31310 Linear Control Design 2

Compulsory Assignment 2015 : Loudspeaker control

by

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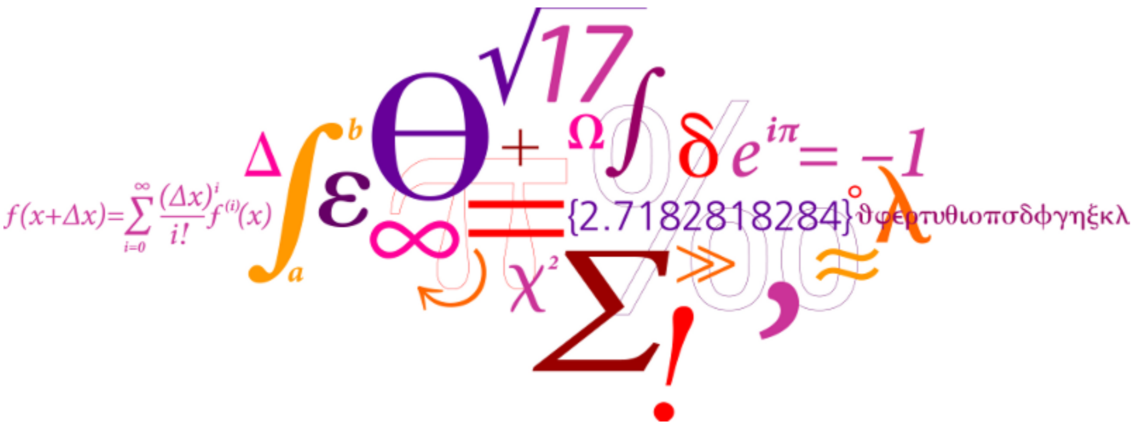

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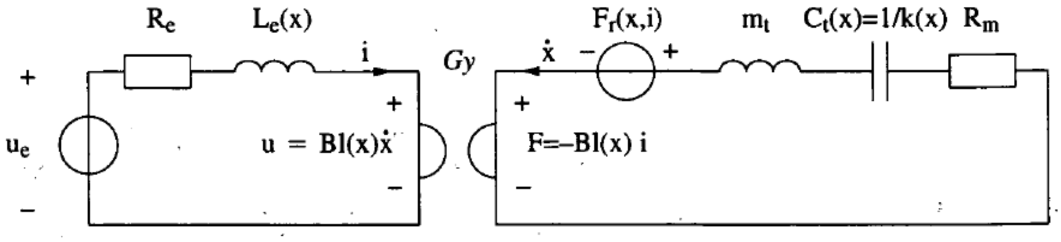

Figure 1: Electrical equivalent lumped element model of the voltage driven electrodynamic loudspeaker for low frequencies. The coupling between the electrical and mechanical domain is performed through the gyrator with gy- ration constant Bl(x).

$$u_e = R_e i + \frac{dL_e(x)}{dx}\frac{dx}{dt}i + L_e(x)\frac{di}{dt} + Bl(x)\frac{dx}{dt} \tag{1}$$

$$Bl(x)i = m_t\frac{d^2x}{dt^2} + R_m\frac{dx}{dt} + k(x)x - \frac{1}{2}\frac{dL_e(e)}{dx}i^2 \tag{2}$$

where

$$Bl(x) = Bl_0 + b_1x + b_2x^2 \tag{3}$$

$$L_e(x) = L_{e0} + l_1x + l_2x^2 \tag{4}$$

$$k(x) = k_0 + k_1x + k_2x^2 \tag{5}$$

1.1 Problem 1

By means of Eqs 1, 2, 3, 4 and 5, we can identify 3 state variables i, x and $\dot{x}$. We can also identify the input u_e.

$\mathrm{x} = \begin{pmatrix} i \\ x \\ \dot{x} \end{pmatrix}$ and $\mathrm{u} = (u_e)$

Then, we can derive the nonlinear dynamical state space model to obtain

$$\dot{i} = \frac{u_e - (R_e + (l_1 + 2l_2 x)\dot{x})i - (Bl_0 + b_1 x + b_2 x^2)\dot{x}}{L_{e0} + l_1 x + l_2 x^2} \tag{6}$$

$$\dot{x} = \dot{x} \tag{7}$$

$$\ddot{x} = \frac{(Bl_0 + b_1 x + b_2 x^2)ß - R_m \dot{x} - (k_0 + k_1 x + k_2 x^2)\mathrm{x}(2) + \frac{1}{2}(l_1 + 2l_2 x)\dot{x} i^2}{m_t} \tag{8}$$

In matrix format, we have

$$\dot{\mathrm{x}} = f(\mathrm{x}) + g(\mathrm{x})\mathrm{u} \tag{9}$$

with

$$f(\mathrm{x}) = \begin{pmatrix} \frac{u_e - (R_e + (l_1 + 2l_2\mathrm{x}(2))\mathrm{x}(3))\mathrm{x}(1) - (Bl_0 + b_1\mathrm{x}(2) + b_2\mathrm{x}(2)^2)\mathrm{x}(3)}{L_{e0} + l_1\mathrm{x}(2) + l_2\mathrm{x}(2)^2} \\ \mathrm{x}(3) \\ \frac{(Bl_0 + b_1\mathrm{x}(2) + b_2\mathrm{x}(2)^2)\mathrm{x}(1) - R_m\mathrm{x}(3) - (k_0 + k_1\mathrm{x}(2) + k_2\mathrm{x}(2)^2)\mathrm{x}(2) + \frac{1}{2}(l_1 + 2l_2\mathrm{x}(2))\mathrm{x}(3)\mathrm{x}(1)^2}{m_t} \end{pmatrix} \tag{10}$$

$$g(\mathrm{x}) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{11}$$

References