

Problem Set 2

Deadline: March 25, 2021

March 11, 2021

Problem 1

Show that if $g_{\mu\nu}$ is diagonal, then $g^{\mu\nu}$ is diagonal with corresponding reciprocal diagonal elements.

(Introducing Einstein's Relativity, Ray D' Inverno, Oxford University Press, 1992)

Problem 2

Two space described by the metric $\tilde{g}_{\mu\nu}$ and $g_{\mu\nu}$ are said to be conformally related if

$$\tilde{g}_{\mu\nu}(x) = \Omega^2(x)g_{\mu\nu} \quad (1)$$

Show that, given two infinitesimal line segments originating from a point, the angle between them is preserved by this conformal transformation.

(Einstein Gravity In A Nutshell, A.Zee, Princeton University Press, 2013)

Problem 3

3 + 1 Minkowski metric is

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2. \quad (2)$$

Find the components $g_{\mu\nu}$ and $g^{\mu\nu}$, of the metric and inverse metric in:

- Rotating coordinates:

$$\begin{aligned} t' &= t \\ x' &= \sqrt{x^2 + y^2} \cos(\phi - \omega t) \\ y' &= \sqrt{x^2 + y^2} \sin(\phi - \omega t) \\ z' &= z \end{aligned} \quad (3)$$

(General Relativity, R.Wald, University of Chicago Press, 1984)

- Boosted (along x) coordinates:
Find the boosted coordinates yourself.