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PR-02 SPSF

a) Hukum Newton:

$$\sum \vec{F} = m.\vec{a}$$

$$q(\vec{v} \times \vec{B}) = m\frac{d\vec{v}}{dt} \quad \ldots (1)$$

pada bidang x-y, $\vec{v} = v_x\hat{i} + v_y\hat{j}$ ... (2)

Substitusikan (2) → (1)

$$q(v_x\hat{i} + v_y\hat{j}) \times \vec{B} = m.\frac{d}{dt}(v_x\hat{i} + v_y\hat{j}) \ldots (3)$$

b) Pers. diferensial tertaopel diperoleh dengan menuntaskan perkalian cross di ruas kiri:

Medan magnet $\vec{B}$ masuk ke bidang, bisa dituliskan:

$$\vec{B} = -B\hat{k} \quad \ldots (4)$$

$$\vec{v} \times \vec{B} = -\hat{i}Bv_y + \hat{j}Bv_x \quad \ldots (5)$$

substitusikan (5) → (3):

$$q(\vec{v} \times \vec{B}) = q(Bv_x\hat{j} - B_y\hat{i}) = m.\left(\frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}\right)$$

untuk arah-y saja:

$$qBv_x = m\frac{dv_y}{dt}$$

$$\frac{dv_y}{dt} - \frac{qB}{m}v_x = 0 \ldots (6).$$

untuk arah-x saja:

$$-qBv_y = m\frac{dv_x}{dt}$$

$$\frac{dv_x}{dt} + \frac{qB}{m}v_y = 0 \ldots (7)$$

c) Solusi analitik

substitusi (7) ke (6)

$$\frac{d}{dt}\left(-\frac{m}{qB}.\frac{dv_x}{dt}\right) = \frac{qB}{m}v_x$$

$$-\frac{m}{qB}.\frac{d^2v_x}{dt^2} = \frac{qB}{m}v_x$$

$$\frac{d^2v_x}{dt^2} + \left(\frac{qB}{m}\right)^2.v_x = 0 \ldots (8)$$

misalkan $k = \frac{qB}{m}$, maka (8) menjadi

$$\frac{d^2v_x}{dt^2} + k^2v_x = 0 \longrightarrow \text{solusinya:}$$

$$v_x = A\sin kt + B\cos kt$$

substitusi (6) ke (7)

$$\frac{d}{dt}\left(\frac{m}{qB}\frac{dv_y}{dt}\right) = -\frac{qB}{m}.v_y$$

$$\frac{m}{qB}.\frac{d^2v_y}{dt^2} = -\frac{qB}{m}v_y$$

$$\frac{d^2v_y}{dt^2} = -\left(\frac{qB}{m}\right)^2v_y$$

$$\frac{d^2V_y}{dt^2} + \left(\frac{qB}{m}\right)^2 V_y = 0$$

karena $k = \frac{qB}{m}$, maka

$$\frac{d^2V_y}{dt^2} + k^2 V_y = 0$$

solusi $\longrightarrow$ $V_y = C \sin kt + D \cos kt$

saat $t = 0$ misalkan partikel hanya gerak horizontal, maka

$t = 0 \longrightarrow V_y = 0$

$0 = C \sin 0 + D \cos 0$

$D = 0$

saat $t = \Delta t$, partikel gerak vertikal saja:

$V_x(\Delta t) = 0$

$0 = A \sin k.\Delta t + B \cos k.\Delta t$

$\tan k.\Delta t = -\frac{B}{A}$ maka:

$V_x = A \sin kt - A \tan k.\Delta t \cos kt$

$V_x = A (\sin kt - \tan k.\Delta t \cos kt) \ldots (9)$

dan $V_y = C \sin kt \ldots (10)$.

sementara saat $t = 0$, V_x harus bernilai, misalkan u

$V_x(0) = u = A (\sin 0 - \tan k.\Delta t . \cos 0)$

$u = A.\tan(k.\Delta t)$, maka:

$$V_x(t) = \frac{u}{\tan(k.\Delta t)} \sin kt - u \cos kt \ldots (11)$$

Persamaan posisi diperoleh dengan integral pers (10) dan (11) terhadap t.

$$y(t) = \int C \sin kt \, dt = -\frac{C}{k} \cos kt + y_0 \ldots (12)$$

$$x(t) = \int \left(\frac{u}{\tan k.\Delta t} \sin kt - u \cos kt\right) dt$$

$$x(t) = -\frac{u}{k \tan k.\Delta t} \cos kt - \frac{u}{k} \sin kt + t_0$$