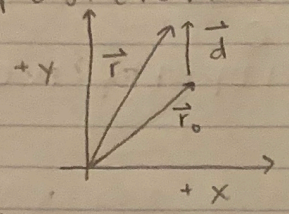
1/14

Lecture 3
Study of Motion

- position vector, $\vec{r}$



$\vec{r}_i$ $\boxed{\vec{r}_0}$ position vector @ $t = 0$

$\vec{r}_f$ $\boxed{\vec{r}}$ position vector @ a later time

- displacement vector, $\vec{d}$: vector from starting position to final position ($\vec{d} = \vec{r} - \vec{r}_0$) (in m)
  $= \Delta r$
- average velocity vector, $\vec{v}_{avg} = \frac{\vec{r} - \vec{r}_0}{\Delta t} \left(\frac{m}{sec}\right)$

ex: $\vec{r}_0 = (2, 0, 0)$
$\vec{r} = (5, 0, 0)$
$t = 3$ sec

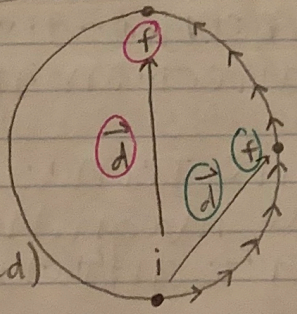
$\vec{v}_{avg} = \frac{(5,0,0) - (2,0,0)}{3}$

$= \frac{(3,0,0)}{3}$

$\boxed{\vec{v}_{avg} = (1, 0, 0)}$

Uniform circular Motion

✓ something is the same as time goes by (ie going @ constant speed)



full circle
$\vec{r} - \vec{r}_0 = 0$
$\therefore \vec{v}_{avg} = 0$

half circle

quarter circle

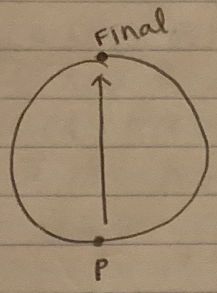
* $\vec{v}_{avg}$ must be same direction as $\vec{d}$

instantaneous velocity vector, $\vec{v} = \lim_{\Delta t \to 0} \frac{\vec{r} - \vec{r}_0}{\Delta t} = \frac{d\vec{r}}{dt}$
* tangent to trajectory in uniform circular motion
$\left(\frac{m}{sec}\right)$

- average acceleration vector, $\vec{a}_{avg} = \frac{\vec{v} - \vec{v}_0}{\Delta t}$
  * direction of $\vec{a}_{avg}$ is same as numerator
  * smaller the segment, the smaller the resultant vector gets

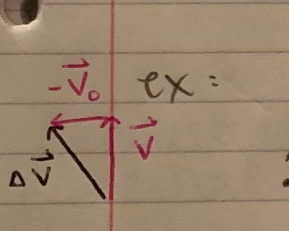
- instantaneous acceleration, $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$ $\left(\frac{m}{sec^2}\right)$

ex:



starts @ P
1.5 laps in 60 sec
$\vec{V}_{avg} = \uparrow$

ex:



$(V_B > V_A > 0)$
as object moves it speeds up
quarter circle
$\vec{a}_{avg} = \uparrow$

1-D MOTION w/ constant ACCELERATION

- a_x = constant
- $a_x = \frac{dv_x}{dt}$ ① $\int_{V_{0x}}^{V_x} dv_x = \int_0^t a_x\, dt'$

② $V_x = V_{0x} + a_x t = \frac{dx}{dt}$

③ $\int_{x_0}^{x} dx = \int_0^t (V_{0x} + a_x t)\, dt'$

↑ x component of initial velocity

* $x = x_0 + V_{0x}t + \frac{1}{2}a_x t^2$
* $V_x^2 = V_{0x}^2 + 2a_x(x - x_0)$
* $V_x = V_{0x} + a_x t$

} only valid in 1-D motion w/ constant acceleration

- free fall (neglect air resistance): acceleration is downward @ a rate of $9.8\ m/sec^2$ ← use on hw/ sections
  * on exams, use $g = 10\ m/sec^2$

Strategy

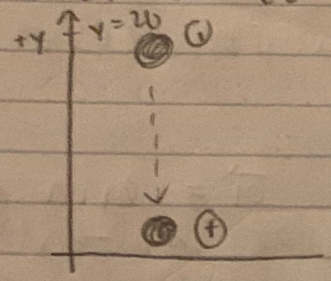
① picture, axis, (i) (f)
② list
③ choose equation/plugin

list (6 different quantities)

$x_0 =$
$x =$
$v_{ox} =$
$v =$
$a_x =$
$t =$

ex: Rock released from rest 20 m above ground:

a) time to hit ground?



$x_0 = 20$
$x = 0$
$v_{ox} = 0$
$v_x = ?$ (solve in pt B)
$a_x = -10$
$t = ?$

$$x = x_0 + v_{ox}t + \frac{1}{2}a_x t^2$$

$$0 = 20 + \frac{1}{2}(-10)t^2$$

$$-20 = -5t^2$$

$$t^2 = 4$$

$$\boxed{t = 2 \text{ sec}}$$

b) speed @ impact?

$$v_x = v_{ox} + a_x t$$

$$v_x = (-10)(2)$$

$$\boxed{v_x = -20 \text{ m/sec}}$$