Game theory

Computationele biologie


LET'S PLAY A
GAME

Rules of prisoner's dilemma

You have committed a serious crime together with your partner. You have been arrested and are now being interrogated separately (no means of communication).

The police offers you a deal: testify against your partner and he will go to jail for 10 years while you go free. Your partner has been offered the same deal. If neither of you confesses, you both go to jail for 1 year. If you both confess, you both go to jail for 5 years.

Payoff matrix

Opponent

You

	Keep quiet	Confess
Keep quiet	**(-1,-1)**	**(-10,0)**
Confess	**(0,-10)**	**(-5,-5)**

The Nash equilibrium

A strategy is a Nash equilibrium if no player can improve their score by unilaterally changing strategy when knowing what their opponent will play

$E(S, S) > E(T, S)$

Where does the equilibrium lie in the prisoner's dilemma?

	Keep quiet	**Confess**
Keep quiet	**(-1,-1)**	**(-10,0)**
Confess	**(0,-10)**	**(-5,-5)**

Nash equilibrium in prisoner's dilemma

Confessing is the dominant strategy here!

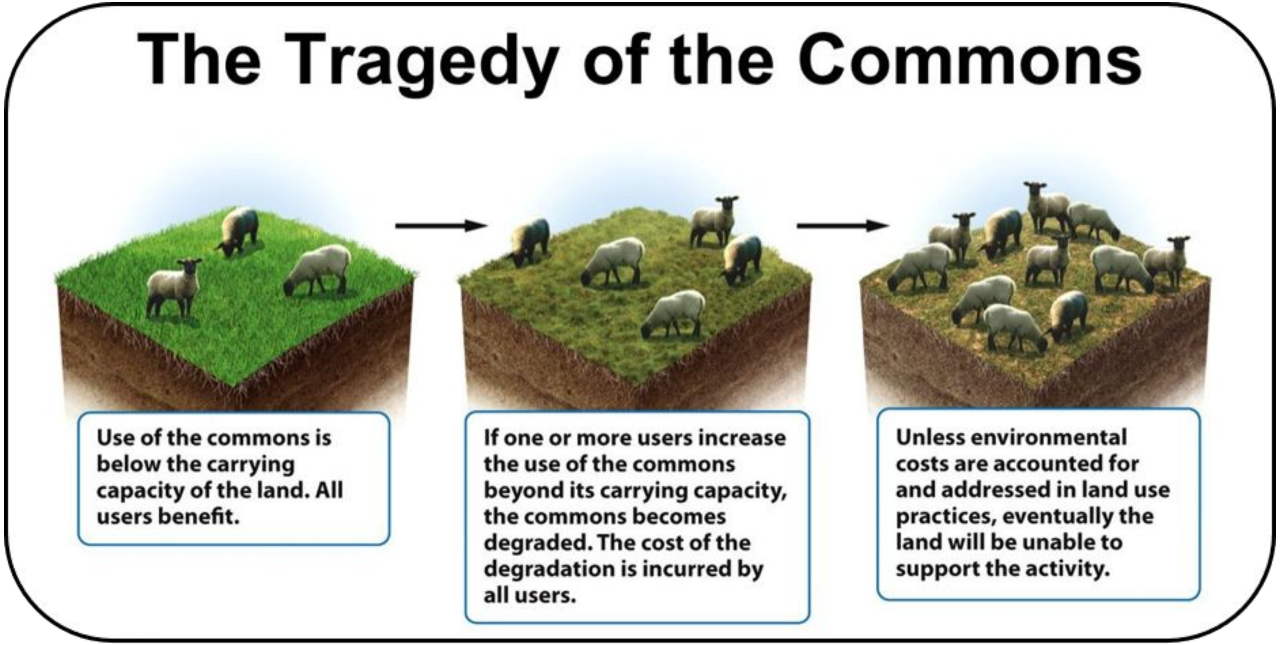
Nash lies at [confess, confess]: the players are always better off by confessing and implicating the other prisoner.

[Keep quiet, keep quiet] is better overall, but is not a Nash equilibrium! If you know that the other player keeps quiet, you can reduce your sentence by one year by switching.

	Keep quiet	Confess
Keep quiet	(-1,-1)	(-10,0)
Confess	(0,-10)	(-5,-5)

Real-life situations analogous to prisoner's dilemma

Ecology: tragedy of the commons, e.g. overfishing



Real-life situations analogous to prisoner's dilemma

Economics: advertising versus no advertising

Real-life situations analogous to prisoner’s dilemma

Ecology: tragedy of the commons, e.g. overfishing

Economics: advertising versus no advertising

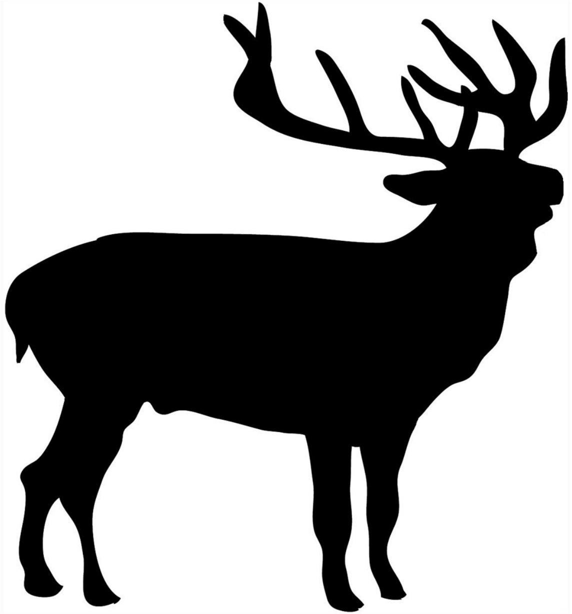
Politics: nuclear arms race during cold war

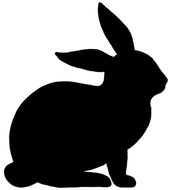
Sport: use of doping

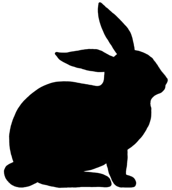
Stag-hunt game

- Two hunters, can either hunt a stag or rabbits
- The catch: the stag can escape easily from one hunter, but not from two
- One stag is bigger than two rabbits

	Hunt stag	Hunt rabbit
Hunt stag	**(2,2)**	**(0,1)**
Hunt rabbit	**(1,0**	**(1,1)**



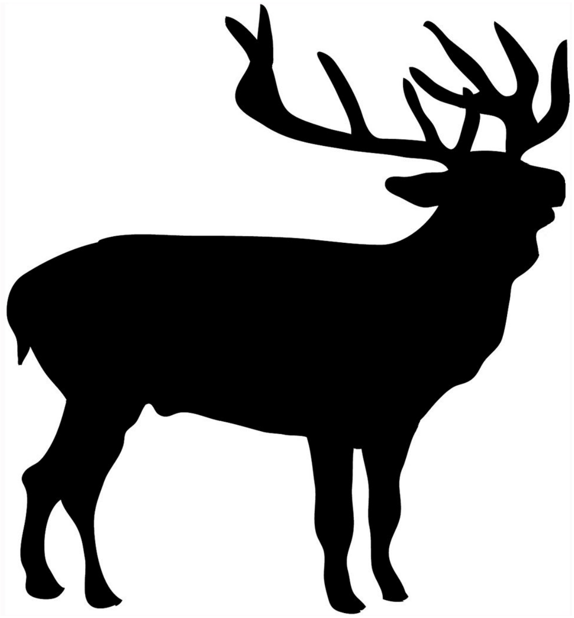


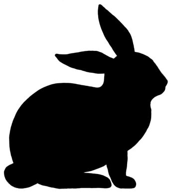


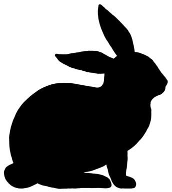
Stag-hunt game

- Where is the Nash equilibrium?
- Remember: E(S, S) > E(T, S)

	Hunt stag	Hunt rabbit
Hunt stag	**(2,2)**	**(0,1)**
Hunt rabbit	**(1,0**	**(1,1)**





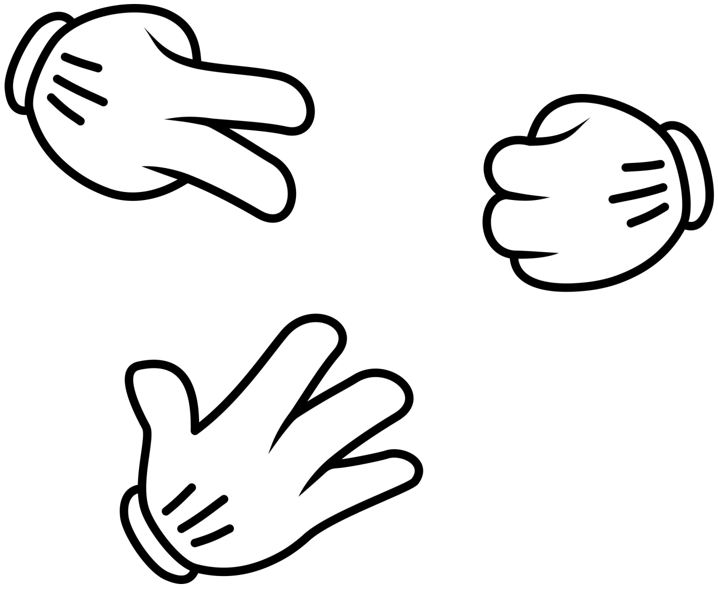


Stag-hunt continued

Stag-hunt has two Nash equilibria!

	Hunt stag	Hunt rabbit
Hunt stag	**(2,2)**	**(0,1)**
Hunt rabbit	**(1,0)**	**(1,1)**

Rock, paper, scissors



	Rock	Paper	Scissors
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissors	(-1,1)	(1,-1)	(0,0)

Rock, paper, scissors

No *pure strategy* Nash equilibrium exists: if you know what your opponent will play, you will always pick what beats him.

But we also have *mixed strategies*: for instance, my strategy could be to roll a die and play rock for 1-2, paper for 3-4 and rock for 5-6.

In this case, we have a mixed strategy Nash equilibrium. Knowing the strategy above, there is no incentive for the other player to switch to another strategy.

Rock, paper, scissors

Or: Lizard, Lizard, Lizard

Large territory
Several females
Loosely guarded

Small territory
Single female
Strictly guarded

No territory,
“Female mimic”
Sneaky

3 morphs of side-blotched lizard

Tournament time!

Partner up with your neighbour.

We will play a variant of prisoner's dilemma. Try to think of a strategy!

We will play five to seven rounds, with three opponents (15-21 rounds total)

Keep track of your own score!

	Cooperate	**Defect**
Cooperate	**(3,3)**	**(0,5)**
Defect	**(5,0)**	**(1,1)**

Strategies?

Did anyone play:

- Tit-for-tat/copycat (start off cooperating, then copy opponents last move)
- Grudger (start off always cooperating, switch to always defect when opponent defects once?)
- Always cooperating?
- Always defecting?
- Something else entirely?

Axelrod tournament

Researchers entered programs that can play prisoner's dilemma

The winner: tit-for-tat

Top scoring programs had 4 things in common:

- Nice: start off cooperating
- Retaliating: do not allow themselves to be taken advantage of
- Forgiving: do not keep on retaliating when opponent has stopped cheating
- Non-envious: do not seek to out-score a single opponent, but instead maximize your own overall score

Link with biology

Axelrod tournament showed that cooperation can spontaneously occur (given certain payoffs).

Reciprocal altruism: help others in the hope that they will help you in the future.

This is frequently observed in biology:

- Monkeys will call out to alert others when they spot predators, even though this may come at personal cost (may lure the predator to them).
- Vampire bats will share food with other hungry vampire bats.

Evolutionary stable strategies (ESS)

A strategy which, if adopted by a population in a given environment, cannot be invaded by any alternative strategy that is initially rare.

Mathematically, for a population consisting of S with alternative strategy T:

1. $E(S,S) > E(T,S)$, **or**
2. $E(S,S) = E(T,S)$ and $E(S,T) > E(T,T)$

ESS vs Nash

Identify Nash equilibria in this game

$E(S, S) > E(T, S)$

	A	B
A	(2,2)	(1,2)
B	(2,1)	(2,2)

ESS vs Nash

Identify Nash equilibria in this game

E(S, S) > E(T, S)

	A	B
A	(2,2)	(1,2)
B	(2,1)	(2,2)

ESS vs Nash

Identify ESS (A or B) in this game

A strategy which, if adopted by a population in a given environment, cannot be invaded by any alternative strategy that is initially rare.

	A	**B**
A	**(2,2)**	**(1,2)**
B	**(2,1)**	**(2,2)**

ESS vs Nash

	A	B
A	(2,~~2~~)	(1,~~2~~)
B	(2,~~1~~)	(2,~~2~~)

[A,A] and [B,B] are both Nash equilibria.

A is not evolutionary stable!

1. E(A,A) > E(B,A) ☒

or

1. E(A,A) = E(B,A) ☑
2. E(A,B) > E(B,B) ☒

Population of A can be invaded by B, because B is better at playing against B then A is.

ESS vs Nash

	A	B
A	(2,~~2~~)	(1,~~2~~)
B	(2,~~1~~)	(2,~~2~~)

[A,A] and [B,B] are both Nash equilibria.

B is evolutionary stable!

1. E(B,B) > E(A,B) ☑

or

1. E(B,B) = E(A,B) ☒
2. E(B,A) > E(A,A) ☒

Replicator equation

Consider a population consisting of animals A and B with a payoff matrix E.

The fitness (ie. how well an element is suited to its environment) for someone playing strategy A is a function of the proportion of A (x_A) and B (x_B)

$$f_A = x_A * E[A,A] + x_B * E[A,B]$$

Average fitness in the system is given by a weighted average over the population:

$$\overline{f} = x_A * f_A + x_B * f_B$$

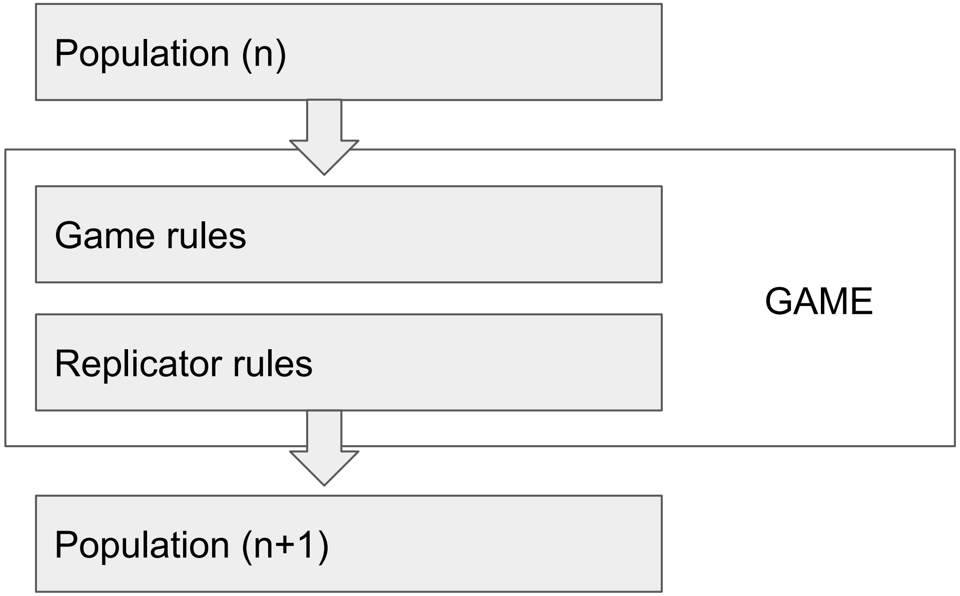
Replicator equation continued

Animals with high fitness (relative to the average fitness) will replicate more readily

Fraction of the population **xi** using strategy **i** with fitness function **fi** can be propagated with the **discrete replicator equation**:

$$x_i(t+1) = x_i(t)\frac{f_i}{\bar{f}}$$

Evolutionary systems modelling



Hawk-dove game

Consider a population consisting of Hawks and Doves competing over a resource:

- Hawks are aggressive and will fight for resources
  - Scare off doves, taking the entire resource
  - Will fight with other hawks, winning the entire resource half of the time and losing the other half, sustaining injuries
- Doves are passive and will not fight
  - Will flee from hawks, gaining nothing
  - Will share with other doves, splitting the resource

Hawk-dove game

Consider a population consisting of Hawks and Doves competing over a resource:

- Hawks are aggressive and will fight for resources
- Doves are passive and will not fight

With resource value **V** and cost of fighting **C**:

	Hawk	**Dove**
Hawk	**(V-C)/2**, **(V-C)/2**	**V**, **0**
Dove	**0**,**V**	**V/2**,**V/2**

Your turn!

Initial population: 20 hawks, 80 doves

	Hawk	Dove
Hawk	(-1,-1)	(2,0)
Dove	(0,2)	(1,1)

$$x_i(t+1) = x_i(t)\frac{f_i}{\bar{f}}$$

$$f_A = x_A * E[A,A] + x_B * E[A,B]$$

$$\bar{f} = x_A * f_A + x_B * f_B$$

Given:

- Value V = 2
- Cost C = 4

Questions:

- What proportion of the population is hawks after one step?
- Can you find the ESS here?

Your turn!

Initial population: 20 hawks, 80 doves

	Hawk	Dove
Hawk	(-1,-1)	(2,0)
Dove	(0,2)	(1,1)

$$x_i(t+1) = x_i(t)\frac{f_i}{\bar{f}}$$

$$f_A = x_A * E[A,A] + x_B * E[A,B]$$

$$\bar{f} = x_A * f_A + x_B * f_B$$

Questions:

- What proportion of the population is hawks after one step?
- Can you find the ESS here?
  - Hint: think in terms of *stable population*, rather than strategy
  - Hint: think about the link between fitness and stability

Solution

After one step we have 29,4% hawks

Pure hawk is not an ESS, nor is pure dove: we are looking at a mixed ESS.

At ESS, the fitness for all animals is equal.

	Hawk	Dove
Hawk	(-1,-1)	(2,0)
Dove	(0,2)	(1,1)

$$f_H = x_H * E[H,H] + x_D * E[H,D]$$

$$f_H = x_H * (-1) + x_D * 2$$

$$f_D = x_D * 1 + x_H * 0$$

$$x_D * 1 + x_H * 0 = x_H * (-1) + x_D * 2$$

$$x_D = -x_H + 2x_D$$

$$x_D = x_H$$

Final project

Build a functional evolutionary game theory tool

Basics are fairly simple, but you can add on a lot of things:

- Add spatial information
- Enable mixed strategies
- Add outside effects
- Make resources limited
- ….

Example

Evolution of trust

https://ncase.me/trust/