31310 Linear Control Design 2

Compulsory Assignment 2015 : Loudspeaker control

by

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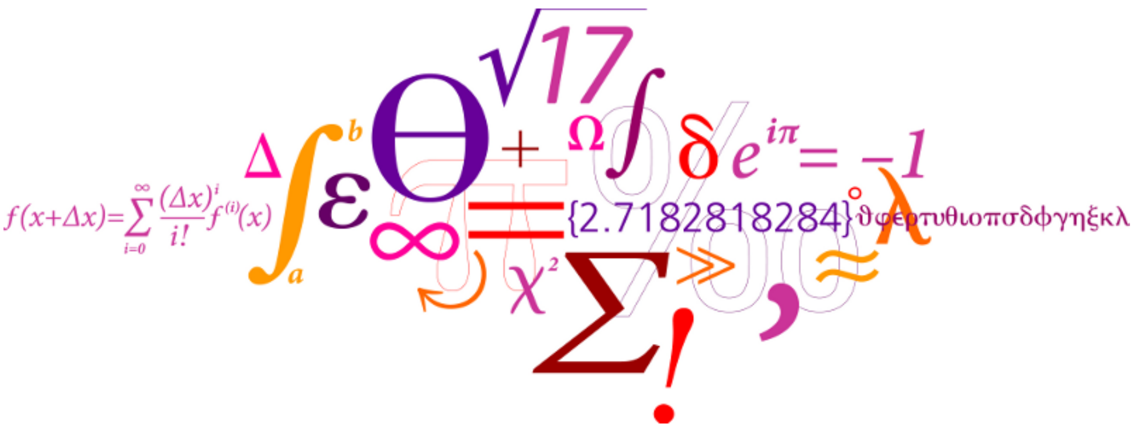

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Distortion Attenuation for Loudspeakers

1.1 Moving-coil Loudspeakers

1.1.1 Loudspeakers electrical equivalent circuit

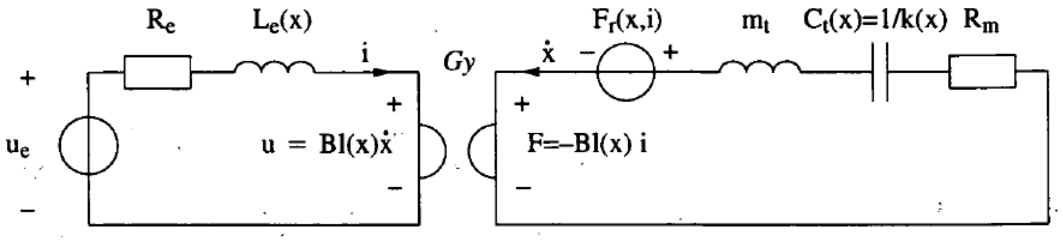

Figure 1.1: Electrical equivalent lumped element model of the voltage driven electrodynamic loudspeaker for low frequencies. The coupling between the electrical and mechanical domain is performed through the gyrator with gyration constant Bl(x).

$$u_e = R_e i + \frac{dL_e(x)}{dx}\frac{dx}{dt} i + L_e(x)\frac{di}{dt} + Bl(x)\frac{dx}{dt} \tag{1.1}$$

$$Bl(x)i = m_t\frac{d^2x}{dt^2} + R_m\frac{dx}{dt} + k(x)x - \frac{1}{2}\frac{dL_e(e)}{dx}i^2 \tag{1.2}$$

where

$$Bl(x) = Bl_0 + b_1 x + b_2 x^2 \tag{1.3}$$

$$L_e(x) = L_{e0} + l_1 x + l_2 x^2 \tag{1.4}$$

$$k(x) = k_0 + k_1 x + k_2 x^2 \tag{1.5}$$

Problem 1

By means of Eqs 1.1, 1.2, 1.3, 1.4 and 1.5, we can identify 3 state variables x, $\dot{x}$ and i. We can also identify the input u_e.

$\mathrm{x} = \begin{pmatrix} x \\ \dot{x} \\ i \end{pmatrix}$ and $\mathrm{u} = (u_e)$

Then, we can derive the nonlinear dynamical state space model to obtain

$$\dot{x} = \dot{x} \tag{1.6}$$

$$\ddot{x} = \frac{(Bl_0 + b_1 x + b_2 x^2)i - R_m \dot{x} - (k_0 + k_1 x + k_2 x^2)x + \frac{1}{2}(l_1 + 2l_2 x)\dot{x}i^2}{m_t} \tag{1.7}$$

$$\dot{i} = \frac{u_e - (R_e + (l_1 + 2l_2 x)\dot{x}^2)i - (Bl_0 + b_1 x + b_2 x^2)\dot{x}}{L_{e0} + l_1 x + l_2 x^2} \tag{1.8}$$

In matrix format, we have

$$\dot{\mathrm{x}} = f(\mathrm{x}) + g(\mathrm{x})\mathrm{u} \tag{1.9}$$

with

$$f(\mathrm{x}) = \begin{pmatrix} \mathrm{x}(2) \\ \frac{(Bl_0+b_1\mathrm{x}(1)+b_2\mathrm{x}(1)^2)\mathrm{x}(3)-R_m\mathrm{x}(2)-(k_0+k_1\mathrm{x}(1)+k_2\mathrm{x}(1)^2)\mathrm{x}(1)+\frac{1}{2}(l_1+2l_2\mathrm{x}(1))\mathrm{x}(2)\mathrm{x}(3)^2}{m_t} \\ \frac{-(R_e+(l_1+2l_2\mathrm{x}(1))\mathrm{x}(2)^2)\mathrm{x}(3)-(Bl_0+b_1\mathrm{x}(1)+b_2\mathrm{x}(1)^2)\mathrm{x}(2)}{L_{e0}+l_1\mathrm{x}(1)+l_2\mathrm{x}(1)^2} \end{pmatrix} \tag{1.10}$$

$$g(\mathrm{x}) = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_{e0}+l_1\mathrm{x}(1)+l_2\mathrm{x}(1)^2} \end{pmatrix} \tag{1.11}$$

Problem 2

Problem 3

The input voltage is set to $u_e = A_u sin(2\pi f_c t)$, where $A_u = 5\ V$ and $f_c = 20\ Hz$. Then the nonlinear system implemented in PB2 is simulated to get the states responses. To visualise the signals in the frequency domain, we used the *power_spectral_density* function given in the assignment [1] with $F_s = \frac{1}{TIME\_SIM} = 10\ kHz$. The function output gives the power spectral density P_{xx} in $[Amplitude^2/\mathrm{Hz}]$. In order to convert it in [dB/Hz], we use:

$$P_{dB} = 10log_{10}(P_{xx})$$

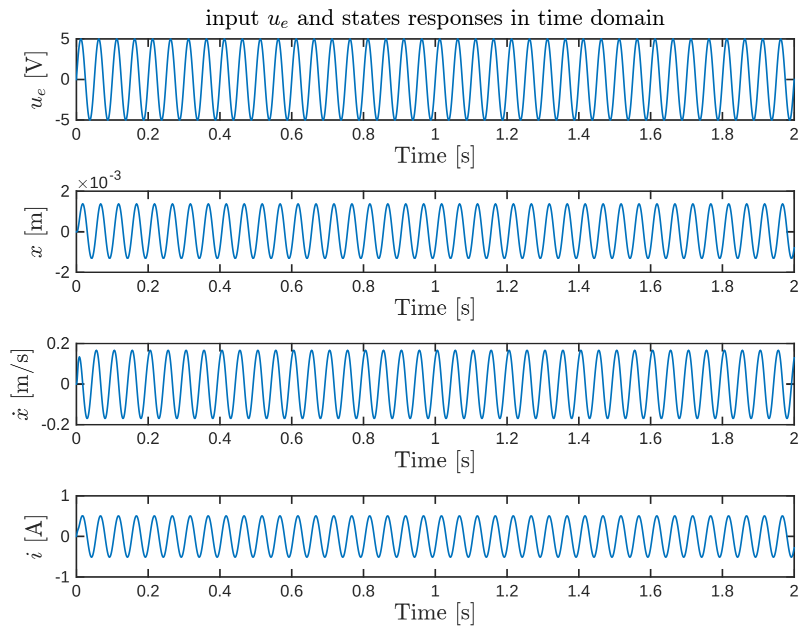

Figure 1.2: Nonlinear Model: input u_e and the states responses in the time domain

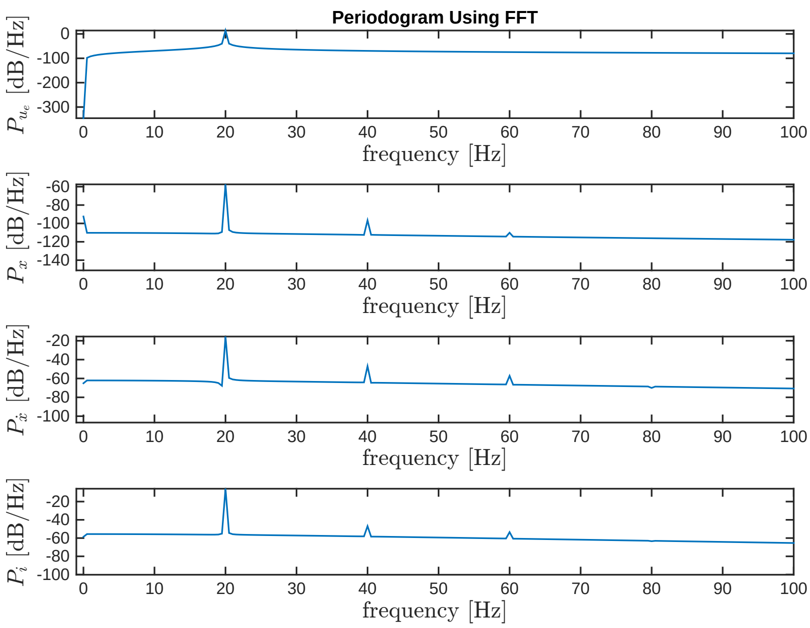

Figure 1.3: Nonlinear Model: input u_e and the states responses in the frequency domain

In the time domain (see figure 1.2), we notice that the states responses are sinusoidal, with the same amplitude. The system looks stable. Moreover, the frequency of the states seems similar to the frequency of the input signal, with slight variations. The explanation for this changes is given in the frequency domain, figure 1.3, where we can see several more spikes at the frequencies $f_n = nf_c$, smaller than the fundamental one at $f_c = 20\ Hz$. The presence of other harmonics in the states responses, which are not in the input, shows the effects of the nonlinearity of the system. It may due to the fact that the voice coil displacement is limited: when it goes too far, the spider prevents it from leaving the magnetic field, which can cause the harmonic distortion.

1.1.2 Harmonic Distortion

We are now going to study the harmonic distortion observed in the states responses in the frequency domain.

Problem 4

The analysis will be restricted to the voice coil velocity $\dot{x}$. The Total Harmonic Distortion (THD) is described by

$$THD = \frac{\sqrt{\sum_{n=2}^{N} A_n^2}}{\sqrt{\sum_{n=1}^{N} A_n^2}}\ 100\% \tag{1.12}$$

where A_1 the amplitude of the fundamental frequency and A_n the amplitudes of the harmonics.

To compute THD for the 5 harmonics after the fundamental frequency f_c we need to find the 6 amplitudes corresponding to the fundamental frequency pikes and the 5 following harmonics. To that purpose, a function *amplitude* was implemented (see below) with x the signal and *fr* the frequency at which we desire to know the amplitude of the spike. The Fast Fourier Transform is computed but not the spectral power density as previously.

```
function [ res ] = amplitude(x, TIME_SIM, fr)
    NFFT = length(x);
    X = fft(x,NFFT)/(NFFT);
    X = 2*abs(X(1:NFFT/2+1));
    res = X(fr*TIME_SIM+1);
end
```

Finally for a TIME_SIM = 5s, for the voice coil velocity $THD = 2.57\ \%$. We then use the equations (1.13) and (1.14) to compute the second and third order harmonic distortion. We obtain $d_2 = 2.48$ % and $d_3 = 0.65$ %.

$$d_2 = \frac{A_2}{\sqrt{A_1^2 + A_2^2}} 100\% \tag{1.13}$$

$$d_3 = \frac{A_3}{\sqrt{A_1^2 + A_3^2}} 100\% \tag{1.14}$$

We can notice that d_2 corresponds to the THD calculated for $N = 2$. Because the amplitudes of the harmonics decrease when the order gets bigger, it is logical that the values of THD and d_2 are quite similar. On the contrary, A_3 is negligible in front of A_1, that why d_3 is small and far from the THD.

Problem 5

In order to study the effect of the variations of the frequency f_c and the amplitude A_u of the input u_e on the second and third order harmonic distortion, we have computed d_2 and d_3 for a range of A_u and f_c. The results can be seen on the figure 1.4. According to the graphic, the variations of the amplitude of the input do not affect the distortion levels. On the opposite, when the fundamental frequency increases, the distortion level

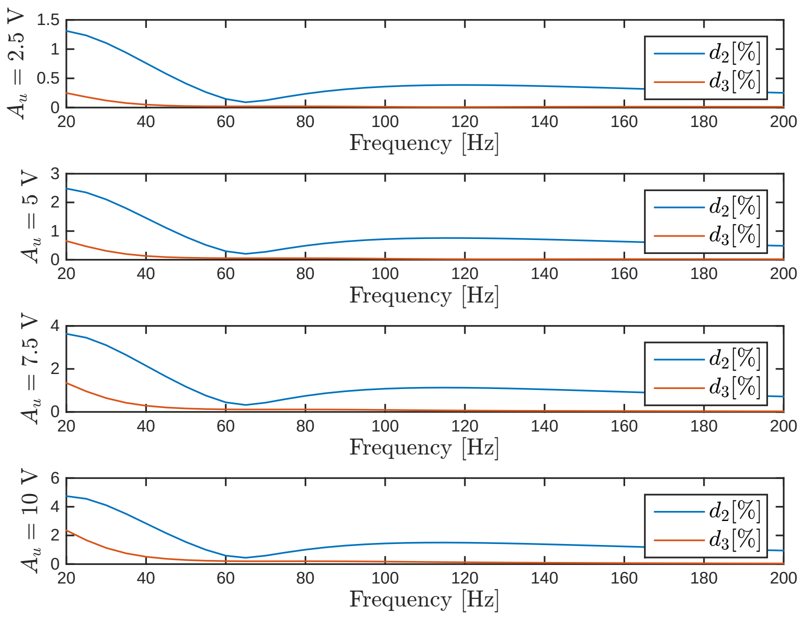

Figure 1.4: Effect of the variations of the frequency f_c and the amplitude A_u of the input u_e on the second and third order harmonic distortion

1.1.3 Linearised Model

The measured output is set to the voice coil current. Therefore $y(t) = i(t)$. We also take $u_e = 0$ for the analysis of the linear and nonlinear model around the resting position of the voice coil.

Problem 6

All time derivatives are set to zero in order to determine the stationary states. Therefore, we have $\frac{dx}{dt} = 0$ and equations (1.1) and (1.2) are rewritten below:

$$u_e = R_e i \tag{1.15}$$

$$Bl(x)i = k(x)x \tag{1.16}$$

As $u_e = 0$, from (1.15) we obtain $i = 0$ and we deduce by substituting in (1.16) that $k(x)x = 0$. Then $k(x) = 0$ or $x = 0$. The discriminant of the polynomial $k(x)$ of degree 2 is $\Delta = k_1^2 - 4k_2k_0 < 0$. The voice coil displacement x being real, we discard this value and get:

$$\mathrm{x}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

We linearise the model $\dot{\mathrm{x}} = h(\mathrm{x}, \mathrm{u})$ with $h(\mathrm{x}) = f(\mathrm{x}) + g(\mathrm{x})\mathrm{u}$ around the stationary states:

$$\begin{aligned} x(t) &= x_0 + \Delta x(t) = \Delta x(t) \\ \dot{x}(t) &= \dot{x}_0 + \Delta \dot{x}(t) = \Delta \dot{x}(t) \\ i(t) &= i_0 + \Delta i(t) = \Delta i(t) \end{aligned}$$

The linear model can then be written in the form:

$$\begin{aligned} \dot{\mathrm{x}} &= A\mathrm{x} + B\mathrm{u} \\ y &= C\mathrm{x} \end{aligned}$$

where

$$A = \begin{pmatrix} \frac{\partial h_1}{\partial \mathrm{x}_1} & \frac{\partial h_1}{\partial \mathrm{x}_2} & \frac{\partial h_1}{\partial \mathrm{x}_3} \\ \frac{\partial h_2}{\partial \mathrm{x}_1} & \frac{\partial h_2}{\partial \mathrm{x}_2} & \frac{\partial h_2}{\partial \mathrm{x}_3} \\ \frac{\partial h_3}{\partial \mathrm{x}_1} & \frac{\partial h_3}{\partial \mathrm{x}_2} & \frac{\partial h_3}{\partial \mathrm{x}_3} \end{pmatrix}_{\mathrm{x}_0} \qquad B = \begin{pmatrix} \frac{\partial h_1}{\partial \mathrm{u}} \\ \frac{\partial h_2}{\partial \mathrm{u}} \\ \frac{\partial h_3}{\partial \mathrm{u}} \end{pmatrix}_{\mathrm{x}_0} \qquad C = \begin{pmatrix} \frac{\partial r}{\partial \mathrm{x}_1} & \frac{\partial r}{\partial \mathrm{x}_2} & \frac{\partial r}{\partial \mathrm{x}_3} \end{pmatrix}_{\mathrm{x}_0}$$

We obtain:

$$\frac{\partial h_1}{\partial \mathrm{x}_1} = 0 \qquad \frac{\partial h_1}{\partial \mathrm{x}_2} = 1 \qquad \frac{\partial h_1}{\partial \mathrm{x}_3} = 0$$

$$\frac{\partial h_2}{\partial \mathrm{x}_1} = \frac{b_1 \mathrm{x}_{30} + 2b_2\mathrm{x}_{10}\mathrm{x}_{30} - (k_0 + 2k_1\mathrm{x}_{10} + 3k_2\mathrm{x}_{10}^2) + l_2\mathrm{x}_{20}\mathrm{x}_{30}^2}{m_t}$$

$$\frac{\partial h_2}{\partial \mathrm{x}_2} = \frac{-R_m + \frac{1}{2} \times (l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{30}^2}{m_t}$$

$$\frac{\partial h_2}{\partial \mathrm{x}_3} = \frac{Bl_0 + b_1\mathrm{x}_{10} + b_2\mathrm{x}_{10}^2 + (l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}\mathrm{x}_{30}}{m_t}$$

$$\frac{\partial h_3}{\partial \mathrm{x}_1} = \frac{(-2l_2\mathrm{x}_{20}^2\mathrm{x}_{30} - (b_1 + 2b_2\mathrm{x}_{10})\mathrm{x}_{20}) \times (L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2)}{(L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2)^2} - (l_1 + 2l_2\mathrm{x}_{10}) \times \frac{(-(R_e + (l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}^2)\mathrm{x}_{30} - (Bl_0 + b_1\mathrm{x}_{10} + b_2\mathrm{x}_{10}^2)\mathrm{x}_{20})}{(L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2)^2}$$

$$\frac{\partial h_3}{\partial \mathrm{x}_2} = -\frac{2(l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}\mathrm{x}_{30} + Bl_0 + b_1\mathrm{x}_{10} + b_2\mathrm{x}_{10}^2}{L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2}$$

$$\frac{\partial h_3}{\partial \mathrm{x}_3} = -\frac{(l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}^2 + R_e}{L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2}$$

$$\frac{\partial h_1}{\partial \mathrm{u}} = 0 \qquad \frac{\partial h_2}{\partial \mathrm{u}} = 0 \qquad \frac{\partial h_3}{\partial \mathrm{u}} = \frac{1}{L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2}$$

$$\frac{\partial r}{\partial \mathrm{x}_1} = 0 \qquad \frac{\partial r}{\partial \mathrm{x}_2} = 0 \qquad \frac{\partial r}{\partial \mathrm{x}_3} = 1$$

We then substitute $\mathrm{x}_{10} = \mathrm{x}_{20} = \mathrm{x}_{30} = 0$. Finally, we get:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -\frac{k_0}{m_t} & -\frac{R_m}{m_t} & \frac{Bl_0}{m_t} \\ 0 & -\frac{Bl_0}{L_{e0}} & -\frac{R_e}{L_{e0}} \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_{e0}} \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

Numerically

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -1.20 \cdot 10^5 & -50.46 & 279.19 \\ 0 & -2.57 \cdot 10^3 & -1.40 \cdot 10^3 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ 177 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \tag{1.17}$$

We can check these results with the matlab function *linmod(model,x_0,u_e)*:

```
[A,B,C,D] = linmod('nonLinearModel',[0;0;0],0);
```

Problem 7

In this problem, we draw the block diagram of the linearised loudspeaker (see figure 1.5) showing the couplings between the different states.

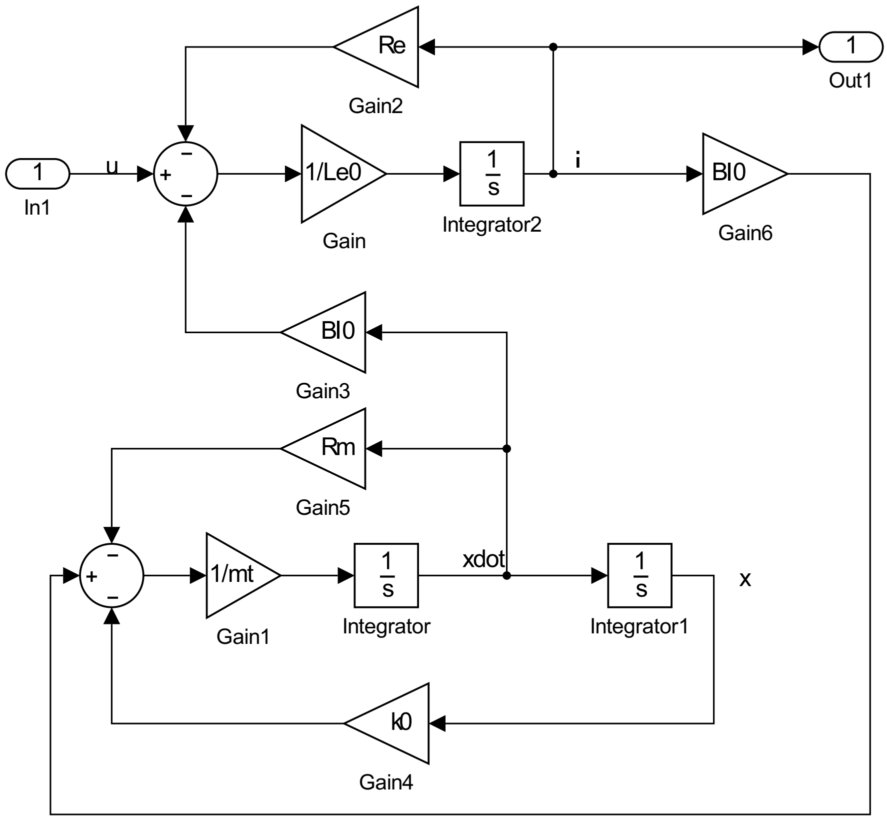

Figure 1.5: block diagram of the linearised loudspeaker

By means of this block diagram, we can make preliminary assessments of the system. Indeed, we can see that the state i seems to be controllable and observable as it is connected to the input u_e and to the output. However, the states x and $\dot{x}$ seems to be controllable as they are also connected to the input u_e but not to be observable because they are not connected to any output. Moreover the 3 states seems to be stable because of the 2 backloops.

We can verify our assumptions regarding the controllability and the observability by calculating the *rank* of M_c and M_o.

```
Mc = [B A*B A^2*B];
```

```
rank(Mc) % = 3 controllable
Mo=[C
    C*A
    C*A^2];
rank(Mo) % = 3 observable
```

Problem 8

We can derive analytically the eigenvalues of the system dynamical matrix A.

$$det(\lambda I - A) = det\begin{pmatrix} \lambda & -1 & 0 \\ \frac{k_0}{m_t} & \lambda + \frac{R_m}{m_t} & -\frac{Bl_0}{m_t} \\ 0 & \frac{Bl_0}{Le_0} & \lambda + \frac{Re}{Le_0} \end{pmatrix} \tag{1.18}$$

$$= \lambda^3 + \left(\frac{Re}{Le_0} + \frac{Rm}{m_t}\right)\lambda^2 + \left(\frac{RmRe + Bl_0}{m_t Le_0} + \frac{k_0}{m_t}\right)\lambda + \frac{k_0 Re}{m_t Le_0} \tag{1.19}$$

We therefore have a third-order polynomial which can be solved with MATLAB but the eigenvalues can also be calculated using

```
lambda = eig(A);
```

Thus, we obtain

$$\lambda = 1,0.10^2 \begin{pmatrix} -2.9573 \\ -5.7648 + 4.8571i \\ -5.7648 - 4.8571i \end{pmatrix} \tag{1.20}$$

We can notice that

$$Re(\lambda_i) < 0, \;\; i = 1, 2, 3 \tag{1.21}$$

which means that the system is asymptotically stable. Moreover, the eigenmodes of the system are $e^{\lambda_1 t}$, $e^{\lambda_2 t}$ and $e^{\lambda_3 t}$. We can notice that

$$\frac{1}{\lambda_1} = \tau_1 > \tau_2, \tau_3$$

which means that the response of the state x to an input u_e will be slower than the one of i and $\dot{x}$. We can also see that as λ_2 and λ_3 have an imaginary part which means that $\dot{x}$ and i will have an oscillatory response.

Problem 9

Using the results of Problem 6 (1.17), we can implement a SIMULINK model (see figure 1.6) of the linear system.

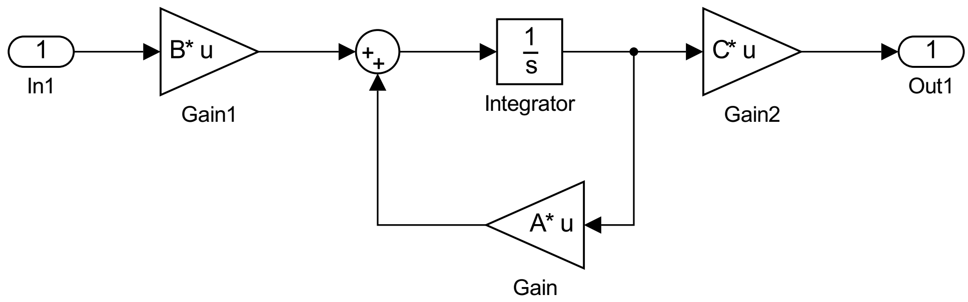

Figure 1.6: SIMULINK model of the linearised loudspeaker

Then, we can simulate the linearised model with an input $u_e = A_u \sin(2\pi f_c t)$, where $A_u = 5V$ and $f_c = 20Hz$. The states response in the time domain is plotted figure 1.7 and the PSD is plotted figure 1.8.

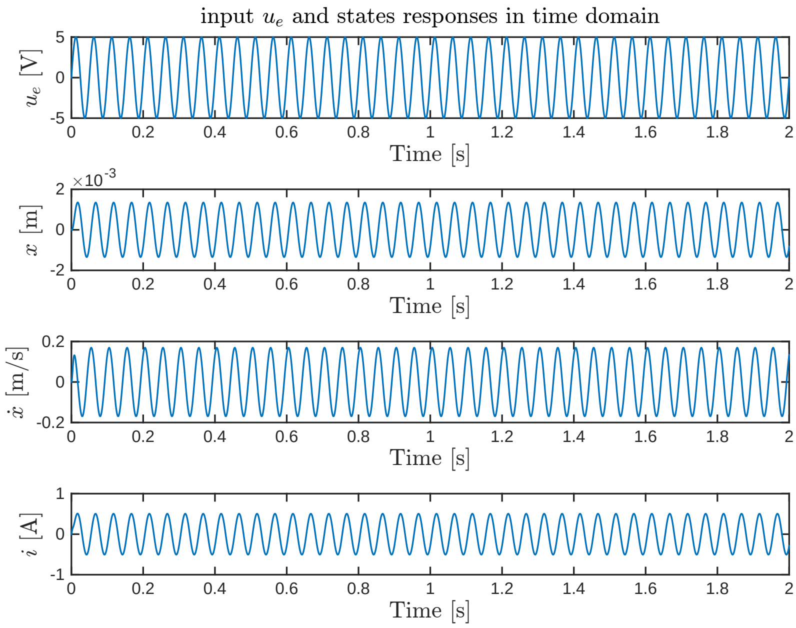

Figure 1.7: u_e and states reponse to u_e in time domain

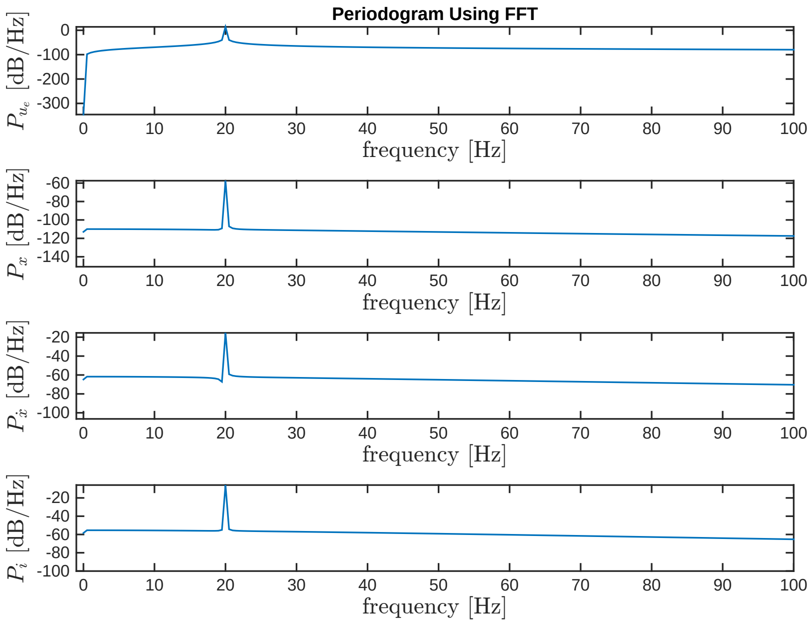

Figure 1.8: PSD of u_e and states

First, comparing the response to u_e in the time domain (figure **??** and figure 1.7), the states response seems to be the same, but comparing the PSD of the states response (figure **??** and figure 1.8), we can see that the states response is not the same. Indeed, the linearised system is non affected by the harmonic distortion, there is only on frequency present in the state response, the one of $20Hz$. This result was expected because the nonlinear distortion only affects non-linear systems. Moreover, we have linearised the system with an input $u_e = 0$ which means that we have a linearised system without harmonic, and then, we used a new input u_e with a frequency $f_c = 20Hz$, which means we only obtain a state response with an harmonic of a frequency of f_c.

1.1.4 Harmonic distortion and fictitious disturbances

In this section, we basically want to add a disturbance in order to obtain the same output with the linearised system than with the nonlinear system (the analysis is restrained to the second and third order harmonics). Thus, the output $i(t)$ should be

$$y_{nl}(t) = A_1 \sin(2\pi f_c t + \psi_1) + A_2 \sin(4\pi f_c t + \psi_2) + A_3 \sin(6\pi f_c t + \psi_3) \tag{1.22}$$

Problem 10

First, we extend our model with two input disturbances such that the linear output will also show the second and third order harmonics. The new linearised model is

$$\dot{\mathrm{x}} = A\mathrm{x} + B\mathrm{u} + B_d\mathrm{d}$$
$$y = C\mathrm{x}$$

with $\mathrm{d} = [d_{i1}, d_{i2}]$.

We know that $d_{i1} = A_{i1}\sin(4\pi f_c t)$ and $d_{i2} = A_{i2}\sin(4\pi f_c t)$ but we have to determine the magnitudes A_{i1} and A_{i2}. As this disturbance can be considered as an input, we choose to take $B_d = [B\ B]$ and to find the right magnitudes to use.

In order to find A_{i1} and A_{i2}, we evaluate the gain of the transfer function for the three different frequencies. Then, knowing the desired amplitude for each harmonic, we can find the magnitudes A_{i1} and A_{i2}.

```
sys=ss(A,B,C,[0]);
w = [2:2:6]*pi*fc;
[MAG,PHASE] = bode(sys,w)
MAG=[MAG(1) MAG(2) MAG(3)];
Amplitude(3,1:3)./MAG % = 5.0212    0.0830    0.0943
```

Problem 11

The disturbance is now added to our SIMULINK model (see figure 1.9).

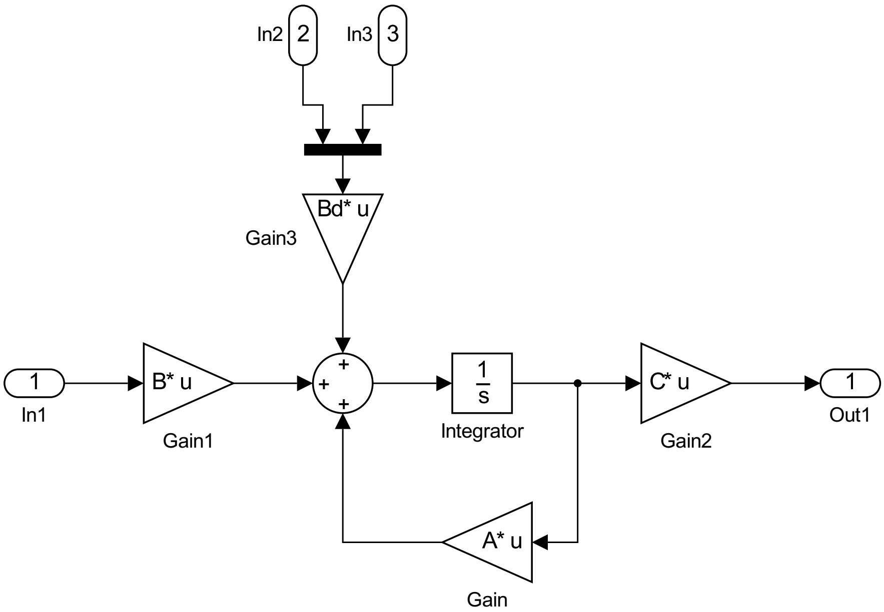

Figure 1.9: SIMULINK model of the linearised loudspeaker with noise

The response to the input can be seen figure 1.10 and the PSD figure 1.11.

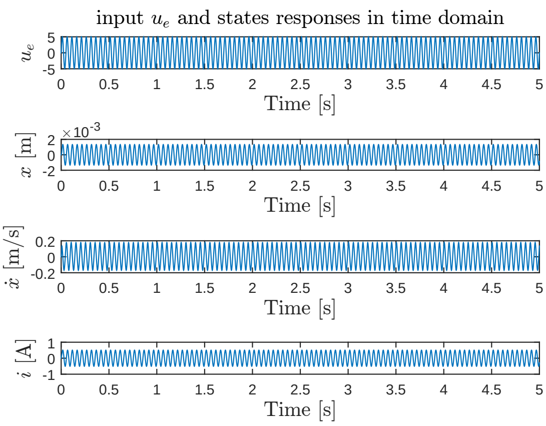

Figure 1.10: u_e and states response to the input u_e using the linearised model extended with disturbances

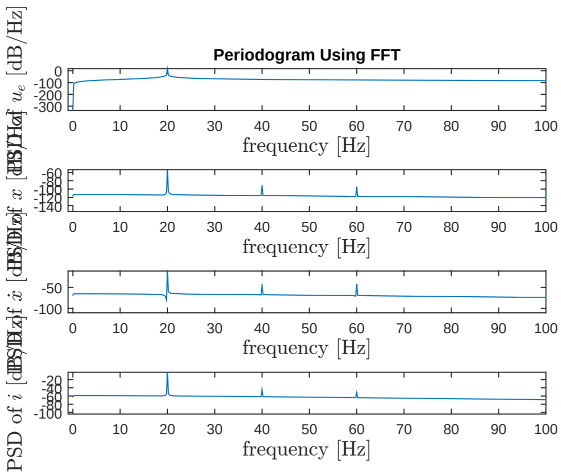

Figure 1.11: PSD of u_e and states using the linearised model extended with disturbances

We can see that the output of the new model permits to have the harmonic distortion at $f = 40$ and 60 Hz desired. However, we can notice that there is a difference of 2 db for the third harmonic on the PSD figure 1.11. Therefore, we will reduce 'manually' the magnitude A_{i2} to 0.075 V in order to delete this error and improve the model.

1.1.5 System discretization

Problem 12

We now want to discretize the continous time linear model. As the smallest constant time is $\tau_2 = 0.001$ s, we will choose a sampling time $T_s = 0.0001$ s, that is to say 10 times faster.

Using

Bibliography

[1] Roberto Galeazzi, *Manual For Compulsory Exercise*, 2015.