Будет Тервер... Но это неточно...

Прекрасный курс по терверу :)

https://web.stanford.edu/class/archive/cs/cs109/cs109.1166//handouts/overview.html

Комбинаторные конструкции

Перестановки	Размещения	Сочетания
n элементов n клеток	n элементов k клеток	n элементов k клеток
Порядок имеет значение	Порядок имеет значение	Порядок не имеет значения
$P_n = n!$	$A_n^k = \frac{n!}{(n-k)!}$	$C_n^k = \frac{n!}{(n-k)!\cdot k!}$

PIGEON HOLE PRINCIPLE

MAGNUS PREP

pigeonhole

The pigeonhole principle states that if n items are put into m containers, with n > m, then at least one container must contain more than one item.

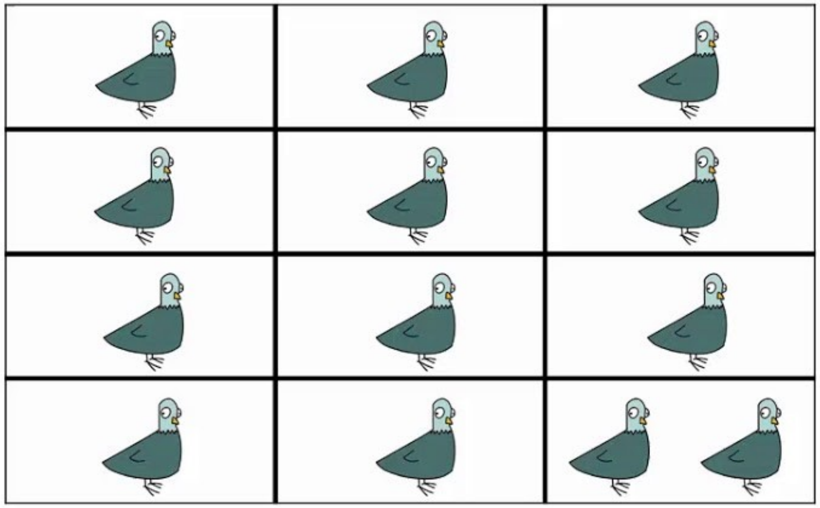
13 pigeons

3 billion.

2 people

highest number

$\geq$1,50,000



The Generalized Pigeonhole Principle

- The Principle

  If n pigeons are allocated into k pigeonholes, then there is at least one pigeonhole which contains at least $\left\lceil \frac{n}{k} \right\rceil$ pigeons.

Вероятность

Event Space and Sample Space

Sample space, S, is set of all possible outcomes of an experiment. For example:

1. Coin flip: S = {Head, Tails}
2. Flipping two coins: S = {(H, H), (H, T), (T, H), (T, T)}
3. Roll of 6-sided die: S = {1, 2, 3, 4, 5, 6}
4. # emails in a day: $S = \{x \mid x \in \mathbf{Z}, x \geq 0\}$ (non-neg. ints)
5. YouTube hrs. in day: $S = \{x \mid x \in \mathbf{R}, 0 \leq x \leq 24\}$

Event Space, E, is some subset of S that we ascribe meaning to. In set notation ($E \subseteq S$).

1. Coin flip is heads: E = {Head}
2. ≥ 1 head on 2 coin flips: E = {(H, H), (H, T), (T, H)}
3. Roll of die is 3 or less: E = {1, 2, 3}
4. # emails in a day ≤ 20: $E = \{x \mid x \in \mathbf{Z}, 0 \leq x \leq 20\}$
5. Wasted day (≥ 5 YT hrs.): $E = \{x \mid x \in \mathbf{R}, 5 \leq x \leq 24\}$

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

Axiom 1: $0 \leq P(E) \leq 1$

Axiom 2: $P(S) = 1$

Axiom 3: If E and F mutually exclusive ($E \cap F = \varnothing$), then $P(E) + P(F) = P(E \cup F)$

Совместные и несовместные события

Independence

Two events E and F are called independent if: $P(EF) = P(E)\,P(F)$. Or, equivalently: $P(E \mid F) = P(E)$. Otherwise, they are called dependent events

Three events E, F, and G independent if:

$P(EFG) = P(E)\,P(F)\,P(G)$, and
$P(EF) = P(E)\,P(F)$, and
$P(EG) = P(E)\,P(G)$, and
$P(FG) = P(F)\,P(G)$

Conditional Independence

Two events E and F are called conditionally independent given G, if
$P(E\,F \mid G) = P(E \mid G)\,P(F \mid G)$
Or, equivalently: $P(E \mid F\,G) = P(E \mid G)$

Условная вероятность

$$P(E|F) = \frac{P(EF)}{P(F)}$$

Here is the general form of the Chain Rule:

$$P(E_1E_2\dots E_n) = P(E_1)P(E_2|E_1)\dots P(E_n|E_1\dots E_{n-1})$$

In the case where the sample space has equally likely outcomes:

$$P(E|F) = \frac{\text{\# outcomes in E consistent with F}}{\text{\# outcomes in S consistent with F}} = \frac{|EF|}{|SF|} = \frac{|EF|}{|F|}$$

approximation of probability we get:

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{\frac{\text{people who liked both}}{\text{people who watched both}}}{\frac{\text{people who liked amelie}}{\text{people who watched amelie}}}$$

Теорема Байеса

Most Common Version:

$$P(F|E) = \frac{P(E|F)P(F)}{P(E)}$$

Expanded Version:

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^c)P(F^c)}$$

Пример (cs109.stanford.edu/demos/naturalBayes.html)

Random Variable and Expectation

- $P(Y=0)=1/8$ (T, T, T)
- $P(Y=1)=3/8$ (H, T, T), (T, H, T), (T, T, H)
- $P(Y=2)=3/8$ (H, H, T), (H, T, H), (T, H, H)
- $P(Y=3)=1/8$ (H, H, H)
- $P(Y\geq 4)=0$

Probability Mass Function

Probability mass functions (PMF) is a function that maps possible outcomes of a random variable to the corresponding probabilities. We can plot PMF graphs:

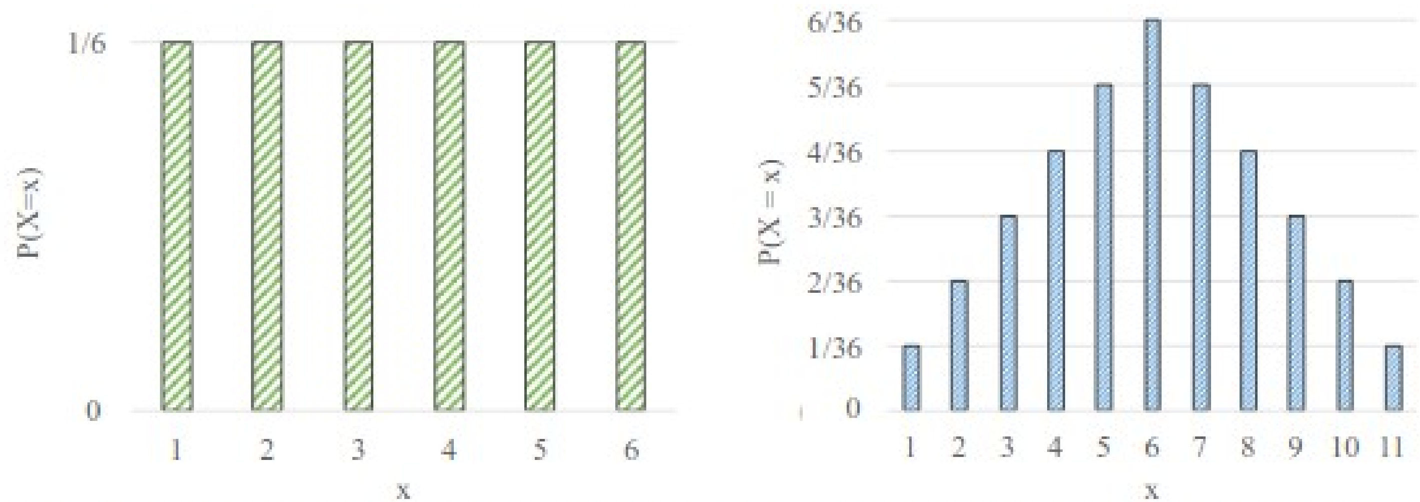


Figure: On the left, the PMF of a single 6 sided die roll. On the right, the PMF of the sum of two dice rolls.

Cumulative Distribution Function

For a random variable X, the Cumulative Distribution Function (CDF) is defined as:

$$F(a) = P(X \leq a) \text{ where } -\infty < a < \infty$$

Expected Value

The Expected Value for a discrete random variable X is defined as:

$$E[X] = \sum_{x:P(x)>0} xP(x)$$

It goes by many other names: Mean, Expectation, Weighted Average, Center of Mass, 1st Moment.

Variance (дисперсия)

Here are some useful identities for variance:

- $Var(aX + b) = a^2 Var(X)$
- Standard deviation is the root of variance: $SD(X) = \sqrt{Var(X)}$

Continuous Random Variables

Cumulative Distribution Function

For a continuous random variable X the Cumulative Distribution Function, written $F(a)$ or as (CDF) is:

$$F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x)dx$$

Expectation and Variance

For continuous RV X:

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)dx$$

$$E[X^n] = \int_{-\infty}^{\infty} x^n f(x)dx$$

For both continuous and discrete RVs:

$$E[aX + b] = aE[X] + b$$

$$Var(X) = E[(X - \mu)^2] = E[X^2] - (E[X])^2$$

Uniform Random Variable

X is a Uniform Random Variable $X \sim Uni(\alpha, \beta)$ if:

$$f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \text{when } \alpha \leq x \leq \beta \\ 0 & \text{otherwise} \end{cases}$$

The key properties of this RV are:

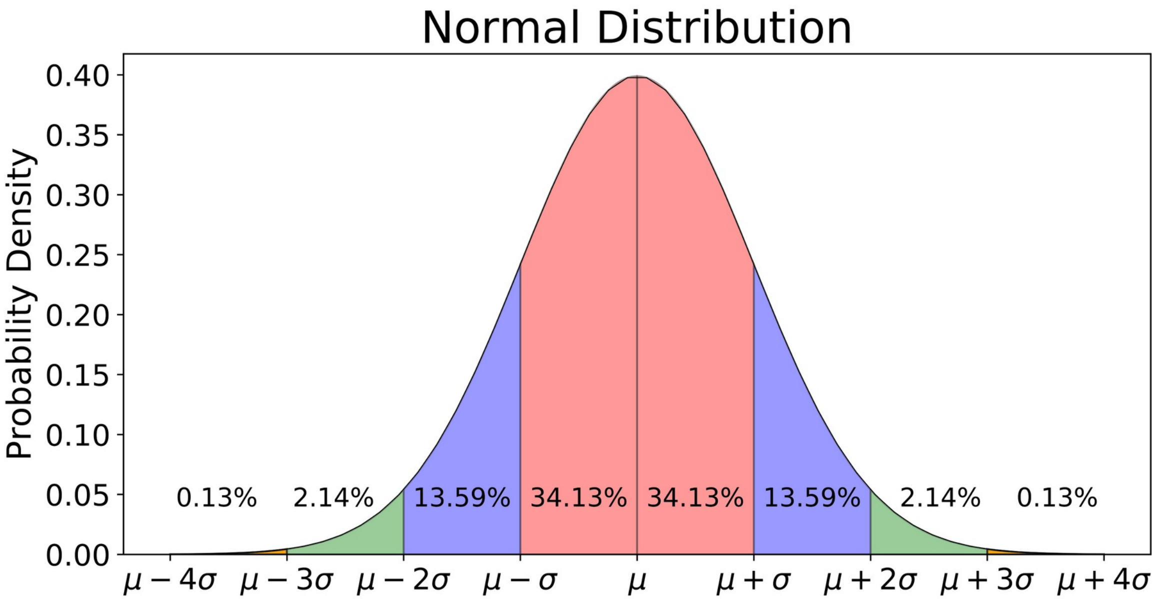
$$P(a \leq X \leq b) = \int_{a}^{b} f(x)dx = \frac{b-a}{\beta - \alpha} \text{ (for } \alpha \leq a \leq b \leq \beta\text{)}$$

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx = \int_{\alpha}^{\beta} \frac{x}{\beta - \alpha}dx = \frac{x^2}{2(\beta - \alpha)}\Bigg|_{\alpha}^{\beta} = \frac{\alpha + \beta}{2}$$

$$Var(X) = \frac{(\beta - \alpha)^2}{12}$$

Normal (aka Gaussian) Random Variable

If X is a normal variable we write $X \sim \mathcal{N}(\mu, \sigma^2)$.



https://www.youtube.com/watch?v=rzFX5NWojp0

Maximizing Likehood

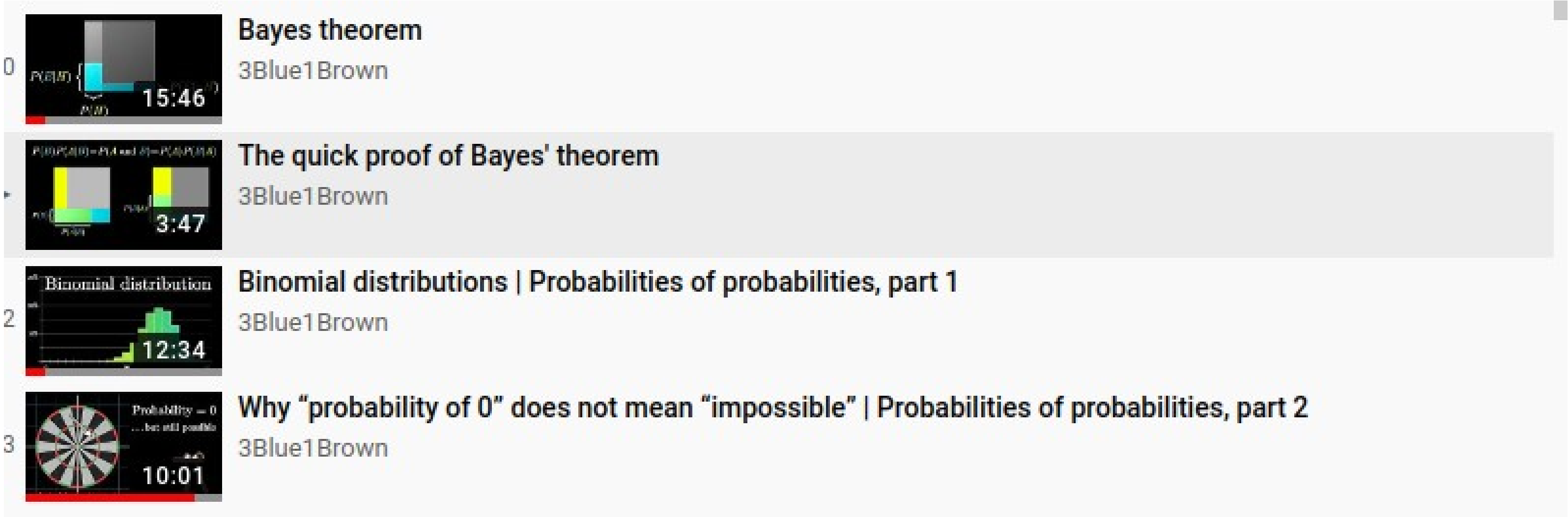
$$L(\theta) = \prod_{i=1}^{n} f(X_i|\theta)$$

$$LL(\theta) = \log \prod_{i=1}^{n} f(X_i|\theta) = \sum_{i=1}^{n} \log f(X_i|\theta)$$

Naive Bayes

https://web.stanford.edu/class/archive/cs/cs109/cs109.1166//pdfs/
39%20NaiveBayes.pdf

Несколько полезных видео к просмотру.



https://www.youtube.com/watch?v=HZGCoVF3YvM&list=WL&index=10

https://www.youtube.com/watch?v=8idr1WZ1A7Q&list=WL&index=12

https://www.youtube.com/watch?v=U_85TaXbeIo&list=WL&index=11

https://www.youtube.com/watch?v=ZA4JkHKZM50&list=WL&index=13