

فیزیک نوترینو

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \sigma^\mu & 0 \end{bmatrix} \quad \sigma^\mu = (1, \vec{\sigma}) \quad \bar{\sigma}^\mu = (1, -\vec{\sigma})$$

notation Moril et al

$$\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

مستقل از نوتراسیون

در نوتراسیون قیاس (-۱)

$m=0 \rightarrow$

هلستی

$$\psi_R = \frac{1+\gamma_5}{2} \psi \quad \psi_L = \frac{1-\gamma_5}{2} \psi$$

مستقل از نوتراسیون

$$\Sigma^{\mu\nu} = \frac{i}{4} [\gamma^\mu, \gamma^\nu] = \frac{i}{2} \begin{bmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \sigma^{\mu\nu} \end{bmatrix}$$

$$\sigma^{\mu\nu} = \sigma^\mu \sigma^\nu - \sigma^\nu \sigma^\mu \quad \bar{\sigma}^{\mu\nu} = \bar{\sigma}^\mu \bar{\sigma}^\nu - \bar{\sigma}^\nu \bar{\sigma}^\mu \quad \mu \neq \nu$$

$$\psi_R = \begin{pmatrix} 0 \\ \chi_a \end{pmatrix} \quad \psi_L = \begin{pmatrix} \xi_a \\ 0 \end{pmatrix} \quad a, \alpha = 1, 2$$

$$\begin{cases} SL(2, \mathbb{C}) \\ SU(2, \mathbb{C}) \end{cases} \quad \begin{matrix} \text{سایر های} \\ \text{مختلط} \end{matrix} \quad [\gamma_5, \Sigma^{\mu\nu}] = 0$$

سودای ...؟
قناب

We easily know that each of chiral fermion forms an irreducible representation of Lorentz transformation $SL(2, \mathbb{C})$, as $[\gamma_5, \Sigma^{\mu\nu}] = 0$

$$m(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R)$$

$$\psi_D = \psi_R + \psi_L = \begin{pmatrix} \xi_a \\ \chi_a \end{pmatrix}$$

$$\psi_R = R \psi_D \quad \psi_L = L \psi_D \quad R = \frac{1+\gamma_5}{2} \quad L = \frac{1-\gamma_5}{2}$$

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$$\psi_D = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\bar{\psi}_D (i\not{\partial} - m) = \bar{\psi}_R i\not{\partial} \psi_R + \bar{\psi}_L i\not{\partial} \psi_L - m(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R)$$

$$(\psi_R)^c = -i\gamma^2 \left(\frac{1+\gamma_5}{2} \psi \right)^*$$

$$R (\psi_R^c) = \frac{1+\gamma_5}{2} \psi_R^c = 0$$

$$L \psi_R^c = \psi_R^c \quad \psi_R^c = \begin{bmatrix} -i\sigma_2 \psi^* \\ 0 \end{bmatrix} = \begin{bmatrix} -\psi_2^* \\ +\psi_1^* \\ 0 \\ 0 \end{bmatrix}$$

$$\psi_R = \begin{pmatrix} 0 \\ \psi_1 \end{pmatrix}$$

$$\bar{\psi}^* = \bar{\psi}^\dagger \gamma^0 \quad \bar{\psi}_i = (\psi_i)^*$$

مادرزادی فقط

$$\psi_L = \begin{bmatrix} \xi \\ 0 \end{bmatrix} \quad \psi_L^c = \begin{bmatrix} 0 \\ i\sigma_2 \xi^* \end{bmatrix} = \begin{bmatrix} 0 \\ \xi_2^* \\ -\xi_1^* \end{bmatrix}$$

$$\psi_{M_1} = \psi_R + \psi_R^c = \begin{bmatrix} -i\sigma_2 \psi^* \\ \psi_1 \end{bmatrix}$$

$$\psi_{M_2} = \psi_L + \psi_L^c = \begin{bmatrix} \xi \\ i\sigma_2 \xi^* \end{bmatrix}$$

$$\psi_{M_1}^c = \psi_{M_1} \quad \psi_{M_2}^c = \psi_{M_2}$$

اسینورهای مایورانا

$$\begin{bmatrix} \xi \\ i\sigma_2 \xi^* \end{bmatrix} = \begin{bmatrix} -i\sigma_2 \psi^* \\ \psi_1 \end{bmatrix}$$

$$\bar{\psi}_D i\not{\partial} \psi_D = \bar{\psi}_R i\not{\partial} \psi_R + \bar{\psi}_L i\not{\partial} \psi_L =$$

$$\frac{1}{2} (\bar{\psi}_{m_1} i \not{\partial} \psi_{m_1} + \bar{\psi}_{m_2} i \not{\partial} \psi_{m_2})$$

جزئیات هست.

تعداد درجات آزادی

جرم صفر \rightarrow انتشارها

جرم دیراک - جرم مایورانا
جرم دیراک:

$$-m_D \bar{\psi}_D \psi_D = -m_D (\bar{\psi}_L \psi_R + H.c.) = m_D (\xi \xi + H.c.)$$

جرم مایورانا:

$$-\frac{1}{2} m_R (\bar{\psi}_R^c \psi_R + H.c.) = \frac{m_R}{2} (\chi \chi + H.c.)$$

$$-\frac{m_L}{2} (\bar{\psi}_L^c \psi_L + H.c.) = \frac{m_L}{2} (\xi \xi + H.c.)$$



$$\bar{\psi}_R i \not{\partial} \psi_R - \frac{m_R}{2} (\bar{\psi}_R^c \psi_R + H.c.) = \frac{1}{2} \bar{\psi}_{m_1} (i \not{\partial} - m_R) \psi_{m_1}$$

$$\bar{\psi}_L i \not{\partial} \psi_L - \frac{m_L}{2} (\bar{\psi}_L^c \psi_L + H.c.) = \frac{1}{2} \bar{\psi}_{m_2} (i \not{\partial} - m_L) \psi_{m_2}$$

انواع ممکن برای جرم های نورینو

ذرات ابردار نمی توانند جرم مایورانا داشته باشند.

جباری به ارتباط

کلی ترین حالت

$$\mathcal{L}_m = -\frac{1}{2} m_R \bar{\nu}_R^c \nu_R - \frac{m_L}{2} \bar{\nu}_L^c \nu_L - m_D \bar{\nu}_R \nu_L + H.c.$$

$$\mathcal{L}_m = -\frac{1}{2} (\bar{\nu}_L^c \quad \bar{\nu}_R) \begin{matrix} \text{M} \\ \begin{bmatrix} m_L & m_D \\ m_D & m_R^* \end{bmatrix} \end{matrix} \begin{bmatrix} \nu_L \\ \nu_R^c \end{bmatrix} + H.c.$$

$$\bar{\nu}_R \nu_L = \bar{\nu}_L^c \nu_R^c \quad * \text{ نشان دهید که}$$

$$\begin{array}{c|c} \nu_L & \nu_R^c \\ \hline L=1 & -1 \end{array}$$

M مایس مخلوط و مقادیر

$$M_\nu = \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix}$$

$$\begin{cases} m_s = \frac{1}{2} \left\{ (m_L + m_R) + \sqrt{(m_R - m_L)^2 + 4m_D^2} \right\} \\ m_a = \frac{1}{2} \left\{ -(m_L + m_R) + \sqrt{(m_R - m_L)^2 + 4m_D^2} \right\} \\ \nu_s = \sin \theta_\nu \nu_L + \cos \theta_\nu \nu_R^c \\ \nu_a = i \left(\cos \theta_\nu \nu_L - \sin \theta_\nu \nu_R^c \right) \end{cases}$$

\downarrow
 i حتماً

$$\tan 2\theta_\nu = \frac{2m_D}{m_R - m_L}$$

$$m_R = m_L \quad \leftarrow \text{pure Dirac} \quad m_a = m_s$$

$$\mathcal{L}_m = -\frac{1}{2} m_s \bar{\nu}_s^c \nu_s - \frac{1}{2} m_a \bar{\nu}_a^c \nu_a + \text{H.c.}$$

$$N_s = \nu_s + \nu_s^c \quad N_a = \nu_a + \nu_a^c$$

$$\mathcal{L}_\nu = \frac{1}{2} \left\{ \bar{N}_s (i\not{\partial} - m_s) N_s + \bar{N}_a (i\not{\partial} - m_a) N_a \right\}$$

$$\varphi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}$$

مختلط

تناظر با میدان اسکالر

$$M^2 |\varphi|^2 + \frac{1}{2} (m^2 \varphi^2 + m^{\star 2} \varphi^{\star 2})$$

$$\mathcal{L}_m = -\frac{1}{2} (\varphi_1 \ \varphi_2) \begin{bmatrix} m_1^2 & m_{12}^2 \\ m_{12}^2 & m_2^2 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}$$

$$m^2 = 0 \quad m_1^2 = m_2^2 \quad , \quad m_{12}^2 = 0$$

$$m_L = m_R = 0 \quad m_S = m_a$$

$$\nu_D = \frac{N_S - i N_a}{\sqrt{2}} = \nu_R + \nu_L$$

$$\mathcal{L}_\nu = \bar{\nu}_D (i \not{\partial} - m_D) \nu_D$$

$$\langle 0 | \psi_m \bar{\psi}_m | 0 \rangle = \frac{i}{\not{p} - m} \quad \leftarrow (6.40) \quad \leftarrow (6.41)$$

$$\langle 0 | \psi_m \psi_m^{Tr} | 0 \rangle = \frac{i}{\not{p} - m} C^{-1} \quad C = i \gamma^0 \gamma^2$$

* تمرین تابش 6.40 را بدینیه

معادله حرکت (اولیه لارانتس) را از لاگرانژی زیر بدست آورید.

$$\bar{\psi} i \not{\partial} \psi - \left(\frac{m}{2} \bar{\psi}^c \psi + \text{H.c.} \right)$$

مکانیزم های تولید جرم نوترینو

Majorana ??

$0 \nu \beta \beta$

Dirac

دیراک خالص

$$N_a \quad N_s \quad m_a = m_s = m_n$$

$$N_a \quad N_s \quad m_a = m_s = m_b$$

$$\Upsilon \quad \bar{\nu}_R \quad H_L^T L \rightarrow \text{مجموعه دریاک خالص}$$

س - س

شبه دریاک

pseudo - Dirac

Wolfenstein, 1981

$$m_R, m_L \ll m_D \quad \theta_v \simeq \frac{\pi}{4}$$

$$\nu_L \rightarrow \nu_R^c \quad \text{oscillation}$$

(Kobayashi Lim, 2001)

$$m_s^2 - m_a^2 \simeq 2 m_D (m_R + m_L)$$

$$P(\nu_L \rightarrow \nu_R^c) = \sin^2 \left(\frac{m_s^2 - m_a^2}{2E} \right) \quad \text{نشان}$$

Seesaw

(Yanagida, 1979 ; Gell-Mann - Ramond - Slansky, 1979)

Peter Minkowski

$$m_R \quad \bar{\nu}_R^c \quad \nu_R$$

$$\underbrace{m \bar{\nu}_L^c \nu_L}_{I_3 = 1} \quad \nu_L^c \quad H_T \quad \nu_L$$

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} \simeq 1$$

$$M_\nu = \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix}$$

ویژگی‌ها

$$m_a \approx -\frac{m_D^2}{m_R} \ll m_D$$

$$m_S = m_R$$

$$\theta_v \approx \frac{m_D}{m_R} \ll 1$$

$$N_S \approx \nu_R + \nu_R^c \quad N_a \approx i(\nu_L - \nu_L^c)$$

N_S decouples

$$m_R \bar{\nu}_R^c \nu_R + Y \bar{\nu}_R \phi \in L$$

$$m_D = Y \langle \phi^0 \rangle \quad Y^2 \phi \in L \quad \frac{1}{m_R} \phi \in L$$

اینجکه در کتاب آمده است.

$$\sum_{a=1}^3 \frac{C_a}{M} \phi^{\dagger} \sigma_a \phi (\nu_L^{\dagger} e_L^{\dagger}) \epsilon^{\dagger} C (\nu_L e_L)$$

charge conjugation

* آیا عبارت فوق تحت $U(1)$ نادرست؟
 $\sim \sim SU(2) \sim \sim$
 آیا پس از شکست تقارن الکتروضعیف عبارت فوق به $U(1)_{em}$
 احترام می‌گذارد؟ آیا به ν_e جرم می‌دهد؟

$$\sigma^M = (1, \vec{\sigma})$$

راهنمایی

$$(\sigma^M)_{\alpha\beta} (\sigma^M)_{\gamma\delta} = 2 \epsilon_{\alpha\gamma} \epsilon_{\beta\delta}$$

$$(\sigma^a)_{\alpha\beta} (\sigma^a)_{\gamma\delta} = ?$$

Lepton sector

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}; \nu_{eR} \nu_{\mu R} \nu_{\tau R} e_R \mu_R \tau_R$$

$$m_D \rightarrow \text{diagonal} \quad \text{دیرک فرم}$$

$$\bar{\nu}_{eR} \quad \nu_{eL} \quad \dots \quad \leftarrow L_e$$

نابت

$$\nu_e \rightarrow \nu_\mu$$

All three neutrino masses are degenerate


 $U(3)$ symmetry

$$m_\nu \leftarrow \text{nondegenerate} \xleftrightarrow{\text{mixing}} \text{GIM mechanism}$$

Flavor Mixing

Dirac scenario

$$\mathcal{L}_m = (m_D)_{\alpha\beta} \bar{\nu}_{\alpha L} \nu_{\beta R} + \text{H.c.}$$

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = U^\dagger m_D U$$

$$\begin{bmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{bmatrix} = U \begin{bmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{bmatrix}$$

$$\mathcal{L} = \frac{g}{\sqrt{2}} (\bar{e}_L \quad \bar{\mu}_L \quad \bar{\tau}_L) U \gamma_\mu \begin{bmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{bmatrix} W^\mu$$

$U \rightarrow$ Maki - Nakagawa - Sakata, 1962

$$(\nu_L)_\alpha \rightarrow (\nu_L)_\beta$$

$$m_D m_D^\dagger = U \text{diag}(m_1^2, m_2^2, m_3^2) U^\dagger$$

See saw

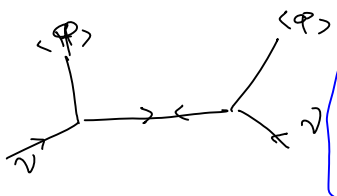
$$\Psi = (\nu_{eL} \ \nu_{\mu L} \ \nu_{\tau L} \ (\nu_{eR})^c \ (\nu_{\mu R})^c \ (\nu_{\tau R})^c)$$

$$m_\nu = \begin{pmatrix} m_L & m_D^{Tr} \\ m_D & m_R \end{pmatrix}$$

$$m_L^{Tr} = m_L \quad m_R^{Tr} = m_R$$

$$- \bar{l}_{\alpha\beta}^D (\bar{\nu}_{\alpha L} \quad \bar{l}_{\alpha L}) \tilde{\Phi} \nu_{\beta R} - (m_R)_{\alpha\beta} \bar{\nu}_{\alpha R}^c \nu_{\beta R}$$

$$(m_D)_{\alpha\beta} = f_{\alpha\beta}^{D\Phi} \times \frac{v_\Phi}{\sqrt{2}}$$



$$\begin{bmatrix} iI & -im_D^{Tr} m_R^{*-1} \\ m_R^{-1} m_D^* & I \end{bmatrix} m_\nu \begin{bmatrix} iI & m_D^+ m_R^{-1} \\ -im_R^{-1} m_D & I \end{bmatrix}$$

$$= \begin{bmatrix} m_D^{Tr} (m_R^*)^{-1} m_D & 0 \\ 0 & m_R^* \end{bmatrix}$$

$$m_{\nu L} = m_D^{Tr} (m_R^*)^{-1} m_D$$

$$U^{Tr} m_{\nu L} U = \text{diag}(m_1, m_2, m_3)$$

$$m_{\nu L}^+ m_{\nu L} = U \text{diag}(m_1^2, m_2^2, m_3^2) U^\dagger$$

magnetic dipole moment

$$(\square + m_i^2) \nu_i = 0 \quad E_i = \sqrt{\vec{p}^2 + m_i^2} \approx |\vec{p}| + \frac{m_i^2}{2E}$$

$$i \frac{d}{dt} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} = \begin{bmatrix} \frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E_2} & \\ & & \frac{m_3^2}{2E_3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$i \frac{d}{dt} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} = \begin{bmatrix} \frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E} & \\ & & \frac{m_3^2}{2E} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$i \frac{d}{dt} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U \begin{bmatrix} \frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E} & \\ & & \frac{m_3^2}{2E} \end{bmatrix} U^\dagger \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}$$

$$\begin{bmatrix} \nu_e(t) \\ \nu_\mu(t) \\ \nu_\tau(t) \end{bmatrix} = e^{-\frac{i}{2E} m_j m_j^\dagger t} \begin{bmatrix} \nu_e(0) \\ \nu_\mu(0) \\ \nu_\tau(0) \end{bmatrix}$$

$$= U \begin{pmatrix} e^{-\frac{i m_1^2 t}{2E}} & 0 & 0 \\ 0 & e^{-\frac{i m_2^2 t}{2E}} & 0 \\ 0 & 0 & e^{-\frac{i m_3^2 t}{2E}} \end{pmatrix} U^\dagger \begin{bmatrix} \nu_e(0) \\ \nu_\mu(0) \\ \nu_\tau(0) \end{bmatrix}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| \sum_i U_{\beta i} e^{-\frac{i m_i^2}{2E} t} U_{\alpha i}^* \right|^2$$

$$= \left| \sum_i U_{\beta i} e^{-\frac{i \Delta m_{i1}^2}{2E} t} U_{\alpha i}^* \right|^2$$

$$\Delta m_{i1}^2 = m_i^2 - m_1^2$$

Dirac mass term

$$\mathcal{L}_c = \frac{g}{\sqrt{2}} \left(\sum_{iL} \gamma_\mu \nu_{jL} U_{ij}^{PMNS} W^{-\mu} \right)$$

non Unitary

$$Z: \frac{(n-1)(n-2)}{2}$$

جذب

$$\bar{\nu}^c m_\nu \nu$$

non symmetric

$$\frac{n(n-1)}{2}$$

$$\left\{ \begin{array}{l} \frac{(n-1)(n-2)}{2} \\ n-1 \end{array} \right. \leftarrow \text{ليجلا}$$

$$m_\nu = U_{PMNS} \underbrace{\text{diag}}_{m_\nu} U_{PMNS}^T$$

$$\text{dia} (m_1 e^{i\alpha_1}, m_2 e^{i\alpha_2}, m_3 e^{i\alpha_3})$$

دع

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{21}^2}{4E} t \right)$$

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m_{21}^2 t}{4E}$$

CPT

$$P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$$

CP

$$P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$$

T

$$P(\nu_\alpha \rightarrow \nu_\beta) = P(\nu_\beta \rightarrow \nu_\alpha)$$

دردونورينو

$$P(\nu_\mu \rightarrow \nu_\mu) = P(\nu_e \rightarrow \nu_e)$$

Averaging

$$\overline{P}(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} \sin^2 2\theta$$

$$\overline{P}(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2} \sin^2 2\theta$$

$$\overline{P}(\nu_e \rightarrow \nu_e) \geq \frac{1}{2}$$

$$\overline{P}(\nu_e \rightarrow \nu_e) \geq \frac{1}{n} \quad \text{نتیجی دهم}$$

$$\overline{P}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2$$

$$\overline{P}(\nu_\alpha \rightarrow \nu_\alpha) = \sum_i |U_{\alpha i}|^4$$

$$\sum_i |U_{\alpha i}|^2 = 1$$

$$\sum_i |U_{\alpha i}|^4 + 2 \sum_{i < j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 = 1$$

$$\sum_{i,j} (|U_{\alpha i}|^2 - |U_{\alpha j}|^2)^2 \geq 0$$

$$(n-1) \left(\sum_i |U_{\alpha i}|^4 \right) - 2 \sum_{i < j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \geq 0$$

$$\sum_i |U_{\alpha i}|^4 \geq \frac{1}{n}$$

معادله اولر - لا لارنر معادله حرکت

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - \frac{1}{2} (m \bar{\psi}^c \psi + \text{H.c.})$$

$$\psi^c = -i \gamma^2 \psi^* \quad \bar{\psi}^c = -i \psi^T \gamma^2 \gamma^0$$

$$\psi^c = -i\gamma^2 \psi^* \quad \bar{\psi}^c = -i\psi^T \gamma^2 \gamma^0$$

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi + \frac{i}{2} (m \psi^T \gamma^2 \gamma^0 \psi + m^* \psi^\dagger \gamma^2 \gamma^0 \psi^*)$$

$$\not{\partial} \psi + m^* \gamma^2 \psi^* = 0$$

$$\psi = \int \frac{d^3 p}{(2\pi)^3} \left(e^{i p \cdot x} u a + e^{-i p \cdot x} v a^\dagger \right)$$

$$a = a^c \quad \swarrow \text{Boost}$$

$$u_s = \begin{bmatrix} \sqrt{p \cdot \sigma} \xi_s \\ \sqrt{p \cdot \bar{\sigma}} \eta_s \end{bmatrix} \quad m^* v_s^* = i\gamma^2 \begin{bmatrix} 0 & p \cdot \sigma \\ p \cdot \bar{\sigma} & 0 \end{bmatrix} u$$

$$m v_s^* = i\gamma^2 \begin{bmatrix} 0 & p \cdot \sigma \\ p \cdot \bar{\sigma} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{p \cdot \sigma} \xi_s^* \\ \sqrt{p \cdot \bar{\sigma}} \eta_s^* \end{bmatrix} = i m \gamma^2 \begin{bmatrix} \sqrt{p \cdot \sigma} \eta_s^* \\ \sqrt{p \cdot \bar{\sigma}} \xi_s^* \end{bmatrix}$$

$$v_s^* = \begin{bmatrix} i\sigma_2 \sqrt{p \cdot \bar{\sigma}} \xi_s^* \\ -i\sigma_2 \sqrt{p \cdot \sigma} \eta_s^* \end{bmatrix} = \begin{bmatrix} \sqrt{p \cdot \sigma} (i\sigma_2 \xi_s^*) \\ -\sqrt{p \cdot \bar{\sigma}} (i\sigma_2 \eta_s^*) \end{bmatrix}$$

$$\xi = \eta \times 2 \quad \text{اسين شغص}$$

$$\psi^c = e^{i\beta} \psi \Rightarrow \eta^* = e^{i\beta} \xi^* \quad \xi^* = e^{i\beta} \eta^*$$

$$\Rightarrow |\eta^*|^2 = |\xi|^2, \quad \eta^* = e^{i\beta} \xi$$

$$\sum_s \xi_s \xi_s^\dagger = 1 \quad \sum_s \eta_s \eta_s^\dagger = 1 \quad \sum_s \xi_s \eta_s^\dagger = e^{i\beta}$$

$$\sum_s \eta_s \xi_s^\dagger = e^{-i\beta}$$

$$u \bar{u} = \begin{bmatrix} m e^{i\beta} & p \cdot \sigma \\ p \cdot \bar{\sigma} & m e^{-i\beta} \end{bmatrix} \quad v \bar{v} = \begin{bmatrix} -m e^{-i\beta} & p \cdot \sigma \\ p \cdot \bar{\sigma} & -m e^{i\beta} \end{bmatrix}$$

$$u v^T = \begin{bmatrix} -i m \sigma_2 & e^{i\beta} p \cdot \sigma i \sigma_2 \\ p \cdot \bar{\sigma} (-i \sigma_2) e^{-i\beta} & i \sigma_2 m \end{bmatrix}$$

$$u v^T = \begin{bmatrix} -i m \sigma_z & e^{i p} p \cdot \sigma i \sigma_z \\ p \cdot \sigma (-i \sigma_z) e^{-i p} & i \sigma_z m \end{bmatrix}$$

$$\langle (\psi \bar{\psi}) \rangle = \frac{\begin{bmatrix} m e^{i \beta} & p \cdot \sigma \\ p \cdot \sigma & m e^{-i \beta} \end{bmatrix}}{p^2 - m^2} \quad \begin{matrix} \psi = \psi^c \\ \bar{\psi} = -\bar{\psi}^c \end{matrix}$$

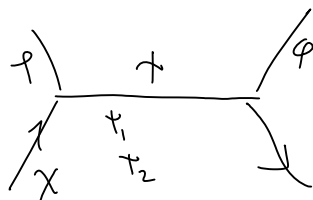
$$\langle T(\psi \psi^T) \rangle = \frac{\begin{bmatrix} m & p \cdot \sigma e^{i \beta} \\ p \cdot \sigma e^{-i \beta} & m \end{bmatrix}}{p^2 - m^2} C \quad \downarrow \quad i \gamma^5$$

$$\begin{aligned} \psi = \psi^c &\rightarrow \beta = 0 \\ \bar{\psi} = -\bar{\psi}^c &\rightarrow \beta = \pi \end{aligned} \quad \mathcal{L} = \bar{\psi} i \not{\partial} \psi - m \frac{\bar{\psi}^c \psi + \text{h.c.}}{2}$$

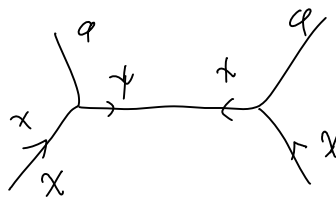
$$\psi^c = C \bar{\psi}^T \quad \psi^T = \pm \psi^c \rightarrow \pm \bar{\psi} C^T$$

آیا فایزهای فیزیکی دارد؟ بله!

$$\underbrace{\varphi \bar{\chi} \psi}_{\text{real}}$$



$$\frac{\bar{\chi} \begin{bmatrix} 0 & p \cdot \sigma \\ p \cdot \sigma & 0 \end{bmatrix} \chi}{p^2 - m^2}$$



$$\frac{\chi^T m C \chi}{p^2 - m^2}$$

می‌توانیم حسب جابجایی و انتگرال بسط دهیم.

$$\longrightarrow \frac{i \not{p}}{p^2}$$

$$\longrightarrow \times \longleftarrow \frac{-i \not{p}}{p^2} i m C \frac{i \not{p}^T}{p^2}$$

$$\rightarrow \times \leftarrow \times \rightarrow \quad \frac{-i \cancel{p}}{p^2} (imc) \frac{i \cancel{p}^T}{p^2} (-imc) \frac{i \cancel{p}}{p^2}$$

⋮

$$\rightarrow \quad \frac{i \cancel{p}}{p^2 - m^2} = \frac{i \cancel{p}}{p^2} + \frac{i \cancel{p}}{p^2} \frac{m^2}{p^2} + \dots$$

$$\rightarrow \times \leftarrow \quad \frac{imc}{p^2 - m^2} = \dots$$

$$\frac{c}{M} \varphi^T \varepsilon \sigma_a \varphi \quad L^T \varepsilon \sigma_a C L$$

$$\varphi = \begin{bmatrix} \varphi^+ \\ \varphi^0 \end{bmatrix} \quad L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}$$

$$(\sigma^a)_{\alpha\beta} (\sigma^a)_{\gamma\delta} = 2 \varepsilon_{\alpha\gamma} \varepsilon_{\beta\delta}$$

$$(\sigma^a)_{\alpha\beta} (\sigma^a)_{\gamma\delta} = \delta_{\alpha\beta} \delta_{\gamma\delta} - (\sigma^a)_{\alpha\gamma} (\sigma^a)_{\beta\delta}$$

$$\sum_a \chi_1^T \sigma_a \chi_2 \quad \chi_3^T \sigma_a \chi_4 = \chi_1^T \chi_2 \quad \chi_3^T \chi_4$$

$$-2 \underbrace{\chi_1^T \varepsilon \chi_3}_{\oplus \chi_3^T \varepsilon \chi_1} \quad \chi_2^T \varepsilon \chi_4$$

$$\chi_1^T = \varphi^T \varepsilon \quad \chi_2 = \varphi \quad \chi_3^T = L^T \varepsilon \quad \chi_4 = C L$$

$$= \frac{c}{M} \left[\underbrace{\varphi^T \varepsilon \varphi}_{\text{singlet}} \underbrace{L^T \varepsilon C L}_{\text{singlet}} + 2 \underbrace{L^T \varepsilon \varphi}_{\text{singlet}} \underbrace{\varphi^T \varepsilon C L}_{\text{singlet}} \right]$$

unitary/gauge 0 $\varphi = \begin{pmatrix} 0 & \frac{H_1 v}{\sqrt{2}} \end{pmatrix}$

همان جبر کدین نوشته بودم.

چند اصطلاح

$P(\nu_\alpha \rightarrow \nu_\beta)$ appearance probability
($\alpha \neq \beta$)

$P(\nu_\alpha \rightarrow \nu_\alpha)$ survival probability

$P(\nu_\alpha \rightarrow \nu_\alpha)$ survival probability

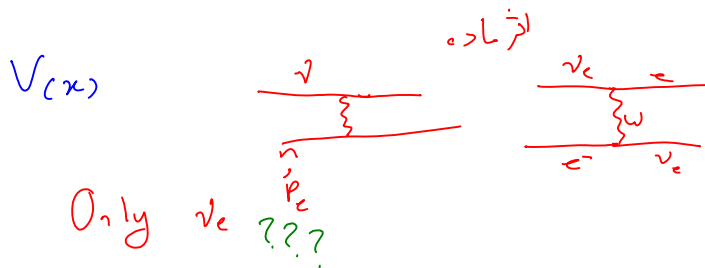
$1 - P(\nu_\alpha \rightarrow \nu_\alpha)$ disappearance probability

پارامتر اول بحث ۴-۴

$$\overline{P(\nu_e \rightarrow \nu_e)} > \frac{1}{n} \quad n=3 \leftarrow SM$$

R. Davis

$$|U_{e1}|^2 = |U_{e2}|^2 = |U_{e3}|^2 = \frac{1}{3}$$



Elastic forward scattering

$$\frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e \quad e^- \gamma^\mu (1 - \gamma_5) \nu_e$$

\Downarrow Fierz transformation
anti-commuting

$$\frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma_\mu (1 - \gamma_5) \nu_e \quad e^- \gamma^\mu (1 - \gamma_5) e$$

* تمرین ۸-۱ جزوه فیزیکی
همان تمرین را برای حالتی که محیط قطبیه است حل کنید.
سبب هفتی بعد صبح حل کنید.

$$\bar{\nu}_L \gamma_\mu \nu_L V_c(x)$$

$$V_c(x) = \sqrt{2} G_F N_e(x)$$

Dispersion relation

معادله پاشنی

$$\mathcal{L} = \bar{\nu}_L \not{\partial} \nu_L - \bar{\nu}_L \gamma_\mu \nu_L V_c(x)$$

$$\mathcal{L} = \bar{\nu}_L \not{\partial} \nu_L - \bar{\nu}_L \gamma_\mu \nu_L V_\mu(x)$$

معادله اولی (لازارز)

$$\not{\partial} \nu_L - V_\mu \gamma_\mu \nu_L = 0$$

$$((E - V_c) \gamma^0 - \vec{p} \cdot \vec{\sigma}) \nu_L = 0$$

$$((E - V_c) \gamma^0 + \vec{p} \cdot \vec{\sigma}) \nu_L = 0$$

$$[(E - V_c)^2 - |\vec{p}|^2] \nu_L = 0$$

$$E = V_c + |\vec{p}| \quad \text{معادله ی پائین}$$

* نشان دهید که در حضور جمله جری (براک - مایورانا)

معادله ی پائین به صورت زیر تبدیل می شود.

$$E = V_c + \sqrt{\vec{p}^2 + m^2}$$

نکته ی - لفظی

Majorana

مایورانا

Jarlskog

بارلسکوف

تقریب فرانسیتی

$$E \approx V_c + |\vec{p}| + \frac{m^2}{2|\vec{p}|}$$

$$i \frac{d}{dt} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \left\{ U \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m_{21}^2}{2E} & 0 \\ 0 & 0 & \frac{\Delta m_{31}^2}{2E} \end{bmatrix} \cdot U^\dagger \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}$$

$$\alpha = \sqrt{2} G_F N_e$$

فاز کلی

تقلیل به آنالیز دو نوترینو

$$\frac{\Delta m_{32}^2}{2E} \gg \frac{\Delta m_{21}^2}{2E}$$

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U_{\text{PMNS}} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$U_{\text{PMNS}} =$$

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \times \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$$

$$\theta_{13} \ll 1$$

$$s_{23} \simeq c_{23} = \frac{\pi}{4}$$

$$\nu_e \simeq c_{12} \nu_1 + \sin \theta_{12} \nu_2$$

$$\nu' = -\sin \theta_{12} \nu_1 + \cos \theta_{12} \nu_2$$

$$i \frac{d}{dt} \begin{bmatrix} \nu_e \\ \nu' \end{bmatrix} = \underbrace{\begin{bmatrix} \sqrt{2} G_F N_e & \frac{\Delta m^2}{4E} \sin 2\theta_{12} \\ \frac{\Delta m^2}{4E} \sin 2\theta_{12} & \frac{\Delta m^2}{2E} \cos 2\theta_{12} \end{bmatrix}}_{H(t)} \begin{bmatrix} \nu_e \\ \nu' \end{bmatrix}$$

mild time dependence \leftrightarrow adiabaticity

$$U_m(t)^\dagger H(t) U_m(t) = \begin{bmatrix} E_1(t) & 0 \\ 0 & E_2(t) \end{bmatrix}$$

$$U_m(t) = \begin{bmatrix} \cos \theta_m(t) & \sin \theta_m(t) \\ -\sin \theta_m(t) & \cos \theta_m(t) \end{bmatrix}$$

$$\begin{bmatrix} \nu_e \\ \nu' \end{bmatrix} = U_m(t) \begin{bmatrix} \nu_{m1} \\ \nu_{m2} \end{bmatrix}$$

$$E_{1,2}(t) = \frac{1}{2} \left(\sqrt{2} G_F N_e(t) + \frac{\Delta m^2}{2E} \cos 2\theta \pm \right)$$

$$\sqrt{\left(\sqrt{2} G_F N_e(t) - \frac{\Delta m^2}{2E} \cos 2\theta\right)^2 + \left(\frac{\Delta m^2}{2E} \sin 2\theta\right)^2}$$

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{2E} \sin 2\theta}{\frac{\Delta m^2}{2E} \cos 2\theta - \sqrt{2} G_F N_e(t)}$$

$$N_e^{\text{res}} \equiv \frac{\Delta m^2 \cos 2\theta}{2E G_F \sqrt{2}} \rightarrow \theta_m = \frac{\pi}{4}$$

$$i \frac{d}{dt} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix} = \left\{ \begin{pmatrix} E_1(t) & 0 \\ 0 & E_2(t) \end{pmatrix} + \begin{pmatrix} 0 & i\dot{\theta}_m \\ -i\dot{\theta}_m & 0 \end{pmatrix} \right\} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}$$

adiabatic condition $|\dot{\theta}_m| \ll |E_2 - E_1|$
 resonance : له خطرناک

منطقه رزنانس

$$\tan 2\theta_m > 1$$

$$\text{پهنای منطقه رزنانس} = \Delta x$$

$$\sqrt{2} G_F \frac{dN_x}{dx} \Delta x = \frac{\Delta m^2}{2E} \sin 2\theta$$

$$N|_{\text{resonance}} = \frac{\Delta m^2 \cos 2\theta}{2E \sqrt{2} G_F}$$

$$\Delta x = \frac{\tan 2\theta}{\frac{1}{N} \frac{dN}{dx}} = \frac{\tan 2\theta}{\frac{d \ln N}{dx}}$$

$$\text{خط آدیاباتیک} \quad \Delta x (E_2 - E_1) \Big|_{\text{resonance}} \gg 1$$

$$\frac{\tan 2\theta}{\frac{d \ln N_e}{dx}} \Big|_{\text{res}} \gg \frac{E}{\Delta m^2 \sin 2\theta}$$

بقای انرژی؟

$$\text{resonance} = \underline{\nu_m}$$

$$\nu_{m1}(t) = T e^{-i \int_0^t E_1(t') dt'} \nu_{m1}(0)$$

$$\nu_{m2}(t) = T e^{-i \int_0^t E_2(t') dt'} \nu_{m2}(0)$$

$$P(\nu_e \rightarrow \nu_e) = \left| \langle \nu_e | T \exp(-i \int_0^t H(t') dt') | \nu_e \rangle \right|^2 =$$

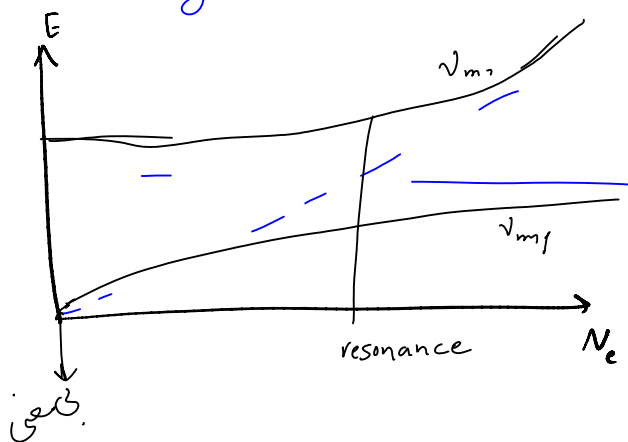
$$\left| \begin{pmatrix} \cos \theta_m(t) & \sin \theta_m(t) \end{pmatrix} \begin{bmatrix} e^{-i \int_0^t E_1(t') dt'} & 0 \\ 0 & e^{-i \int_0^t E_2(t') dt'} \end{bmatrix} \begin{pmatrix} \cos \theta_m(0) \\ \sin \theta_m(0) \end{pmatrix} \right|^2 =$$

$$\overline{P}(\nu_e \rightarrow \nu_e) = \cos^2 \theta_m(t) \cos^2 \theta_m(0) + \sin^2 \theta_m(t) \sin^2 \theta_m(0)$$

MSW

Wolfenstein, 1978

Mikheyev & Smirnov, 1985



$$\overline{P}(\nu_e \rightarrow \nu_e) = (1 - P_{\text{jump}}) (\cos^2 \theta_m(t) \cos^2 \theta_m(0) + \sin^2 \theta_m(t) \sin^2 \theta_m(0)) + P_{\text{jump}} (\sin^2 \theta_m(t) \cos^2 \theta_m(0) + \cos^2 \theta_m(t) \sin^2 \theta_m(0))$$

$$+ \cos^2 \theta_m(t) \sin^2 \theta_m(0))$$

$$\theta_m(t) = \theta \quad \leftarrow \text{خدا}$$

$$\theta_m(0) = \frac{\pi}{2} \quad \leftarrow \text{بسیار غلط}$$

$$\bar{P}(\nu_e \rightarrow \nu_e) \approx \sin^2 \theta + P_{\text{jump}} \cos^2 \theta$$

پیش‌بینی در منطقه شیبی شود:

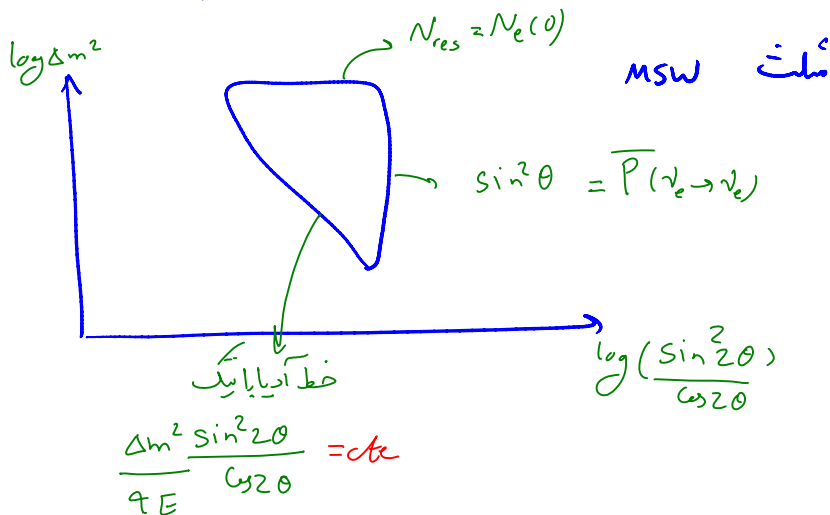
$$N_e = N_e^{\text{res}} + \frac{dN_e}{dx} \quad \leftarrow \text{خطی}$$

$$P_{\text{jump}} = \exp\left(-\frac{\pi}{4} \frac{\Delta m^2 \sin^2 2\theta}{4E \cos 2\theta} \frac{d \ln N_e}{dx} \Big|_{\text{res}}\right)$$

Landau-Zener formula

Landau, 1932

Zener, 1932



پایه فرم‌های

اسکی دیند

$$(\psi_1^T C \psi_2)^T = -\psi_1^T C \psi_2$$

$$= -\psi_2^T C \psi_1$$

symmetric anti-symmetric

$$(m_\nu)_{\alpha\beta} = \frac{\overbrace{(m_\nu)_{\alpha\beta} + (m_\nu)_{\beta\alpha}}^{\nu}}{2} + \frac{\overbrace{(m_\nu)_{\alpha\beta} - (m_\nu)_{\beta\alpha}}^{\bar{\nu}}}{2}$$

$$(M_\nu)_{\alpha\beta} \quad (A_\nu)_{\alpha\beta}$$

$$(m_\nu)_{\alpha\beta} \quad \nu_{L\alpha}^T C \nu_{L\beta} = (M_\nu)_{\alpha\beta} \quad \nu_{L\alpha}^T C \nu_{L\beta}$$

ماتریس جرمی مایورانا متناظر است.

$$m_\nu = U_{PMNS} \begin{bmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{bmatrix} U_{PMNS}^T$$

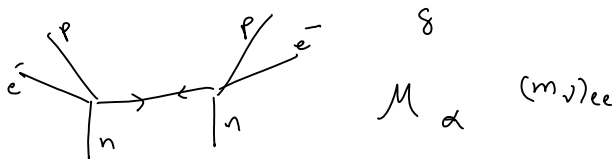
$$U_{PMNS} = V_{23} V_{13} V_{12} \Phi$$

$$V_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \quad V_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{bmatrix}$$

$$V_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \Phi = \text{diag}(1, e^{i\alpha_2}, e^{i\alpha_3})$$

۹

$$\text{Oscillation} \quad \begin{cases} \Delta m_{21}^2, \Delta m_{31}^2 \\ \theta_{12}, \theta_{13}, \theta_{23} \end{cases}$$

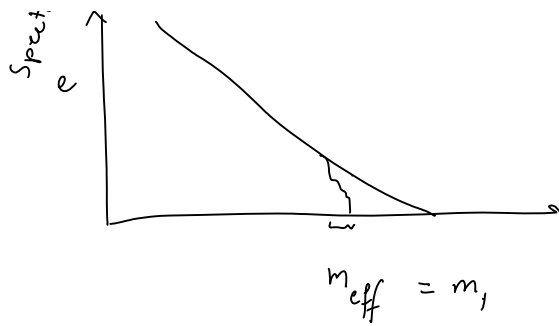


$$= m_1 e^{i\alpha_1} U_{e1}^2 + m_2 e^{i\alpha_2} U_{e2}^2 + m_3 e^{i\alpha_3} U_{e3}^2$$

$$|\mathcal{M}|^2$$

Kuric plot





Katrin $\rightarrow m_1 \rightarrow$; 6

{ Solar neutrino Δm_{12}^2
KamLAND θ_{12}

atmospheric Δm_{31}^2
 $\theta_{23} \approx \frac{\pi}{4}$

ICZK
MINOS

{ CERN - Granassasso $\leftarrow \nu_2$
T2K

Nova

CHOOZ \rightarrow { Daya Bay
Double-CHOOZ
Reno
 $\theta_{13} < 10^\circ$
 $\theta_{13} = ?$
 $\delta = ??$

منابع نوترینو

پُر انرژی $\rightarrow \bar{\nu}_e$ راکتور

پُر $\rightarrow \nu_e$ خدایگی

پُر $\rightarrow \nu_e, \nu_\mu, \nu_\tau$ اتمسفری

ابرنوا

relic neutrino

AGN - GRBs
نوترینوی زمینی

آلکارسازها

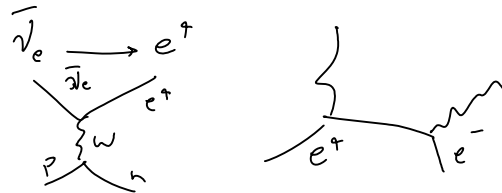
super Kamio kande SK

↑
Kamio kande - IMP
SNO ← D₂O

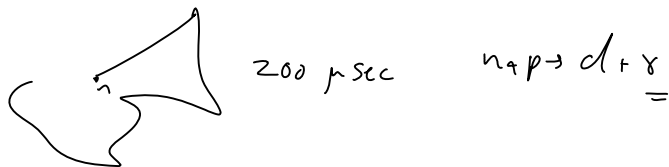
ICECUBE ANITA
BAICAL
ANTARES

فوتونهای خورشیدی را چه گونه آشکارا می کنند؟

~ ~ ~ ~ راتوری ~ ~
KamLAND

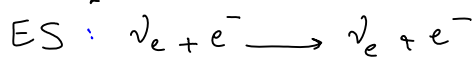


ν̄_e در سوز
پ. در تابش

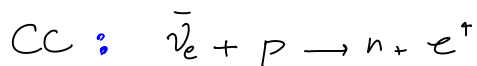
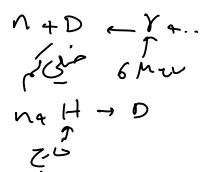
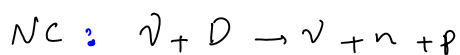


SNO in Canada D₂O

SK متری با



نیاز رام هاراکبشید



Seesaw

type I ??

type II ?

type III ?

Neutrino Oscillation in Three Generation Scheme

$$\Delta m_{21}^2, \Delta m_{31}^2$$

$$\theta_{12}, \theta_{13}, \theta_{23}, \delta$$

$$\Delta m_{31}^2 = \Delta m_{\text{atm}}^2 \simeq 2 \times 10^{-3} \text{ eV}^2$$

$$\Delta m_{21}^2 = \Delta m_{\text{solar}}^2 \simeq 7 \times 10^{-5} \text{ eV}^2$$

Vacuum neutrino oscillation due to Δm_{31}^2

$$\frac{\Delta m_{21}^2}{E} L = 3.6 \times 10^{-2} \frac{\Delta m_{21}^2 / 7 \times 10^{-5} \text{ eV}^2}{E / 1 \text{ GeV}} \frac{L}{100 \text{ km}} \ll 1$$

$$\frac{\Delta m_{31}^2}{E} L = 1.0 \times \frac{\frac{\Delta m_{31}^2}{2 \times 10^{-3} \text{ eV}^2}}{\frac{E}{1 \text{ GeV}}} \frac{L}{100 \text{ km}} \gtrsim 1$$

K2K

KEK to Kamiokande

{ OPERA
ICARUS

MINOS

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| \underbrace{U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2}}_{\delta_{\alpha\beta} U_{\alpha 3}^* U_{\beta 3}} + U_{\alpha 3}^* U_{\beta 3} e^{-i \frac{\Delta m_{31}^2}{2E} t} \right|^2$$

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - 4(1 - |U_{\mu 3}|^2) |U_{\mu 3}|^2 \sin^2\left(\frac{\Delta m_{31}^2}{4E} t\right)$$

$$= 1 - 4(1 - \sin^2 \theta_{23} \cos^2 \theta_{13}) \sin^2 \theta_{23} \cos^2 \theta_{13} \sin^2 \frac{\Delta m_{31}^2 t}{4E}$$

$$\approx 1 - \sin^2 \theta_{23} \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

$$P(\nu_e \rightarrow \nu_e) = 1 - 4 |U_{e3}|^2 |U_{e3}|^2 \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) \\ = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_{31}^2 L}{4E} \approx 1$$

CHOOZ $E \sim \text{few MeV}$ $P(\nu_e \rightarrow \nu_e) \approx 1$
 $L \sim 1 \text{ km}$

$$\sin^2 2\theta_{13} \leq 0.2$$

$$P(\nu_\mu \rightarrow \nu_e) = 4 |U_{\mu 3}|^2 |U_{e3}|^2 \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) \\ = \sin^2 2\theta_{23} \cos^4 \theta_{13} \sin^2 \frac{\Delta m_{31}^2 L}{4E} \\ \approx \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

$$P(\nu_\mu \rightarrow \nu_e) = 4 |U_{e3}|^2 |U_{\mu 3}|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E} \\ \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \frac{\Delta m_{31}^2 L}{4E} \approx 0$$

$\theta_{13} > \Delta m_{31}^2$ زیریندهای صری $\left\{ \begin{array}{l} \nu_e \\ \nu_\mu \rightarrow \nu_e \end{array} \right.$

اثر δ ؟

Vacuum oscillation due to Δm_{21}^2

$$\frac{\Delta m_{31}^2 L}{2E} \gg 1 \quad \leftarrow \text{averaging}$$

$$\frac{\Delta m_{21}^2}{E} L = 7.2 \frac{\frac{\Delta m_{31}^2 / 7 \times 10^{-5} \text{ eV}^2}{E}}{5 \text{ MeV}} \left(\frac{L}{100 \text{ km}} \right) \approx 1$$

$$\frac{\Delta m_{31}^2}{E} L = 2 \times 10^2 \frac{\frac{\Delta m_{31}^2}{2 \times 10^3 \text{ eV}^2}}{\frac{E}{5 \text{ MeV}}} \frac{L}{100 \text{ km}} \gg 1$$

Kam LAND

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| U_{\beta 1} U_{\alpha 1}^* + U_{\beta 2} U_{\alpha 2}^* e^{-i \frac{\Delta m_{21}^2 t}{2E}} \right|^2 + |U_{\beta 3}|^2 |U_{\alpha 3}|^2$$

$$P(\nu_\alpha \rightarrow \nu_\alpha) = (1 - |U_{\alpha 3}|^2)^2 P_{\text{eff}}(\nu_\alpha \rightarrow \nu_\alpha) + |U_{\alpha 3}|^4$$

$$P_{\text{eff}}(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4 \frac{|U_{\alpha 2}|^2 (1 - |U_{\alpha 2}|^2 - |U_{\alpha 3}|^2)}{(1 - |U_{\alpha 3}|^2)^2} \sin^2 \frac{\Delta m_{21}^2 t}{4E}$$

$$S = \cos^4 \theta_{13} S_{\text{eff}}(\theta_{12}, \Delta m_{21}^2) + \sin^4 \theta_{13} \\ \approx \cos 2\theta_{13} S_{\text{eff}}(\theta_{12}, \Delta m_{21}^2)$$

$$S_{\text{eff}}(\theta_{12}, \Delta m_{21}^2) = 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{21}^2 t}{4E}$$

برای $\bar{\nu}_e$ همین

CP violation in neutrino oscillation

$$A_{\alpha\beta}^{\text{CP}} = P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$$

$$A_{\alpha\beta}^{\text{T}} = P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha)$$

$$A_{\alpha\beta}^{\text{CP}} = A_{\alpha\beta}^{\text{T}}$$

$$\text{CP: } U_{\alpha i} \longrightarrow U_{\alpha i}^*$$

$$A_{\alpha\beta}^{\text{T}} = -A_{\beta\alpha}^{\text{T}}$$

$$A_{\alpha\beta}^{\text{CP}} = -A_{\beta\alpha}^{\text{CP}} \implies A_{\alpha\alpha}^{\text{CP}} = 0$$

در ...

$$P(\nu_\alpha \rightarrow \nu_\alpha) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha)$$

$$\sum_{\beta=e,\mu,\tau} P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{\beta=e,\mu,\tau} P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = 1$$

$$A_{e\mu}^{CP} = A_{\tau e}^{CP} \quad A_{\mu\tau}^{CP} = A_{e\mu}^{CP}$$

$$A_{e\mu}^{CP} = A_{\mu\tau}^{CP} = A_{\tau e}^{CP} \equiv A^{CP}$$

8

$$A_{\gamma\beta}^{CP} = P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = -4 \sum_{i < j}$$

$$\text{Im} [U_{\alpha i} U_{\beta i}^* U_{\beta j} U_{\alpha j}^*] \sin \frac{\Delta m_{ij}^2 t}{2E}$$

$$J_{\alpha\beta,ij} = \text{Im} [U_{\alpha i} U_{\beta i}^* U_{\beta j} U_{\alpha j}^*]$$

$$J_{\alpha\beta,ij} = -J_{\beta\alpha,ij} \quad J_{\alpha\beta,ji} = -J_{\alpha\beta,ij}$$

$$J_{\alpha\beta,12} + J_{\alpha\beta,22} + J_{\alpha\beta,32} = J_{\alpha\beta,12} + J_{\alpha\beta,32} = 0$$

$$J_{\alpha\beta,13} = J_{\alpha\beta,32} \quad \text{همین طور}$$

همین طور

$$J_{e\mu,ij} = J_{\mu\tau,ij} = J_{\tau e,ij}$$

Jarlskog پارامتر

$$J \equiv J_{e\mu,12}$$

$$J = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta$$

$$A^{CP} = -4J \left\{ \sin \frac{\Delta m_{12}^2 t}{2E} + \sin \frac{\Delta m_{23}^2 t}{2E} + \sin \frac{\Delta m_{31}^2 t}{2E} \right\}$$

$$\left\{ \sin \frac{\Delta m_{21}^2 L}{2E} \right\}$$

Re-phasing invariant

$$\begin{aligned} \nu_\alpha &\rightarrow U_{\alpha\beta} \nu_\beta \\ L_\alpha &\rightarrow U_{\alpha\mu} L_{L\beta} \end{aligned} \quad \rightarrow \quad \bar{L}_\alpha \gamma^{\frac{1-\gamma_5}{2}} \nu_\alpha$$

$$m_\nu^\dagger m_\nu \rightarrow U^\dagger m_\nu^\dagger m_\nu U$$

$$m_l^\dagger m_l \rightarrow U^\dagger m_l^\dagger m_l U$$

$$\begin{aligned} &\bar{L}_{\alpha\beta} m_l L_{L\beta} \\ &[m_\nu^\dagger m_\nu, m_l^\dagger m_l] \rightarrow U^\dagger [m_\nu^\dagger m_\nu, m_l^\dagger m_l] U \\ &\text{Det}[[m_\nu^\dagger m_\nu, m_l^\dagger m_l]] \quad \leftarrow \begin{array}{l} \text{فازها} \\ \text{تاریخچه} \end{array} \\ &\quad \times \text{تکلیف: به کار گرفتن پارامترهای استاندارد} \\ &\text{Det}[[m_\nu^\dagger m_\nu, m_l^\dagger m_l]] \\ &\quad \leftarrow \text{را محاسبه کنید.} \end{aligned}$$

Observing CP-violation

$$\Delta m_{31}^2 \neq 0 \quad \Delta m_{21}^2 = 0 \quad (1)$$

$$\sin \delta = 0 \quad \text{or} \quad \theta_{12} = 0 \quad (2)$$

no time averaging (3)

Resonant Matter Oscillation

اثر ماده + سه ضلع \rightarrow حل تحلیلی ندارد

$$S = P(\nu_\mu \rightarrow \nu_e) = \cos^4 \theta_{13} S_{\text{or}}(\theta_{13}, \Delta m_{21}^2; a_m) + \sin^4 \theta_{13}$$

$$S = P(\nu_e \rightarrow \nu_e) = \cos^4 \theta_{13} S_{\text{eff}}(\theta_{12}, \Delta m_{21}^2; a_{\text{eff}}) + \sin^4 \theta_{13}$$

$$a_{\text{eff}} \equiv \cos^2 \theta_{13} a(x) = \cos^2 \theta_{13} \sqrt{2} G_F N_e(x)$$

Lim, 1987; Smirnov, 1992; Shi and Schramm 1992

$$\frac{\Delta m_{21}^2}{2E}, \sqrt{2} G_F N_e \ll \frac{\Delta m_{31}^2}{2E}$$

$$H \simeq \text{diag}(0, 0, \frac{\Delta m_{31}^2}{2E})$$

$$\bar{\nu}_e \equiv \cos \theta_{12} \bar{\nu}_1 + \sin \theta_{12} \bar{\nu}_2$$

$$\bar{\nu}_\mu = -\sin \theta_{12} \bar{\nu}_1 + \cos \theta_{12} \bar{\nu}_2$$

$$\xi = \begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_3 \end{pmatrix} = \begin{pmatrix} \cos \theta_{12} \bar{\nu}_1 + \sin \theta_{12} \bar{\nu}_2 \\ -\sin \theta_{12} \bar{\nu}_1 + \cos \theta_{12} \bar{\nu}_2 \\ \bar{\nu}_3 \end{pmatrix}$$

$$\xi = \begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta_{13} \bar{\nu}_e - \sin \theta_{13} e^{-i\delta} \bar{\nu}_\tau \\ \bar{\nu}_\mu \\ \sin \theta_{13} e^{i\delta} \bar{\nu}_e + \cos \theta_{13} \bar{\nu}_\tau \end{pmatrix}$$

$$\bar{\nu}_\mu \equiv \cos \theta_{23} \bar{\nu}_\mu - \sin \theta_{23} \bar{\nu}_\tau \simeq \frac{1}{\sqrt{2}} (\bar{\nu}_\mu - \bar{\nu}_\tau)$$

$$\bar{\nu}_\tau \equiv \sin \theta_{23} \bar{\nu}_\mu + \cos \theta_{23} \bar{\nu}_\tau \simeq \frac{1}{\sqrt{2}} (\bar{\nu}_\mu + \bar{\nu}_\tau)$$

$$\theta_{23} \simeq \frac{\pi}{4}$$

فقط $\bar{\nu}_e$ اشباعه را می بیند.

$$i \frac{d\xi}{dt} = \left\{ V_{12} \begin{pmatrix} 0 & & \\ & \Delta_{12} & \\ & & \Delta_{13} \end{pmatrix} V_{12}^\dagger + V_{13}^\dagger \begin{pmatrix} a(t) & & \\ & 0 & \\ & & 0 \end{pmatrix} V_{13} \right\} \xi$$

$$= \left\{ \begin{pmatrix} \Delta_{12} S_{12}^2 + a C_{13}^2 & \Delta_{12} S_{12} C_{12} & 0 \\ \Delta_{12} S_{12} C_{12} & \Delta_{12} C_{12}^2 & 0 \end{pmatrix} \right\} \xi$$

$$\begin{pmatrix} 0 & 0 & a s_{13} c_{13} \\ 0 & 0 & 0 \\ a s_{13} c_{13} & 0 & 0 \end{pmatrix} \left\{ \right\}$$

$$\Delta_{12} = \frac{\Delta m_{21}^2}{2E} \quad \Delta_{13} = \frac{\Delta m_{31}^2}{2E}$$

$$c_{ij} = \cos \theta_{ij} \quad s_{ij} = \sin \theta_{ij} \quad \frac{\Delta m_{21}^2}{2E}, \sqrt{2} G_F N_e \ll \frac{\Delta m_{31}^2}{2E}$$

$$i \frac{d\xi}{dt} \simeq \begin{pmatrix} \Delta_{12} s_{12}^2 + a c_{13}^2 & \Delta_{12} s_{12} c_{12} & 0 \\ \Delta_{12} s_{12} c_{12} & \Delta_{12} c_{12}^2 & 0 \\ 0 & 0 & \Delta_{13} + a s_{13}^2 \end{pmatrix} \left\{ \right\}$$

$$a_{eff} = \cos^2 \theta_{13} a(t)$$

$$S = \left| c_{13}^2 A_{eff}(\bar{\nu}_e \rightarrow \bar{\nu}_e) + s_{13}^2 e^{-i(\Delta_{13} + a s_{13}^2)t} \right|$$

$$= \cos^2 \theta_{13} S_{eff}(\theta_{12}, \Delta m_{21}^2; a_{eff})$$

$$e^{-i\Delta_{13}t} \rightarrow 0$$

$$\frac{a \sin \theta_{13}}{\frac{\Delta m_{31}^2}{2E}} < 10^{-2} \quad \text{خطای کوچک}$$

$$\pi^+ \rightarrow \mu^+ + \bar{\nu}_\mu$$

$$\mu^+ \rightarrow e^+ + \bar{\nu}_e + \bar{\nu}_\mu$$

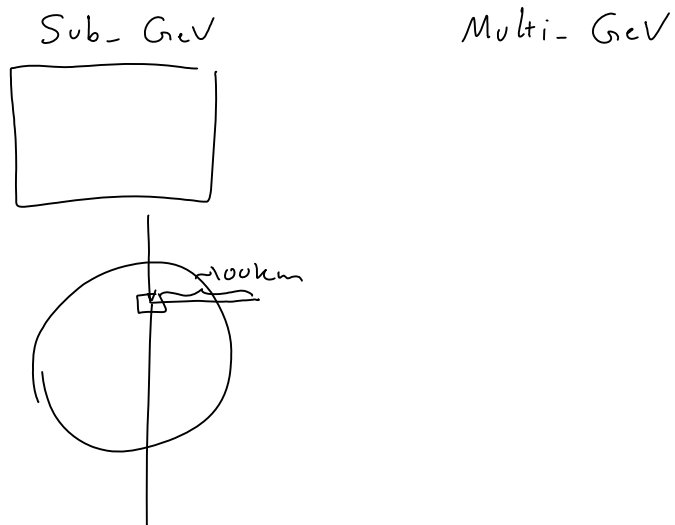
$$R = \frac{(\bar{\nu}_\mu + \bar{\nu}_\mu)_{obs} / (\bar{\nu}_e + \bar{\nu}_e)_{obs}}{(\bar{\nu}_\mu + \bar{\nu}_\mu)_{pred} / (\bar{\nu}_e + \bar{\nu}_e)_{pred}} \sim 0.6$$

$\bar{\nu}_\mu$ کمتر
 له سائیدار باشد
 $\bar{\nu}_e$ حیر
 له سائیدار نیست

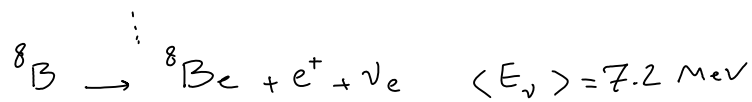
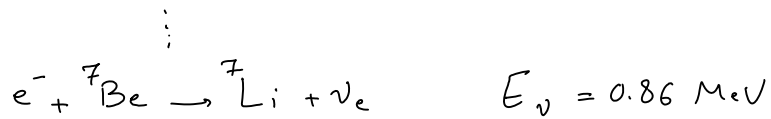
$$\Delta m_{31}^2 = (1.3 - 3.0) \times 10^{-3} \text{ eV}^2$$

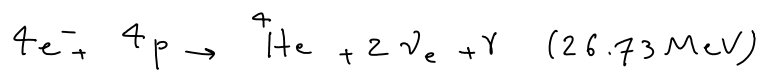
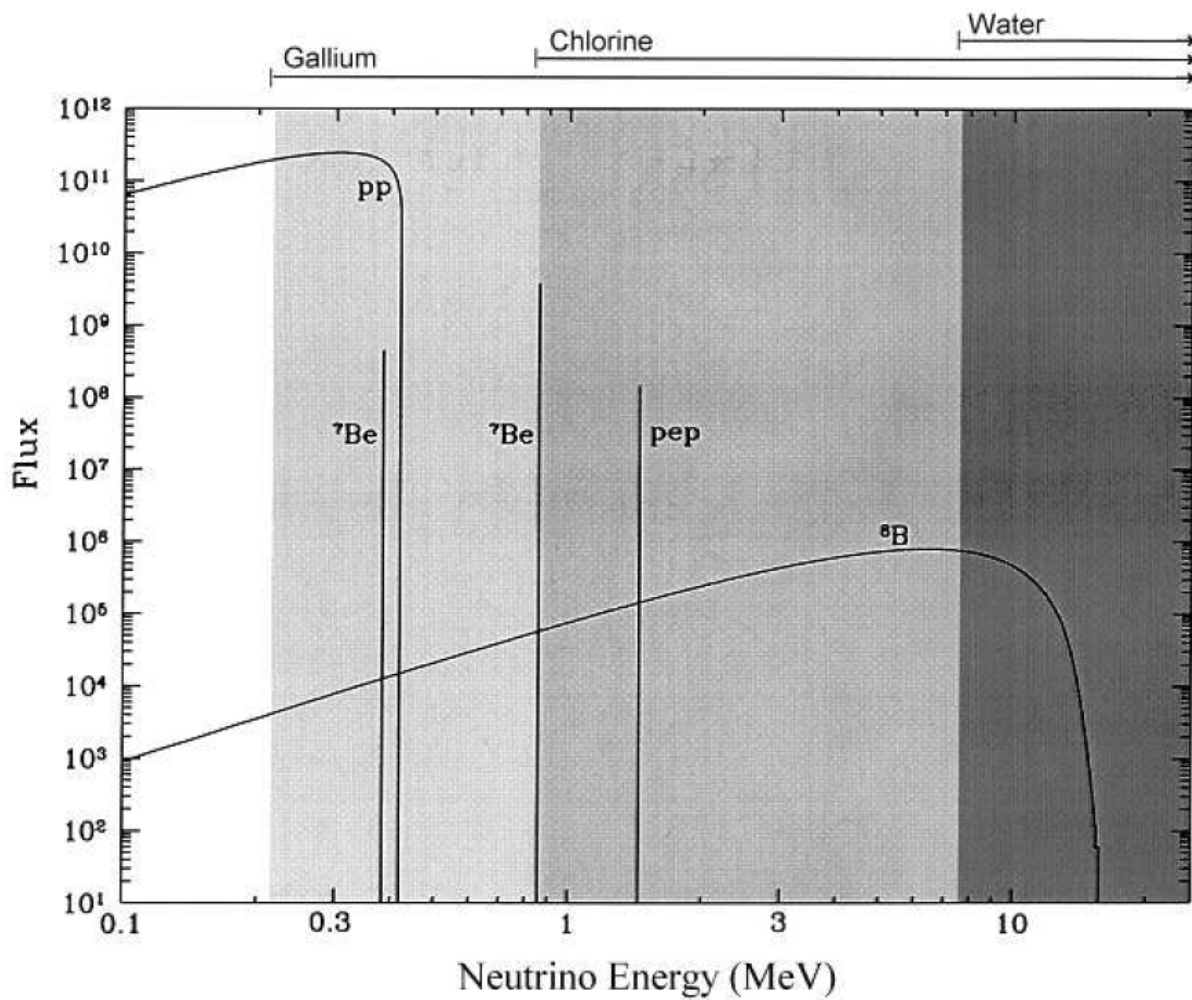
$$\Delta m_{31}^2 = (1.3 - 3.0) \times 10^{-3} \text{ eV}^2$$

$$\sin^2 2\theta_{23} > 0.9$$



Solar neutrino oscillation





R. Davis Cl experiment

C_2Cl_4 Homestake mine

cl experiment



$$E_{th} = 0.81 \text{ MeV}$$

B, Be

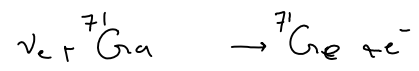
SK



Ga experiment

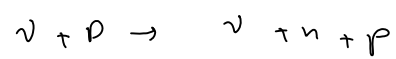
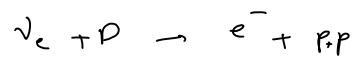
SAGE GALL

Ga experiment SAGE, GNO



pp Be B

SNO

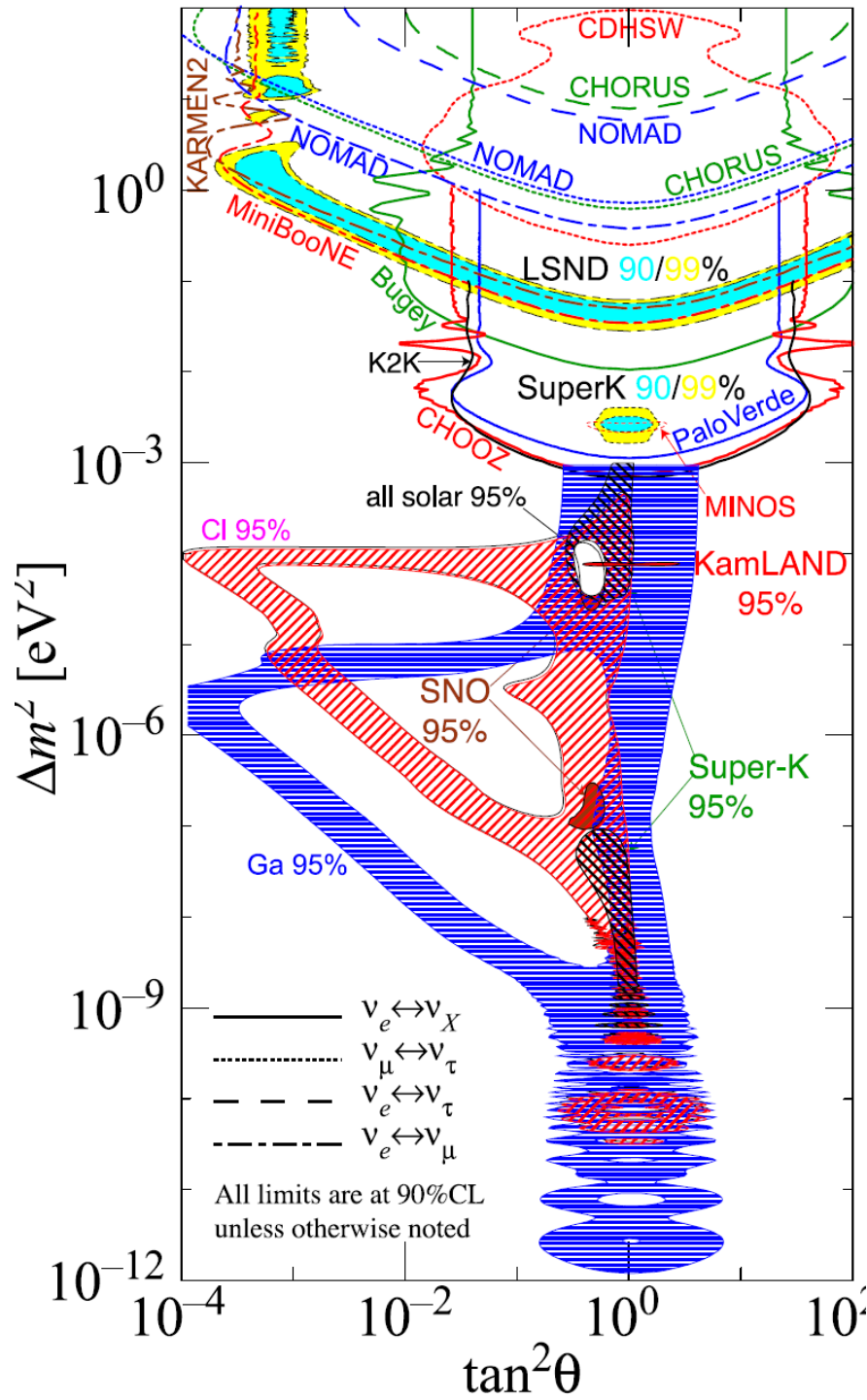


\uparrow
B neutrinos

radio-chemical Cl & Ga experiments

observed / predicted < 1

NC SNO open



$$\mathcal{P}_{SK} = \mathcal{P}_{SSM} \left\{ S + (1-S) R_{\tau} \right\}$$

$$R_{\tau} \sim \frac{1}{2}$$

$$\mathcal{P}_{SNO}(CC) = \mathcal{P}_{SSM} \times S$$

$$\nu_{\mu}, \nu_e$$

$$ES \quad ?$$

$$\mathcal{P}_{SK} \simeq 2.4 \times 10^6 \text{ cm}^{-2} \text{ sec}^{-1}$$

$$\mathcal{P}_{SNO}(CC) \simeq 1.8 \times 10^6 \text{ cm}^{-2}$$

$$\theta_{ssm} = 5.4 \times 10^4 \text{ } \mu\text{m}^{-2} \text{ sec}^{-1}$$

$$\cancel{\nu_e \rightarrow \nu_s}$$

$$\Delta m_{21}^2, \Delta m_{31}^2 \text{ على}$$