Natural Language Processing MEMMs and CRFs

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- The goal of sequence labeling is to assign tags to words, or more generally, to assign discrete labels to discrete elements in a sequence [Eisenstein, 2018].
- Well known examples of this problem are: part-of-speech tagging (POS) and Named Entity Recognition (NER).
- Maximum-entropy Markov models (MEMMs) make use of log-linear multi-class models for sequence labeling tasks [McCallum et al., 2000].
- In the early NLP literature, logistic regression was often called maximum entropy classification [Eisenstein, 2018].
- Hence, MEMMs will look very similar to the multi-class softmax models seen in the lecture about linear models.
- In contrast to HMMs, here we rely on parameterized functions.

The goal of MEMMs is model the following conditional distribution:

$$P(s_1, s_2 \ldots, s_m | x_1, \ldots, x_m)$$

- Where each x_j for j = 1...m is the j-th input symbol (for example the j-th word in a sentence), and each s_i for j = 1...m is the j-th tag.¹
- We would expect P(DET,NOUN,VERB|the,dog,barks) to be higher than P(VERB,VERB,VERB|the,dog,barks) in a model trained from a POS-tagging training dataset.

¹These slides are based on lecture notes of Michael Collins http://www.cs.columbia.edu/~mcollins/crf.pdf. The notation and terminology has been adapted to be consistent with the rest of the course.

- We use S to denote the set of possible tags.
- We assume that S is a finite set.
- For example, in part-of-speech tagging of English, S would be the set of all
 possible parts of speech in English (noun, verb, determiner, preposition, etc.).
- Given a sequence of words x_1, \ldots, x_m , there are k^m possible part-of-speech sequences s_1, \ldots, s_m , where k = |S| is the number of possible parts of speech.
- We want to estimate a distribution over these k^m possible sequences.

In a first step, MEMMs use the following decomposition (s_0 has always a special tag *):

$$P(s_{1}, s_{2}..., s_{m}|x_{1},..., x_{m}) = \prod_{i=1}^{m} P(s_{i}|s_{1}..., s_{i-1}, x_{1},..., x_{m})$$

$$= \prod_{i=1}^{m} P(s_{i}|s_{i-1}, x_{1},..., x_{m})$$
(1)

- The first equality is exact (it follows by the chain rule of conditional probabilities).
- The second equality follows from an independence assumption, namely that for all i,

$$P(s_i|s_1\ldots,s_{i-1},x_1,\ldots,x_m)=P(s_i|s_{i-1},x_1,\ldots,x_m)$$

- Hence we are making a first order Markov assumption similar to the Markov assumption made in HMMs².
- The tag in the *i*-th position depends only on the tag in the (i-1)-th position.
- Having made these independence assumptions, we then model each term using a multiclass log-linear (softmax) model:

$$P(s_{i}|s_{i-1},x_{1},...,x_{m}) = \frac{\exp(\vec{w} \cdot \vec{\phi}(x_{1},...,x_{m},i,s_{i-1},s_{i}))}{\sum_{s' \in S} \exp(\vec{w} \cdot \vec{\phi}(x_{1},...,x_{m},i,s_{i-1},s'))}$$
(2)

²We actually made a second order Markov assumption in HMMs. MEMMs can also be extended to second order assumptions.

Here $\vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i)$ is a feature vector where:

- x_1, \dots, x_m is the entire sentence being tagged.
- *i* is the position to be tagged (can take any value from 1 to *m*).
- s_{i-1} is the previous tag value (can take any value in S).
- s_i is the new tag value (can take any value in S).

The scope of the feature vector is **restricted** to the whole input sequence x_1, x_m , and only the previous and current tag values. This restriction allows efficient training of both MEMMs and CRFs.

Example of Features used in Part-of-Speech Tagging

- 1. $\vec{\phi}(x_1, \cdots, x_m, i, s_{i-1}, s_i)_{[1]} = 1$ if $s_i = \mathsf{ADVERB}$ and word x_i ends in "-ly"; 0 otherwise. If the weight $\vec{w}_{[1]}$ associated with this feature is large and positive, then this feature is essentially saying that we prefer labelings where words ending in -ly get labeled as ADVERB.
- 2. $\vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i)_{[2]} = 1$ if $i = 1, s_i = VERB$, and $x_m = ?$; 0 otherwise. If the weight $\vec{w}_{[2]}$ associated with this feature is large and positive, then labelings that assign VERB to the first word in a question (e.g., "Is this a sentence beginning with a verb?") are preferred.
- 3. $\vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i)_{[3]} = 1$ if $s_{i-1} = \text{ADJECTIVE}$ and $s_i = \text{NOUN}$; 0 otherwise. Again, a positive weight for this feature means that adjectives tend to be followed by nouns.
- 4. $\vec{\phi}(x_1, \cdots, x_m, i, s_{i-1}, s_i)_{[4]} = 1$ if s_{i-1} = PREPOSITION and s_i = PREPOSITION. A negative weight $\vec{w}_{[4]}$ for this function would mean that prepositions don't tend to follow prepositions.

³Source: https://blog.echen.me/2012/01/03/ introduction-to-conditional-random-fields/

Feature Templates

It is possible to define more general feature templates covering unigrams, bigrams, n-grams of words as well as tag values of s_{i-1} and s_i .

- 1. A word unigram and tag unigram feature template:
 - $\vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i)_{[index(j,z)]} = 1$ if $s_i = \mathsf{TAG}_{[j]}$ and $x_i = \mathsf{WORD}_{[z]}$; 0 otherwise $\forall j, z$.
 - Notice that j is and index spanning all possible tags in S and z is another index spanning the words in the vocabulary V.
- 2. A word bigram and tag bigram feature template:

$$\vec{\phi}(x_1,\cdots,x_m,i,s_{i-1},s_i)_{[index(j,z,u,v)]}=1$$
 if $s_{i-1}=\mathsf{TAG}_{[j]}$ and $s_i=\mathsf{TAG}_{[z]}$ and $x_{i-1}=\mathsf{WORD}_{[u]}$ and $x_i=\mathsf{WORD}_{[v]}$; 0 otherwise $\forall j,z,u,v$.

The function index(j, k, ...) will map each different feature to a unique index in the feature vector.

Notice that the resulting vector will be very high-dimensional and sparse.

Example

$$P(s_i|s_{i-1}, x_1, \dots, x_m) = \frac{\exp(\vec{w} \cdot \vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i))}{\sum_{s' \in S} \exp(\vec{w} \cdot \vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s'))}$$

Example:

1 2 3 4 The dog barks loudly DT NN VB ADV

Let's check that $P(s_4 = \text{ADV}|s_3 = \text{VB}, \text{the,dog,barks,loudly}) > P(s_4 = \text{VB}|s_3 = \text{VB}, \text{the,dog,barks,loudly})$

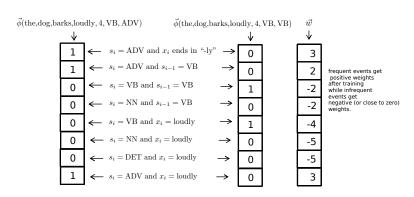
$$P(s_4 = \text{ADV} | s_3 = \text{VB}, \text{the,dog,barks,loudly}) = \frac{\exp(\vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB}, \text{ADV}))}{\sum_{s' \in S} \exp(\vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB}, s'))}$$

$$P(s_4 = \text{VB} | s_3 = \text{VB}, \text{the,dog,barks,loudly}) = \frac{\exp(\vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB}, \text{VB}))}{\sum_{s' \in S} \exp(\vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB}, s'))}$$

Example

This is the same as checking if:

 $\vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB, ADV}) > \vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB, VB})$



$$\begin{split} &\vec{w}\cdot\vec{\phi}(\text{the,dog,barks,loudly},4,\text{VB,ADV}) = 1*3+1*2+1*3=6\\ &>\vec{w}\cdot\vec{\phi}(\text{the,dog,barks,loudly},4,\text{VB,VB}) = 1*-2+1*-4=-6 \end{split}$$

MEMMs and Multi-class Softmax

- Notice that the log-linear model from above is very similar to the multi-class softmax model presented in the lecture about linear models.
- A general log-linear model has the following form:

$$P(y|x; \vec{w}) = \frac{\exp(\vec{w} \cdot \vec{\phi}(x, y))}{\sum_{y' \in Y} \exp(\vec{w} \cdot \vec{\phi}(x, y'))}$$

A multi-class softmax model has the following form:

$$\hat{\vec{y}} = \operatorname{softmax}(\vec{x} \cdot W + \vec{b})
\hat{\vec{y}}_{[i]} = \frac{e^{(\vec{x} \cdot W + \vec{b})_{[i]}}}{\sum_{j} e^{(\vec{x} \cdot W + \vec{b})_{[j]}}}$$
(3)

MEMMs and Multi-class Softmax

- Difference 1: in the log-linear model we have a fixed parameter vector \vec{w} instead of having multiple vectors (one column of W for each class value).
- Difference 2: the feature vector of the log-linear model $\vec{\phi}(x,y)$ includes information of the label y, whereas the input vector \vec{x} of the softmax model is independent of y.
- Log-linear models allow using features that consider the interaction between x and y (e.g., x ends in "ly" and y is an ADVERB).

Training MEMMs

- Once we've defined the feature vectors $\vec{\phi}$, we can train the parameters \vec{w} of the model in the usual way linear models are trained.
- We set the negative log-likelihood as the loss function and optimize parameters using gradient descent from the training examples.
- This is equivalent as using the cross-entropy loss.
- "Any loss consisting of a negative log-likelihood is a cross-entropy between the empirical distribution defined by the training set and the probability distribution defined by model" [Goodfellow et al., 2016].

- The decoding problem is as follows.
- We are given a new test sequence x_1, \ldots, x_m .
- Our goal is to compute the most likely state sequence for this test sequence,

$$\arg\max_{s_1,\ldots,s_m} P(s_1,\ldots,s_m|x_1,\ldots,x_m). \tag{4}$$

- There are k^m possible state sequences, so for any reasonably large sentence length m brute-force search through all the possibilities will not be possible.
- We can use the Viterbi alogrithm in a similar way as used for HMMs.

- The basic data structure in the algorithm will be a dynamic programming table π with entries π[j, s] for j = 1,..., m, and s ∈ S.
- π[j, s] will store the maximum probability for any state sequence ending in state s
 at position j.
- · More formally, our algorithm will compute

$$\pi[j,s] = \max_{s_1,\ldots,s_{j-1}} \left(P(s|s_{j-1},x_1,\ldots,x_m) \prod_{k=1}^{j-1} P(s_k|s_{k-1},x_1,\ldots,x_m) \right)$$

for all j = 1, ..., m, and for all $s \in S$.

The algorithm is as follows:

• Initialization: for $s \in S$

$$\pi[1,s] = P(s|s_0,x_1,\ldots,x_m)$$

where s_0 is a special "initial" state.

• For $j \in \{2, ..., m\}$, $s \in \{1, ..., k\}$

$$\pi[j, s] = \max_{s' \in S} [\pi[j-1, s'] \times P(s|s', x_1, \dots, x_m)]$$

• Finally, having filled in the $\pi[j, s]$ values for all j, s, we can calculate

$$\max_{s_1,\ldots,s_m}=\max_s \ \pi[m,s].$$

- The algorithm runs in $O(mk^2)$ time (i.e., linear in the sequence length m, and quadratic in the number of tags k).
- As in the Viterbi algorithm for HMMs, we can compute the highest-scoring sequence using backpointers in the dynamic programming algorithm.

Comparison between MEMMs and HMMs

- So what is the motivation for using MEMMs instead of HMMs?
- Note that the Viterbi decoding algorithms for the two models are very similar.
- In MEMMs, the probability associated with each state transition s_{i-1} to s_i is

$$P(s_i|s_{i-1},x_1,...,x_m) = \frac{\exp(\vec{w} \cdot \vec{\phi}(x_1,...,x_m,i,s_{i-1},s_i))}{\sum_{s' \in S} \exp(\vec{w} \cdot \vec{\phi}(x_1,...,x_m,i,s_{i-1},s'))}$$

In HMMs, the probability associated with each transition is:

$$P(s_i|s_{i-1},x_1,\ldots,x_m) = P(s_1|s_{i-1})P(x_i|s_i)$$

Comparison between MEMMs and HMMs

- The key advantage of MEMMs is that the use of feature vectors $\vec{\phi}$ allows much richer representations than those used in HMMs.
- For example, the transition probability can be sensitive to any word in the input sequence x₁,..., x_m.
- In addition, it is very easy to introduce features that are sensitive to spelling features (e.g., prefixes or suffixes) of the current word x_i, or of the surrounding words.
- These features are useful in many NLP applications, and are difficult to incorporate within HMMs in a clean way.

- We now turn to Conditional Random Fields (CRFs) [Lafferty et al., 2001].
- Notation: for convenience, we'll use $x_{1:m}$ to refer to an input sequence x_1, \ldots, x_m , and $s_{1:m}$ to refer to a sequence of tags s_1, \ldots, s_m .
- The set of all possible tags is again S.
- The set of all possible tag sequences is S^m .
- In conditional random fields we'll again build a model of

$$P(s_1,...,s_m|x_1,...,x_m) = P(s_{1:m}|x_{1:m})$$

 A first key idea in CRFs will be to define a feature vector that maps an entire input sequence x_{1:m} paired with an entire tag sequence s_{1:m} to some d-dimensional feature vector:

$$\vec{\Phi}(x_{1:m}, s_{1:m}) \in \mathcal{R}^d$$

- We'll soon give a concrete definition for $\vec{\Phi}$.
- For now just assume that some definition exists.
- We will often refer to $\vec{\Phi}$ as being a "global" feature vector.
- It is global in the sense that it takes the entire state sequence into account.

In CRFs we build a giant log-linear model:

$$P(s_{1:m}|x_{1:m}; \vec{w}) = \frac{\exp(\vec{w} \cdot \vec{\Phi}(x_{1:m}, s_{1:m}))}{\sum_{s'_{1:m} \in S^m} \exp(\vec{w} \cdot \vec{\Phi}(x_{1:m}, s'_{1:m}))}$$

- This is "just" another log-linear model, but it is "giant".
- The space of possible values for s_{1:m} is huge S^m.
- The normalization constant (denominator in the above expression) involves a sum over all possible tag sequences S^m.
- These issues might seem to cause severe computational problems.
- Under appropriate assumptions we can train and decode efficiently with this type of model.

• We define the global feature vector $\vec{\Phi}(x_{1:m}, s_{1:m})$ as follows:

$$\vec{\Phi}(x_{1:m}, s_{1:m}) = \sum_{j=1}^{m} \vec{\phi}(x_{1:m}, j, s_{j-1}, s_j)$$

where $\vec{\phi}(x_{1:m}, j, s_{j-1}, s_j)$ are the same as the feature vectors used in MEMMs.

- Example: $\vec{\Phi}(\text{[the,dog,barks]}, \text{DET,NOUN,VERB]}) = \vec{\phi}(\text{[the,dog,barks]}, 1, *, \text{DET}) + \vec{\phi}(\text{[the,dog,barks]}, 2, \text{DET, NOUN}) + \vec{\phi}(\text{[the,dog,barks]}, 3, \text{NOUN, VERB})$
- Essentially, we are adding up many sparse vectors.

Example

x_i =will $\land y_i = NN$				
y_{i-1} =START $\wedge y_i = NN$				
x_i =will $\land y_i = MD$				
y_{i-1} =START $\wedge y_i = MD$				
$x_i=to \land y_i = TO$				
$y_{i-1}=NN \wedge y_i = TO$				
$y_{i-1}=MD \land y_i = TO$				
x_i =fight $^ y_i = VB$				
V: 1=TO ∧ V: = VB				

will φ(x, 1, y ₁ , y ₀)	tο φ(x, 2, y ₂ , y ₁)	fight _(x, 3, y3, y2)	Φ(x, NN TO VB)
1	0	0	1
1	0	0	1
0	0	0	0
0	0	0	0
0	1	0	1
0	1	0	1
0	0	0	0
0	0	1	1
0	0	1	1

⁴source:

http://people.ischool.berkeley.edu/~dbamman/
nlpF18/slides/12_neural_sequence_labeling.pdf

• We are assuming that for any dimension of $\vec{\Phi}_{[k]}, k=1,\ldots,d$, the k'th global feature is:

$$\vec{\Phi}(x_{1:m}, s_{1:m})_{[k]} = \sum_{j=1}^{m} \vec{\phi}(x_{1:m}, j, s_{j-1}, s_j)_{[k]}$$

- Thus $\vec{\Phi}(x_{1:m}, s_{1:m})_{[k]}$ is calculated by summing the "local" feature vector $\vec{\phi}(x_{1:m}, j, s_{j-1}, s_j)_{[k]}$ over the m different tag transitions in s_1, \ldots, s_m .
- We would expect each local vector to encode relevant information about the tag transition by turning on some vector dimensions (setting the value to one).
- We now turn to two critical practical issues in CRFs: first, decoding, and second, parameter estimation (training).

Decoding with CRFs

- The decoding problem in CRFs is as follows.
- For a given input sequence x_{1:m} = x₁, x₂,..., x_m, we would like to find the most likely underlying state sequence under the model, that is,

$$arg \max_{s_{1:m} \in S^{m}} P(s_{1:m}|x_{1:m}; \vec{w}) = arg \max_{s_{1:m} \in S^{m}} \frac{\exp(\vec{w} \cdot \vec{\Phi}(x_{1:m}, s_{1:m}))}{\sum_{s'_{1:m} \in S^{m}} \exp(\vec{w} \cdot \vec{\Phi}(x_{1:m}, s'_{1:m}))}$$

$$= arg \max_{s_{1:m} \in S^{m}} \exp(\vec{w} \cdot \vec{\Phi}(x_{1:m}, s_{1:m}))$$

$$= arg \max_{s_{1:m} \in S^{m}} \vec{w} \cdot \vec{\Phi}(x_{1:m}, s_{1:m})$$

$$= arg \max_{s_{1:m} \in S^{m}} \vec{w} \cdot \sum_{j=1}^{m} \vec{\phi}(x_{1:m}, j, s_{j-1}, s_{j})$$

$$= arg \max_{s_{1:m} \in S^{m}} \sum_{j=1}^{m} \vec{w} \cdot \vec{\phi}(x_{1:m}, j, s_{j-1}, s_{j})$$

Decoding with CRFs

 We have shown that finding the most likely sequence under the model is equivalent to finding the sequence that maximizes:

$$arg \max_{s_{1:m} \in S^m} \sum_{j=1}^m \vec{w} \cdot \vec{\phi}(x_{1:m}, j, s_{j-1}, s_j)$$

- This problem has a clear intuition. Each transition from tag s_{j-1} to tag s_j has an associated score: w̄ · φ̄(x_{1:m}, j, s_{i-1}, s_j)
- This score could be positive or negative.
- Intuitively, this score will be relatively high if the state transition is plausible, relatively low if this transition is implausible.
- The decoding problem is to find an entire sequence of states such that the sum
 of transition scores is maximized.
- We can again solve this problem using a variant of the Viterbi algorithm, in a very similar way to the decoding algorithm for HMMs or MEMMs.

Parameter Estimation in CRFs (training)

- For parameter estimation, we assume we have a set of n labeled examples, $\{(x_{1:m}^i, s_{1:m}^i)\}_{i=1}^n$. Each $x_{1:m}^i$ is an input sequence x_1^i, \ldots, x_m^i each $s_{1:m}^i$ is a tag sequence s_1^i, \ldots, s_m^i .
- We again set the negative log-likelihood (or cross-entropy) as the loss function L
 as optimize parameters using gradient descent.
- The main challenge here is that gradient calculations $\frac{\partial L}{\partial \vec{w}_{[k]}}$ involve summing over S^m (a very large set containing all possible tag sequences).
- This sum can be computed efficiently using the Forward-backward algorithm⁵.
- This is another dynamic programming algorithm that is closely related to the Viterbi algorithm.

⁵http://www.cs.columbia.edu/~mcollins/fb.pdf

CRFs and MEMMs

- CRFs and MEMMS are discriminative sequence labeling models: they model the conditional probability directly via a parameterized log-linear multi-class function (softmax).
- HMMs, on the other hand, are generative models.
- In MEMM the normalization (denominator of the softmax) is local: it happens at each tag step (the sum runs over all possible tag values S).
- In CRFs the normalization is global: the sum runs over all possible tag sequences S^m.
- Training a MEMM is quite easy: just train a multi-class log-linear model for for a given word to the label. This classifier is used at each word step to predict the whole sequence.
- Training CRF is more complex. The objective is to maximize the log probability of the most likely sequence.

CRFs and MEMMs: the label bias problem

- MEMMs end up making up decision at each time step independently.
- This leads to a problem called label bias: in some tag space configurations, MEMMs essentially completely ignore important aspects of the context.
- Example: The right POS labeling of sentence "will to fight" (la voluntad de pelear) is "NN TO VB". 6
- Here NN stands for "noun", TO stands for "infinitive to", and VB stands for "verb base form".
- Modals (MD) show up much more frequently at the start of the sentence than nouns do (e.g., questions).
- Hence, tag "MD" will receive a higher score than tag "NN" when x_0 ="will" : $P(s_1 = MD|s_0 = *, x_1 = \text{`will''}, ...) > P(s_1 = NN|s_{i-1} = *, x_1 = \text{`will''}).$
- But we know that MD + TO is very rare: "... can to eat", "... would to eat".

CRFs and MEMMs: the label bias problem

- The word "to" is relatively deterministic (almost always has tag TO) so it doesn't
 matter what tag precedes it.
- Because of the local normalization of MEMMs, $P(s_i = TO | s_{i-1}, x_1, \dots, x_i = \text{"to"}, \dots, x_n)$ will always be 1 when $x_i = \text{"to"}$ regardless of the value of s_{i-1} (MD or NN).
- That means our prediction for "to" can't help us disambiguate "will".
- We lose the information that MD + TO sequences rarely happen.
- As a consequence: a MEMMS would likely label the first word to "MD".
- CRF overcomes this issue by doing a global normalization: it considers the score
 of the whole sequence before normalizing to make it a probability distribution.

Links

- http://people.ischool.berkeley.edu/~dbamman/nlpF18/slides/ 11_memm_crf.pdf
- http://people.ischool.berkeley.edu/~dbamman/nlpF18/slides/ 12_neural_sequence_labeling.pdf
- https://www.depends-on-the-definition.com/ sequence-tagging-lstm-crf/
- https://www.quora.com/
 What-are-the-pros-and-cons-of-these-three-sequence-models-MaxEnt
- https:
 //people.cs.umass.edu/~mccallum/papers/crf-tutorial.pdf
- http://www.davidsbatista.net/blog/2017/11/13/Conditional_ Random_Fields

Questions?

Thanks for your Attention!

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