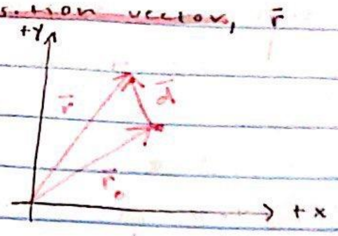
Lecture #3 - 1/14/19 - Motion

2-D Motion

- Position vector, $\bar{r}$



initial position: $\bar{r}_0$

final position: $\bar{r}_f$, $\bar{r}$

- vector for one instant in time

- displacement vector, $\bar{d}$: vector from starting position to the final position
  - $\bar{r}_0 + \bar{d} = \bar{r}$
  - $\bar{d} = \bar{r} - \bar{r}_0$

    $= \Delta\bar{r}$

★ SI units: meters

- Average velocity, $\bar{v}_{avg}$: exists during some time interval, not an instantaneous vector

  $\bar{v}_{avg} = \frac{\bar{r} - \bar{r}_0}{\Delta t}$

  - to define an axis, we need:
    1. origin
    2. positive direction

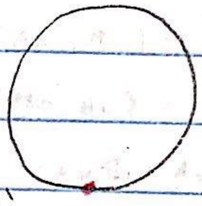
  - Ex: $\bar{r}_0 = (2, 0, 0)$

    $\bar{r} = (5, 0, 0)$

    $\bar{v}_{avg} = \frac{(3, 0, 0)}{3} = (1, 0, 0)\ \frac{m}{s}$

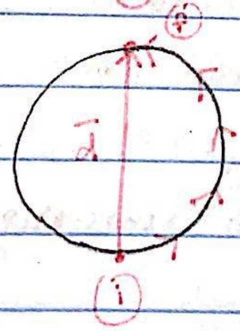
  - Ex (more difficult):

    Uniform circular motion: speed is constant over time

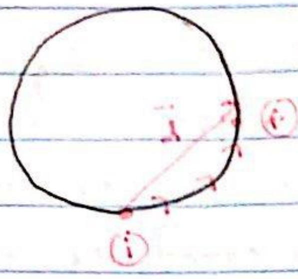


initial point ⇒ (i)

- full circle
  - $\bar{r} - \bar{r}_0 = 0$
  - $\bar{v}_{avg} = 0$



- half circle
  - direction & magnitude of $\bar{v}_{avg}$ seem to correlate more with actual.



- quarter circle:
  - even closer to actual $\bar{v}_{avg}$

- As we shrink time interval, $\bar{v}_{avg}$ correlates exactly with what exactly was happening at that instant of time

· **Instantaneous velocity**, $\bar{v}$: velocity as time interval shrinks to 0

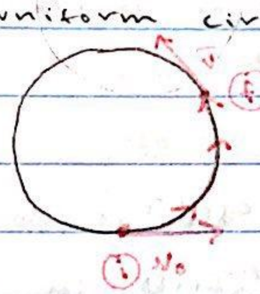
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\vec{r} - \vec{r}_0}{\Delta t} \quad m/s = \frac{d\vec{r}}{dt}$$ ← time derivative

★ direction is tangent to the trajectory

· **Average acceleration**, $\bar{a}_{avg}$

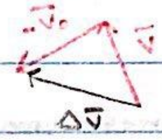
$$a_{avg} = \frac{\bar{v} - \bar{v}_0}{\Delta t}$$

- numerator is vectors ($\bar{v}$, $\bar{v}_0$ are generally not the same vector)
- uniform circular motion (constant speed)



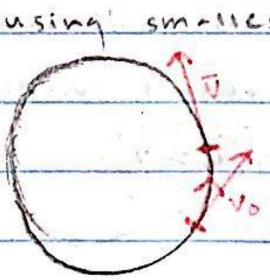
- $\bar{v}$ and $\bar{v}_0$ are the same magnitude, but have different directions. they are not the same

- so, resultant vector is:

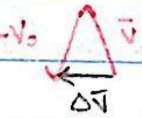


- direction is in direction of $\Delta\vec{v}$

• using smaller time interval



- smaller segment means $\Delta\vec{v}$ goes to 0
- resultant $\Delta\vec{v}$:



★ a_{avg} during any arc of circle will be pointing towards center of the circle

- Instantaneous acceleration:

$\bar{a} = \lim_{\Delta t \to 0} \frac{\Delta \bar{v}}{\Delta t} = \frac{d\bar{v}}{dt}$

- SI units: $\frac{m}{s^2}$

1-D Motion with Constant Acceleration

- a_x = constant

$= \frac{dV_x}{dt}$

- numerator: small change in velocity
- denominator: small change in time
- think of a_x as a fraction, not a derivative

$dV_x = a_x dt$

$\int_{V_{ox}}^{V_x} dV_x = \int_0^t a_x dt'$

- V_{ox}: x-component of initial velocity
- V_x: x-component of final velocity

- $\boxed{V_x = V_{ox} + a_x t}$

$= \frac{dx}{dt}$

- $\int_{x_0}^{x} dx' = \int_0^t (V_{ox} + a_x t')\, dt'$

↳ $x = x_0 + V_{ox} t + \frac{1}{2} a_x t^2$

- $\boxed{V_x^2 = V_{ox}^2 + 2a_x(x - x_0)}$

- this eq. is convenient b/c we don't have time

- these equations are only valid in 1-D motion which has constant acceleration

· Free fall:

- neglect air resistance (we do not consider it)
- acceleration is downward at a rate of $9.8 \frac{m}{s^2} = g$
  - number does vary, but not in Physics 1
  - number represents any object on/near Earth's surface
- object is released from rest: $\boxed{\bar{V}_0 = 0}$

** on exams, $g = 10 \frac{m}{s^2}$

** all constants are non-negative in physics 1

- depends on your choice of axis

↑ +y

$a_y = -10$

↳ g-component of acceleration would still be positive

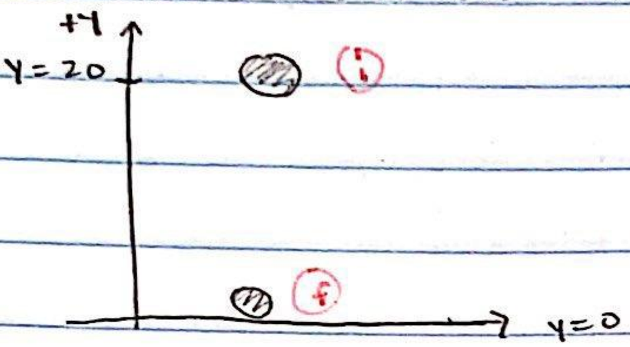
· **Strategy (1-D Motion w/ constant acceleration):**

① picture, axis, label initial & final

② list (x_0, x, v_{0x}, v_x, a_x, t)

③ choose equation / plug in

- **Ex:** rock released from rest 20 m above ground



$y_0 = 20$

$y = 0$

$v_{0y} = 0$

$v_y =$

$a_y = -10$

$t =$

a) time to hit ground?

want t, don't know/care v_y, so we use:

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$0 = 20 + 0 + \frac{1}{2}(-10) t^2$$

$$t^2 = 4 \rightarrow t = \pm 2$$

$$\boxed{t = 2\,s}$$

b) speed