31310 Linear Control Design 2

Compulsory Assignment 2015 : Loudspeaker control

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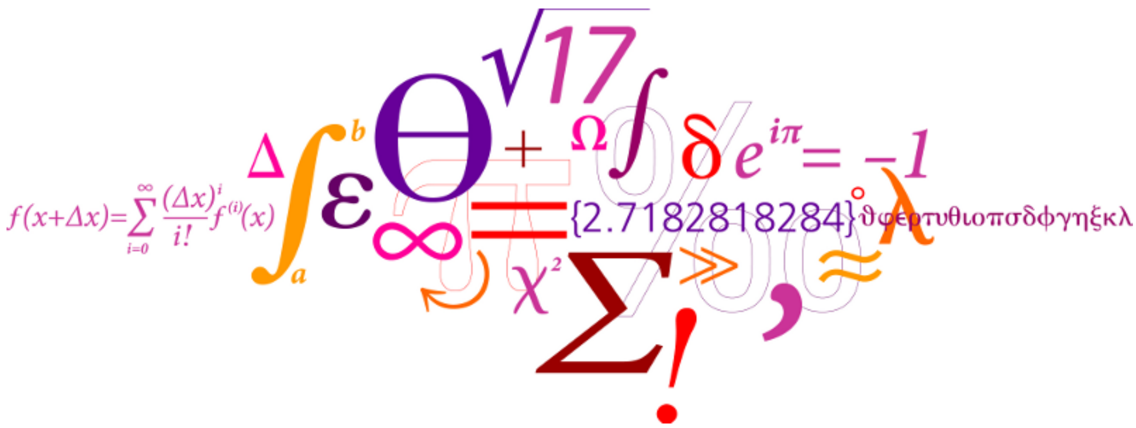

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Distortion Attenuation for Loudspeakers

1.1 Moving-coil Loudspeakers

1.1.1 Loudspeakers electrical equivalent circuit

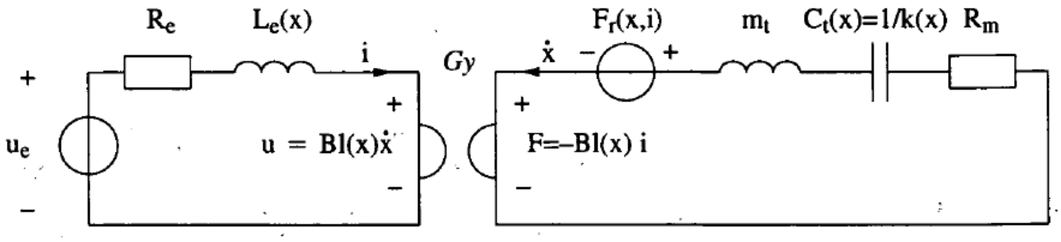

Figure 1.1: Electrical equivalent lumped element model of the voltage driven electrodynamic loudspeaker for low frequencies. The coupling between the electrical and mechanical domain is performed through the gyrator with gyration constant Bl(x).

$$u_e = R_e i + \frac{dL_e(x)}{dx}\frac{dx}{dt} i + L_e(x)\frac{di}{dt} + Bl(x)\frac{dx}{dt} \tag{1.1}$$

$$Bl(x)i = m_t \frac{d^2x}{dt^2} + R_m \frac{dx}{dt} + k(x)x - \frac{1}{2}\frac{dL_e(e)}{dx} i^2 \tag{1.2}$$

where

$$Bl(x) = Bl_0 + b_1 x + b_2 x^2 \tag{1.3}$$

$$L_e(x) = L_{e0} + l_1 x + l_2 x^2 \tag{1.4}$$

$$k(x) = k_0 + k_1 x + k_2 x^2 \tag{1.5}$$

Problem 1

By means of Eqs 1.1, 1.2, 1.3, 1.4 and 1.5, we can identify 3 state variables x, $\dot{x}$ and i. We can also identify the input u_e.

$\mathrm{x} = \begin{pmatrix} x \\ \dot{x} \\ i \end{pmatrix}$ and $\mathrm{u} = (u_e)$

Then, we can derive the nonlinear dynamical state space model to obtain

$$\dot{x} = \dot{x} \tag{1.6}$$

$$\ddot{x} = \frac{(Bl_0 + b_1 x + b_2 x^2)i - R_m \dot{x} - (k_0 + k_1 x + k_2 x^2)x + \frac{1}{2}(l_1 + 2l_2 x)\dot{x} i^2}{m_t} \tag{1.7}$$

$$\dot{i} = \frac{u_e - (R_e + (l_1 + 2l_2 x)\dot{x}^2)i - (Bl_0 + b_1 x + b_2 x^2)\dot{x}}{L_{e0} + l_1 x + l_2 x^2} \tag{1.8}$$

In matrix format, we have

$$\dot{\mathrm{x}} = f(\mathrm{x}) + g(\mathrm{x})\mathrm{u} \tag{1.9}$$

with

$$f(\mathrm{x}) = \begin{pmatrix} \mathrm{x}(2) \\ \frac{(Bl_0+b_1\mathrm{x}(1)+b_2\mathrm{x}(1)^2)\mathrm{x}(3)-R_m\mathrm{x}(2)-(k_0+k_1\mathrm{x}(1)+k_2\mathrm{x}(1)^2)\mathrm{x}(1)+\frac{1}{2}(l_1+2l_2\mathrm{x}(1))\mathrm{x}(2)\mathrm{x}(3)^2}{m_t} \\ \frac{-(R_e+(l_1+2l_2\mathrm{x}(1))\mathrm{x}(2)^2)\mathrm{x}(3)-(Bl_0+b_1\mathrm{x}(1)+b_2\mathrm{x}(1)^2)\mathrm{x}(2)}{L_{e0}+l_1\mathrm{x}(1)+l_2\mathrm{x}(1)^2} \end{pmatrix} \tag{1.10}$$

$$g(\mathrm{x}) = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_{e0}+l_1\mathrm{x}(1)+l_2\mathrm{x}(1)^2} \end{pmatrix} \tag{1.11}$$

Problem 2

Problem 3

1.1.2 Harmonic Distortion

Problem 4

Problem 5

1.1.3 Linearised Model

The measured output is set to the voice coil current. Therefore $y(t) = r(\mathrm{x}, t) = i(t)$. We also take $u_e = 0$ for the analysis of the linear and nonlinear model around the resting position of the voice coil.

Problem 6

All time derivatives are set to zero in order to determine the stationary states. Therefore, we have $\frac{dx}{dt} = 0$ and equations (1.1) and (1.2) are rewritten below:

$$u_e = R_e i \tag{1.12}$$

$$Bl(x)i = k(x)x \tag{1.13}$$

As $u_e = 0$, from (1.12) we obtain $i = 0$ and we deduce by substituting in (1.13) that $k(x)x = 0$. Then $k(x) = 0$ or $x = 0$. The discriminant of the polynomial $k(x)$ of degree 2 is $\Delta = k_1^2 - 4k_2k_0 < 0$. The voice coil displacement x being real, we discard this value and get:

$$\mathrm{x}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

We linearise the model $\dot{\mathrm{x}} = h(\mathrm{x}, \mathrm{u})$ with $h(\mathrm{x}) = f(\mathrm{x}) + g(\mathrm{x})\mathrm{u}$ around the stationary states:

$$\begin{aligned} x(t) &= x_0 + \Delta x(t) = \Delta x(t) \\ \dot{x}(t) &= \dot{x}_0 + \Delta \dot{x}(t) = \Delta \dot{x}(t) \\ i(t) &= i_0 + \Delta i(t) = \Delta i(t) \end{aligned}$$

The linear model can then be written in the form:

$$\dot{\mathrm{x}} = A\mathrm{x} + B\mathrm{u}$$
$$y = C\mathrm{x}$$

where

$$A = \begin{pmatrix} \frac{\partial h_1}{\partial \mathrm{x}_1} & \frac{\partial h_1}{\partial \mathrm{x}_2} & \frac{\partial h_1}{\partial \mathrm{x}_3} \\ \frac{\partial h_2}{\partial \mathrm{x}_1} & \frac{\partial h_2}{\partial \mathrm{x}_2} & \frac{\partial h_2}{\partial \mathrm{x}_3} \\ \frac{\partial h_3}{\partial \mathrm{x}_1} & \frac{\partial h_3}{\partial \mathrm{x}_2} & \frac{\partial h_3}{\partial \mathrm{x}_3} \end{pmatrix}_{\mathrm{x}_0} \qquad B = \begin{pmatrix} \frac{\partial h_1}{\partial \mathrm{u}} \\ \frac{\partial h_2}{\partial \mathrm{u}} \\ \frac{\partial h_3}{\partial \mathrm{u}} \end{pmatrix}_{\mathrm{x}_0} \qquad C = \begin{pmatrix} \frac{\partial r}{\partial \mathrm{x}_1} & \frac{\partial r}{\partial \mathrm{x}_2} & \frac{\partial r}{\partial \mathrm{x}_3} \end{pmatrix}_{\mathrm{x}_0}$$

We obtain:

$$\frac{\partial h_1}{\partial \mathrm{x}_1} = 0 \qquad \frac{\partial h_1}{\partial \mathrm{x}_2} = 1 \qquad \frac{\partial h_1}{\partial \mathrm{x}_3} = 0$$

$$\frac{\partial h_2}{\partial \mathrm{x}_1} = \frac{b_1\mathrm{x}_{30} + 2b_2\mathrm{x}_{10}\mathrm{x}_{30} - (k_0 + 2k_1\mathrm{x}_{10} + 3k_2\mathrm{x}_{10}^2) + l_2\mathrm{x}_{20}\mathrm{x}_{30}^2}{m_t}$$

$$\frac{\partial h_2}{\partial \mathrm{x}_2} = \frac{-R_m + \frac{1}{2} \times (l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{30}^2}{m_t}$$

$$\frac{\partial h_2}{\partial \mathrm{x}_3} = \frac{Bl_0 + b_1\mathrm{x}_{10} + b_2\mathrm{x}_{10}^2 + (l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}\mathrm{x}_{30}}{m_t}$$

$$\frac{\partial h_3}{\partial \mathrm{x}_1} = \frac{(-2l_2\mathrm{x}_{20}^2\mathrm{x}_{30} - (b_1 + 2b_2\mathrm{x}_{10})\mathrm{x}_{20}) \times (L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2)}{(L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2)^2} - (l_1 + 2l_2\mathrm{x}_{10}) \times \frac{(-(R_e + (l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}^2)\mathrm{x}_{30} - (Bl_0 + b_1\mathrm{x}_{10} + b_2\mathrm{x}_{10}^2)\mathrm{x}_{20})}{(L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2)^2}$$

$$\frac{\partial h_3}{\partial \mathrm{x}_2} = -\frac{2(l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}\mathrm{x}_{30} + Bl_0 + b_1\mathrm{x}_{10} + b_2\mathrm{x}_{10}^2}{L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2}$$

$$\frac{\partial h_3}{\partial \mathrm{x}_3} = -\frac{(l_1 + 2l_2\mathrm{x}_{10})\mathrm{x}_{20}^2 + R_e}{L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2}$$

$$\frac{\partial h_1}{\partial \mathrm{u}} = 0 \qquad \frac{\partial h_2}{\partial \mathrm{u}} = 0 \qquad \frac{\partial h_3}{\partial \mathrm{u}} = \frac{1}{L_{e0} + l_1\mathrm{x}_{10} + l_2\mathrm{x}_{10}^2}$$

$$\frac{\partial r}{\partial \mathrm{x}_1} = 0 \qquad \frac{\partial r}{\partial \mathrm{x}_2} = 0 \qquad \frac{\partial r}{\partial \mathrm{x}_3} = 1$$

We then substitute $\mathrm{x}_{10} = \mathrm{x}_{20} = \mathrm{x}_{30} = 0$. Finally, we get:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -\frac{k_0}{m_t} & -\frac{R_m}{m_t} & \frac{Bl_0}{m_t} \\ 0 & -\frac{Bl_0}{L_{e0}} & -\frac{R_e}{L_{e0}} \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_{e0}} \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

Numerically

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -1.20 \cdot 10^5 & -50.46 & 279.19 \\ 0 & -2.57 \cdot 10^3 & -1.40 \cdot 10^3 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ 177 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

We can check these results with the matlab function *linmod(model,x_0,u_e)*:

```
[A,B,C,D] = linmod('nonLinearModel',[0;0;0],0);
```

Bibliography