31310 Linear Control Design 2
Manual for Compulsory Exercise

Roberto Galeazzi

Automation & Control Group
Department of Electrical Engineering
Technical University of Denmark

Contents

Preface iii

1 Distortion Attenuation for Loudspeakers 1
1.1 Moving-coil Loudspeaker . . . 2
1.1.1 Loudspeaker electrical equivalent circuit . . . 3
1.1.2 Harmonic Distortion . . . 7
1.1.3 Linearized model . . . 8
1.1.4 Harmonic distortion and fictitious disturbances . . . 8
1.2 Distortion Attenuation . . . 10
1.2.1 System discretization . . . 10
1.2.2 Voice coil position reference tracking . . . 11
1.2.3 Output feedback voice coil position tracking control with disturbance rejection . . . 12

Preface

This is a manual for the compulsory assignment for the course 31310 Linear Control Design 2. The assignment consists of one large exercise, which must be solved and handed in to the course instructor.

The exercise covers all the major topics faced during the course. Theory and practice clearly intersect into this exercise, which makes the student to face all the different stages of control system design.

Background material can be found in the textbook for the course. It may also be a good idea to have a textbook for the first course in control engineering (31300 Linear Control Design 1) within reach. For Danish speaking students the following textbook is suggested:

- Jannerup, O., Sørensen, P. H., *Reguleringsteknik - 3. udgave*, Polyteknisk Forlag, 2004.

For English speaking students the following textbooks are suggested:

- Ogata, K., *Modern Control Engineering - Fifth Edition*, Prentice Hall, 2009
- Franklin, G. F., Powell, J. D., and Emani-Naeimi, A., *Feedback Control of Dynamical Systems - Sixth Edition*, Prentice Hall, 2009

or earlier versions.

Report

All the results of the given exercise must be documented in the final report. The report must include calculations, derivations and simulation results in the form of plots from MATLAB and SIMULINK. The SIMULINK models must also be included in the report as an Appendix; whereas the MATLAB files must only be delivered in electronic format. It is strongly suggested that MATLAB windows and print outs are not included in the report text.

The report may contain a set of appendices. However it is essential that **the report can be read/understood without frequent references to the appendices for important information**. Usually it is sensible to include the most important calculations, plots and/or graphs in the report. SIMULINK models, additional graphs and other material should be placed in the appendices.

Do not forget coordinate grid, units and necessary legends in the graphs. You may use the editing facilities in the MATLAB graphic windows. All plots should have a clear caption and be referred to in the report text. Comments on all important calculation results, simulations and plots are required.

The report must have a clear introduction explaining the control problem at hand, the control object and the methods you will use. It is also important that the results achieved with the different designed controllers are summarized in the conclusions of the report. In order to make it possible for the instructor to read the report as easily as possible, the sections in the report should have the same numbering as the Problems in the *Manual for Compulsory Exercise*.

The report should not be longer than 25 pages (excluding the appendices) with normal line spacing and 12 pt. Times or Times New Roman type. All material included in the report in excess of the 25 pages (excluding the appendices) will not be considered for the evaluation/grading.

Delivery

The report must be handed in not later than

Wednesday 2 December 2015, at 13:00 CET

The report must be delivered **both in paper and electronic version**. **In the report must be clearly stated who did what** in order to assess the specific contribution of each group member.

The paper version must be delivered directly to the course instructor, or in his absence the report can be left on the table placed in front of room 122 (Building 326).

The electronic version must be delivered through the 31310 course page in CampusNet (look under Assignments/Compulsory Assignment). Beware that the server will close on Wednesday 2 December at 13:00 sharp, and that **late deliveries will not be taken into consideration for the evaluation/grading**. The electronic version must include the following material:

- pdf file of the final report named as

  GroupMember1_GroupMember2_2015.pdf

  Due to the constraint imposed by the plagiarism checking system **only pdf files will be accepted for evaluation/grading**.

- all the MATLAB and SIMULINK files implemented to solve the compulsory assignment. The instructor must be able to run those files at any time without the need of contacting one or more group members. MATLAB and SIMULINK files must be delivered within a zip folder named as

  GroupMember1_GroupMember2_Matlab_2015.zip

The pdf file (containing the report) and the zip folder (containing the MATLAB/SIMULINK files) must be uploaded separately.

All group members are requested to sign the front page of the report in paper form.

DTU Electrical Engineering, November 2015
Roberto GALEAZZI

Exercise 1

Distortion Attenuation for Loudspeakers

The speaker turns 100! In November 1915 in Napa Valley, California, the Danish engineer Peter L. Jensen presented the technology that now, 100 years later, remains the basis of speakers in everything from classic hi-fi systems to hearing aids and smart-phones: the electrodynamic speaker. His *Magnavox* ("Great Voice") speaker laid the foundation for what evolved into a Danish export success of an industry that today has a turnover of around 30 billion kroner, of which 80-85% is exported.

Figure 1.1: Jensen and Pridham together with their first Magnavox. (Source:)

Jensen's first electrodynamic loudspeaker consisted of a three inches diaphragm made of a nickel-silver alloy. On the membrane was fitted a coil, which was placed in a constant magnetic field generated by an electromagnet. When the amplified alternating current from a microphone run through the coil, moving the coil, the diaphragm moved along back and forth in pace with the magnetic attraction and repulsion forces generated by the current variation. A long horn over the diaphragm contributed to amplify the sound, and the world's first electrodynamic speaker was born. Although new materials and

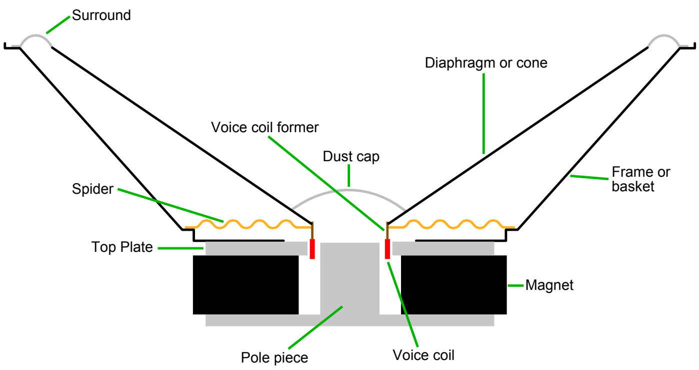

Figure 1.2: Cross section of the moving-coil loudspeaker (source:)

shapes have been introduced in the past 100 years, the fundamental engineering principle has hardly changed (text translated from [1]). Figure 1.1 shows Peter L. Jensen together with Edwin S. Pridham working on their Magnavox.

1.1 Moving-coil Loudspeaker

The first part of the compulsory exercise deals with the derivation, analysis and simulation of the nonlinear and linearized dynamical models of a moving-coil loudspeaker. The following overview of the basic working principles of a moving-coil loudspeaker is based on [2] and [3].

A loudspeaker is an electromechanical device utilized to convert an electric signal into an acoustic wave form. The design based on the moving-coil principle is, by far, the most common. Figure 1.2 shows a schematic cross-sectional view of the moving-coil loudspeaker. The loudspeaker consists of a voice coil that is placed inside a radial magnetic field generated by a permanent magnet, which is driven by an alternating current supplied by an amplifier. The voice coil is mostly winded on a paper or aluminium cylinder, which in turn is connected to the diaphragm. The diaphragm is centred through two flexible connections to the frame: the rim (rather flexible) and a spider (rather stiff), where the latter prevents the voice coil from running out of the magnet.

At low frequencies, where the sound wavelength is much larger than the dimensions of the transducer, the dynamical behaviour of the diaphragm can be represented as that of a rigid circular piston, and the loudspeaker can be modelled using lumped elements. At higher frequencies this modelling approach is not valid as the diaphragm behaves no longer as a rigid piston any more.

The dynamical behaviour of the loudspeaker changes also with the amplitude of the input signal. When the amplitude is small the loudspeaker dynamics can be accurately described by a linear model. However, as the amplitude of the input signal increases, all loudspeakers are strongly affected by nonlinear characteristics, which generate distortion not present in the input. The nonlinearities arise from the mechanical and electrical properties of the components utilized in different parts of the loudspeaker.

Three major sources of nonlinearities will be consider in this assignment:

- **Force factor** $Bl(x)$: The electromagnetic conversion from electrical to mechanical energy is described by the force factor $Bl(x)$, where B is the effective air gap magnetic flux density, l is the effective length of the coil wire within the magnetic field, and x is the current position of the coil. The force factor decreases when the coil moves away from the gap, which makes the force factor displacement dependent. The force factor is measured in [T·m].

- **Voice coil inductance** $L_e(x)$: The inductance of the voice coil is also dependent on current displacement of the voice coil itself. This is caused by the difference in surrounding materials, which are air for the part of the coil outside the gap and steel for the part of the coil inside the gap. The inductance is measured in [H].

- **Suspension compliance** $C_t(x)$ The elements the suspension consists of are typically made of rubber or polymer. The suspension behaviour is similar to a normal linear spring for small displacements, but increases quicker for larger excursions causing a nonlinear force dependent on the voice coil displacement. The compliance $C_t(x) = 1/k(x)$, where $k(x)$ is the equivalent stiffness associated with the same elements. The compliance is measured in [m/N].

1.1.1 Loudspeaker electrical equivalent circuit

The electrical equivalent circuit with lumped parameters describing the dynamics of the loudspeaker at low frequencies is shown in Figure 1.3. The system parameters displayed in the circuit are (all quantities are measured in SI units)

- u_e driving voltage at the loudspeaker terminals [V]

- R_e electrical resistance of the voice coil [Ω]

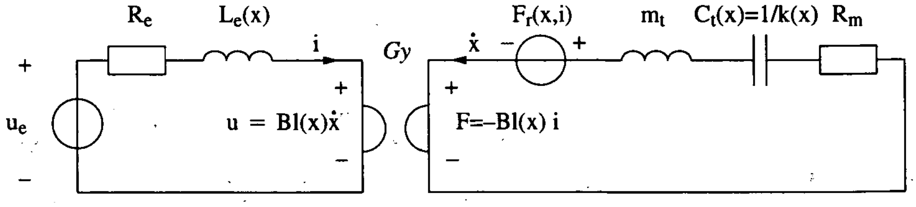

Figure 1.3: Electrical equivalent lumped element model of the voltage driven electrodynamic loudspeaker for low frequencies. The coupling between the electrical and mechanical domain is performed through the gyrator with gyration constant $Bl(x)$. (Source: [3])

- $L_e(x)$ nonlinear self-inductance of the voice coil [H] coupled to the mechanical subsystem through the gyrator with constant given by the nonlinear force factor $Bl(x)$
- i voice coil current [A]
- u voltage induced in the voice coil [V]
- $Bl(x)$ nonlinear force factor coupling the electrical and mechanical subsystems [T·m]
- F Lorentz force acting on the voice coil [N]
- x voice coil displacement [m]
- $\dot{x}$ velocity of the voice coil [m/s]
- $C_t(x)$ nonlinear suspension compliance [m/N]
- m_t mechanical moving mass [kg]
- R_m mechanical damping [N·s/m]
- $F_r(x, i)$ reluctance force caused by the nonlinear self-inductance and the displacement dependent total stiffness $k(x) = 1/C_t(x)$ [N]

The fully coupled nonlinear differential equations describing the dynamics of the electrical and mechanical subsystems are

$$u_e = R_e i + \frac{\mathrm{d}L_e(x)}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t} i + L_e(x)\frac{\mathrm{d}i}{\mathrm{d}t} + Bl(x)\frac{\mathrm{d}x}{\mathrm{d}t} \tag{1.1}$$

$$Bl(x)i = m_t\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + R_m\frac{\mathrm{d}x}{\mathrm{d}t} + k(x)x - \frac{1}{2}\frac{\mathrm{d}L_e(x)}{\mathrm{d}x}i^2 \tag{1.2}$$

where

$$Bl(x) = Bl_0 + b_1 x + b_2 x^2 \tag{1.3}$$
$$L_e(x) = L_{e0} + l_1 x + l_2 x^2 \tag{1.4}$$
$$k(x) = k_0 + k_1 x + k_2 x^2 \tag{1.5}$$

P1 By means of Eqs. (1.1), (1.2), (1.3), (1.4), and (1.5) first identify the state variables x_i, $i = 1, \ldots, n$ and the control inputs u_j, $j = 1, \ldots, m$, then derive the nonlinear dynamical state space model of the loudspeaker in the form

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})\mathbf{u} \tag{1.6}$$

P2 Draw the block diagram of the nonlinear loudspeaker model showing how the different subsystems are interconnected. Then implement the nonlinear model in MATLAB/SIMULINK using the numerical values of the parameters given in Table 1.1. In order to simulate the system set the *Model Configuration Parameters* according to Fig. 1.4, where `TIME_SIM` is a value of your choice properly selected, and `STEP_SIZE` is 0.0001 seconds.

P3 Set the input voltage $u_e = A_u \sin(2\pi f_c t)$, where $A_u = 5$V and $f_c = 20$Hz. Simulate the nonlinear system and plot the input u_e and the states responses in the time and frequency domain. In the frequency domain signals have to be visualized by means of their power spectral density $S(f)$ [dB/Hz], which can be computed by means of a Fast Fourier Transform (FFT). The following function should be implemented in order to get the power spectral density, where `x` is the signal in the time domain, and `Fs` is the sampling frequency.

```
function [Pxx,f] = power_spectral_density(x,Fs)

% Generate the frequency vector
NFFT = length(x);
f = Fs*(0:NFFT/2)/NFFT;

% Calculate the one sided power spectral density
X = fft(x,NFFT);
Px = abs(X).^2/(Fs*NFFT);
Pxx = Px(1:NFFT/2+1);
Pxx(2:end-1) = 2*Pxx(2:end-1);
```

Compare your findings in the time and frequency domain and discuss the effect of the nonlinearities on the state variables.

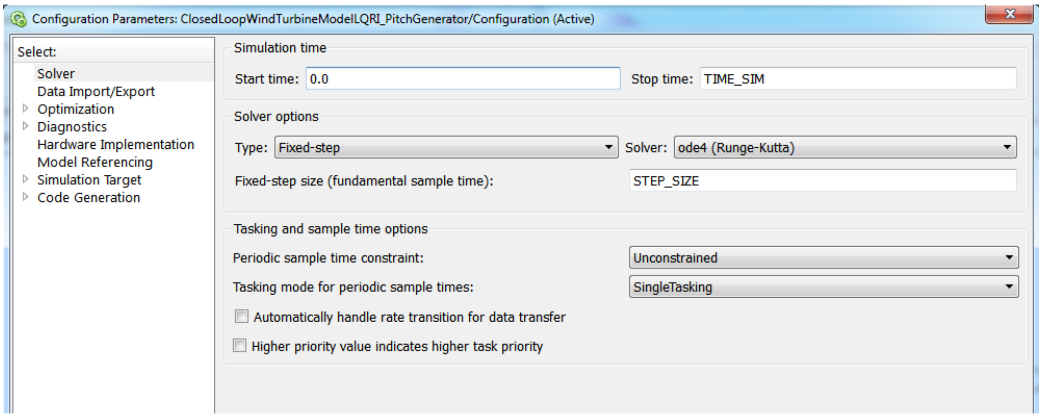

Figure 1.4: Model configuration parameter settings for SIMULINK.

Parameter	Value	Unit
R_e	7.9	Ω
L_{e0}	5.65	mH
Bl_0	14.54	T·m
k_0	6259.3	N/m
R_m	2.628	N·s/m
m_t	52.08	g
b_1	2.6826	T
b_2	$-4.0269 \cdot 10^3$	T/m
k_1	$-1.4562 \cdot 10^5$	N/m^2
k_2	$2.6106 \cdot 10^7$	N/m^3
l_1	$-5.2937 \cdot 10^{-1}$	H/m
l_2	$-1.2012 \cdot 10^2$	H/m^2

Table 1.1: Woofer Loudspeaker Parameters

1.1.2 Harmonic Distortion

The nonlinear characteristics of several components of the loudspeaker cause the generation of additional frequency components, which are not present in the input signal u_e. This phenomenon is called **nonlinear distortion**. There are several types of nonlinear distortion, but in this assignment we will focus on the so-called harmonic distortion.

Harmonic distortion: The case of harmonic distortion occurs when, having only one frequency f_c present at the input, the loudspeaker will generate additional frequency components (harmonic frequencies) which are multiples of the input frequency, i.e. $f_n = nf_c$ with $n \in \mathbb{N}$. The percentage of *Total Harmonic Distortion* (THD) is defined as

$$\text{THD} = \frac{\sqrt{\sum_{n=2}^{N} A_n^2}}{\sqrt{\sum_{n=1}^{N} A_n^2}} 100\% \tag{1.7}$$

where A_1 is the amplitude of the fundamental frequency f_c that is input to the loudspeaker, and A_n for $n = 2, \ldots, N$ are the amplitudes of the higher order harmonics. In general, second and third order harmonics are of significant magnitude. Therefore only two harmonic frequencies ($n = 3$) will be used to compute the harmonic distortion. Distortion levels calculated with only the second or the third harmonic are called, respectively, second and third order harmonic distortion. These distortion levels are defined as

$$d_2 = \frac{A_2}{\sqrt{A_1^2 + A_2^2}} 100\% \tag{1.8}$$

$$d_3 = \frac{A_3}{\sqrt{A_1^2 + A_3^2}} 100\% \tag{1.9}$$

Since the sound pressure is proportional to the velocity of the voice coil the harmonic distortion will be evaluated on this signal.

P4 Starting from the power spectral density of the voice coil velocity computed in **P3**, use Eqs. (1.7)-(1.9) to calculate the THD (include the first 5 harmonics after the fundamental frequency f_c) and d_2, and d_3. Discuss the significance of d_2 and d_3 with respect to THD.

P5 Now we would like to investigate how the second and third order harmonic distortion depend on the amplitude and frequency of the input signal u_e. Make a frequency sweep for $f_c \in [20\,,\,200]$Hz and $A_u \in [2.5\,,\,10]$V with a $\Delta f = 5$Hz and $\Delta A = 2.5$V. For each pair

$(\bar{f}_c, \bar{A}_u)$ compute d_2 and d_3. Present the results of your analysis graphically and discuss the effect of amplitude and frequency variations on the harmonic distortion.

1.1.3 Linearized model

The loudspeaker is an electrodynamic device that produces sound waves when at its terminal inputs there is an alternating voltage signal. Therefore the overall analysis of the nonlinear and linear model is to be carried out around the resting position of the voice coil for $u_e = 0$. Moreover, from now onwards the measured output is the voice coil current, that is $y(t) = i(t)$.

P6 Determine the stationary state associated with $u_e = 0$; then linearise the nonlinear model obtained in **P1** both analytically and numerically. Present the linear model in the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{1.10}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{1.11}$$

P7 Draw the block diagram of the linearised loudspeaker showing the couplings between the different states. By means of the block diagram make your preliminary assessment of system stability, controllability, and observability.

P8 Derive analytically and calculate numerically the eigenvalues of the system dynamical matrix $\mathbf{A}$. Assess the stability of linearised model based on the computed eigenvalues. Determine the eigenmodes of the system and discuss how much each eigenmode affects the individual states.

P9 Implement a SIMULINK model of the linear system. Set the input voltage $u_e = A_u \sin(2\pi f_c t)$, where $A_u = 5\text{V}$ and $f_c = 20\text{Hz}$. Simulate the linearised loudspeaker and plot the input u_e and the states responses in the time and frequency domain (power spectral densities). Compare the results with those obtained in **P3**. Does the harmonic distortion affect the states of the linearised system? Was this result expected? If yes, why? If no, why?

1.1.4 Harmonic distortion and fictitious disturbances

The harmonic distortion appears as additional spectral lines in the power spectral densities of the system states. Figure 1.5 shows the power spectral density of the voice coil velocity when the loudspeaker is excited with a tone

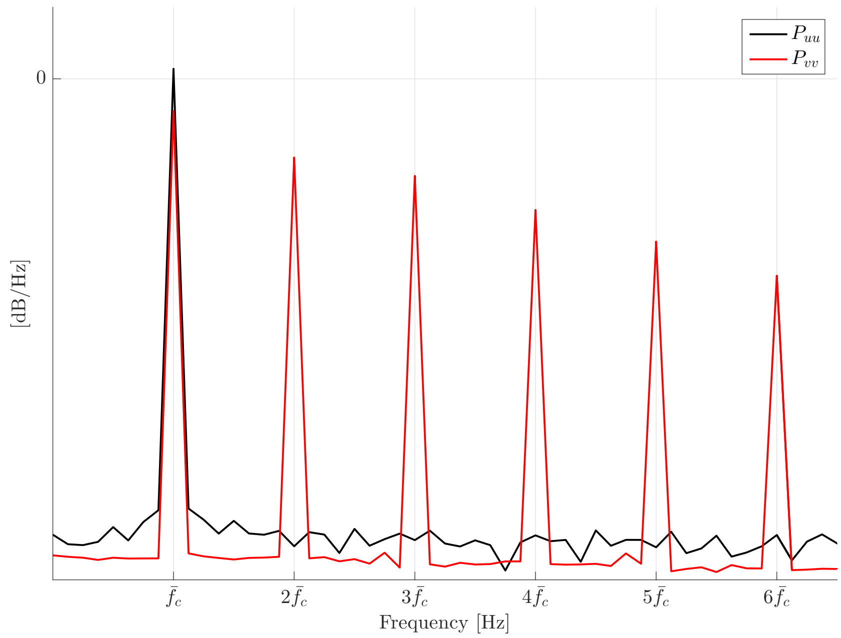

Figure 1.5: Power spectral density $P_{vv}(f)$ of the voice coil velocity showing harmonic distortion as spectral lines at integer multiples of the fundamental frequency f_c.

at $f_c = \bar{f}_c$Hz. Therefore the output of the nonlinear system (1.6) can be written as

$$y_{\mathrm{nl}}(t) = i(t) \tag{1.12}$$

$$= A_1 \sin(2\pi f_c t + \psi_1) + \sum_{n=2}^{N} A_n \sin(2\pi n f_c t + \psi_n) \tag{1.13}$$

where (A_1, ψ_1) are amplitude and phase of the voice coil current in correspondence of the fundamental frequency, and the pairs (A_n, ψ_n) are amplitude and phase of the higher-order harmonics. If we restrain the analysis to the second and third order harmonics then Eq. (1.13) reduces to

$$y_{\mathrm{nl}}(t) = A_1 \sin(2\pi f_c t + \psi_1) + \underbrace{A_2 \sin(4\pi f_c t + \psi_2)}_{2^{\mathrm{nd}}\text{ order harm. dist.}} + \underbrace{A_3 \sin(6\pi f_c t + \psi_3)}_{3^{\mathrm{rd}}\text{ order harm. dist.}} \tag{1.14}$$

The effect of the harmonic distortion could be included in the linearised model (1.10)-(1.11) by introducing fictitious sinusoidal input disturbances properly dimensioned acting at the frequencies $f = \{2f_c, 3f_c\}$Hz.

P10 The linearised model needs to be extended with two input disturbances such that the linear output will also show the second and third order harmonics, that is determine the disturbance input matrix $\mathbf{B}_d$ and the disturbance vector $\mathbf{d} = [d_{i1}\,,\, d_{i2}]^{\mathrm{T}} \in \mathbb{R}^2$ such that

$$S_{y_{\mathrm{ln}}}(f) = S_{y_{\mathrm{nl}}}(f) \tag{1.15}$$

at $f = \{f_c, 2f_c, 3f_c\}$, where $y_{\mathrm{ln}}(t)$ is the output of the linearised model. The linearised model will then read

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{B}_d\mathbf{d} \tag{1.16}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{1.17}$$

P11 Add the disturbance input to the SIMULINK model of the linearised system implemented in **P9**, and repeat the simulation run in **P9**. Compare the obtained state and output responses of the linear model (1.16)-(1.17) with those of the nonlinear loudspeaker model obtained in **P3** and verify that the modelling requirement (1.15) is fulfilled. Is the introduction of the fictitious sinusoidal input disturbances a viable way to model harmonic distortion in the linear framework? If yes, why? If no, why?

1.2 Distortion Attenuation

Audible sound distortion is clearly an undesired effect of the electrical and mechanical nonlinearities present in the loudspeaker. Therefore it is desired to attenuate the amount of harmonic distortion in the frequency range where the loudspeaker is expected to be active, in particular around the second and third order harmonics where most of the distortion power content is located. The loudspeaker is generally excited with inputs in the frequency range $f_c \in [20\,,\,200]$Hz; hence distortion attenuation should be achieved at least in the range of frequencies up to $3f_{c,\max}$ by means of feedback control.

1.2.1 System discretization

P12 The first step in designing the regulator to achieve distortion attenuation is to discretize the continuous time linear model (1.16)-(1.17). Select a proper sampling time T_s and motivate, if possible analytically, your choice. Then discretize the linearised system in order to obtain the following discrete time representation

$$\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{G}\mathbf{u}(k) + \mathbf{G}_d\mathbf{d}(k) \tag{1.18}$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \tag{1.19}$$

Calculate the eigenvalues of the discrete time system and compare them with those obtained for the continuous time in **P8**. Does discretization change the stability properties of the open loop system? If yes, why? If no, why?

P13 Simulate the dynamical behaviour of discrete time system in response to the voltage input $u_e(k) = A_u \sin(2\pi f_c k T_s)$ and the disturbance vector $\mathbf{d}(k)$, $k = 0, 1, 2, \ldots$ Compare the state and output responses in discrete time with those in continuous time obtained in **P11**. The comparison should be carried out in the time and frequency domain. Do the simulation results support the chosen sampling time T_s? Use the MATLAB function `stairs` to plot discrete time signals in the time domain.

1.2.2 Voice coil position reference tracking

At first we will try to diminish the distortion effects by designing a reference tracking controller for the voice coil position. The control objective can be stated as follows:

Objective 1 *Consider the linear loudspeaker model with fictitious disturbances*

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u_e + \mathbf{B}_d\mathbf{d} \tag{1.20}$$

$$\mathbf{y} = \mathbf{x} \tag{1.21}$$

where the state is fully accessible to the measurement. Given the voice coil position reference $x_{\text{ref}}(k) = A_x \sin(2\pi f_c k T_s)$ *with* $A_x \in [0.001\,,\, 0.005]\text{m}$ *and* $f_c \in [20\,,\, 200]\text{Hz}$ *design a feedback controller* $u_e(k) = \boldsymbol{\kappa}(\mathbf{x}(k), x_{\text{ref}}(k))$ *such that*

$$x_{\text{ln}}(t - \tau) = x_{\text{ref}}(t) \quad \text{for}\, t \geq T_t > 0 \tag{1.22}$$

where T_t *is the time after which the transient is finished, and* $\tau > 0$ *is the time delay.*

P14 Evaluate numerically the controllability of the system. If the system is controllable, propose a control system architecture that can achieve the position reference tracking, and motivate your choice.

P15 Design the feedback controller $u_e(k) = \boldsymbol{\kappa}(\mathbf{x}(k), x_{\text{ref}}(k))$ by means of *Arbitrary Eigenvalue Assignment*. Motivate the choice of the closed loop eigenvalues in relation to the control requirements. Show through the Bode diagram of the closed loop system that the control objective is achieved and numerically quantify the time delay τ when $f_c = 20\text{Hz}$.

P16 Implement the discrete time feedback control law on the continuous time linear system (1.16)-(1.17) in SIMULINK. Evaluate the controller performance in these two scenarios:

- Set the disturbance vector $\mathbf{d}$ to zero and evaluate the reference tracking performance of the closed loop system in the given range of amplitudes A_x and frequencies f_c.
- Enable the disturbance vector $\mathbf{d}$ and set the voice coil position reference $x_{\text{ref}}(k) = x_{\text{ln}}(k)$, where x_{ln} is the timeseries recorded in **P9**.

When testing and assessing the performance of the controller on the nonlinear loudspeaker model hard limits on the input voltage should be taken into account. The given loudspeaker has a rated power of 100W and a rated input impedance of 8Ω. This implies that the input voltage u_e has a maximum amplitude of 40V.

P17 Implement the discrete time feedback control law on the SIMULINK model of the nonlinear loudspeaker built in **P2**. Evaluate the reference tracking and the distortion attenuation capabilities of the designed controller when the reference signal is $x_{\text{ref}}(k) = x_{\text{ln}}(k)$. Report and discuss your results in the time and frequency domain. Focus should be placed both on the control input u_e and on the state variables. Assess THD (include the first 5 harmonics after the fundamental frequency f_c), d_2 and d_3 on the voice coil velocity and discuss if the controller succeeded in attenuating it.

P18 Make a frequency sweep for $f_c \in [20\,,\,200]$Hz and $A_x \in [0.001\,,\,0.005]$m with a $\Delta f = 5$Hz and $\Delta A_x = 0.002$m. For each pair $(\bar{f}_c, \bar{A}_x)$ compute d_2 and d_3 for the voice coil velocity. Present the results of your analysis graphically, compare them with those obtained in **P5** and discuss the effect of the designed controller.

1.2.3 Output feedback voice coil position tracking control with disturbance rejection

An alternative approach for attenuating the distortion around the second and third order harmonics is to directly compensate for it through a disturbance feedforward action in the controller. This requires the implementation of an observer, which can provide an estimate of the fictitious disturbances acting on the linear system and hence of the distortion effects on the nonlinear loudspeaker. At the same time feedback shall be established relying only the measured output $y(t) = i(t)$. The control objective can be formulated as follows:

Objective 2 *Consider the linear loudspeaker model with fictitious disturbances*

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u_e + \mathbf{B}_d\mathbf{d} \tag{1.23}$$

$$y = \mathbf{C}\mathbf{x} \tag{1.24}$$

Given the voice coil position reference $x_{\text{ref}}(k) = A_x \sin(2\pi f_c k T_s)$ *with* $A_x \in [0.001\,,\,0.005]\text{m}$ *and* $f_c \in [20\,,\,200]\text{Hz}$ *design a feedback controller* $u_e(k) = \boldsymbol{\kappa}(\hat{\mathbf{x}}(k), \hat{\mathbf{d}}(k), x_{\text{ref}}(k))$ *such that*

$$x_{\text{ln}}(t-\tau) = x_{\text{ref}}(t) \quad \text{for}\, t \geq T_t > 0 \tag{1.25}$$

where T_t *is the time after which the transient is finished, and* $\tau > 0$ *is the time delay.*

To achieve the control objective the problem can be decomposed in two sub-problems: an estimation problem and a reference tracking with disturbance rejection problem. First we address the estimation problem through the design and implementation of a Kalman filter.

P19 In order to design a Kalman filter capable of reconstructing the unmeasured states and the fictitious disturbances the model (1.10)-(1.11) has to be expanded. Let $\mathbf{x}_k = [\mathbf{x}^{\text{T}}\,,\,\mathbf{x}_d^{\text{T}}]^{\text{T}} \in \mathbb{R}^7$ be the new state vector including the loudspeaker state $\mathbf{x} \in \mathbb{R}^3$ and the disturbance model state $\mathbf{x}_d \in \mathbb{R}^4$; $\mathbf{n}_1 = [\mathbf{n}_x^{\text{T}}\,,\,\mathbf{n}_d^{\text{T}}]^{\text{T}} \in \mathbb{R}^6$ be the process noise vector with $\mathbf{n}_x = [n_i\,,\,n_v]^{\text{T}}$; $n_2 \in \mathbb{R}$ be the measurement noise; u_e be the control input, and $y = i + n_2$ the measured output. Then the state space model representing the combined loudspeaker-disturbance model is

$$\dot{\mathbf{x}}_k = \mathbf{A}_k\mathbf{x}_k + \mathbf{B}_k u_e + \mathbf{B}_{kn}\mathbf{n}_1 \tag{1.26}$$

$$y = \mathbf{C}_k\mathbf{x}_k + n_2 \tag{1.27}$$

Propose a suitable dynamical model to represent the disturbance vector $\mathbf{d}$ in state space form. Then analytically determine the matrices $\mathbf{A}_k$, $\mathbf{B}_k$, $\mathbf{B}_{kn}$, $\mathbf{C}_k$ under the assumption that the process noise $\mathbf{n}_x$ enters the system in the feedback path of the open loop system. Assess the observability of the system (1.26)-(1.27).

Std. Dev.	Value	Unit
σ_i		A
σ_v		m/s
σ_{n_2}		A

Table 1.2: Noise Characteristics

The loudspeaker process noise $\mathbf{n}_x = [n_i, n_v]^{\mathrm{T}}$ and measurement noise n_2 are band-limited noise characterized by the following autocorrelation function

$$R(\tau) = \sigma^2 e^{-\beta|\tau|} \tag{1.28}$$

where the values of σ for each noise component is given in Table 1.2, while $\beta = XX$ for both of them. The process noise $\mathbf{n}_d$ is band-limited **white** Gaussian noise and its covariance matrix

$$\mathbf{V}_{n_d} = \begin{bmatrix} \sigma_{x_{d1}}^2 & 0 & 0 & 0 \\ 0 & \sigma_{x_{d2}}^2 & 0 & 0 \\ 0 & 0 & \sigma_{x_{d3}}^2 & 0 \\ 0 & 0 & 0 & \sigma_{x_{d4}}^2 \end{bmatrix} \tag{1.29}$$

is a tuning parameter in the design of the Kalman filter.

P20 Augment the system (1.26)-(1.27) such that the proper noise models are included. Then discretize the obtained system and assess once again the observability.

P21 Design a discrete time Kalman filter for the model derived in **P20**. The design should be done using the MATLAB function `dlqe`. Beware that the provided noise variances are for the continuous time system. Motivate your choice of the covariance matrix $\mathbf{V}_{n_d}$.

P22 Implement the noise sources ($\mathbf{n}_1$ and n_2) and the Kalman filter in the SIMULINK diagram built in **P11** and assess its performance in simulation when $u_e = A_u \sin(2\pi f_c t)$ whit $A_u = 5\text{V}$ and $f_c = 20\text{Hz}$. The performance assessment should take into account

- the estimation error covariance in comparison with the theoretical value
- the state estimate covariance in comparison with the theoretical value

- the innovation process variance in comparison with the theoretical value
- the level of whiteness of the innovation process
- graphic comparison of the true states with the estimated ones
- graphic comparison of the disturbance $\mathbf{d}$ with its estimate $\hat{\mathbf{d}}$

P23 Implement the noise sources ($\mathbf{n}_1$ and n_2) and the Kalman filter in the SIMULINK diagram of the nonlinear loudspeaker built in **P2** and assess its performance in simulation when $u_e = A_u \sin(2\pi f_c t)$ whit $A_u = 5$V and $f_c = 20$Hz. The following measures of estimation quality should be compared with those obtained in **P22** from simulating the linear system

- the estimation error covariance
- the state estimate covariance
- the innovation process covariance
- the level of whiteness of the innovation process

Moreover the following comparison should be carried out

- graphic comparison of the true states with the estimated ones
- graphic comparison of the power spectral density of the estimated distortion $\hat{d}_{i1} + \hat{d}_{i2}$ with the power spectral density of the voice coil current.

Discuss your findings and conclude about the suitability of using the Kalman filter to obtain an estimate of the second and third order harmonic distortion.

Now that estimates of the state vector and of the second and third order harmonic distortion components are available, the reference tracking controller with disturbance feedforward can be designed, implemented and tested.

P24 Design the disturbance feedforward component of the controller such that the second and third order harmonic distortion is reduced by approximately a factor of 10.

P25 Implement the reference tracking controller with disturbance rejection in the SIMULINK model built in **P23**. The controller is to be fed with the estimates provided by the Kalman filter. Evaluate the reference tracking and the distortion attenuation capabilities of the design controller when the reference signal is $x_{\text{ref}}(k) = x_{\text{ln}}(k)$. Report and discuss your results in the time and frequency domain. Have the second and third order harmonic distortions been attenuated?

P26 Make a frequency sweep for $f_c \in [20\,,\, 200]$Hz and $A_x \in [0.001\,,\, 0.005]$m with a $\Delta f = 5$Hz and $\Delta A_p = 0.002$m. For each pair $(\bar{f}_c, \bar{A}_p)$ compute d_2 and d_3 for the voice coil velocity. Present the results of your analysis graphically, compare them with those obtained in **P5** and in **P18** and discuss the effect of this control system in relation to the controller designed in **P15**.

Bibliography

[1] R. Kornum. (2015, November) Højttaleren fylder 100 år – og fødselaren er dansk. Web article. Ingeniøren. [Online]. Available: http://ing.dk/artikel/hoejttaleren-fylder-100-aar-og-foedselaren-er-dansk-179868

[2] H. Schurer, "Linearization of electroacoustic transducers," Ph.D. dissertation, University of Twente Enschede, 1997.

[3] H. Schurer, C. H. Slump, and O. Herrmann, "Theoretical and experimental comparison of three methods for compensation of electrodynamic transducer nonlinearity," *J. Audio Eng. Soc.*, vol. 46(9), pp. 723–740, 1998.