Modeling and Optimizing Household Energy Consumption

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Contents

1 Abstract

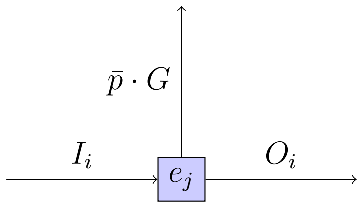
Across the world, within corporations and general public is the issue of minimizing energy consumption. As there are various types of energy that are consumed, there are various prices that fluctuate with the market associated with the costs of energy. These costs can be problematic for people and some corporations as there are plenty of other costs of living to allocate money for. Saving money is everyone's goal, thus our mathematical model will help a city determine what age group is using the most energy and how the energy consumption for each age group will change over the years.

2 Introduction

The definition of energy usage is very ambiguous as most might simply associated it with electric energy usage. However, the physical realm that we exist in energy usage can be defined to be anything that involves the exchange of heat. Thus when considering energy usage in a city or residential community, we don't have to exclusively consider a form of energy over another.
There are many factors that go into a households energy consumption for any given frame of time. For one, the energy usage to heat and cool a home, maintain a home's homeostasis, will consume energy relative to the area the house is located, the time of year, and the severity of weather. Households usually consist of numerous appliances that are useful for day to day function but also appliances that aren't necessities but commodities. Depending on how old someone is can play a factor in determining how many necessary and unnecessary appliances one owns. Thus it is important to study energy consumption on an age class basis. Also, when going the lengths to model energy consumption in a city/residential community, it is worth considering items that are disjoint from household energy consumers such as automobiles. This could help in modeling pollution and climate change but also aide in figuring out energy consumption overall as a whole. Thus we also have to consider the number of gas and electric powered cars in a given age class as different generations might tend to prefer gas or electric cars. Another major contributor to a area's energy consumption is of course the number of people in the area. More specifically, the number of people in each age class in a given area so we can apply what was previously stated. It is also important to see population trends for this reason as well. So when considering a population model that is continuous, the Beverton-Holt Model jumps out. There is always inflow and outflow within a city as people come and go and also people being born and dying. Thus we can consider inflow and outflow to be model by periodic functions as sometimes there might be a lot of people moving and other tomes not so much. A big factor in population dynamics and population retention is the density factor as humans like space and privacy. Thus if a cities population gets too high people might tend to move. Thus population can be viewed as a balance scale essentially hovering around equilibrium.

Schematic Diagram

As we are concerned with energy consumption, in classification of age groups, we will omit the population aged 0-9 and anyone older than 80 years. Thus the classifications of age groups is as follows: 1:[10-19], 2:[20-29], 3:[30-39], 4:[40-49], 5:[50-59], 6:[60-69], 7:[70-79]. The population of our city will be model by the inflow and outflow of the city regarding different age classes. (See Below)



3 Parameter Table

Parameter	Description	Value
$\bar{p}$	Probability of general population moving to city	$\frac{1}{19495}$(19495 number of cities in the US)
p_j for $j = 1, \ldots, 7$	Probability of age class j moving to city	(See Appendix)
k	Capacity of city	1000000
G	General Population	$331900000 - k$ (331900000 people in the US)
m_j	Population dependent departure rate for age class j	(See Appendix)
K_{c_j}	Proportion of people in age class j that own an electric car	(See Appendix)
K_{g_j}	Proportion of people in age class j that own a gas powered car	$1 - K_{c_j}$
υ_C	Monthly energy usage of an electric car (avg)	600kW
υ_G	Monthly energy usage of a gas powered car (avg)	450kW
Λ_j	Appliances own by age class j (avg)	(See Appendix)
E_{Λ_j}	Monthly energy usage based off appliances owned for age class j	(See Appendix)
μ_{I_j}	Mean inflow rate for age class j	(See appendix)
μ_{O_j}	Mean outflow rate for age class j	(See appendix)
b	Number of months for simulation	36
ω	Frequency	$\frac{2\pi}{b}$
$\beta(t)$	Seasonal Energy Consumption	(See Appendix)
e_j	Population of age class j	(varies)
η_j	Energy consumption of age class j	(varies)

Table 1: Parameter Table

4 Model

Note that the subscript j here is still denoting the age classes that were outlined in *Schematic Diagram*. Energy consumption of the city directly relates to the population of the city and hence the energy consumption of each age class depends on the population of each age class in the city.

$$I_j(t) = \mu_{I_j} + \sin(\omega \cdot t) \tag{1}$$

$$O_j(t) = \mu_{O_j} + \cos(\omega \cdot t) \tag{2}$$

$$\frac{de_j}{dt} = \bar{p} \cdot G + p_j \cdot (I_j - O_j) + (1 - \frac{e_j(t)}{k}) - (\frac{m_j \cdot e_j(t)}{(1 + \frac{e_j(t)}{k})}) \tag{3}$$

$$\eta_j(t + \Delta t) \approx \eta_j(t) + \Delta t \cdot [\beta(t) \cdot e_j(t) + K_{c_j} \cdot \upsilon_C + K_{g_j} \cdot \upsilon_G + E_{\Lambda_j}] \tag{4}$$

The derivation and rationale for Eq(3) is the common biological continuous population Beverton-Holt Model. As the population increases in a city, many people might feel that it is time to move to a new and smaller city, hence m_j which will decrease the population faster alluding to the fact just stated. The discrete $\eta_j(t)$ equation is used determine energy usage of each age class which is simply the sum of the terms impacting energy usage outlined in *Table 1: Parameter Table.*

5 Results

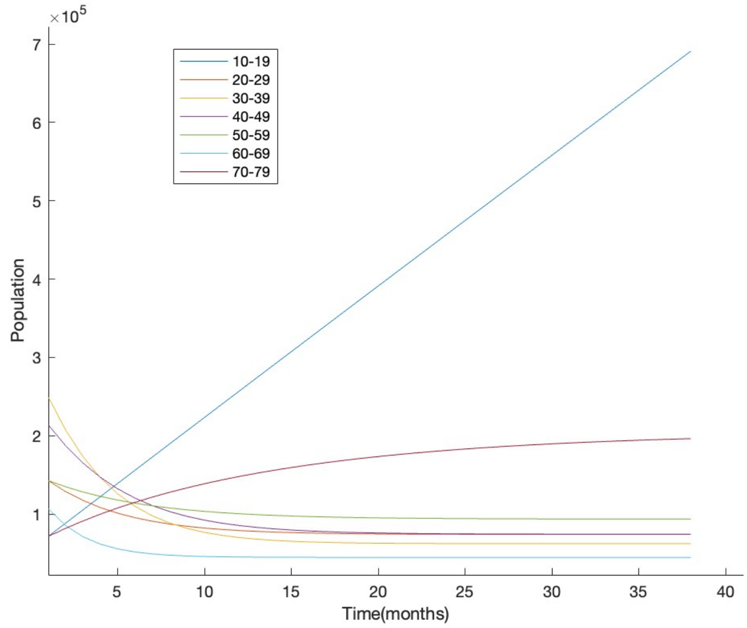
Upon iterating through the model for 36 months the following results were obtained.

From Figure 1(a), we see that the population of the age class 10-19 grows linearly at a slope around 1. While this result is odd, it most likely has to do with inadequate data and thus parameters for the equations. However, it is not surprising that the population of the arbitrary city for that age class is increasing as college students and younger people are more likely to change living situations compared to older people who are more settled down. This can also be see by the fact that the age class 20-29 also is the second highest among the 36 months.

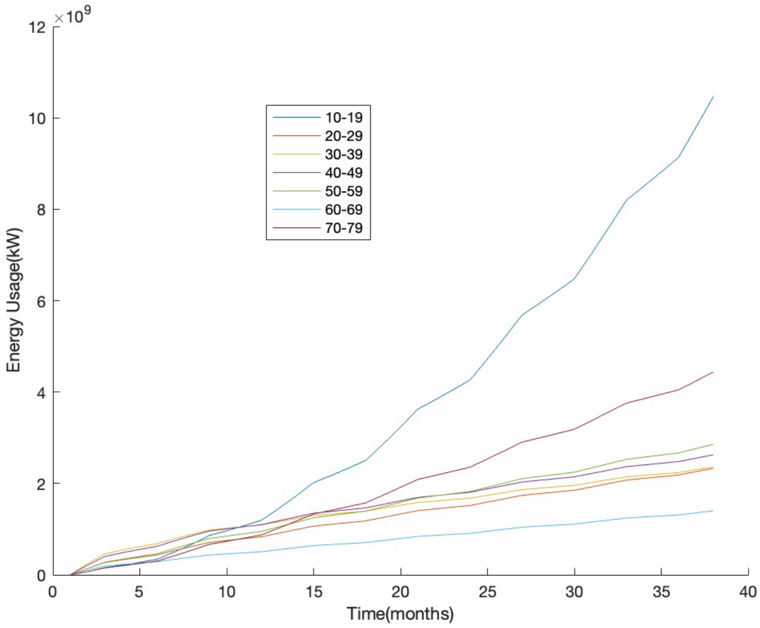
As for Figure 1(b), the energy consumption of each age class, we again see a general increase in energy consumption. Notably, the graphs that are similar in Figure 1(a) correlate to similar graphs to each other in Figure 1(b). We see that following suit with the highest population increase over the three years, the age class 10-19 would consume the most energy in the time period. This again can perhaps be justified by the fact that younger people are prone to buy electric vehicles which consume more energy than gas powered cars and also perhaps watch more television and have more electronic devices. Again, this could all be validated with precise data for parameter fitting.

6 Conclusion

Building off the observations from *Results*, we can conclude that the younger populations are more susceptible to increased energy usage. This could be from the fact that technological literacy tends towards younger aged people and technology consistently uses a decent amount of energy. Also note, that this means that the cities population will tend to get younger which is good for the economy but perhaps the city should attempt to improve catering towards older people as a balance in age classes is important for stability and function. But to cut back on energy consumption of younger age classes, they could employ some more outdoor recreation in the city such as parks and fields which would encourage people to spend more time outdoors. Even just more recreation not necessarily outdoors could suffice. This would reduce the residents energy bill (population retention) as well as the cities energy bill.



((a)) Change in Population of each age class j over 36 months



((b)) Energy Consumption of each age class j over 36 months

Figure 1: Combined Figure

7 Limitations & Improvements

Limitations	**Improvements**
- The model fails to use real data thus biases in parameter values can be present.	- The model includes population dependent energy consumption which is key and supports findings.
- There could be numerous factors omitted from the model.	- The age class distinction in the model allows for more precision in determining solutions.
- The consistency of the parameters is not realistic, stochastics could be introduced to support uncertainty in energy consumption in different months and seasons.	- The model provides appropriate tendencies for certain populations such as younger people tending more towards electric car ownership and moving cities.

8 Future Works

Further directions of this model include incorporating real data for parameter fitting and bifurcation analysis. Finding stability of the model is also something that could be explored. Overall just finding better numbers for parameters will enhance model accuracy and function. Expanding on more energy usage factors would be of interest as well.

9 Appendix

Here we will formally define the values from the parameter table that were used to perform the simulation:

First we will define the number of people in each age class. For convenience, denote the number of people in age class j by ppl_j Thus, $ppl_1 = \frac{1}{14}k, ppl_2 = \frac{2}{14}k, ppl_3 = \frac{3.5}{14}k, ppl_4 = \frac{3}{14}k, ppl_5\frac{2}{14}, ppl_6\frac{1.5}{14}k, ppl_7 = \frac{1}{14}k$. Most of the population in cities tends towards the middle age classes thus the reasoning behind calculating as such. Real population data for a city could of course be used as these are good for initial conditions.

Probability of age class j moving to city: $p_1 = 0.1, p_2 = 0.25, p_3 = 0.3, p_4 = 0.25, p_5 = 0.2, p_6 = 0.4, p_7 = 0.1$ (younger people and retirement age people more likely to move).

Population dependent departure rate for age class j: $m_1 = 0.001, m_2 = 0.25, m_3 = 0.3, m_4 = 0.25.m_5 = 0.2, m_6 = 0.4, m_7 = 0.1$ (relates somewhat to p_j).

Proportion of people in age class j that own an electric car: $K_{c_1} = 0.25, K_{c_2} = 0.6, K_{c_3} = 0.55, K_{c_4} = 0.5, K_{c_5} = 0.38, K_{c_6} = 0.2, K_{c_7} = 0.1$.

Appliances own by age class j (avg): $\Lambda_1 = 5, \Lambda_2 = 13, \Lambda_3 = 17, \Lambda_4 = 22, \Lambda_5 = 19, \Lambda_6 = 12, \Lambda_7 = 10$.

The monthly energy usage based off appliances owned for age class j can be calculated by

$$E_{\Lambda_j} = \begin{cases} 175, \text{ if } 0 \leq \Lambda_j \leq 6 \\ 325, \text{ if } 6 < \Lambda_j \leq 12 \\ 650, \text{ if } 12 < \Lambda \leq 18 \\ 1000, \text{ if } 18 < \Lambda \end{cases}$$

Mean inflow rate for age class j: $\mu_{I_1} = 357.1429, \mu_{I_2} = 1142.9, \mu_{I_3} = 2250, \mu_{I_4} = 1500, \mu_{I_5} = 857.1429, \mu_{I_6} = 535.7143, \mu_{I_7} = 357.1429$.

Mean outflow rate for age class j: $\mu_{O_1} = 71.4286, \mu_{O_2} = 142.8571, \mu_{O_3} = 250, \mu_{O_4} = 214.2857, \mu_{O_5} = 142.8571, \mu_{O_6} = 107.1429, \mu_{71.4286}$.

Seasonal energy consumption defined by $\beta(t)$. Note we will say that January corresponds to 1 such that if it is January and t denotes the time in months, $t \equiv \pmod{12}$. This follows up to November denoted by 11 and December denoted by 12. Thus it follows that if i denotes the number representing a month, that if it is that month then the remainder when taking modulo 12 is the number that represents the month. Thus we will define the function as:

$$\beta(t) = \begin{cases} 1000, \text{ if } t mod\ 12 \ \in \{0, 1, 2, 6, 7, 8\} \\ 500, \text{ if } t mod\ 12 \ \in \{3, 4, 5, 9, 10, 11\} \end{cases}$$

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Hence, there is higher and equal energy consumption in the winter as in the summer and lower in the fall and spring months.

10 References

MatLab code below:

```
% 1-Jan, 2-Feb, 3-Mar, 4-Apr, 5-May, 6-June, 7-July, 8-Aug, 9-Sept, 10-Oct,
% 11-Nov, 0-Dec

%age classes: (10-19),(20-29),(30-39),(40-49),(50-59),(60-69),(70-79)

%SIMULATION IS IN MONTHLY INCREMENTS

clear; clc;
format long;

pbar = 1/19495; %prob of general population moving to city (1/number of cities in US)

%prob's of age class j moving to city
p(1) = 0.1;
p(2) = 0.25;
p(3) = 0.3;
p(4) = 0.25;
p(5) = 0.2;
p(6) = 0.4;
p(7) = 0.1;

k = 1000000; %capacity of city (negligable)

%number of people in each age class
ppl(1) = (1/14)*k;
ppl(2) = (2/14)*k;
ppl(3) = (3.5/14)*k;
ppl(4) = (3/14)*k;
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```
ppl(5) = (2/14)*k;
ppl(6) = (1.5/14)*k;
ppl(7) = (1/14)*k;

G = 331900000-k; %pool of people in the general population (minus
    the 'city')

%population dependent departure rate of age class j
m(1) = 0.001;
m(2) = 0.25;
m(3) = 0.3;
m(4) = 0.25;
m(5) = 0.2;
m(6) = 0.4;
m(7) = 0.1;

%proportion of people in age class j that own an electric car
ec(1) = (0.25*ppl(1))/ppl(1);
ec(2) = (0.6*ppl(2))/ppl(2);
ec(3) = (0.55*ppl(3))/ppl(3);
ec(4) = (0.5*ppl(4))/ppl(4);
ec(5) = (0.38*ppl(5))/ppl(5);
ec(6) = (0.2*ppl(6))/ppl(6);
ec(7) = (0.1*ppl(7))/ppl(7);

%proportion of people in age class j that own a gas powered car
gc(1) = 1-ec(1);
gc(2) = 1-ec(2);
gc(3) = 1-ec(3);
gc(4) = 1-ec(4);
gc(5) = 1-ec(5);
gc(6) = 1-ec(6);
gc(7) = 1-ec(7);

eC = 600; %energy usage of electric car
gC = 450; %energy usage of gas car

%appliances owned by age class j (avg)
apl(1) = 5;
apl(2) = 13;
apl(3) = 17;
apl(4) = 22;
apl(5) = 19;
apl(6) = 12;
apl(7) = 10;

%mean inflow/outflow rate for age class j
Ia(1) = 0.005*ppl(1);
Ia(2) = 0.008*ppl(2);
Ia(3) = 0.009*ppl(3);
Ia(4) = 0.007*ppl(4);
```

```
Ia(5) = 0.006*ppl(5);
Ia(6) = 0.005*ppl(6);
Ia(7) = 0.005*ppl(7);
Oa(1) = 0.001*ppl(1);
Oa(2) = 0.001*ppl(2);
Oa(3) = 0.001*ppl(3);
Oa(4) = 0.001*ppl(4);
Oa(5) = 0.001*ppl(5);
Oa(6) = 0.001*ppl(6);
Oa(7) = 0.001*ppl(7);

for j=1:7
    if apl(j)>=0 && apl(j)<=6
        E_a(j) = 175;
    end
    if apl(j)>6 && apl(j)<=12
        E_a(j) = 325;
    end
    if apl(j)>12 && apl(j)<=18
        E_a(j) = 650;
    end
    if apl(j)>18
        E_a(j) = 1000;
    end
end

a = 0;
b = 36; %three years in months
h = 1; %one month

t = length(a:h:b);

w = (2*pi)/b;

e(1,1) = ppl(1);
e(2,1) = ppl(2);
e(3,1) = ppl(3);
e(4,1) = ppl(4);
e(5,1) = ppl(5);
e(6,1) = ppl(6);
e(7,1) = ppl(7);

eta(1,1) = 1000;
eta(2,1) = 3000;
eta(3,1) = 4000;
eta(4,1) = 5000;
eta(5,1) = 4500;
eta(6,1) = 3000;
eta(7,1) = 2000;

for j=1:7
```

```
    for i=1:t
        I(j,i)= Ia(j)+sin(w*i);
        O(j,i) = Oa(j)+cos(w*i);
        e(j,i+1) = e(j,i)+h*(pbar*G + p(j)*(I(j,i)-O(j,i))+(1-(e(j,i)/k))-(m(j)*e(j,i)/(1+(e(j,i)/k))));
        if ismember(mod(i,12), [0,1,2])
            beta = 1000; %energy consumption in winter
            eta(j,i+1) = eta(j,i)+h*(beta*e(j,i)+ec(j)*eC + gc(j)*gC + E_a(j));
        end
        if ismember(mod(i,12), [3,4,5])
            beta = 500; %energy consumption in spring
            eta(j,i+1) = eta(j,i)+h*(beta*e(j,i)+ec(j)*eC + gc(j)*gC + E_a(j));
        end
        if ismember(mod(i,12), [6,7,8])
            beta = 1000; %energy consumption in summer
            eta(j,i+1) = eta(j,i)+h*(beta*e(j,i)+ec(j)*eC + gc(j)*gC + E_a(j));
        end
        if ismember(mod(i,12), [9,10,11])
            beta = 500; %energy consumption in fall
            eta(j,i+1) = eta(j,i)+h*(beta*e(j,i)+ec(j)*eC + gc(j)*gC + E_a(j));
        end
    end
end

figure(1);
hold on;
for j=1:7
    plot(eta(j,:));
end

legend('10-19','20-29','30-39','40-49','50-59','60-69','70-79');
xlabel('Time(months)');
ylabel('Energy Usage(kW)')
hold off;

figure(2);
hold on;
for j=1:7
    plot(e(j,:));
end

legend('10-19','20-29','30-39','40-49','50-59','60-69','70-79');
xlabel('Time(months)');
ylabel('Population')
hold off;
```

Listing 1: Energy.m