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Data of the hierarchical decision model are determined by a group of experts, on condition that the judgments are evaluated to find suitable alternatives based on the values and preferences of the decision maker to estimate the associated absolute numbers from 1 to 9, the fundamental scales of the AHP. The pairwise comparison matrix between criterion i and criterion j of each expert k is shown below:

$$A^k = {a_{ij}}^k = \begin{bmatrix} 1 & {a_{12}}^k & \cdots & {a_{1n}}^k \\ {a_{21}}^k & 1 & \cdots & {a_{2n}}^k \\ \vdots & \vdots & \vdots & \vdots \\ {a_{n1}}^k & {a_{n2}}^k & \cdots & 1 \end{bmatrix}$$

This opinion on the importance of j compared with i is the inverse of a_{ij} :

$${a_{ij}}^k = \frac{1}{{a_{ji}}^k} \qquad i > j \qquad \text{Eq. 2}$$

where n are number of criteria and ${a_{ij}}^k$ is a digit from 1 to 9 which corresponds to the opinion of expert k on the importance of criterion i and criterion j; it means that all terms of subdiagonal are the inverse of the coefficients of below diagonal.

Development of Fuzzy pairwise matrix

To aggregate the experts' judgments, Buckley's method is applied here (Buckley, 1985; Parviz et al., 2013).

In order to calculate fuzzy numbers $\tilde{a}_{ij}$, maximum, minimum and geometrical mean values of expert opinions (from all pairwise comparison matrix among k experts) were determined based on the coefficients l_{ij},m_{ij} and u_{ij}. They have been computed as defined by:

$$l_{ij} = \min {a_{ij}}^k$$

$$m_{ij} = \sqrt[k]{\prod {a_{ij}}^k}$$

$$u_{ij} \quad = \quad \max a_{ij}{}^{k}$$

Eq. 3

where k is number of experts.

Let Ã be the fuzzy judgment matrix of experts, independent of k, defined by:

$$\tilde{A} = \tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij}): \quad l_{ij} \leq m_{ij} \leq u_{ij}; \; l_{ij}, m_{ij}, u_{ij} \in [1,9]$$

which can be written explicitly as

$$\tilde{A} = \tilde{a}_{ij} = \begin{bmatrix} 1,1,1 & l_{12}, m_{12}, u_{12} & \cdots & l_{1n}, m_{1n}, u_{1n} \\ l_{21}, m_{21}, u_{21} & 1,1,1 & \cdots & l_{2n}, m_{2n}, u_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ l_{n1}, m_{n1}, u_{n1} & l_{n2}, m_{n2}, u_{n2} & \cdots & 1,1,1 \end{bmatrix}$$

where

$\tilde{a}_{ij}$= (l_{ij}, m_{ij}, u_{ij}); $\tilde{a}_{ji}$ =1/$\tilde{a}_{ij}$ = ($1/l_{ij}, 1/m_{ij}, 1/u_{ij}$) for *i,j* = 1,...,*n* and *i*< *j*=*1*,...,*n*

Defuzzification of the fuzzy pairwise comparison matrix

As it is mentioned in (Rezaei et al., 2013), we define a new function:

$$g_{\alpha,\mu}\left(\tilde{a}_{ij}\right) = \left[\mu. f_{\alpha}\left(l_{ij}\right) + (1-\mu). f_{\alpha}\left(u_{ij}\right)\right], \quad 0 \leq \alpha, \mu \leq 1 \quad \text{for} \quad i < j$$

$$g_{\alpha,\mu}\left(\tilde{a}_{ij}\right) = \frac{1}{g_{\alpha,\mu}\left(\tilde{a}_{ij}\right)} \qquad 0 \leq \alpha, \mu \leq 1: \qquad i > j$$

where

$$f_{\alpha}\left(l_{ij}\right) \quad = \quad \left(m_{ij} - l_{ij}\right).\alpha + l_{ij}$$

$$f_{\alpha}\left(u_{ij}\right) \quad = \quad u_{ij} - \left(u_{ij} - m_{ij}\right).\alpha$$

$$\alpha \quad = \quad Index\ to\ define\ stable\ or\ unstable\ conditions$$

$$\lambda \quad = \quad Index\ of\ the\ degree\ of\ pessimism\ of\ a\ decision\ maker\ for\ the\ judgment\ matrix\ \tilde{A}$$

Calculating the Consistency Ratio (CR)

In this step, the consistency ratio (CR) indicating the relative importance of different criteria was calculated (Saaty, 2008; Triantaphyllou, 2000). The consistency property of matrices is checked to ensure that the judgments of decision-makers are consistent. First, eigenvalue λ_{max} of the single pairwise comparison matrix $g_{\alpha.\beta}$ (Ã) can be calculated using the Matlab software.

$$\det\left(g_{\alpha,\beta}\,(\tilde{A}) - \lambda_{max}\mathrm{I}\,\right) = 0$$

Verifying the conformity of $CR < 0.1$

To determine whether the level of inconsistency is reasonable, Saaty developed the following methodology. Then, he used what he calls the Consistency Index (CI) of the matrix Ã. He defined (CI) as follows (Saaty, 1980; Triantaphyllou, 2000). As a rule of thumb, if CR is equal to or less than 0.10, the pairwise comparison results are acceptable, otherwise, they should be rejected and revised.

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

$$CR = \frac{CI}{RC}$$

where (RC) is the Random Consistency of the matrix Ã that can be estimated using a standard table (Saaty, 2012).

Table 1: The random consistency index

n=criteria	2	3	4	5	6	7	8	9	10
RC	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.46	1.49

Calculating corresponding criteria weights

The weight (W) was obtained from the eigenvector of the matrix $g_{\alpha.\beta}$ (Ã) using the Matlab software.

$$\left[\, g_{\alpha,\beta}\,(\tilde{A}) - \lambda_{max} I\right] W = 0$$