tutashtiruvchi ixtiyoriy o’zaro kesishmaydigan egri chiziq olaylik. Tutash muhit harakati tufayli ko’rilayotgan onda chiziqlarning barcha nuqtalarida $\vec{V}$ tezlik vektorini chizish mumkin. Kuch maydonida bajariladigan ishni hisoblash kabi, ushbu $\int_A^B \left(\vec{V} \cdot d\vec{s}\right)$ ifodani tuzaylik. Bu ifodaning qiymati skalyar miqdor va u tezlik vektoridan berilgan chiziq bo’yicha olingan sirkulyatsiya deyiladi. Agar ko’rilayotgan chiziq yopiq chiziqdan iborat bo’lsa, (ya’ni *A* va *B* nuqtalar ustma-ust tushsa) tezlik vektori sirkulyatsiyasi quyidagicha yoziladi:

$$\oint_C \left(\vec{V} \cdot d\vec{s}\right)$$

bu yerda *S* - yopiq konturdir.

Endi potensialli maydon tushunchasini eslaylik. Agar ushbu $\vec{a} = \vec{a}\left(x^1, x^2, x^3, t\right)$ vektor maydoni berilgan bo’lib, bu maydon uchun shunday $\varphi\left(x^1, x^2, x^3, t\right)$ funksiya topish mumkin bo’lsaki, natijada $\vec{a}\left\{a_x, a_y, a_z\right\}$ uchun

$$a_x = \frac{\partial\varphi}{\partial x^1},\ a_y = \frac{\partial\varphi}{\partial x^2},\ a_z = \frac{\partial\varphi}{\partial x^3}$$

o’rinli bo’lsa, berilgan maydon potensialli maydon deyiladi. U holda $\varphi\left(x^1,x^2,x^3,t\right)$ funksiya maydon potensiali deyiladi. Ko’rish qiyin emaski, potensialli maydon uchun quyidagi tenglikning bajarilishini ko’rish qiyin emas:

$$\frac{\partial a_x}{\partial x^2}=\frac{\partial a_y}{\partial x^1},\ \frac{\partial a_x}{\partial x^3}=\frac{\partial a_z}{\partial x^1},\ \frac{\partial a_y}{\partial x^3}=\frac{\partial a_z}{\partial x^2}$$

Agar potensialli tezlik maydoni uchun sirkulyatsiya ko’riladigan bo’lsa, quyidagini yoza olamiz:

$$\int_A^B \vec{V}\cdot d\vec{s}=\int_A^B grad\varphi\cdot d\vec{s}=\int_A^B d\varphi=\varphi_B-\varphi_A .$$

Bu yerda potensialli maydonda yopiq kontur bo’yicha olingan tezlik vektori sirkulasiyasi nolga tengligi, A va B nuqtalarni tutashtiruvchi chiziq esa bu chiziqning tuzilishiga bog’liq bo’lmay, sirkulyatsiya ko’rilayotgan nuqtalardagi potensiallar ayirmasiga tengligi isbotlanadi.

Endi tutash muhit tezligi bilan bog’liq bo’lgan tok chiziqlarini o’rganaylik. Tok chizig’i deb tutash muhit harakati jarayonida biror t onda o’tkazilgan shunday chiziqqa aytiladiki, bu chiziq har bir nuqtasiga o’tkazilgan urinma yo’nalishi shu nuqtasi uchun t onda tezlik yo’nalishi bilan bir xil bo’lishi kerak. Agar bu ta’rifimizda $\vec{V}$ o’rniga $\vec{\omega}=2rot\vec{V}$ ko’rilsa, ko’rilayotgan chiziq uyurma chizig’i deyiladi. Tok chizig’i va uyurma chiziqlaridan tuzilgan (ular joylashgan) sirtlar mos ravishda tok sirtlari va uyurma sirtlari deyiladi.

Tok chiziqlarini berilgan tezlik maydoni bo’yicha tuzish uchun t va $t+\Delta t$ onlarni olaylik. Bunda $\vec{V}\left(x^1,x^2,x^3,t\right)$ uchun $d\vec{r}=\vec{V}\left(x^1,x^2,x^3,t\right)dt$ bo’ladi. Umuman olganda, tok chizig’i uchun dt o’rniga $d\lambda$ ni ham olish mumkin.

Agar $\vec{V}$ vaqtga bog’liq bo’lmasa, maydon statsionar, aks holda statsionar bo’lmagan maydon deyiladi. Demak, statsionar bo’lgan maydonda tok chiziqlari tutash muhit zarralari trayektoriyalaridan farq qilmaydi. Statsionar bo’lmagan harakat uchun esa ular umuman olganda turlicha chiziqlardan iborat bo’ladi.

Endi dekart koordinatalari sistemasida biror ondagi tezlik maydoni gradiyenti $\frac{\partial V_i}{\partial x_j}$ bilan ish ko'raylik. Bu tenzorni simmetrik va antisimmetrik tenzorlar yig'indisi sifatida yozish mumkin:

$$\frac{\partial V_i}{\partial x_j}=\frac{1}{2}\left(\frac{\partial V_i}{\partial x_j}+\frac{\partial V_j}{\partial x_i}\right)+\frac{1}{2}\left(\frac{\partial V_i}{\partial x_j}-\frac{\partial V_j}{\partial x_i}\right)=D_{ij}+\mathrm{V}_{ij}$$

$D_{ij}=\frac{1}{2}\left(\frac{\partial V_i}{\partial x_j}+\frac{\partial V_{ji}}{\partial x_i}\right)$-simmetrik tenzor bo'lib, deformatsiya tezligi tenzoridir,

$\mathrm{V}_{ij}=\frac{1}{2}\left(\frac{\partial V_i}{\partial x_j}-\frac{\partial V_j}{\partial x_i}\right)$-antisimmetrik tenzor uyurma tenzori deyiladi.

Masalalar yechishga doir namuna

1. Tutash muhit harakati Lagranj o'zgaruvchilarida

$$x_1=\xi_1e^t+\xi_3(e^t-1);\ \ x_2=\xi_2+\xi_3(e^t-e^{-t});\ \ x_3=\xi_3$$

ko'rinishda berilgan. Uni Eyler ko'rinishga o'tkazing. Lagranj va Eyler koordinatalarida ko'chish vektori komponentalarini toping. Lagranj koordinatalarida tezlik, tezlanish, deformatsiya va deformatsiya tezliklari tenzori komponentalarini toping.

Yechish. Harakatni Eyler ko'rinishida ifodalash uchun dastlab Yakobianning noldan farqli ekanligini quyidagicha tekshiramiz

$$J=\left|\frac{\partial x_i}{\partial \xi_j}\right|=\begin{vmatrix}e^t & 0 & e^t-1\\ 0 & 1 & e^t-e^{-t}\\ 0 & 0 & 1\end{vmatrix}=e^t\neq 0$$

Demak, Yakobian noldan farqli Lagranj koordinatalaridan Eyler koordinatalariga o'tish mumkin.

$x_1=\xi_1e^t+\xi_3(e^t-1);\ \ x_2=\xi_2+\xi_3(e^t-e^{-t});\ \ x_3=\xi_3$ tenglamalarni e_j larga nisbatan quyidagicha yechamiz,

$$\xi_1=x_1e^{-t}-x_3(1-e^{-t});$$
$$\xi_2=x_2-x_3(e^t-e^{-t});$$
$$\xi_3=x_3.$$

Endi ko’chish vektori komponentalarini topamiz. $w_i = x_i - \xi_i$ formulaga ko’ra Lagranj koordinatalarida

$$w_1 = \xi_1(e^t - 1) + \xi_3(e^t - 1);$$
$$w_2 = \xi_3(e^t - e^{-t});$$
$$w_3 = 0.$$

Eyler koordinatalarida esa quyidagiga teng bo’ladi.

$$w_1 = x_1(1 - e^{-t}) + x_3(1 - e^{-t});$$
$$w_2 = x_3(e^t - e^{-t});$$
$$w_3 = 0.$$

Endi tezlik, tezlanish komponentalarini topamiz:

Tezlik vektori komponentalari

$$v_1 = \xi_1(e^t + 1) + \xi_3(e^t - 1);$$
$$v_2 = \xi_3(e^t + e^{-t});$$
$$v_3 = 0.$$

Tezlanish vektori komponentalari

$$a_1 = x_1(e^t + 1) + x_3(e^t - 1);$$
$$a_2 = x_3(e^t + e^{-t});$$

$a_3 = 0$ ga teng bo’ladi.

Deformatsiya tenzori komponentalari quyidagiga,

$$\varepsilon_{11} = (e^t + 1), \varepsilon_{22} = 0, \varepsilon_{33} = 0, \varepsilon_{12} = \varepsilon_{21} = 0, \varepsilon_{13} = \varepsilon_{31} = (e^t - 1), \varepsilon_{23} = \varepsilon_{32} = (e^t - e^{-t}).$$

Deformatsiya tezliklari tenzori komponentalari esa shunday aniqlanadi,

$$e_{11} = (e^t + 1), e_{22} = 0, e_{33} = 0, e_{12} = e_{21} = 0, e_{13} = e_{31} = (e^t - 1), e_{23} = e_{32} = (e^t - e^{-t}).$$

III bobga doir masalalar.

2-8 masalalar uchun ko’chish vektori komponentalarini, tezlik vektori komponentalarini, tezlanish vektori komponentalarini va deformatsiya tezliklari tenzori komponentalarini toping.

2. $x_1 = \xi_1;\ x_2 = \xi_2 e^{-t} + A\xi_3;\ x_3 = \xi_3 e^{-t} + A\xi_2$,
3. $x_1 = \xi_1 A + \xi_3 e^{-t};\ x_2 = \xi_2 e^{kt};\ x_3 = \xi_3 + A\xi_1,$
4. $x_1 = \xi_1 e^t + A\xi_2;\ x_2 = A\xi_1;\ x_3 = \xi_3 e^{-t},$
5. $x_1 = \xi_1 + \xi_3(e^t - 1);\ x_2 = \xi_2 + \xi_3(e^{2t} - e^{-2t});\ x_3 = e^t\xi_3 + A\xi_2,$
6. $x_1 = 3\xi_1\xi_3 e^{-t^2};\ x_2 = 2\xi_1\xi_2 e^{-t^2};\ x_3 = 5\xi_1\xi_2 e^{-t^2},$
7. $x_1 = (3\xi_2 - 4\xi_3)e^{-t};\ x_2 = (2\xi_1 - \xi_3)e^{-t};\ x_3 = (4\xi_2 - \xi_1)e^{-t},$
8. $x_1 = k\xi_1\xi_3 e^{-t}\ ;\ x_2 = k\xi_1\xi_2 e^t\ ;\ x_3 = k\xi_2\xi_3.$

9. Zarrachalarning fazoviy koordinatalar sistemasini, Lagranj koodinatalarini kiriting hamda quyidagi hollarda harakat qonunini toping:

a) Qattiq jism yo'nalishi va son qiymati o'zgarmas bo'lgan $\boldsymbol{v}$ tezlik bilan harakatlanmoqda;

b) Qattiq jism qo'zg'almas o'q atrofida o'zgarmas $\boldsymbol{\omega}$ burchak tezlik bilan aylanmoqda.

10. Qattiq jismni ilgarilanma harakati uchun Lagranj tavsifidagi tezlik maydonini va harakat qonunini umumiy ko'rinishini keltiring.

11. Muxitning harakati quyidagi qonun bo'yicha sodir bo'lmoqda:

$$x_1 = \xi_1\left(1 + \frac{t}{\tau}\right),\ \ x_2 = \xi_2 + bt\xi_1,\ \ x_3 = \xi_3,\ \ a, b = const$$

Xususiy zarra uchun (ξ_1, ξ_2, ξ_3) sonlari t=0 bo'lganda zarracha turgan nuqtaning fazoviy koordinatalari (x_1, x_2, x_3) ga mos kelishini tekshiring. Tezlik va tezlanish maydonlarini Lagranj tavsifida toping. Vaqt t_0 bo'lganda qanday zarracha fazoning (x_{01}, x_{02}, x_{03}) koordinatasida joyashgan bo'ladi.

12. Muxit quyidagi qonun boyicha harakatlanmoqda

$$x_1 = \xi_1\left(1 + \frac{t}{\tau}\right), x_2 = \xi_2\left(1 + 2\frac{t}{\tau}\right), x_3 = \xi_3\left(1 + \frac{t^2}{\tau^2}\right), \tau = const$$

$a)$ Lagranj ta'rifida tezlik va tezlanish maydonlarini toping;

b) $t = \tau$ vaqtda fazoning koordinatalari (a, b, c) bo'lgan nuqtasidagi zarracha $t = 3\tau$ vaqtda qayerda joylashadi.

13. Vaqtning *t* momentida harakat qonuni

$$x_i = f_i(\xi_1, \xi_2, \xi_3, t), \qquad i = 1,2,3$$

ga teskari funksiya

$$\xi_\alpha = g_\alpha(x_1, x_2, x_3, t),\ \ \alpha = 1,2,3$$

ko'rilyapti. ξ_1, ξ_2, ξ_3 larning ma'nosi nimadan iborat? $\frac{d\xi_\alpha}{dt}$ xususiy xosilalar nimaga teng?

14. Agar muxit harakati quyidagi qonunlar boyicha sodir bo'layotgan bo'lsa, Lagranj va Eyler tavsifida tezlik va tezlanish maydonlarini toping:

a) $x_1 = a(t)\xi_1,\ \ x_2 = b(t)\xi_2,\ x_3 = c(t)\xi_3;$

b) $x_1 = \xi_1 + b(t)\xi_2,\ x_2 = \xi_2,\ \ x_3 = \xi_3;$

d) $x_i = A_{i1}(t)\xi_1 + A_{i2}(t)\xi_2 + A_{i3}(t)\xi_3,\ det\|A_{ij}\| \neq 0.$

15. Agar harakat quyidagi tezlik maydoni bilan sodir bo'layotgan bo'lsa,

$$v_1 = \frac{x_1}{t+\tau}, \qquad v_2 = \frac{2tx_2}{t^2+\tau^2}, \qquad v_3 = \frac{3t^2x_3}{t^3+\tau^3}, \qquad \tau = const, \tau > 0$$

Lagranj koordinatasini kiriting va muxitning harakat qonunini toping.

16. Agar tezlik maydoni quyidagi ko'rinishda bo'lsa,

a) $v_1 = \frac{Q(t)x_1}{2\pi r^2}, v_2 = \frac{Q(t)x_2}{2\pi r^2}, v_3 = 0, r = \sqrt{x_1^2 + x_2^2}, Q(t) > 0;$

b) $v_i = \frac{Q(t)x_i}{4\pi R^3}, i = 1,2,3, R = \sqrt{x_1^2 + x_2^2 + x_3^2}, Q(t) > 0;$

d) $v_1 = -Ax_1, v_2 = Bx_2, v_3 = 0, A = const > 0, B = const > 0.$

Lagranj koordinatasini kiriting va tutash muxit harakat qonunini, oqim chizig'ini hamda traektoriyasini toping.

17. Agar tutash muxit harakati quyidagi tezlik maydoni bilan sodir bo'lsa,

$v_1 = -A(t)x_1, v_2 = B(t)x_2, v_3 = 0, A(t) > 0, B(t) > 0.$

Lagranj koordinatalarini kiriting, harakat qonunini toping. Oqim chizig'ini toping va uni *A*, *B*=*const* bo'lgan xususiy holdagi oqim chizig'i bilan taqqoslang. *A(t)* va *B(t)* funksiyalar uchun oqim chizig'i va zarralar trayektoriyasi mos kelmaydigan ko'rinishlariga misollar keltiring.

18. a) Zarralarni ma'lum traektoriyalari orqali ularni harakat qonunlarini topish mumkinmi?

b) Vaqtni shu ondagi oqim chiziqlarini bilgan holda oniy tezlik maydonini toppish mumkinmi?

19. Muxit harakati quyidagi tezlik maydoni bilan sodir bo'lsa, oqim chizig'i va trayektoriyani toping.

a) $v_1 = -\omega x_2, v_2 = \omega x_1, v_3 = u, \omega, u = const;$

b) $v_1 = -Ax_2, v_2 = Bx_1, v_3 = 0, A = const > 0, B = const > 0;$

d) $v_1 = -V sin\omega t, v_2 = V cos\omega t, v_3 = 0, \omega, V = const.$

20. Agar,

a) barcha zarralar tezligi bir xil bo'lsa;

b) fazoning xar bir nuqtasida tezlik vaqt o'tishi bilan o'zgarmasa;

muxit zarralari tezlanish bilan xarakatlana oladimi?

21. Siqilmaydigan muxitning xar bir xususiy zarrasi zichligi doimiy bo'ladi. Fazoning biror nuqtasida zichlik vaqt bo'yicha ozgarishi mumkinmi?

22. Agar muxitning tezlik maydoni quyidagiga teng bo'lsa, tezlanish maydoni toping:

a) 7-masaladagidek;

b) $v_1 = A(t)x_2, v_2 = B(t)x_1, v_3 = 0$ qiymatlarga ega bo'lsa.

23. Muxitning xarakati tezlik maydoni

$$v_1 = -\omega x_2, v_2 = \omega x_1, v_3 = 0, \omega = const,$$

bo'lgan holda fazoda quyidagi xarorat maydoni xosil qilingan;

$$T = T_0 e^{-\frac{t}{\tau}-\left(\frac{x_1}{a}\right)^2-\left(\frac{x_2}{b}\right)^2-\left(\frac{x_2}{c}\right)^2}, T_0, \tau, a, b, c = const.$$

Agar zarracha t_0 vaktda fazoning kordinatalari $x_1 = a, x_2 = b, x_3 = c$ nuqtasida bo'lsa, xususiy zarradagi haroratni ozgarish tezligini toping.

24. Muxitning xarakati

$$v_1 = kx_1, v_2 = -kx_2, v_3 = 0, k = const$$

tezlik maydoni va

$$\rho = \rho_0 + Ax_2 e^{kt}, \quad \rho_0, A = const$$

zichlik maydoni bilan sodir bo'moqda. Muxitning xar bir zarrasining zichligini o'zgarish tezligini toping.

25. Xususiy zarrachaning (ξ_1, ξ_2, ξ_3) vaqtning xar bir *t* ondagi holati quyidagi munosabat bilan aniqlanadi:

$$x_i = f_i(\xi_1 + Ut, \xi_2, \xi_3), i = 1, 2, 3, U = const,$$

a) xarakat barqarorligini ko'rsating;

b) oqim chiziqlari parametrik tenglamalari

$$x_i = f_i(\tau, \xi_2^0, \xi_3^0), i = 1,2,3,$$

Korinishdagi egri chiziqlardan iborat ekanligini ko'rsating. Bunda $\tau -$ egri chiziq bo'ylab o'zgaruvchi parametr, ξ_2^0, ξ_3^0 xar bir egri chiziqlar uchun ma'lum bir aniq sonlar.

26. Muxit xarakati shunday sodir bo'lmoqdaki, bunda barcha zarralar trayektoriyasi O nuqtadan chiquvchi nurlarda yotibdi.Tezlik ϑva zichlik ρ ning qiymatlari faqat t vaqt momenti va O nuqtagacha bolgan masofa r ga bogliq. Bunday sferik simmetriyaga ega bo'lgan harakatni o'rganishda moddiy nuqtani biror Lagranj koordinatasi ξ sifatida *t=0* vaqtda markazi *O* nuqtada bo'lgan shu nuqtada bo'lgan shu nuqtadan o'tuvchi sfera ichidagi muxit massasi olinadi. Vaqtning *t* onida *O* nuqtadan r masofada joylashgan moddiy nuqtaning Lagranj koordinatasi ξ uchun quyidagi ifoda o'rinli ekanligini ko'rsating:

$$\xi = \int_0^r 4\pi R^2 \rho(R,t)dR$$

Lagranj tavsifida tezik va zichik qiymati faqat ξ va *t* ga bog'iqligini ko'rsating. $r(\xi,t)$-funksiyani o'z ichiga oluvchi $\tilde{\vartheta}(\xi,t),\tilde{\rho}(\xi,t)$ funksiyalar uchun tenglamalarni toping. Buning uchun Eyler tavsifidagi massani saqlanish qonunini ifodolovchi tenglamani $\frac{\partial\rho}{\partial t} + \vartheta\frac{\partial\rho}{\partial r} + \rho\frac{\partial\vartheta}{\partial r} + 2\frac{\rho\vartheta}{r} = 0$ o'zgartiring.

27. Muxit harakati quyidagi qonun bo'yicha sodir bo'lmoqda:

$$x_1 = \xi_1, x_2 = \xi_2\left(1 + \frac{t}{\tau}\right), x_3 = \xi_3\frac{1}{1 + \frac{t}{\tau}}, \tau = const$$

a) tezlik va tezlanish maydonlarini toping

b) Vaqtning $t = \tau$ momentida fazoviy (a,a,a) koordinatalarda joylashgan zarra, $t = 2\tau$ vaqtda qayerda joylashadi.

28. Agar muxit harakati quyidagi qonun boyicha sodir bo'lsa,

$x_1 = \xi_1 + c(t)\xi_2, x_2 = \xi_2 + c(t)\xi_3, x_3 = \xi_3$

Lagranj va Eyler tavsifida tezlik va tezlanish maydonini toping.

29. Muxit xarakati quyidagi tezlik va temperatura maydonlari bilan sodir bo'lmoqda:

$$v_1(x,t) = at, v_2(x,t) = -u\frac{x_2}{x_1}, v_3(x,t) = 0, u = const$$

$$T = T_0\left(1 + \frac{t^2}{\tau^2}\right)\left(1 + \frac{R^2}{x_1^2 + x_2^2}\right), \qquad T_0, \tau, R = const.$$

Koordinatalari

$$x_1 = \frac{u^2}{a}, x_2 = 2\frac{u^2}{a}, x_3 = 3\frac{u^2}{a}$$

bo’lgan nuqtada joylashgan xususiy zarrachani $t = \tau$ momentidagi tempeaturasini o’zgarish tezligini toping.

IV BOB. DEFORMATSIYALAR NAZARIYASI

1 §. Uzayish va siqilish.

Tutash muhitlar mehanikasida deformatsiya tushunchasini kiritamiz. Yevklid fazosidagi tutash muhitning t_0 - dastlabki holati ma'lum, ya'ni $\vec{X} = X^k \cdot \vec{e}_k$ vektor ma'lum deylik. U holda $t \geq 0$ onda ko'rilayotgan tutash muhit barcha nuqtalari holati ma'lum bo'lishi uchun deformatsiya tushunchasidan foydalanamiz. Biz tutash muhit nuqtasi deganimizda, oldinroq ta'kidlanganidek, shu geometrik nuqta bilan bog'liq tutash muhitning cheksiz kichik bo'lagini fikran ajratib qaraymiz. Geometrik nuqtani va uni xarakterlovchi ($t = t_0$ dagi $\vec{X} = X^k \cdot \vec{e}_k$ vektor va $t \geq 0$ da $\vec{x} = x^i \vec{e}_i$ vektor) vektorlarning uchidagi moddiy zarralar turlicha kichik hajm va uni qamrovchi sirtlariga ega bo'lib, ular harakat davomida deformatsiyalanish imkoniyatiga ega. Tutash muhit uchun deformatsiyalanish uning ixtiyoriy ikki nuqtasi orasidagi masofa o'zgarishi tufayli paydo bo'ladi. Agar bu shart bajarilmasa, bunday muhit absolut qattiq jismdan farq qilmaydi. Deformatsiyalanish tutash muhit turli qismlarida miqdori va sifati bo'yicha turlicha bo'lishi mumkin. Tushinish qiyin emaski, bir-biriga cheksiz kichik masofadagi yaqin nuqtalar olinishi bilangina nuqtadagi deformatsiyalanishni o'rganish mumkin. Bu vazifani Lagranj koordinatalarida tekshiramiz. Albatta, bu deformatsiyalanishning mazmunini ravshanlashtirish va miqdorini aniqlash ahamiyatlidir. Tutash muhitning ixtiyoriy $\vec{X} = X^k \cdot \vec{e}_k$ vektor bilan berilgan nuqtasi barcha ixtiyoriy $t \geq t_0$ onda deformatsiya ma'lum bo'lsa, tutash muhit deformatsiyasi ma'lum deyiladi.

Ixtiyoriy nuqta atrofi deformatsiyasi ma'lum bo'lishi uchun shu nuqtada olingan ixtiyoriy yo'nalishdagi cheksiz kichik $\vec{X}_N(\vec{X} + \vec{\xi}) - \vec{X}_M(\vec{X})$ vektor – tola $d\vec{X} = \vec{\xi}$ ning deformatsiyasi ma'lum bo'lishi zarur va yetarliligi geometrik nuqtayi nazardan ravshandir. Ya'ni $t = t_0$ da

$$(d\vec{X})^2 = (\vec{\xi})^2 = \xi^i \xi^j \cdot \vec{e}_i \cdot \vec{e}_j = \dot{g}_{ij}\, d X^i \cdot d X^j$$

Lekin $t = t_0$ da to'g'riburchakli Dekart koordinatalar sistemasi uchun $\dot{g}_{ij} = \delta_{ij}$ bo'lgani tufayli $(d\vec{X})^2 = \xi^i \xi^j \cdot \delta_{ij}$ bo'ladi. Ma'lumki:

$$(\vec{x})_M = \vec{x}(X^i, t)$$
$$(\vec{x})_N = \vec{x}(X^i + \xi^i, t)$$

lekin $\vec{\rho} = (\vec{x})_N - (\vec{x})_M$ ni X^i nuqta atrofida qatorga yoysak, quyidagini topamiz:

$$\vec{\rho} = \frac{\partial\vec{x}}{\partial X^k}\xi^k = \frac{\partial\vec{x}}{\partial X^i}dX^i + ...$$

Ko'p nuqtalar bilan ko'rsatilgan hadlar birinchi hadga nisbatan yuqori tartibli cheksiz kichik o'zgaruvchilar deb olsak va $\vec{\ni}_k = \frac{\partial\vec{x}}{\partial X^k}$ Lagranj koordinatalariga bog'liq ifodaligini eslab yoza olamiz:

$$\vec{\rho} = \vec{\ni} \cdot dX^k = \vec{\ni}_k \cdot \xi^k$$

Demak, $t = t_0$ dagi $\vec{\xi} = \xi^k \cdot \vec{e}_k$ tola $t \geq t_0$ da $\vec{\rho} = \xi^k \cdot \vec{\ni}_k$ bo'ladi.

$\vec{\rho} = \frac{\partial\vec{x}}{\partial X^k}\xi^k$ dan $\rho^i = \frac{\partial x^i}{\partial X^k} \cdot \xi^k$ orqali bu almashtirishda $A^i_k = \frac{\partial x^i}{\partial X^k}$ matrisa $\vec{\xi}$ ni $\vec{\rho}$ ga qaralayotgan nuqta atrofida affin (chiziqli) almashtiradi. Uni $\vec{\rho} = \tilde{A}\vec{\xi}$ sifatida yozish mumkin. $\tilde{A}$ operatori uchun quyidagilar o'rinli:

1. $\tilde{A}(\vec{\xi} + \vec{\eta}) = \tilde{A}\vec{\xi} + \tilde{A}\vec{\eta}$
2. $\tilde{A}(k\vec{\xi}) = k \cdot \tilde{A} \cdot \vec{\xi}$
3. $\tilde{A}(\vec{X}, t)$ operatori $\vec{\xi}$ ga bog'liq emas, bunday operator affinor deyiladi.

Faraz qilaylik, tutash muhitning qaralayotgan nuqtasi atrofi uchun ($\left\| A^i_k \right\| \neq 0$ bo'lsin deylik. U holda $\xi^i = B^i_k \cdot \rho^k$ bo'ladi.

Bu yerda $B^j_k = \frac{\partial X^j}{\partial x^k}$ va $A^i_k \cdot B^k_j = \delta^i_j$ bo'ladi.

Affin almashtirishning xossalari:

a) tekislik tekislikka o'tadi (ya'ni akslanadi);
b) nuqta nuqtaga almashinadi, ya'ni akslanadi;

c) biror chiziq ikkinchi bir chiziqqa almashinadi;

d) parallel tekisliklar parallel tekisliklarga almashinadi.

Yuqoridagi xossalardan ko'rish mumkinki, $\vec{\xi}$ tola (vektor) $\vec{\rho}$ tolaga (vektorga) o'tadi. Masalan, $\xi^i \cdot \xi^i = R^2$ sfera $B_k^i \cdot B_j^i \cdot \rho^k \cdot \rho^j = R^2$ ellipsoidga o'tadi.

Shunday qilib, biz yuqorida

$$\vec{\xi} = \xi^k \cdot \vec{e}_k \to \vec{\rho} = \xi^k \cdot \vec{\ni}_k \quad \left(\vec{\ni}_k = \frac{\partial \vec{x}}{\partial X^k} \right)$$

$\bar{\rho} = \rho^k \cdot \vec{e}_k$ ifodalarni hosil qildik. Quyidagicha yozishimiz mumkin:

$$\vec{\xi}^2 = \vec{\xi} \cdot \vec{\xi} = \xi^2 = \delta_{ij} \cdot \xi_i \cdot \xi_j, \; \left(\delta_{ij} = g_{ij}\big|_{t=t_0} = \vec{e}_i \cdot \vec{e}_j \right)$$

$$\vec{\rho}^2 = \vec{\rho} \cdot \vec{\rho} = g_{ij} \cdot \xi_i \xi_j, \qquad \left(g_{ij} = \vec{\ni}_i \cdot \vec{\ni}_j \right)$$

$$\rho^2 - \xi^2 = \left(g_{ij} - \delta_{ij} \right) \cdot \xi^i \cdot \xi^j$$

Endi tola uchun quyidagi nisbiy o'zgarishni xarakterlovchi miqdorni

$$e_\xi = \frac{\rho - \xi}{\xi} = \frac{\rho}{\xi} - 1$$

va

$$\varepsilon_{ij} = \frac{1}{2} \left(g_{ij} - \delta_{ij} \right) \tag{4.1.1}$$

2-rangi tenzor elementlari ifodasini kiritaylik. U holda:

$$\rho^2 - \xi^2 = 2\varepsilon_{ij} \cdot \xi^i \cdot \xi^j$$

va

$$\frac{\xi^i}{\left| \vec{\xi} \right|} = l^i$$

desak,

$$\frac{\rho^2 - \xi^2}{\left| \vec{\xi} \right|^2} = 2\varepsilon_{ij} \cdot l^i \cdot l^j$$

bo'ladi.

U holda

$$e_{\xi} = \sqrt{1 + 2\varepsilon_{ij} \cdot l^{i} \cdot l^{j}} - 1$$

ga ega bo’lamiz.

ε_{ij} metrik tenzorlar elementlari orqali ifodalanganligi tufayli, shubhasiz tenzor elementlari bo’la oladi va uni E bilan belgilaymiz:

$$E = \varepsilon_{IJ} \left(\vec{\ni}^{I} \otimes \vec{\ni}^{j} \right) \tag{4.1.2}$$

(2.19) bilan birga qaralayotgan Lagranj koordinatalarida ushbu $E = \varepsilon^{ij} \left(\vec{\ni}_{i} \otimes \vec{\ni}_{j} \right)$, $E = \varepsilon^{i\cdot}_{\cdot j} (\vec{\ni}_{i} \otimes \vec{\ni}^{j})$ larga ham ega bo’lamiz.

Bu tenzor deformatsiya tenzori deyiladi va tutash muhit nuqtasi atrofi deformatsiyasini aniqlaydi.

Shunday qilib, (4.1.1) formula Lagranj koordinatalarida aniqlandi va ixtiyoriy dastlabki $t = t_0$ da olingan $\vec{X} = X^{k} \cdot \vec{e}_{k}$ nuqtadagi cheksiz kichik $\vec{\xi}$ tola (o’zaro tik koordinatalar sistemasida-Dekart koordinatalar sistemasida aniqlangan) $\bar{\rho} = \xi^{i} \cdot \vec{\ni}_{i}$ tola bo’lib o’zgaradi. O’zaro tik birlik bazis e_i vektorlar umumiy holda $\left| \vec{\ni}_{i} \right| \neq 1$ ga mos ravishda o’tadi. Ixtiyoriy $t \geq t_0$ uchun $\vec{\ni}_{i}$ lar Lagranj koordinatalarida aniqlanadi va nuqta ko’chgan yangi holat uchun o’zaro tik bo’lishi shart bo’lmagan bazis vektorlarni tashkil etadi. Agar dastlabki $\left(t = t_0 \right)$ da nuqta atrofida olingan tola uchun o’zaro tik $\vec{e}_{i}$ bazislar o’zaro tikligi $t \geq t_0$ da ham saqlansa va ularning uzunliklari (ular birga teng) ham saqlansa, qaralayotgan tutash muhit zarrasi deformatsiyalanmaydi va u absolyut qattiq jism uchun olingan $\vec{\xi}$ tola kabi fazoda Lagranj koordinatalarida ilgarilanma va aylanma harakatni (yoki ular yig’indisi bo’lgan vint harakatini) sodir etishi mumkin. U holda $t = t_0$ da olingan $\dot{g}_{ij}$ (bu ifoda xususiy holda, biz ko’rgandek $\dot{g}_{ij} = \delta_{ij}$ bo’lishi mumkin va bunday olinishi Yevklid fazosida umumiyatga zid emas) $\dot{g}_{ij}$ ga teng bo’lganicha (yoki δ_{ij} bo’lganicha) qoladi va deformatsiya sodir bo’lmaydi.

2 §. Deformasiya tenzori, uning bosh o'qlari va bosh qiymatlari.

Biz yuqorida ko'rdikki, deformatsiya tenzori - simmetrikdir. Shuning uchun $\varepsilon_{ij} = \varepsilon_{ji}$ tenzor elementlari bilan tutash muhitning har bir nuqtasida $\varepsilon_{ij}\, dx^i dx^j = C$ kvadratik formani tuzish mumkin. Ma'lumki, agar $\varepsilon_{ij}\, dx^i dx^j$ kvadratik forma tutash muhit biror nuqtasiga tegishli bo'lsa, shu nuqtada uni kanonik ko'rinishga keltirish mumkin, ya'ni

$$\varepsilon_1 \left(d\eta^1\right)^2 + \varepsilon_2 \left(d\eta^2\right)^2 + \varepsilon_3 \left(d\eta^3\right)^2 = C_1$$

Bu yerda η^1, η^2, η^3 koordinata o'qlari - o'zaro tik o'qlardir. Bunda ε_{ij} lar koordinata o'qlari o'zgarishiga qarab o'zgaradi va η^1, η^2, η^3 larga nisbatan $\varepsilon_{ij} = 0 \ (i \neq j)$ bo'ladi. O'zaro tik η^1, η^2, η^3 lar ma'lum bo'lsa, kvadratik forma $\varepsilon_{ij}\, dx^i dx^j$ bilan birga bu koordinata sistemasida $\varepsilon_{ij}\, dx^i dx^j$ ham kanonik ko'rinishga keladi.

Shunday qilib, deformatsiyalanuvchi tutash muhit istalgan nuqtasi atrofi deformatsiyasi ε_{ij} bilan beriladigan bo'lsa, shu nuqtada deformatsiya jarayoni davomida o'zaro tik bo'lib uzluksiz harakatlanadigan (yoki o'z yo'nalishlarini o'zgartirmaydi) 3 ta o'qlarni tuzish mumkin. Bu o'qlarga nisbatan $\varepsilon_{ij}\, dx^i dx^j$ kvadratik forma ko'rilganda, u eng sodda holda bo'lib, deformatsiyani xarakterlovchi tenzor elementlaridan tuzilgan matrisa diagonal matrisadan iborat bo'ladi.

Shunday qilib, bosh o'qlarga nisbatan ushbu matrisalarga ega bo'lamiz:

$$\begin{pmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix}, \begin{pmatrix} \varepsilon^{11} & 0 & 0 \\ 0 & \varepsilon^{22} & 0 \\ 0 & 0 & \varepsilon^{33} \end{pmatrix}, \begin{pmatrix} \varepsilon^{1.}_{.1}, & 0, & 0 \\ 0, & \varepsilon^{2.}_{.2}, & 0 \\ 0, & 0, & \varepsilon^{3.}_{.3} \end{pmatrix}$$

Bu holda o'qlar bo'ylab olingan tola uchun faqat cho'zilish yoki siqilish deformatsiyasigina mavjud bo'lishi mumkin.

Endi deformatsiya tenzorining bosh komponentalari haqida fikr yuritaylik. Dastlabki $t = t_0$ vaqttda, ya'ni boshlang'ich paytda tutash muhit holati ma'lum deyishimiz kerak. Bu vaqtda, asosan, eng tabiiy holat sifatida tutash muhit deformatsiyalanmagan va uning holatini uch o'lchovli o'zaro tik Dekart koordinata sistemasida berish mumkin. Lekin shunday hol ko'rilishi mumkinki, biz ko'rayotgan $t = t_0$ deb olingan dastlabki vaqtda tutash muhit deformatsiyalangan bo'lishi va tekshirishlarni $t = t_0$ vaqt uchun ana shu ma'lum holat asosida o'rganilishi talab qilinishi mumkin. U holda $t = t_0$ vaqtda tutash muhit barcha nuqtalari uchun dastlabki deformatsiyalanganlik holati ma'lum va shu asosda $\varepsilon_{ij} = \frac{1}{2}(g_{ij} - \delta_{ij})$ dan $t = t_0$ da $\dot{g}_{ij}$ (va demak $\dot{\varepsilon}_{ij}$) ma'lum deb olamiz.

Shunday qilib, $t \geq t_0$ holat uchun ε_{ij} quyidagicha aniqlanadi:

$$\varepsilon_{ij} = \frac{1}{2}(g_{ij} - \dot{g}_{ij}) \tag{4.2.1}$$

Agar biror $t \geq t_0$ moment uchun bosh o'qlar ma'lum bo'lsa va bu o'qlarni η^1, η^2, η^3 bilan belgilab, bu o'qlar bo'ylab $d\vec{x}$ vektorni olsak $d\vec{x}\,(ds_1, ds_2, ds_3)$. U holda quyidagi tenglikni yozishimiz mumkin:

$$(ds_\alpha)^2 = g_{\alpha\alpha}\left(d\eta^\alpha\right)^2.$$

Ixtiyoriy yo'nalishdagi $d\vec{x}$ vektor uchun

$$|d\,\vec{x}|^2 = (ds)^2 = (ds_1)^2 + (ds_2)^2 + (ds_3)^2.$$

$$t = t_0 \text{ da esa: } (ds_{0\alpha})^2 = \dot{g}_{\alpha\alpha}\left(d\eta^\alpha\right)^2.$$

$$(ds_0)^2 = (ds_{01})^2 + (ds_{02})^2 + (ds_{03})^2.$$

Bu yerdan

$$(ds)^2 - (ds_0)^2 = (g_{11} - \dot{g}_{11})(d\eta^1)^2 + (g_{22} - \dot{g}_{22})(d\eta^2)^2 + (g_{33} - \dot{g}_{33})(d\eta^3)^2$$

$$2\varepsilon_{ij} = g_{ij} - \dot{g}_{ij} \text{ dan}$$

$$(ds)^2 - (ds_0)^2 = 2\left[\varepsilon_1(ds_1)^2 + \varepsilon_2(ds_2)^2 + \varepsilon_3(ds_3)^2\right] =$$

$$2 \cdot \sum_{i=1}^{3} \frac{\varepsilon_{ii}}{g_{ii}}(ds_i)^2 = 2\sum_{i-1}^{3} \frac{\varepsilon_{ii}}{\dot{g}_{ii}}(ds_{0i})^2$$

Lekin

$$\frac{\varepsilon_{ii}}{g_{ii}} = \varepsilon_{ii} \cdot g^{ii} = \varepsilon_i^i = \varepsilon_i \ , \ \frac{\varepsilon_{ii}}{\dot{g}_{ii}} = \dot{\varepsilon}_i$$

Shunday qilib,

$$(ds)^2 - (ds_0)^2 = 2 \cdot \sum \varepsilon_i (ds_i)^2 = 2 \sum \dot{\varepsilon}_i (ds_{oi})^2$$

ε_i va $\dot{\varepsilon}_i$ lar deformatsiya tenzori bosh komponentalari deyiladi.

3 §. Deformatsiya tenzori sirti. Invariantlar

Deformatsiyalangan tutash muhit har bir nuqtasida shu nuqtaning funksiyalari sifatida 6 ta ε_{ij} ga egamiz. Ular nuqtadan nuqtaga o'tishda, umuman olganda, o'zgarishi bilan birga vaqtga ham bog'liq bo'lishi mumkin. Deformatsiya tenzori bosh o'qlari va bosh qiymatlari ustida fikr yuritilganda har bir ondagi yo'nalish va qiymatlar nazarda tutiladi. Olingan har bir nuqta uchun to'g'ri burchakli Dekart koordinata sistemasi kiritib, ushbu $\varepsilon_{ij} \, x^i \cdot x^j = c^2$ kvadratik forma orqali tuzilgan ikkinchi tartibli sirt deformatsiya tenzori sirti deyiladi. Deformatsiya tenzori bosh o'qlari va bosh qiymatlari shu sirtning bosh o'qlari va bosh qiymatlari bilan bir xil qilib olinishi mumkin. Analitik geometriyadan ma'lumki, ikkinchi tartibli sirtlar o'z invariantlariga ega bo'ladilar, ya'ni shu sirtni xarakterlovchi shunday 3 ta skalyar miqdor ko'rsatish mumkinki, deformatsiyalangan tutash muhit har bir nuqtadagi to'g'ri burchakli koordinatalar sistemasini almashtirganda bu miqdorlar o'zgarmaydilar va ular, tabiiy holda, ε_{ij} lar orqali aniqlanadilar.

Agar bosh o'qlarni x^i lardan farqlash uchun η^i lar bilan belgilasak, ularga nisbatan deformatsiya tenzori sirti quyidagicha bo'ladi:

$$\varepsilon_1 (\eta^1)^2 + \varepsilon_2 (\eta^2)^2 + \varepsilon_3 (\eta^3)^2 = c_1^2 \qquad (4.3.1)$$

Endi yuqorida aytilgan bosh qiymatlar va bosh o'qlarni topish bilan shug'ullanamiz.

Agar $2F(x^1, x^2, x^3) = \varepsilon_{ij} x^i x^j = c^2$ desak, bosh yo'nalishlar uchun $gradF = \lambda \vec{x}$ bo'lishi kerak. Bu yerda λ -skalyar miqdor.

Agar $\vec{e}_i$ - birlik bazis vektorlar kiritsak, yoza olamiz:

$$\frac{\partial F}{\partial x^i} \cdot \vec{e}_i = \lambda x^j \, \vec{e}_j$$

$$(\varepsilon_{ij} \cdot \vec{e}_i - \lambda \cdot \vec{e}_j) \cdot x^j = 0$$

Bundan

$$(\varepsilon_{i1} \cdot \vec{e}_i - \lambda \cdot \vec{e}_1) = 0$$

$$(\varepsilon_{i2} \cdot \vec{e}_i - \lambda \cdot \vec{e}_2) = 0$$

$$(\varepsilon_{i3} \cdot \vec{e}_i - \lambda \cdot \vec{e}_3) = 0$$

$\vec{e}_i$ lar o'zaro tik birlik bazislar bo'lganligi tufayli

$$\begin{vmatrix} \varepsilon_{11} - \lambda & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} - \lambda & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} - \lambda \end{vmatrix} = 0$$

bo'lishi kerak.

Bundan

$$-\lambda^3 + J_1 \lambda^2 - J_2 \lambda + J_3 = 0$$

bu yerda

$$J_1 = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

$$J_2 = \left(J_1^2 - \varepsilon_{ij} \cdot \varepsilon_{ij}\right) \cdot \frac{1}{2} = \begin{vmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{vmatrix} + \begin{vmatrix} \varepsilon_{11} & \varepsilon_{13} \\ \varepsilon_{31} & \varepsilon_{33} \end{vmatrix} + \begin{vmatrix} \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{32} & \varepsilon_{33} \end{vmatrix} \qquad (4.3.2)$$

$$J_3 = det\left|\varepsilon_{ij}\right|$$

Bu miqdorlar - deformatsiya tenzori sirtining invariantlaridir.

Bu tenglamaning haqiqiy, o'zaro teng bo'lmagan $\lambda_1, \lambda_2, \lambda_3$ ildizlari bosh miqdorlarni belgilaydi. Masalan, xususiy holda, $\lambda_1 = \lambda_2 = \lambda_3$ bo'lsa, deformatsiya tenzori sirti sferadan iborat bo'ladi va barcha o'zaro tik qilib olingan (tekshirilayotgan nuqta uchun) yo'nalishlar bosh yo'nalishlar bo'ladi va bosh miqdorlar o'zaro teng bo'ladi.

Deformatsiya tenzori sirtining bosh yo'nalishlarini topaylik. Buning uchun yuqoridagi

$$\left(\varepsilon_{ij}\cdot\vec{e}_i-\lambda\cdot\vec{e}_j\right)\cdot x^j=0$$

ifodaning har ikkala tomonini birlik bazis vektor $\vec{e}_i$ ga skalyar ravishda ko'paytiramiz:

$$\left(\varepsilon_{ij}-\lambda\cdot\delta_{ij}\right)\cdot x^j=0$$

Agar

$$l^i=\frac{x^i}{|\vec{x}|}=cos\left(\vec{l}\cdot\vec{e}_i\right)\qquad \text{kiritsak}$$

$$\left(\varepsilon_{ij}-\lambda\,\delta_{ij}\right)\cdot l^j=0 \tag{4.3.3}$$

va $(l^1)^2+(l^2)^2+(l^3)^2=1$ ifodalarga ega bo'lamiz va bulardan har bir λ_i ga mos ravishda to'g'ri keladigan 3 ta bosh yo'nalishlarni topa olamiz.

Agar $t=t_0$ da metrik fazo tenzori $\dot{g}_{ij}$ va $t>t_0$ da g_{ij} bo'lsa, ma'lumki:

$$\varepsilon_{ij}=\frac{1}{2}(g_{ij}-\dot{g}_{ij})$$

$t=t_0$ dagi bazis vektorlar $\dot{\vec{э}}_i$, $t>t_0$ dagi bazis vektorlar esa $\vec{э}_i$ bo'lsa, $\vec{x}=\vec{X}+\vec{u}(\vec{X},t)$ dan $\vec{u}=\vec{x}-\vec{X}$

$$\vec{r}=\vec{x},\ \vec{r}_0=\vec{X},\ \vec{r}=\vec{r}_0+\vec{u}$$

$$\frac{\partial\vec{u}}{\partial\alpha^i}=\frac{\partial\vec{r}}{\partial\alpha^i}-\frac{\partial\vec{r}_0}{\partial\alpha^i}=\vec{э}_i-\dot{\vec{э}}_i$$

Bundan

$$\vec{э}_i=\dot{\vec{э}}_i+\frac{\partial\vec{u}}{\partial\alpha^i} \tag{4.3.4}$$

$$\dot{\vec{э}}_i=\vec{э}_i-\frac{\partial\vec{u}}{\partial\alpha^i}$$

Shuning uchun

$$g_{ij}=(\vec{э}_i\cdot\vec{э}_j)=(\dot{\vec{э}}_i\cdot\dot{\vec{э}}_j)+\dot{\vec{э}}_i\cdot\frac{\partial\vec{u}}{\partial\alpha^j}+\dot{\vec{э}}_j\cdot\frac{\partial\vec{u}}{\partial\alpha^i}+\frac{\partial\vec{u}}{\partial\alpha^i}\cdot\frac{\partial\vec{u}}{\partial\alpha^j} \tag{4.3.5}$$

$$\dot{g}_{ij}=(\dot{\vec{э}}_i\cdot\dot{\vec{э}}_j)=(\vec{э}_i\cdot\vec{э}_j)-\vec{э}_i\cdot\frac{\partial\vec{u}}{\partial\alpha^j}-\vec{э}_j\cdot\frac{\partial\vec{u}}{\partial\alpha^i}+\frac{\partial\vec{u}}{\partial\alpha^i}\cdot\frac{\partial\vec{u}}{\partial\alpha^j} \tag{4.3.6}$$

Bundan

$$\varepsilon_{ij} = \frac{1}{2}(\vec{\text{э}}_i \cdot \frac{\partial \vec{u}}{\partial \alpha^j} + \vec{\text{э}}_j \cdot \frac{\partial \vec{u}}{\partial \alpha^i} - \frac{\partial \vec{u}}{\partial \alpha^i} \cdot \frac{\partial \vec{u}}{\partial \alpha^j}) =$$

$$= \frac{1}{2}(\dot{\vec{\text{э}}}_i \cdot \frac{\partial \vec{u}}{\partial \alpha^j} + \dot{\vec{\text{э}}}_j \cdot \frac{\partial \vec{u}}{\partial \alpha^i} + \frac{\partial \vec{u}}{\partial \alpha^i} \cdot \frac{\partial \vec{u}}{\partial \alpha^j})$$

Lekin ixtiyoriy egri chiziqli koordinata sistemasida $\vec{u} = \dot{u}^k \cdot \dot{\vec{\text{э}}}_k = u^k \cdot \vec{\text{э}}_k$ dan

$$\frac{\partial \vec{u}}{\partial \alpha^i} = \left(\frac{\partial u^k}{\partial \alpha^i} + \Gamma_{ij}^k \cdot u^j \right) \cdot \vec{\text{э}}_k = \nabla_i u^k \cdot \vec{\text{э}}_k$$

$$\frac{\partial \vec{u}}{\partial \alpha^i} = \nabla_i \dot{u}^k \cdot \dot{\vec{\text{э}}}_k$$

deb yozish mumkin.

$$\varepsilon_{ij} = \frac{1}{2}(\nabla_j u^k \cdot \vec{\text{э}}_k \cdot \vec{\text{э}}_i + \nabla_i u^k \cdot \vec{\text{э}}_k \cdot \vec{\text{э}}_j - \nabla_i u^k \cdot \vec{\text{э}}_k \cdot \nabla_j u^l \cdot \vec{\text{э}}_l) =$$

$$= \frac{1}{2}\left[\nabla_j u_i + \nabla_i u_j - \nabla_i u_k \cdot \nabla_j u^k\right] \tag{4.3.7}$$

$$\varepsilon_{ij} = \frac{1}{2}\left[\nabla_i \dot{u}_j + \nabla_j \dot{u}_i + \nabla_i \dot{u}_k \nabla_j \dot{u}^k\right] \tag{4.3.8}$$

Yuqoridagi hisoblashlarda metrik tenzordan kovariant hosila olish bo’yicha Richchi teoremasidan va indekslarni tushirish amalidan foydalaniladi. Masalan:

$$\nabla_j u^k \, \vec{\text{э}}_k \vec{\text{э}}_i = \nabla_i (u^k g_{ki}) = \nabla_j u_i$$

(4.3.8) formula qiymati Lagranj koordinitalarida, (4.3.7) esa Eyler koordinatalarida hisoblanganligini e’tiborga olib, ularni mos ravishda L_{ij} va E_{ij} bilan belgilab yoza olamiz:

$$L_{ij} = \frac{1}{2}\left[\nabla_i \dot{u}_j + \nabla_j \dot{u}_i + \nabla_i \dot{u}_k \cdot \nabla_j \dot{u}^k\right]$$

$$E_{ij} = \frac{1}{2}\left[\nabla_j \dot{u}_i + \nabla_i \dot{u}_j - \nabla_i u_k \cdot \nabla_j u^k\right].$$

4 §. Deformasiya tenzori komponentalarini ko'chish orqali ifodalash.

To'g'ri burchakli Dekart koordinatalar sistemasi amalda ko'p ishlatilishi tufayli deformatsiya tenzori E ning ε_{IJ} elementlarini ko'chish vektori $\vec{u}(\vec{X},t)$ orqali ifodasini keltirish maqsadga muvofiqdir. ε_{IJ} larning ko'chish vektori komponentalari orqali Lagranj koordinatalarda ifodalaymiz. Buning uchun oldingi paragrafda keltirilgan formulalarda $\dot{g}_{ij} = \delta_{ij}$ ekanligini nazarda tutib, izlanayotgan ifodalar formulalarini xususiy hol sifatida yoza olishimiz mumkin. Lekin biz bu yerda izlanayotgan formulalarni oddiy hisoblashlar orqali keltiramiz. Ma'lumki:

$$\varepsilon_{IJ} = \frac{1}{2}(g_{ij} - \delta_{ij})$$

$$g_{ij} = (\vec{\ni}_i \cdot \vec{\ni}_j), \quad \vec{\ni}_i = \frac{\partial \vec{x}}{\partial X^i}\left\{\frac{\partial x^1}{\partial X^i}, \frac{\partial x^2}{\partial X^i}, \frac{\partial x^3}{\partial X^i}\right\}$$

$$\vec{x} = \vec{X} + \vec{u}, \quad \vec{x} = \vec{x}(\vec{X},t).$$

Yuqoridagilardan foydalanib quyidagicha yozishimiz mumkin:

$$\frac{\partial x^k}{\partial X^i} = \frac{\partial X^k}{\partial X^i} + \frac{\partial u^k(X^1, X^2, X^3, t)}{\partial X^i} = \delta_{ki} + \frac{\partial u^k}{\partial X^i}$$

bundan:

$$g_{ij} = \left(\frac{\partial \vec{x}}{\partial X^i}\frac{\partial \vec{x}}{\partial X^j}\right) = \left(\frac{\partial x^k}{\partial X^i}\frac{\partial x^k}{\partial X^j}\right) =$$

$$= \delta_{ki}\cdot\delta_{kj} + \delta_{ki}\cdot\frac{\partial u^k}{\partial X^j} + \delta_{kj}\cdot\frac{\partial u^k}{\partial X^i} + \frac{\partial u^k}{\partial X^i}\cdot\frac{\partial u^k}{\partial X^j} =$$

$$= \delta_{ij} + \frac{\partial u^i}{\partial X^j} + \frac{\partial u^j}{\partial X^i} + \frac{\partial u^k}{\partial X^i}\cdot\frac{\partial u^k}{\partial X^j}$$

Bu ifodani ε_{IJ} ifodasiga qo'yib topsak, quyidagi formula hosil bo'ladi:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u^i}{\partial X^j} + \frac{\partial u^j}{\partial X^i} + \frac{\partial u^k}{\partial X^i}\cdot\frac{\partial u^k}{\partial X^j}\right) \tag{4.4.1}$$

5 §. Grin va Al'mansi tenzorlari.

Ta'rif: Agar tutash muhit harakati Dekart koordinatalarida, $x_i = x_i(X_1, X_2, X_3, t) = x_i(\vec{X}, t)$ ko'rinishida berilgan bo'lsa, $\dfrac{\partial x_i}{\partial X_j}$ tenzori deformatsiya moddiy gradienti tenzori deb ataadi.

Ta'rif: Aga harakat Eyler koordinatalarida berilgan bo'lsa, $\dfrac{\partial X_i}{\partial x_j}$ deformatsiyaning fazoviy gradienti deb ataadi.

Ta'rif: $c_{ij} = \dfrac{\partial X_k}{\partial x_i} \cdot \dfrac{\partial X_k}{\partial x_j}$-Koshining deformatsiya tenzori, $G_{ij} = \dfrac{\partial x_k}{\partial X_i} \cdot \dfrac{\partial x_k}{\partial X_j}$-Grinning deformatsiya tenzori deb ataladi.

(4.3.8) formula ko'chish vektorining Lagranj koordinatalari orqali ifodalash orqali olinganligi tufayli uni Lagranjning chekli deformatsiya formulasi ham deyiladi. Bu tenzorni Grinning chekli deformatsiya tenzori deb ham ataladi. Chekli deformatsiyaning Eyler koordinatalaridagi ifodasi (4.3.7) asosida olinadi va uning ifodasi Dekart koordinatalarida quyidagicha bo'ladi:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{\partial u_k}{\partial x_i} \cdot \frac{\partial u_k}{\partial x_j}\right) \tag{4.5.1}$$

Bu tenzorni Eylerning chekli deformatsiya tenzori (yoki Almansining chekli deformatsiya tenzori ham deyiladi). (4.4.1) va (4.5.1) formulalar tutash muhit ixtiyoriy nuqtasidagi mazmun jihatdan yagona bo'lgan deformatsiya o'lchovlarining mos ravishda Lagranj va Eyler koordinatalaridagi ifodalaridir, ularni, yuqorida ta'kidlangandek, L_{ij} va E_{ij} deb ham belgilanadi.

6 §. Fazoning yevklidlilik sharti. Riman-Kristofell tenzori. Deformasiyaning birgalikdagi tenglamalari.

Lagranj koordinata sistemasida berilgan 6 ta ε_{IJ} larni, ya'ni $\varepsilon_{IJ} = \varepsilon_{ij}(X^k, t)$ ni ko'raylik. Tutash muhit uchun ε_{IJ} lar turlicha erkin holda berilsa, tutash muhit deformatsiyasi birgalikda bo'lmay qolishi mumkin. Ya'ni tutash muhitda bo'shliq mavjud bo'lib qolishi yoki bir nuqtada bir nechta tutash muhit zarrasi joylashishiga to'g'ri kelardi. Lekin bu hol bo'lishi mumkin emas, chunki $\vec{x} = \vec{X} + \vec{u}$ formulamizga ko'ra tutash muhitning har bir ikki vaqtdagi holati, biri ikkinchisidan uzluksiz va bir qiymatli ko'chish tufayli hosil bo'ladi. Bunda ε_{IJ} lar shu $\vec{u}(u^1, u^2, u^3)$ lar orqali (3 ta komponentasi orqali) ifodalangan. Demak, ε_{IJ} lar o'rtasida ko'chishga shartlar - tenglamalar mavjud bo'lishi kerakki, bu asosda deformatsiyaning birgalikda bo'lish sharti topiladi.

Har bir nuqtada ε_{IJ} lar soni 6 ta bo'lib, ular ko'chish vektori komponentalari hosilalari orqali ifodaga ega. Demak, ε_{IJ} lar ixtiyoriy ravishda, bir biriga bog'liq bo'lmagan holda berilishi mumkin emas. Agar ular ixtiyoriy bo'lganda, tutash muhit deformatsiyalangan paytda g'ovaklar yoki bir nuqtaga ko'chishning ikki xil qiymati mos kelishi mumkin edi. ε_{IJ} lar o'rtasidagi bunday munosabatni fazoning Yevklidligi bajarilishi shartidan hosil qilamiz.

Ma'lumki,

$$\frac{\partial \vec{\ni}_i}{\partial \alpha^j} = \Gamma_{ij}^k \cdot \vec{\ni}_k \qquad (4.6.1)$$

Shuningdek, $d\,\vec{\ni}_i = \frac{\partial \vec{\ni}_i}{\partial \alpha^j} d\alpha^j$ bo'lib, bu yerda $\frac{\partial^2 \vec{\ni}_i}{\partial \alpha^l \partial \alpha^j} = \frac{\partial^2 \vec{\ni}_i}{\partial \alpha^j \partial \alpha^l}$ shart bajarilishi kerak. Yoza olamiz:

$$\frac{\partial^2 \vec{\ni}_i}{\partial \alpha^j \partial \alpha^l} - \frac{\partial^2 \vec{\ni}_i}{\partial \alpha^l \partial \alpha^j} = R_{jli}^m \cdot \vec{\ni}_m$$

Ushbu R_{jlik} tenzor kovariant egrilik tenzori deyiladi.

$$R_{jli}^m g_{mk} = R_{jlik}$$

(4.6.1) ning integrallanish sharti $R_{jli}^m = 0$ dir.

Quyidqgicha yoza olamiz:

$$\frac{\partial}{\partial\alpha^j}\left(\frac{\partial \vec{\text{Э}}_i}{\partial\alpha^l}\right)=\frac{\partial}{\partial\alpha^j}\left(\Gamma_{il}^m\cdot\vec{\text{Э}}_m\right)=\frac{\partial\Gamma_{il}^m}{\partial\alpha^j}\cdot\vec{\text{Э}}_m+\Gamma_{il}^k\cdot\frac{\partial\vec{\text{Э}}_k}{\partial\alpha^j}=$$

$$=\frac{\partial\Gamma_{il}^m}{\partial\alpha^j}\cdot\vec{\text{Э}}_m+\Gamma_{il}^k\cdot\Gamma_{ki}^m\ \vec{\text{Э}}_m=\left(\frac{\partial\Gamma_{il}^m}{\partial\alpha^j}+\Gamma_{il}^k\cdot\Gamma_{kj}^m\right)\cdot\vec{\text{Э}}_m$$

$$\frac{\partial}{\partial\alpha^l}\left(\frac{\partial \vec{\text{Э}}_i}{\partial\alpha^j}\right)=\frac{\partial}{\partial\alpha^l}\left(\Gamma_{ij}^m\cdot\vec{\text{Э}}_m\right)=\frac{\partial\Gamma_{ij}^m}{\partial\alpha^l}\cdot\vec{\text{Э}}_m+\Gamma_{ij}^m\cdot\frac{\partial\vec{\text{Э}}_m}{\partial\alpha^l}=$$

$$=\frac{\partial\Gamma_{ij}^m}{\partial\alpha^l}\cdot\vec{\text{Э}}_m+\Gamma_{ij}^m\cdot\Gamma_{ml}^k\ \vec{\text{Э}}_k=\left(\frac{\partial\Gamma_{ij}^m}{\partial\alpha^l}+\Gamma_{ij}^k\cdot\Gamma_{kl}^m\right)\cdot\vec{\text{Э}}_m$$

Endi deformatsiyani birgalikda bo’lish shartini topamiz:

$$\frac{\partial\Gamma_{il}^m}{\partial\alpha^j}-\frac{\partial\Gamma_{ij}^m}{\partial\alpha^l}+\Gamma_{il}^k\cdot\Gamma_{kj}^m-\Gamma_{ij}^k\cdot\Gamma_{kl}^m=R_{jli}^m=0 \qquad (4.6.2)$$

Agar

$$R_{jlik}=g_{km}\left(\frac{\partial\Gamma_{il}^m}{\partial\alpha^j}-\frac{\partial\Gamma_{ij}^m}{\partial\alpha^l}+\Gamma_{il}^k\cdot\Gamma_{ki}^m-\Gamma_{ij}^k\cdot\Gamma_{kl}^m\right)$$

deb olsak, R_{jlik} kovariant komponentali 4 rang egrilik tenzori bo’ladi.

Ushbu munosabatni yoza olamiz:

$$\frac{\partial g_{ij}}{\partial\alpha^k}=\frac{\partial\vec{\text{Э}}_i}{\partial\alpha^k}\cdot\vec{\text{Э}}_j+\vec{\text{Э}}_i\cdot\frac{\partial\vec{\text{Э}}_i}{\partial\alpha^k}=\Gamma_{ik}^m g_{mj}+\vec{\text{Э}}_i\cdot\Gamma_{jk}^m\cdot\vec{\text{Э}}_m=$$

$$=\Gamma_{ik}^m g_{mj}+\Gamma_{jk}^m\cdot g_{im}=\Gamma_{ik,j}+\Gamma_{jk,i}$$

U holda

$$\frac{\partial\left(\Gamma_{il}^m g_{km}\right)}{\partial\alpha^j}=\frac{\partial\Gamma_{il}^m}{\partial\alpha^j}g_{km}+\Gamma_{il}^m\cdot\frac{\partial g_{km}}{\partial\alpha^j}$$

Yuqoridagi ifodalardan foydalanib yoza olamiz:

$$\frac{\partial\Gamma_{il,k}}{\partial\alpha^j}-\Gamma_{il}^m(\Gamma_{kj,m}+\Gamma_{mj,k})-\left[\frac{\partial\Gamma_{ij,l}}{\partial\alpha^l}-\Gamma_{ij,k}^m(\Gamma_{kl,m}+\Gamma_{ml,k})\right]+$$

$$+\Gamma_{il}^k\cdot\Gamma_{nj,k}-\Gamma_{ij}^n\cdot\Gamma_{nl,k}=0$$

Shunday qilib:

$$\frac{\partial \Gamma_{ij,k}}{\partial \alpha^j} - \frac{\partial \Gamma_{ij,l}}{\partial \alpha^k} + \Gamma_{ij}^m \cdot \Gamma_{kl,m} - \Gamma_{il}^m \cdot \Gamma_{kj,m} = 0 .$$

Kristoffelning 1- va 2- tur simvollari metrik tenzor orqali, ya'ni deformatsiya tenzori orqali ifodalab yozish mumkin. Haqiqatan ham:

$$\Gamma_{ij,k} = \frac{1}{2}\left(\frac{\partial g_{ik}}{\partial \alpha^j} + \frac{\partial g_{jk}}{\partial \alpha^i} - \frac{\partial g_{ij}}{\partial \alpha^k}\right) = \frac{\partial \varepsilon_{ik}}{\partial \alpha^j} + \frac{\partial \varepsilon_{jk}}{\partial \alpha^i} - \frac{\partial \varepsilon_{ij}}{\partial \alpha^k}$$

(chunki Dekart koordinatasi $g_{ij} = 2\varepsilon_{ij} + \delta_{ij}$). U holda tutash muhit deformatsiyasi birgalikda bo'lish sharti quyidagicha bo'ladi:

$$\frac{\partial^2 \varepsilon_{lk}}{\partial \alpha^j \partial \alpha^i} + \frac{\partial^2 \varepsilon_{ij}}{\partial \alpha^l \partial \alpha^k} - \frac{\partial^2 \varepsilon_{il}}{\partial \alpha^j \partial \alpha^k} - \frac{\partial^2 \varepsilon_{jk}}{\partial \alpha^l \partial \alpha^i} + \Gamma_{kl,m} \cdot \Gamma_{ij}^m - \Gamma_{kj,m} \cdot \Gamma_{il}^m = 0 \quad (4.6.3)$$

Tekshirish mumkinki, egrilik tenzori uchun:

$$R_{jlik} = -R_{ljik}, \qquad R_{jlik} = -R_{jlik}, \qquad R_{jlik} = -R_{ikjl} \qquad (4.6.4)$$

bo'ladi.

Masalalar yechishga doir namuna

1. Quyidagi harakat qonunini qanoatlantiruvchi tutash muhit deformatsiyasi oddiy siljish deb ataladi:

$$x_1 = \xi_1 + a(t)\xi_2, x_2 = \xi_2, x_3 = \xi_3$$

Bu yerda (x_i)- fazodagi dekart koodinatalar sistemasi, (ξ_α)- lagranj koordinatalar sistemasi, $a(t)$-vaqtga bog'liq funksiya bo'lib, $a(0) = 0$. $a(t)$ funsiyani berilgan deb hisoblab, Grin va Al'mansi deformatsiya tenzorlarini toping. Ushbu tenzorlarni bosh o'qlari va tashkil etuvchilarini toping.

Yechish.

Grin deformatsiya tenzori quyidagiga teng bo'ladi:

$$\acute{\varepsilon} = \frac{a}{2}(e_1 e_2 + e_2 e_1) + \frac{a^2}{2} e_2 e_2$$

Uning bosh o'qlari va tashkil etuvchilari esa quyidagilarga teng bo'ladi:

$$\lambda_{1,2} = \frac{a^2}{4} \pm \sqrt{\frac{a^2}{4} + \frac{a^4}{16}}, \quad \lambda_3 = 0;$$

$$e_1 + \frac{2}{a}\lambda_{1,2}e_2, \qquad e_3.$$

Almansi deformatsiya tenzori quyidagiga teng bo'ladi:

$$\acute{\varepsilon} = \frac{a}{2}(e_1 e_2 + e_2 e_1) - \frac{a^2}{2} e_2 e_2,$$

Uning bosh o'qlari va tashkil etuvchilari esa quyidagilarga teng bo'ladi:

$$\lambda_{1,2} = -\frac{a^2}{4} \pm \sqrt{\frac{a^2}{4} + \frac{a^4}{16}}, \quad \lambda_3 = 0;$$

$$e_1 + \frac{2}{a}\lambda_{1,2}e_2, \qquad e_3.$$

IV bobga doir masalalar.

1. Koordinatalari (ξ_1, ξ_2, ξ_3) bo'lgan muhitning nuqtasi siljish natijasida koordinatalari

   $x_1 = \xi_1 + a\xi_1, x_2 = \xi_2, x_3 = \xi_3, a = const$

bo'lgan dekart koordinatalar sistemasidagi (x_i) nuqtaga ko'chdi. Bunday deformatsiya bir o'qli, bir jinsli x_1 o'qi bo'yicha cho'zilish deb ataladi.

Dastlab muhitning x_1 o'qqa nisbatan parallel va perpendikulyar joylashgan boshqa elementlari bilan nima sodir bo'ladi? $(a > 0$ va $-1 < a < 0$ holatlarda.)

2. Biro'qli cho'zilish uchun Lagranj va Eyler tavsifidagi maydon siljishini toping. Grin va Al'mansi deformatsiya tenzori komponentalarini hisoblang.
3. a) Modda elementi boshlang'ich koordinatasi ξ bo'lgan zarrachaga $d\xi$ vektor mos keladi. Ushbu zarracha uchun Grin deformatsiya tenzori $\varepsilon_{\alpha\beta}$ komponentelarini bilgan holda deformatsiya natijasida modda elementning nisbiy uzayishini toping;

b) Biro'qli cho'zilish uchun deformatsiya holatida x_3 koordinataga perpendikulyar va x_1 koordinata bilan $\pm\pi/4$ burchak hosil qiluvchi modda elementlarini nisbiy cho'zilishini toping.

4. a) Boshlang'ich koordinatasi ξ bo'lgan zarrachaga to'gri keluvchi ikkita moddiy element $d\xi^{(1)}$ va $d\xi^{(2)}$ vektorlarga mos keladi. Grin deformatsiya tenzori $\varepsilon_{\alpha\beta}$ komponentalarini bilgan holda material deformatsiyadan so'ng qanday burchak hosil qilishini toping;

b) Biro'qli deformatsiya uchun elementar deformatsiyadan oldin x_3 o'qiga perpendikulyar va x_1 o'qi bilan $\pm\pi/4$ burchak hosil qilgan bo'lsa, deformatsiyadan so'ng ular orasidagi burchak qanday bo'lishini toping.

5. Biro'qli cho'zilishda hajmni nisbiy o'zgarishini toping.

6. Dastlabki koordinatasi (ξ_1, ξ_2, ξ_3) bo'lgan muhitning nuqtasi siljish natijasida fazoviy dekart koordinatalarining

$$x_1 = \xi_2, x_2 = -(1+b)\xi_1, x_3 = \xi_3, b = const > -1$$

nuqtasiga ko'chadi.

a) Koordinata o'qlariga parallel joylashgan modda elementlari bilan deformatsiya natijasida nima sodir bo'ladi?

b) Grin va Al'mansi tenzorlarini toping;

c) $|b| \ll 1$ bo'lganda Grin va Al'mansi tenzorlarini aniqlasa bo'ladimi?

7. Siljish natijasida dastlabki koordinatalari (ξ_1, ξ_2, ξ_3) bo'lgan zarra dekart koorrdinatalari

$$x_1 = \xi_1 + \alpha\sin(k\xi_1), x_2 = \xi_2, x_3 = \xi_3$$

nuqtaga ko'chadi. Bu yerda $\alpha = const, |\alpha| < 1, k = const$. Muhitning har bir nuqtasining juda kichik atrofida biro'qli cho'zilish sodir bo'lishini ko'rsating. Boshlang'ich nuqtasi berilgan va deformatsiyadan oldin x_1 o'qqa parallel bo'lgan modda elementining nisbiy cho'ziishi nimaga teng. Grin tenzorini hisoblang.

8. Dastlabki (ξ_1, ξ_2, ξ_3) holatdan siljish natijasida muhit nuqtasi dekart koordinatalar sistemasida

$$x_i = \xi_i + a\xi_i, i = 1,2,3, a = const > -1$$

koordinatalarga ega. Barcha modda elementlarining nisbiy cho'zilishi bir xil bo'lishini ko'rsating. a ning qanday qiymatlarida siqilish va qanday qiymatlarida cho'zilish sodir bo'ladi?

9. Quyidagi harakat qonunini qanoatlantiruvchi tutash muhit deformatsiyasi oddiy siljish deb ataladi:

$$x_1 = \xi_1 + a(t)\xi_2, x_2 = \xi_2, x_3 = \xi_3$$

Bu yerda (x_i)- fazodagi dekart koodinatalar sistemasi, (ξ_α)- lagranj koordinatalar sistemasi, $a(t)$-vaqtga bog'liq funksiya bo'lib, $a(0) = 0$. $a(t)$ funsiyani berilgan deb hisoblab, Grin va Al'mansi deformatsiya tenzorlarini toping. Ushbu tenzorlarni bosh o'qlari va tashkil etuvchilarini toping.

10. Lagranj va Eyler tavsifida oddiy siljish uchun ko'chish maydoni komponentalaini toping. Grin va Al'mansi deformatsiya tenzorlari komponentalarini siljish maydoni hosilasi orqali ifodalangan holda aniqlang. Kichik deformatsiyalar tenzorini aniqlang.

11. Oddiy siljish uchun quyidagilarni toping:

   a) Moddiy elementlarni boshlanishi ξ mumkin bo'lgan barcha zarrachalarda bo'lgan holat uchun va x_1, x_2, x_3 o'qlarga parallel deformatsiyalargacha nisbiy cho'zilishini;

   b) Vaqt *t=0* bo'lganida nisbiy cho'zilishi barcha moddiy elementlarni aniqlang.

12. Oddiy siljishda muhitning kichik hajmining nisbiy o'zgarishini toping. Xisoblashni ikki xil usulda olib boring. Bunda Grin va Al'mansi tenzori invariantlaridan foydalaning.

13. Muhitning biror nuqtasida kichik deformatsiya sodir bo'ldi. Dekart koordinatalar sistemasida kichik deformatsiyalar tenzori quyidagi matritsaga ega:

$$\begin{pmatrix} 0{,}01 & 0{,}03 & 0 \\ 0{,}03 & 0{,}01 & 0 \\ 0 & 0 & 0{,}01 \end{pmatrix}$$

Ushbu nuqtadagi moddiy elementlarni eng kata va eng kichik nisbiy uzayishini toping.

14. Tutash muhitning ikkilanma siljishi deb muhitning quyidagi harakat qonuniga bo'ysunuvchi deformatsiyaga aytiladi;

$$x_1 = \xi_1 + b(t)\xi_2, x_2 = \xi_2 + b(t)\xi_3, x_3 = \xi_3$$

Bu yerda (x_i)-fazoviy dekart va (ξ_α)-lagranj koordinatalari, *b(t)*-vaqtga bog'liq funksiya bo'lib, b(0)=0. Ushbu *b(t)* funksiyani berilgan deb xisoblab, Grin va Al'mansi deformatsiya tenzorlarini toping.

15. Ikkilanma siljishda Eyler tavsifidagi siljish maydoni komponentalarini toping. Kichik deformatsiyalar tenzorini toping.

16. Uchta moddiy elementlarni holati deformatsiyalangan holatda quyidagi vektorlar bilan berilmoqda

$$d(x)^i = dse_i, i = 1,2,3,$$

bu yerda e_i-ortogonal koordinatalar sistemasini bazis vektorlari. Ularni "teskari" nisbiy uzayishi $ds_0^{(i)}/ds - 1$, bu yerda $ds_0^{(i)}$-elementlarning deformatsiyagacha bo'lgan uzunliklari l_i. Deformatsiyalangan holatda $d(x)^i$ va $d(x)^j$ vektorlar bilan xarakterlanuvchi elementlar deformatsiyagacha ψ_{ij} burchak hosil qiladi. Al'mansi tenzori komponentalarini mexanik ma'nosini ifodolovchi formulani isbotlang:

$$\varepsilon_{ij} = \frac{1}{2}\left[-(1 + l_i)(1 + l_j)cos\psi_{ij} + \delta_{ij}\right]$$

i va *j* bo'yicha yig'indi olinmaydi.

17. Grin va Al'mansi deformatsiya tenzorlarining asosiy qiymatlari λ_α va λ_i lar quyidagi tengsizliklarni qanoatlantirishini isbotlang

$$1+2\lambda_\alpha>0, 1-2\lambda_i>0.$$

18. Grin tenzori ε^0 va Al'mansi tenzori ε bilan deformatsiyalangan holat uchun isbotlang:

a) Agar moddiy element deformatsiyagacha ε^0 tenzorini asosiy o'qi bo'ylab yo'nalgan bo'lsa, deformatsiyalangan holatda ε tenzori asosiy o'qi bo'ylab yo'naladi;

b) Aksincha moddiy element deformatsiyalangan holatda ε tenzorining asosiy o'qi bo'ylab yo'naltirilsa, deformatsiyalangan holatda ε^0 tenzorining asosiy o'qi bo'ylab yo'naltiriladi.

c) Grin tenzori va Al'mansi tenzorlarining asosiy qiymatlari λ^0 va λ quyidagi munosabat bilan bog'langan

$$1 + 2\lambda^0 = \frac{1}{1-2\lambda}.$$

GLOSSARIY

O’zbekcha	Ruscha	Inglizcha	Izoh
Tutash muhitlar mexanikasi	Механика сплошной среды	Continuum mechanics	mexanikaning gazlar, suyuqliklar, plazma va deformatsiyalanuvchi qattiq jismlarning harakati hamda muvozanatini o‘rganuvchi bo‘limidir.
nazariy mexanika	теоретическая механика	the theoretical mechanics	muvozanat va mexanik harakat qonuniyatlarini o’rganuvchi fan.
tenzor	тензор	tensor	vektor tushunchasini umumlashtirib, bir necha ma’noda qo‘llanib kelinayotgan matematik atama. Tutash muhitlar mexanikasida qayishqoq (egiluvchan) jismning holatini ifodalovchi miqdor (deformatsiya tenzori) yoki qattiq jism massasining olingan nuqtaga nisbatan taqsimlanishini ifodalovchi miqdor (inersiya tenzori), shuningdek, tutash jismning biror nuktasida kuchlanishlarni ifodalovchi miqdorlar (kuchlanishlar tenzori).
Dekart koordinatalar sistemasi	система Декартовых координат	system of Dekart coordinates	tekislik yoki fazodagi to‘g‘ri chiziq koordinatalar tizimi.
Lame koeffitsientlari	коэффициенты Ламе	coefficient of Lame	kovariant bazis vektorlarning moduli $Э_\alpha = \left\lvert \bar{Э}_\alpha \right\rvert = \sqrt{\bar{Э}_\alpha \cdot \bar{Э}_\alpha} = \sqrt{g_{\alpha\alpha}}$
kовариант координата базиси	базисы ковариантных координат	covariant coordinate bases	$\bar{Э}_\alpha\left(x^1,\ x^2,\ x^3\right) = \frac{\partial \bar{r}}{\partial x^\alpha} = \frac{\partial y^\beta}{\partial x^\alpha}\bar{k}_\beta$ x_α-koordinata chizig’iga urinma bo’ylab musbat tomonga yo’nalgan va $d\bar{r}_\alpha$ ga kolleniar bo’lgan vektordir.

kontrava-riant koordinata bazisi	базиси контравариант-ных координат	bases of controvariant	$\nabla x^{\alpha} \equiv grad\ x^{\alpha} = \dfrac{\partial x^{\alpha}}{\partial y^{\beta}} \bar{k}_{\beta}$ $\left(\alpha =\right.$
asosiy kvadratik forma	основная квадратная форма	basis square shape	$ds^2 = g_{\alpha\beta}\, dx^{\alpha} dx^{\beta}$ ifoda
invariantlar	инварианты	invariants	u yoki bu matematik ob’yekt bilan bog’liq bo’lgan va bu ob’yektlar bilan yoki ular qaralayotgan hisoblash siste-malari bilan ma’lum almash-tirishlar bajarilganda o’zgar-maydigan sonlar, algebraik ifodalar va boshqa miq-dorlardir.
skalyar maydon	скалярные поля	scalar fields	har bir nuqtasida skalyar miq-dor aniqlangan soha
vektor maydon	векторные поля	vector fields	har bir nuqtasida vektor miq-dor aniqlangan soha.
deformasiya	деформация	deformation	tashqi kuch, temperatura, elektr va magnit maydonlari ta’sirida jism shakli va o‘lchamlarining o‘zgarishi.
kinematika	кинематика	kinematics	mexanik harakatni shu hara-katni vujudga keltiruvchi kuchlarga bog’lamasdan o’r-ganadigan bo’lim
dinamika	динамика	dynamics	mexanik harakatni shu hara-katni vujudga keltiruvchi kuchlarni hisobga olib o’r-ganadigan bo’lim.
kuchlanish-lar	напряженности	tensions	ma’lum kuch ta’sirida sodir bo‘ladigan holat.
bazis vektorlar	векторы базиса	basis vectors	koordinata o’qlarida joylash-gan, musbat yo’nalgan va mo-dullari birga teng bo’lgan vektor.

moddiy nuqta	материальная точка	material point	shaklini, o’lchamlarini hisob-ga olmaslik mumkin bo’lgan jism.
muvozanat shartlari	условия баланса	terms of balance	jismni kuchlar ta’siridan o’zini holatini o’zgartir-masligi uchun zarur bo’lgan qoidalar.
nisbiy harakat	относительное движение	relative movement	moddiy nuqtaning qo’zg’a-luchi o’qlarga nisbatan qila-digan harakati.
og’irlik markazi	центр тяжести	centr of gravity	jismning og’irlik kuchi qo’-yiladigan nuqta.
oniy aylanish markazi	мгновенный центр вращения	instant rotation center	tezligi shu onda nolga teng bo’lgan nuqta.
muvozanat	равновесие	equilibrium	biror qo’zg’almas sanoq sistemasiga sistemasiga nis-batan tinch vaziyat.
mexanik harakat	механическое движение	mechanical movement	nuqta yoki jismlarning o’zaro ko’chishi.
absolyut qattiq jism	абсолютное твёрдое тело	absoluteri gidbody	kuch ta’siridan ikki nuqtasi orasidagi masofa o’zgar-masdan qoladigan jism.
erkin jism	свободное тело	free skewfield	ixtiyoriy yo’nalishda harakat-lana oladigan jism.
kuch	сила	force	jismlarning o’zaro ta’siri.
kuchlar sistemasi	система сил	system of forces	ta’sir etuvchi bir nechta kuch.

nuqta harakati	движение точки	point movement	nuqtaning boshlang'ich hola-tidan ohirgi holatiga vaqtga bog'liq holda aniq bir holda o'tishi.
nuqta traektoriyasi	траектории точки	point trajectory	vaqt o'tishi bilan nuqtaning fazoda qoldirgan izi.
nuqta tezligi	скорость точки	point velocity	harakat qonunidan vaqt bo'-yicha olingan birinchi tartibli hosila.
qattiq jismning ilgarilanma harakati	поступательное движение твердого тела	translational rigid body movements	jismda olingan har qanday kesma jism harakati davo-mida hamma vaqt o'z-o'ziga parallel qolishi.
nuqtaning murakkab harakati	абсолютное (сложное) движение точки	absolute (complicated) movement of a point	nuqtaning bir necha sanoq sistemalariga nisbatan hara-kati.
moddiy nuqta harakatining differensial tenglamasi	уравнение движение материальной точки	the equation mass point movement	harakat tenglamasining dif-ferensial ko'rinishi.
boshlang'ich shartlar	начальные условия	entryconditions	harakatning boshlanish vaqti-ga mos keluvchi koordinata va tezlik. (t=0).
harakat miqdori	количество движения	momentum	nuqta yoki jism massasini o'z tezligi ko'paytmasiga teng vektor kattalik.
diffuziya	диффузия	diffusion	issiqlik harakati natijasida idishdagi ikki aralashuvchan suyuqliklar molekulalarining ajralish sirti orqali asta sekin biridan ikkinchisiga o'tishi hodisasi (natijada suyuq-liklar o'zaro aralashishadi). Bu ho-

			disa boshqa agregan holatidagi moddalarda ham sodir bo'ladi.
suyuqlik	жидкость	liquid	moddaning agregat holatlaridan biri bo'lib, xoxlagancha kichik kuch ta'sirida o'z shaklini o'zgartirish xususiyatiga ega uzluksiz muhit (fizik jism), ya'ni oquvchanlik xossasiga ega va o'z shakliga ega bo'lmagan ixtiyoriy muhit. Izoh: gaz «siqiluvchan suyuqlik» (havo, kislorod, azot, propan va hokazo) deb atalgan holda suyuqlikni gazdan ajratish maqsadida «tomchili suyuqlik» (suv, neft, kerosin, yog' va hokazo) atamasi ishlatiladi. Tomchili suyuqliklar (sodda qilib, suyuqliklar) va gazsimon suyuqliklar (gazlar) bir biridan siqiluvchanlik (hajmini o'zgartiruvchanlik) xususiyati bilan ajralib turadi.
divergensiya	дифергенция	divergence	tarqalish (berilgan nuqtadagi vektor maydon oqimining o'zgarishini tavsiflovchi kattalik).
yopishqoqlik (qovushqoqlik) dinamik koeffisiyenti yoki yopishqoqlik (qovushqoqlik) koeffisiyenti	динамический коэффициент вязкости или коэффициент вязкости	dynamic viscosity coefficient or dynamic coefficient	Nyuton suyuqligi urinma kuchlanishlarining deformatsiya tezliklari tenzori ifodasiga kiruvchi propor-sionallik koeffisiyenti. Bu koeffisiyent temperaturaga kuchli darajada bog'liq, bosimga esa deyarli bog'liq emas.
atmosfera	атмосфера	athmosphere	Yer yuzidagi og'irlik kuchi ta'sir qiluvchi havo qatlami.

vakuum	вакуум	vacuum	berk idishdagi havoning yoki gazning atmosfera bosimiga nisbatan siyraklashgan holati: $P_v = P_{atm} - P_t$ bu yerda P_{atm} – atmosfera havo bosimi; P_t– to'la bosim.
qovushoq suyuqlik	вязкая жидкость	viscous fluid	harakati jarayonida suyuqlik zarrachalarining deformatsi-yalanishidan bog'liq ham normal va ham urinma kuchlanishlari paydo bo'la-digan suyuqlik.
gidrostatika	гидростатика	hydrostatics	bu suyuqlik va gazlar mexanikasi fanining tan-langan koordinata boshiga nisbatan suyuqlik muvozanati va suyuqlikka to'la yoki qisman botirilgan qattiq jism muvozanatini o'rganuvchi bo'limi.
gidrostatik bosim	гидростатичес-кое давление	hydrostatic pressure	gidrostatik kuchning u ta'sir qilayotgan yuzaga nisbating shu yuza nolga intilgandagi limiti gidrostatik bosim deyiladi yoki tanlangan sanoq sistemasiga nisbatan muvo-zanat (tinch) holatda turgan suyuqlikdagi bosim
bosh kuchlanish-lar	главные напряжения	principal stresses	qaralayotgan nuqtada kucla-nishning bosh o'qlariga per-pendikulyar yuzalardagi nor-mal kuchlanishlar.
deformatsi-yalarning bosh o'qlari;	главные оси деформации	principal axes of deformation	berilgan nuqta orqali o'tuvchi va suyuqlik zarrachalarining deformatsiyasi natijasida o'zaro perpendikulyar bo'lib qola-digan uchta chiziqli elementlari bilan mos keluv-chi uchta o'zaro perpendi-kulyar to'g'ri chiziqlar.
kuchlanish-larning bosh o'qlari	главные оси напряжений	major stress axes	fazoning berilgan nuqtasi orqali o'tuvchi, urinma kuchlanishlari nolga teng bo'lgan o'zaro perpen-

			dikulyar uchta tekisliklar normallari bo'yicha yo'nalgan uchta to'g'ri chiziq
vektor	вектор	vector	koordinata sistemasini tanlanishiga bog'liq bo'lmagan va oddiy ob'yektlarning ushbuu $\bar{a} = a^{\alpha}\bar{Э}_{\alpha} = a_{\beta}\bar{Э}^{\beta}$ chiziqli ifodasi ko'rinishida beriladigan miqdor
ikkinchi rang tenzor	тензор 2-го ранга	2nd rank tensor	koordinata sistemasini tanlanishiga bog'liq bo'lmagan va diadalarning ushbu $T = T^{\alpha\beta}\bar{Э}_{\alpha}\bar{Э}_{\beta} = T_{\alpha\beta}\bar{Э}^{\alpha}\bar{Э}^{\beta} = T^{\alpha\cdot}_{\cdot\beta}\bar{Э}_{\alpha}Э^{\beta} = T^{\cdot\beta}_{\alpha\cdot}\bar{Э}^{\alpha}\bar{Э}_{\beta}$ chiziqli ifodasi ko'rinishida beriladigan miqdor
n–rang tenzor	тензор n-го ранга	nth rank tensor	Muayyan koordinata sistemasida 3^n ta son - tashkil etuvchilari berilgan va koordinatalar sistemasi almashtirilganda mazkur komponentalar $\hat{T}^{\cdot\beta_2\ldots\beta_n}_{\beta_1} = T^{\cdot\alpha_2\ldots\alpha_n}_{\alpha_1}\frac{\partial\xi^{\alpha_1}}{\partial\eta^{\beta_1}}\frac{\partial\eta^{\beta_2}}{\partial\xi^{\alpha_2}}\ldots\frac{\partial\eta^{\beta_n}}{\partial\xi^{\alpha_n}}$ va $T^{\cdot\alpha_2\ldots\alpha_n}_{\alpha_1} = \hat{T}^{\cdot\beta_2\ldots\beta_n}_{\beta_1}\frac{\partial\eta^{\beta_1}}{\partial\xi^{\alpha_1}}\frac{\partial\xi^{\alpha_2}}{\partial\eta^{\beta_2}}\ldots\frac{\partial\xi^{\alpha_n}}{\partial\eta^{\beta_n}}$ formulalarga binoan o'zgaradigan miqdor
elastik jism	эластическое тело	elastic body	kuch olib tashlangandan so'ng jism o'z holatiga to'liq qaytsa.
ideal suyuqlik	идеальная жидкость	ideal fluid	harakat paytida faqat normal kuchlanishlar paydo bo'ladigan suyuqlik.
siqiluvchan suyuqlik	сжимающаяся жидкость	compressible fluid	zichligi bosimga bog'liq bo'lgan suyuqlik
siqilmaydigan suyuqlik	несжимающаяся жидкость	incompressible fluid	zichligi bosimga bog'liq bo'lmagan suyuqlik.
deformatsiya tezligi	скорость деформации	strain rate	suyuqlik zarrachasining shakli va hajmining, zarrachaning

			berilgan nuqtadan o'tuvchi barcha chiziqli elementlarining o'zgarish tezliklari bilan aniqlanuvchi o'zgarish tezligi
siljish tezligi	скорость сдвига	shear rate	bir nuqtadan chiquvchi suyuqlik chiziqlarining ikkita dastlabki o'zaro perpendikulyar kesmalari orasidagi burchakning o'zgarish tezligi.
notekis harakat	неравномерное движение	uneven movement	oqimning turlicha kesimida tezlik miqdori turlicha bo'lgan harakat
beqaror (nobarqaror) harakat	неустойчивое движение	erratic movement	harakatlanayotgan suyuqlik zarrachalarining tezligi miqdori va uning yo'nalishi vaqt bo'yicha o'zgarib turadigan ho
tekis yoki parallel oqimlari harakat	равномерное или параллелное движение струи	uniform or parallel jet movement	bu shunday harakatki, unda harakat kesimi tezlik epyurasining shakli va o'lchamlari berilgan vaqtda oqim bo'yicha o'zgarmaydi
ideal suyuqlik	идеальная жидкость	ideal fluid	yopishqoqligi (ichki ishqalanishi) yo'q va harorat o'zgarganda hajmi sira o'zgarmaydi deb faraz qilingan (ideallashtirilgan) suyuqlik
harakat miqdori o'garishi qonun	закон изменения количество движения	law of change of quantity motion	individual hajm harakat miqdorining o'zgarish tezligi unga ta'sir etayotgan tashqi kuchlar yig'indisiga teng
massaning saqlanish qonuni;	закон сохранения массы	mass conservation law	individual hajmning massasi ozgarmaydi, ya'ni massaning vaqt bo'yicha o'zgarishi nolga teng
energiyaning saqlanish qonuni	закон сохранения энергии	law of energy conservation	individual hajm to'la energiyasining o'zgarish tezligi vaqt birligi ichida unga tashqaridan kelayotgan energiya oqimiga (tashqi kuchlar, issiqlik va boshqalar ishi shaklida) teng
entropiyanin	закон	entropy	individual hajm entropiya-

g saqlanish qonuni	сохранения энтропии	conservation law	sining o'zgarish tezligi vaqt birligi ichida unga tashqaridan kelayotgan entropiya oqimi va hajm ichida ishlab chiqilgan entropiya yig'indisiga teng (faqat qaytarilmaydigan jarayonlar uchun).
yopishqoqlik (qovushoqlik)ning kinematik koeffisiyenti	кинематический коэффициент въязкости	kinematic viscosity coefficient	miqdor jihatidan μ-qovushoqlik dinamik koeffisiyentining ρ-suyuqlik zichligiga nisbati, yani $v = {}^{\mu}/_{\rho}$
oqim chizig'i	линия тока	current line	urinmasi tezlik vektori bilan mos keluvchi chiziq, boshqacha aytganda, har bir nuqtasiga o'tkazilgan urinma suyuqlik tezligiga parallel bo'lgan chiziq
Lagranj o'zgaruvchilari	переменные Лагранжа	Lagrange variables	suyuqlik harakatini tavsiflashda erkin o'zgaruvchilar sifatida qo'llaniladigan fazoviy nuqtalar koordinalari va vaqt.
Eyler o'zgaruvchilari	переменные Эйлера	Eyler variables	suyuqlik harakatini tavsiflashda erkin o'zgaruvchilar sifatida qo'llaniladigan fazoviy nuqtalar koordinalari va vaqt.
muvozanat holati	состояние равновесия	equilibrium condition	tashqi shartlar saqlanganda tizimning holat parametrlari uzoq vaqt o'zgarmasdan o'zgarmas qiymatlarni qabul qilib turadigan holat
ichki kuchlar	внутренние силы	inner forces	suyuqlik moddiy zarrachalarining o'zaro ta'sir kuchlari
massaviy kuchlar	массовые силы	mass forces	qaralayotgan hajm birligi massasiga proporsional kuchlar
hajmiy kuchlar	объёмные силы	bulk forces	hajmni tashkil etuvchi barcha moddiy zarrachalarga qo'yilgan kuchlar

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Aliyeva Jamila Rayimjonovna

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