Problem Set 1

Deadline: March 10, 2021

February 24, 2021

Problem 1

The familiar Mercator map of of the world is obtained by transforming spherical coordinates θ, ϕ to coordinates x, y given by $x = \frac{W}{2\pi}\phi$, $y = -\frac{W}{2\pi}\ln(\tan\frac{\theta}{2})$. (This was first derived by the English mathematician Edward Wright in 1599.) Show that $ds^2 = \Omega^2(x,y)(dx^2+dy^2)$. Determine Ω.
(Einstein Gravity In A Nutshell, A.Zee, Princeton University Press, 2013)

Problem 2

Consider a $(d+1)$-dimensional Minkowski space ($g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1,1,\dots,1)$). We define a hypersurface in this space by the equation

$$-X_0^2 + X_1^2 + \cdots + X_d^2 = l^2,$$

where l is a parameter with unit of length. for the two following parameterizations obtain the metric on the hypersurface:

1. (τ, ω^i)

$$\begin{aligned} X^0 &= \sinh\tau, \\ X^i &= \omega^i \cosh\tau, \qquad i = 1,\dots,d, \end{aligned}$$

   where $-\infty < \tau < \infty$ and ω^i are the parameters used to define a $(d-1)$-dimension sphere embedded in a d-dimensional Euclidean space.

2. (T, ω^i)

$$\cosh\tau = \frac{1}{\cos T},$$

   where $-\frac{\pi}{2} < T < \frac{\pi}{2}$ and ω^i are the same as the previous case.

(Hint: For further information about ω^i see problem 9 of part 1.6 of Zee's book)