31310 Linear Control Design 2

Compulsory Assignment 2015 : Loudspeaker control

by

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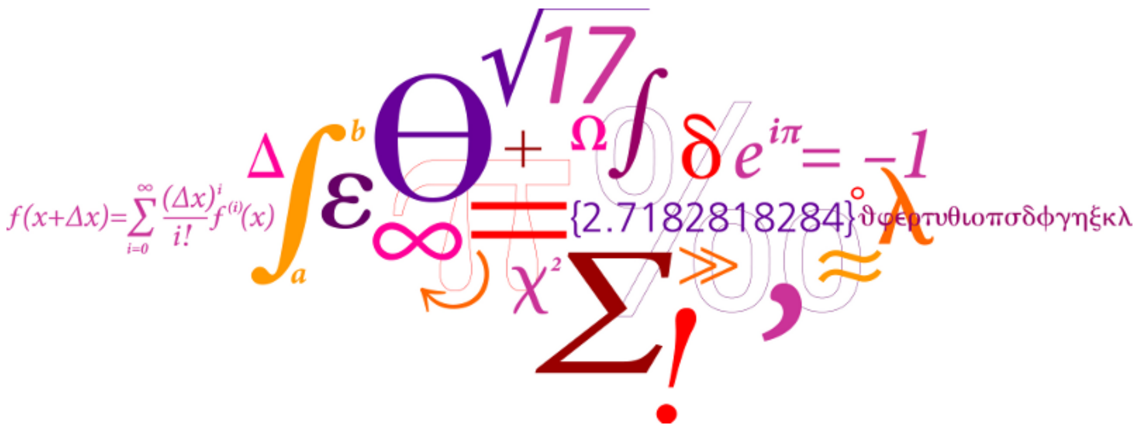

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Distortion Attenuation for Loudspeakers

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1.1.1 Loudspeakers electrical equivalent circuit

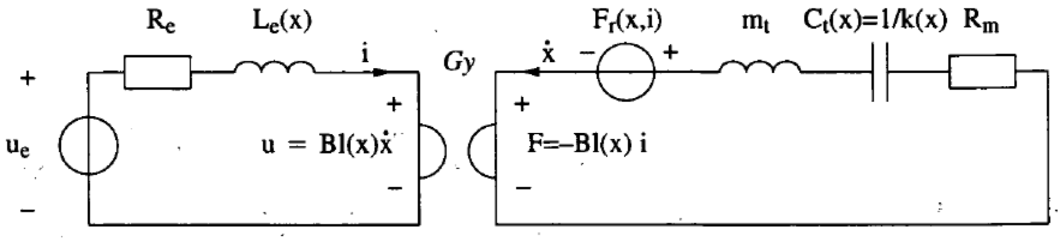

Figure 1.1: Electrical equivalent lumped element model of the voltage driven electrodynamic loudspeaker for low frequencies. The coupling between the electrical and mechanical domain is performed through the gyrator with gyration constant Bl(x).

$$u_e = R_e i + \frac{dL_e(x)}{dx}\frac{dx}{dt}i + L_e(x)\frac{di}{dt} + Bl(x)\frac{dx}{dt} \tag{1.1}$$

$$Bl(x)i = m_t\frac{d^2x}{dt^2} + R_m\frac{dx}{dt} + k(x)x - \frac{1}{2}\frac{dL_e(e)}{dx}i^2 \tag{1.2}$$

where

$$Bl(x) = Bl_0 + b_1x + b_2x^2 \tag{1.3}$$

$$L_e(x) = L_{e0} + l_1x + l_2x^2 \tag{1.4}$$

$$k(x) = k_0 + k_1x + k_2x^2 \tag{1.5}$$

Problem 1

By means of Eqs 1.1, 1.2, 1.3, 1.4 and 1.5, we can identify 3 state variables x, $\dot{x}$ and i. We can also identify the input u_e.

$$\mathrm{x} = \begin{pmatrix} x \\ \dot{x} \\ i \end{pmatrix} \text{ and } \mathrm{u} = (u_e)$$

Then, we can derive the nonlinear dynamical state space model to obtain

$$\dot{x} = \dot{x} \tag{1.6}$$

$$\ddot{x} = \frac{(Bl_0 + b_1 x + b_2 x^2)i - R_m \dot{x} - (k_0 + k_1 x + k_2 x^2)x + \frac{1}{2}(l_1 + 2l_2 x)\dot{x}i^2}{m_t} \tag{1.7}$$

$$\dot{i} = \frac{u_e - (R_e + (l_1 + 2l_2 x)\dot{x}^2)i - (Bl_0 + b_1 x + b_2 x^2)\dot{x}}{L_{e0} + l_1 x + l_2 x^2} \tag{1.8}$$

In matrix format, we have

$$\dot{\mathrm{x}} = f(\mathrm{x}) + g(\mathrm{x})\mathrm{u} \tag{1.9}$$

with

$$f(\mathrm{x}) = \begin{pmatrix} \mathrm{x}(2) \\ \frac{(Bl_0 + b_1\mathrm{x}(1) + b_2\mathrm{x}(1)^2)\mathrm{x}(3) - R_m\mathrm{x}(2) - (k_0 + k_1\mathrm{x}(1) + k_2\mathrm{x}(1)^2)\mathrm{x}(1) + \frac{1}{2}(l_1 + 2l_2\mathrm{x}(1))\mathrm{x}(2)\mathrm{x}(3)^2}{m_t} \\ \frac{-(R_e + (l_1 + 2l_2\mathrm{x}(1))\mathrm{x}(2)^2)\mathrm{x}(3) - (Bl_0 + b_1\mathrm{x}(1) + b_2\mathrm{x}(1)^2)\mathrm{x}(2)}{L_{e0} + l_1\mathrm{x}(1) + l_2\mathrm{x}(1)^2} \end{pmatrix} \tag{1.10}$$

$$g(\mathrm{x}) = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_{e0} + l_1\mathrm{x}(1) + l_2\mathrm{x}(1)^2} \end{pmatrix} \tag{1.11}$$

Problem 2

Problem 3

1.1.2 Harmonic Distortion

Problem 4

Problem 5

1.1.3 Linearized Model

The measured output is set to the voice coil current. Therefore $y(t) = i(t)$. We also take $u_e = 0$ for the analysis of the linear and nonlinear model around the resting position of the voice coil.

Problem 6

All time derivatives are set to zero in order to determine the stationary states. Therefore, we have $\frac{dx}{dt} = 0$ and equations (1.1) and (1.2) are rewritten below:

$$u_e = R_e i \tag{1.12}$$
$$Bl(x)i = k(x)x \tag{1.13}$$

As $u_e = 0$, from (1.12) we obtain $i = 0$ and we deduce by substituting in (1.13) that $k(x)x = 0$. Then $k(x) = 0$ or $x = 0$. The discriminant of the polynomial $k(x)$ of degree 2 is $\Delta = k_1^2 - 4k_2k_0 < 0$. The voice coil displacement x being real, we discard this value and get:

$$\mathrm{x}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

We linearise the model $\dot{\mathrm{x}} = h(\mathrm{x}, \mathrm{u})$ with $h(\mathrm{x}) = f(\mathrm{x}) + g(\mathrm{x})\mathrm{u}$ around the stationary states:

$$\begin{aligned} x(t) &= x_0 + \Delta x(t) = \Delta x(t) \\ \dot{x}(t) &= \dot{x}_0 + \Delta \dot{x}(t) = \Delta \dot{x}(t) \\ i(t) &= i_0 + \Delta i(t) = \Delta i(t) \end{aligned}$$

The linear model can then be written in the form:

$$\dot{\mathrm{x}} = A\mathrm{x} + B\mathrm{u}$$
$$y = C\mathrm{x}$$

where

$$A = \begin{pmatrix} \frac{\partial h_1}{\partial \mathrm{x}_1} & \frac{\partial h_1}{\partial \mathrm{x}_2} & \frac{\partial h_1}{\partial \mathrm{x}_3} \\ \frac{\partial h_2}{\partial \mathrm{x}_1} & \frac{\partial h_2}{\partial \mathrm{x}_2} & \frac{\partial h_2}{\partial \mathrm{x}_3} \\ \frac{\partial h_3}{\partial \mathrm{x}_1} & \frac{\partial h_3}{\partial \mathrm{x}_2} & \frac{\partial h_3}{\partial \mathrm{x}_3} \end{pmatrix}_{\mathrm{x}_0} \qquad B = \begin{pmatrix} \frac{\partial h_1}{\partial \mathrm{u}} \\ \frac{\partial h_2}{\partial \mathrm{u}} \\ \frac{\partial h_3}{\partial \mathrm{u}} \end{pmatrix}_{\mathrm{x}_0} \qquad C = \begin{pmatrix} \frac{\partial r}{\partial \mathrm{x}_1} & \frac{\partial r}{\partial \mathrm{x}_2} & \frac{\partial r}{\partial \mathrm{x}_3} \end{pmatrix}_{\mathrm{x}_0}$$

We obtain:

$$\frac{\partial h_1}{\partial \mathrm{x}_1} = 0 \qquad \frac{\partial h_1}{\partial \mathrm{x}_2} = 1 \qquad \frac{\partial h_1}{\partial \mathrm{x}_3} = 0$$

$$\frac{\partial h_2}{\partial \mathrm{x}_1} = \frac{b_1\mathrm{x}_{30} + 2b_2\mathrm{x}_{10}\mathrm{x}_{30} - (k_0 + 2k_1\mathrm{x}_{10} + 3k_2\mathrm{x}_{10}^2) + l_2\mathrm{x}_{20}\mathrm{x}_{30}^2}{m_t}$$

$$\frac{\partial h_2}{\partial \mathrm{x}_2} = \frac{-R_m + \frac{1}{2} \times (l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{30}^2}{m_t}$$

$$\frac{\partial h_2}{\partial \mathrm{x}_3} = \frac{Bl_0 + b_1\mathrm{x}_{10} + b_2\mathrm{x}_{10}^2 + (l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}\mathrm{x}_{30}}{m_t}$$

$$\frac{\partial h_3}{\partial \mathrm{x}_1} = \frac{(-2l_2\mathrm{x}_{20}^2\mathrm{x}_{30} - (b_1 + 2b_2\mathrm{x}_{10})\mathrm{x}_{20}) \times (L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2)}{(L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2)^2} - (l_1 + 2l_2\mathrm{x}_{10}) \times \frac{(-(R_e + (l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}^2)\mathrm{x}_{30} - (Bl_0 + b_1\mathrm{x}_{10} + b_2\mathrm{x}_{10}^2)\mathrm{x}_{20})}{(L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2)^2}$$

$$\frac{\partial h_3}{\partial \mathrm{x}_2} = -\frac{2(l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}\mathrm{x}_{30} + Bl_0 + b_1\mathrm{x}_{10} + b_2\mathrm{x}_{10}^2}{L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2}$$

$$\frac{\partial h_3}{\partial \mathrm{x}_3} = -\frac{(l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}^2 + R_e}{L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2}$$

$$\frac{\partial h_1}{\partial \mathrm{u}} = 0 \qquad \frac{\partial h_2}{\partial \mathrm{u}} = 0 \qquad \frac{\partial h_3}{\partial \mathrm{u}} = \frac{1}{L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2}$$

$$\frac{\partial r}{\partial \mathrm{x}_1} = 0 \qquad \frac{\partial r}{\partial \mathrm{x}_2} = 0 \qquad \frac{\partial r}{\partial \mathrm{x}_3} = 1$$

We then substitute $\mathrm{x}_{10} = \mathrm{x}_{20} = \mathrm{x}_{30} = 0$. Finally, we get:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -\frac{k_0}{m_t} & -\frac{R_m}{m_t} & \frac{Bl_0}{m_t} \\ 0 & -\frac{Bl_0}{L_{e0}} & -\frac{R_e}{L_{e0}} \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_{e0}} \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

Numerically

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -1.20 \cdot 10^5 & -50.46 & 279.19 \\ 0 & -2.57 \cdot 10^3 & -1.40 \cdot 10^3 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ 177 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \tag{1.14}$$

We can check these results with the matlab function *linmod(model,x_0,u_e)*:

```
[A,B,C,D] = linmod('nonLinearModel',[0;0;0],0);
```

Problem 7

In this problem, we draw the block diagram of the linearised loudspeaker (see figure 1.2) showing the couplings between the different states.

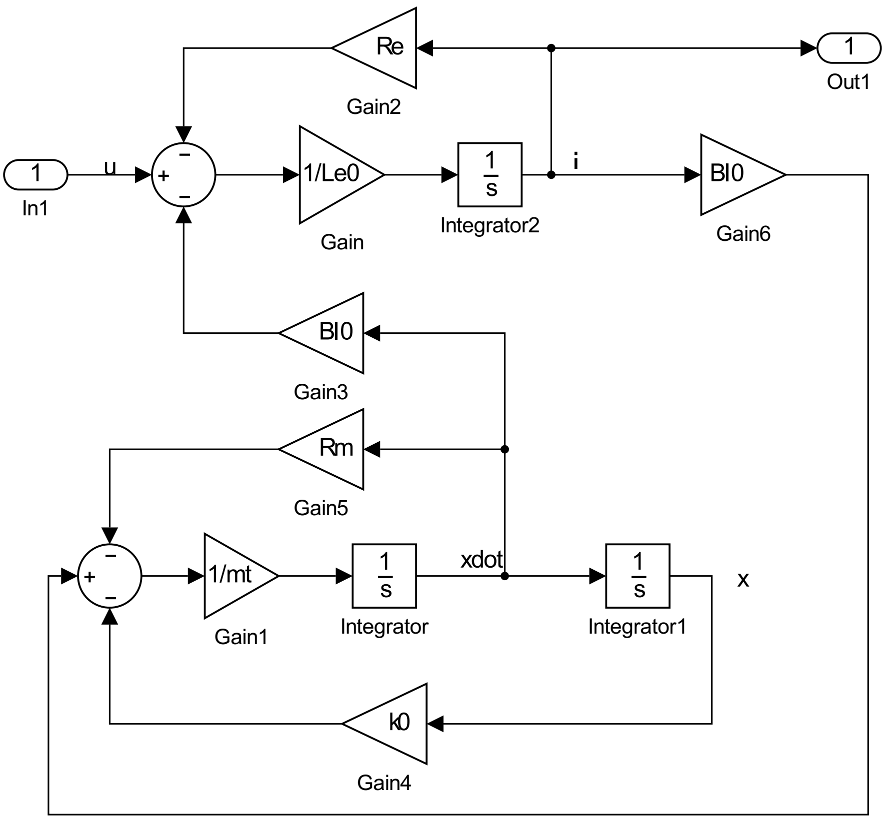

Figure 1.2: block diagram of the linearised loudspeaker

By means of this block diagram, we can make preliminary assessments of the system. Indeed, we can see that the state i seems to be controllable and observable as it is connected to the input u_e and to the output. However, the states x and $\dot{x}$ seems to be controllable as they are also connected to the input u_e but not to be observable because they are not connected to any output. Moreover the 3 states seems to be stable because of the 2 backloops.

We can verify our assumptions regarding the controllability and the observability by calculating the *rank* of M_c and M_o.

```
Mc = [B A*B A^2*B];
rank(Mc) % = 3 controllable
Mo=[C
    C*A
    C*A^2];
rank(Mo) % = 3 observable
```

Problem 8

We can derive analytically the eigenvalues of the system dynamical matrix A.

$$det(\lambda I - A) = det\begin{pmatrix} \lambda & -1 & 0 \\ \frac{k_0}{m_t} & \lambda + \frac{R_m}{m_t} & -\frac{Bl_0}{m_t} \\ 0 & \frac{Bl_0}{Le_0} & \lambda + \frac{Re}{Le_0} \end{pmatrix} \tag{1.15}$$

$$= \lambda^3 + \left(\frac{Re}{Le_0} + \frac{Rm}{m_t}\right)\lambda^2 + \left(\frac{RmRe + Bl_0}{m_t Le_0} + \frac{k_0}{m_t}\right)\lambda + \frac{k_0 Re}{m_t Le_0} \tag{1.16}$$

We therefore have a third-order polynomial which can be solved with MATLAB but the eigenvalues can also be calculated using

```
lambda = eig(A);
```

Thus, we obtain

$$\lambda = 1,0.10^2 \begin{pmatrix} -2.9573 \\ -5.7648 + 4.8571i \\ -5.7648 - 4.8571i \end{pmatrix} \tag{1.17}$$

We can notice that

$$Re(\lambda_i) < 0, \; i = 1, 2, 3 \tag{1.18}$$

which means that the system is asymptotically stable. Moreover, the eigenmodes of the system are $e^{\lambda_1 t}$, $e^{\lambda_2 t}$ and $e^{\lambda_3 t}$. We can notice that

$$\frac{1}{\lambda_1} = \tau_1 > \tau_2, \tau_3$$

which means that the response of the state x to an input u_e will be slower than the one of i and $\dot{x}$. We can also see that as λ_2 and λ_3 have an imaginary part which means that $\dot{x}$ and i will have an oscillatory response.

Problem 9

Using the results of Problem 6 (1.14), we can implement a SIMULINK model (see figure 1.3) of the linear system.

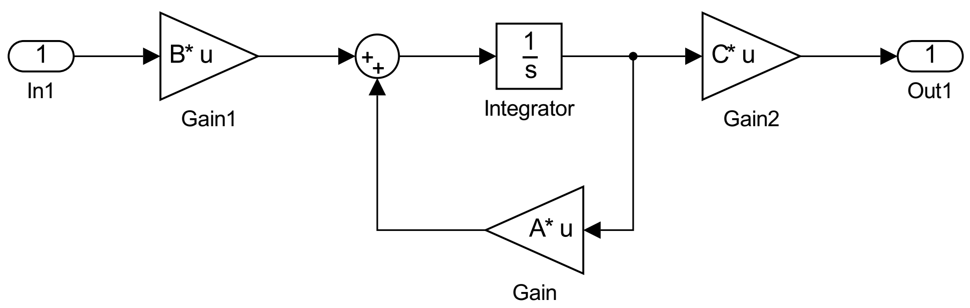

Figure 1.3: SIMULINK model of the linearised loudspeaker

Then, we can simulate the linearised model with an input $u_e = A_u \sin(2\pi f_c t)$, where $A_u = 5V$ and $f_c = 20Hz$. The states response in the time domain is plotted figure 1.4 and the PSD is plotted figure 1.5.

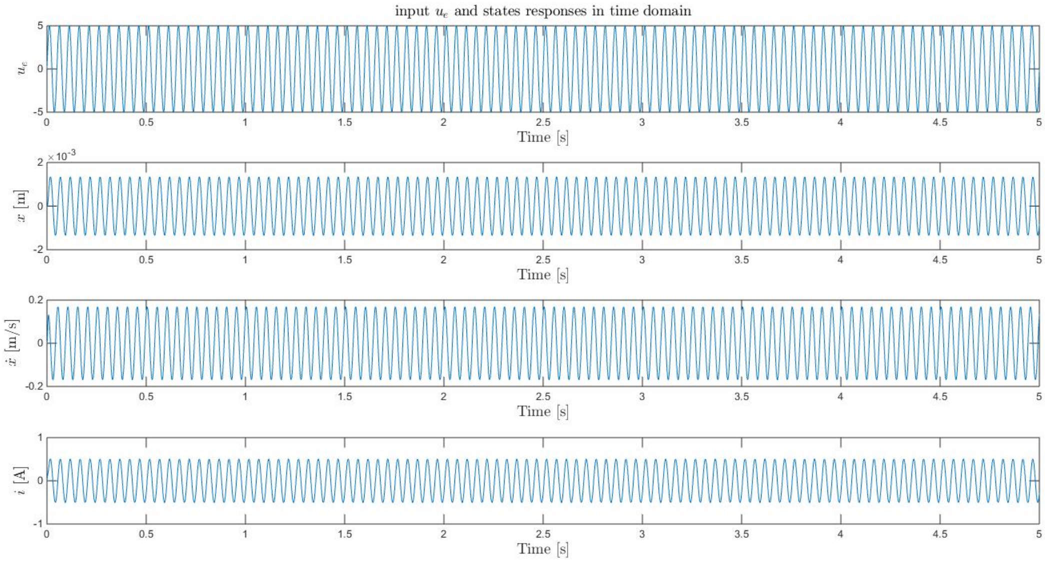

Figure 1.4: u_e and states reponse to u_e in time domain

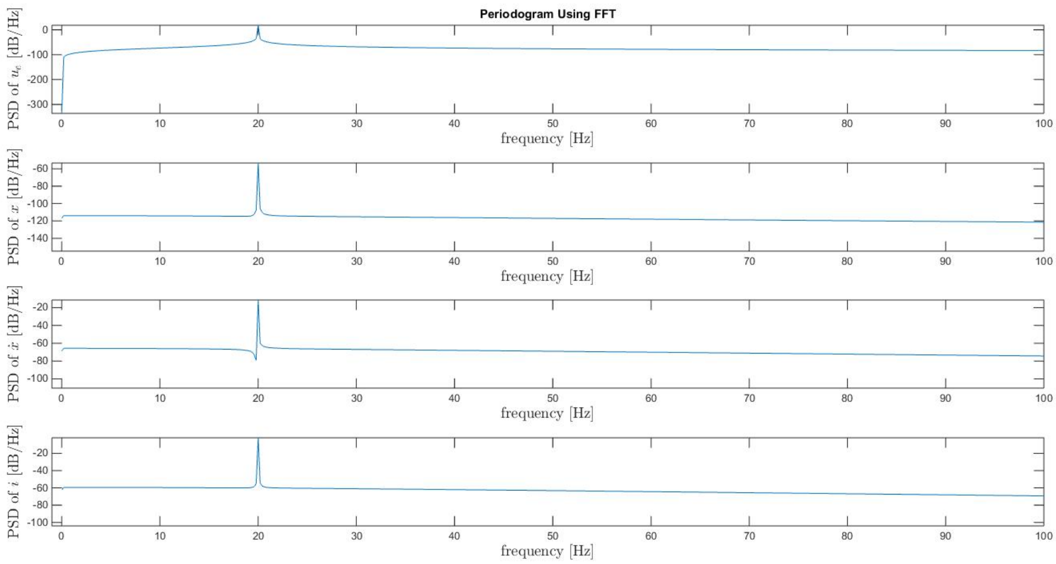

Figure 1.5: PSD of u_e and states

First, comparing the response to u_e in the time domain (figure **??** and figure 1.4), the states response seems to be the same, but comparing the PSD of the states response (figure **??** and figure 1.5), we can see that the states response is not the same. Indeed, the linearised system is non affected by the harmonic distortion, there is only on frequency present in the state response, the one of $20Hz$. This result was expected because the nonlinear distortion only affects non-linear systems. Moreover, we have linearised the system with an input $u_e = 0$ which means that we have a linearised system without harmonic, and then, we used a new input u_e

with a frequency $f_c = 20Hz$, which means we only obtain a state response with an harmonic of a frequency of f_c.

1.1.4 Harmonic distortion and fictitious disturbances

In this section, we basically want to add a disturbance in order to obtain the same output with the linearised system than with the nonlinear system (the analysis is restrained to the second and third order harmonics). Thus, the output $i(t)$ should be

$$y_{nl}(t) = A_1 \sin(2\pi f_c t + \psi_1) + A_2 \sin(4\pi f_c t + \psi_2) + A_3 \sin(6\pi f_c t + \psi_3) \quad (1.19)$$

Problem 10

First, we extend our model with two input disturbances such that the linear output will also show the second and third order harmonics. The new linearised model is

$$\dot{\mathrm{x}} = A\mathrm{x} + B\mathrm{u} + B_d\mathrm{d}$$
$$y = C\mathrm{x}$$

with $\mathrm{d} = [d_{i1}, d_{i2}]$.

We know that $d_{i1} = A_{i1} \sin(4\pi f_c t)$ and $d_{i2} = A_{i2} \sin(4\pi f_c t)$ but we have to determine the magnitudes A_{i1} and A_{i2}. As this disturbance can be considered as an input, we choose to take $B_d = [B\ B]$ and to find the right magnitudes to use.

Problem 11

In order to find A_{i1} and A_{i2}, we started to implement the disturbance to the SIMULINK model (see figure 1.6).

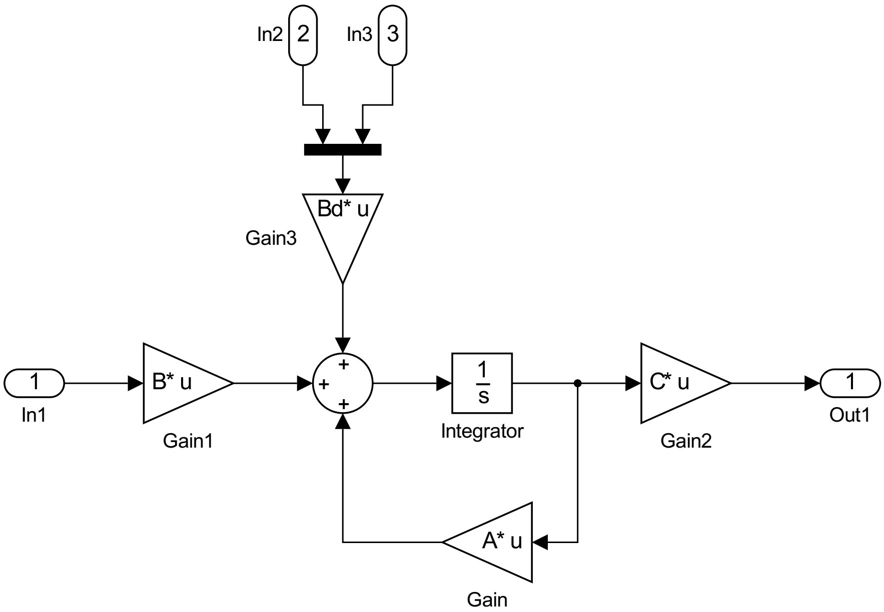

Figure 1.6: SIMULINK model of the linearised loudspeaker with noise

Then, we have chosen $A_{i1} = 5V$ and run the simulation to see the magnitude of the speak at $40Hz$: $magnitude_{40Hz} = 0.2288\ V$. Knowing that the magnitude wanted is $0.0038\ V$ (determined with the nonlinear Model), we can find $A_{i1} = \frac{5*0.0038}{0.2288} = 0.0830\ V$. In the same way, we find $A_{i2} =?\ V$.

Bibliography