

Team Project: Octave Band Filtering

ECE 6530: Digital Signal Processing

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4 Bandpass Filter Design

- a) Generate a bandpass filter that will pass a frequency component at $\omega = 0.4\pi$. Make the filter length $L = 40$ equal to 40. Make a plot of the frequency response magnitude and phase.
- b) Use the plot of the frequency response for the length-40 band-pass filter from part (a), and determine the passband width using the 0.5 level to define the pass band.
- c) Make two other plots of BPFs for L D 20 and L D 80 with the same

4.1 Simple Bandpass Filter Design

4.2 A Better Bandpass Filter Design

5 Detecting Octave Bands

5.1 Piano Octaves

This bit was taken care of in Matlab and this is the function to set up the filter bands table. The results are below and a copy of the code is included in appendix A: Since we have a limit

val [units]	O_1	O_2	O_3	O_4	O_5	O_6	O_7	O_8
Lower (Hz)	32.703	65.406	130.81	261.63	523.25	1046.5	2093	4186
Lower (Rad)	0.025685	0.05137	0.10274	0.20548	0.41096	0.82192	1.6438	3.2877
Upper (Hz)	61.735	123.47	246.94	493.88	987.77	1975.5	3951.1	7902.1
Upper (Rad)	0.048487	0.096974	0.19395	0.3879	0.77579	1.5516	3.1032	6.2063
Center (Hz)	47.219	94.439	188.88	377.75	755.51	1511	3022	6044.1
Center (Rad)	0.037086	0.074172	0.14834	0.29669	0.59338	1.1868	2.3735	4.747

Table 1: Frequency Ranges for Octaves 1 to 8 starting with O_1

on the bands we can recognize that arises from the use of the sampling freq of $8kHz$, we can only obtain unique detection for the set of Octaves whose frequencies are below the Nyquist rate of $\frac{f_s}{2}$ or $4kHz$. Or, O_7 in the table above.

6 Band-pass Filter Bank Design

Comment on the selectivity of the bandpass filters, i.e., use the frequency response (passbands and stopbands) to explain how the filter passes one octave while rejecting the others. Are the filters passbands narrow enough so that only one octave lies in the passband and the others are in the stop-band? The band pass filter bank was put together in python and has the added benefit of being more portable than the use of Matlab for this application. These are the results from the creation of the x signal needed to pass on to the filter bank. Defining the expression below:

$$x(t) = \begin{cases} \cos(2\pi \cdot 220t) & 0.0 < t \leq 0.25, \\ \cos(2\pi \cdot 880t) & 0.3 < t \leq 0.55, \\ \cos(2\pi \cdot 440t) + \cos(2\pi \cdot 880t) & 0.60 < t \leq 0.85 \\ 0 & \text{otherwise} \end{cases}$$

and now the code to do this along with the plot showing the data is what we expect:

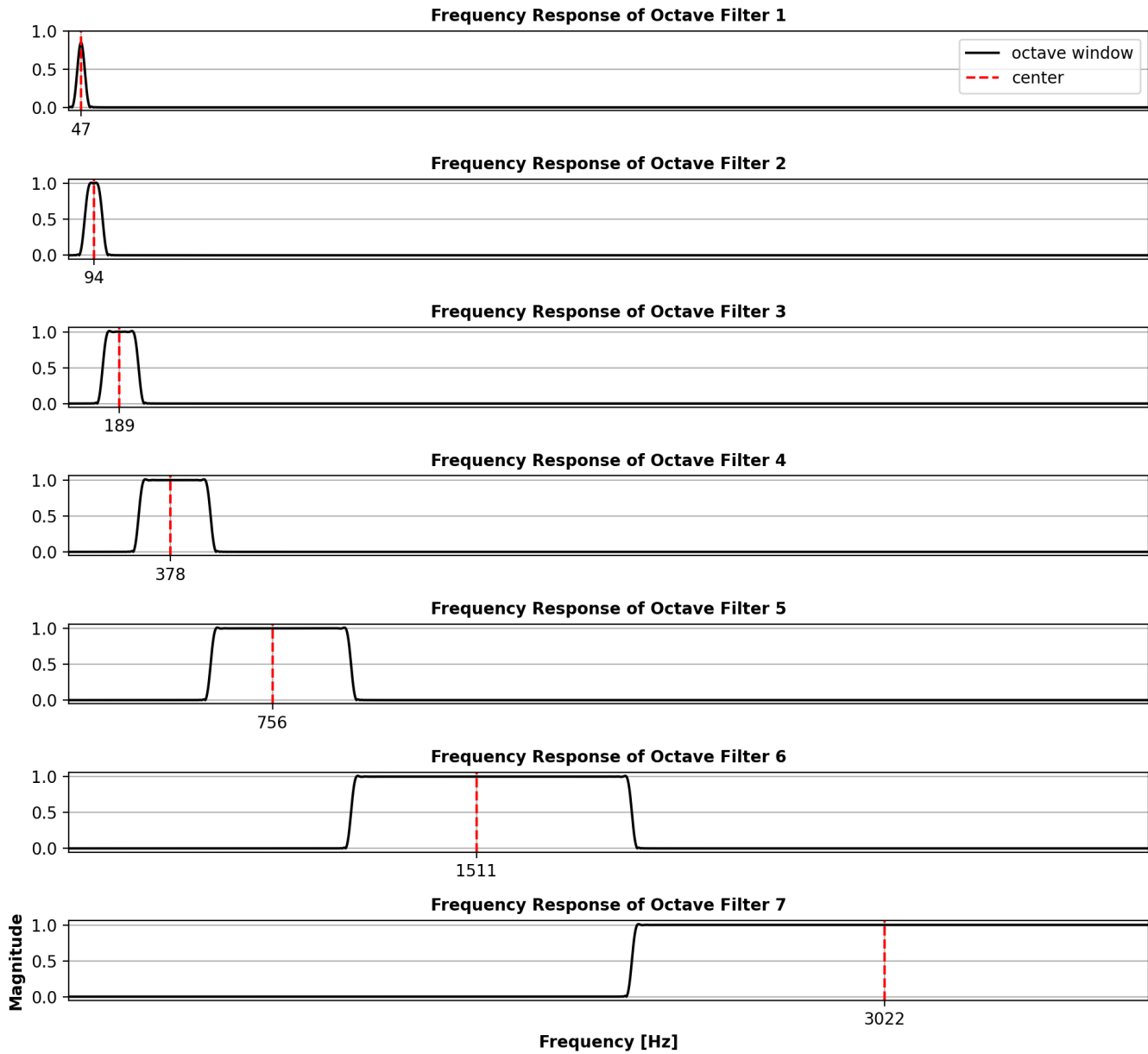


Figure 1: plot for all BPF's

The filters above in fig. 1 begin at a center of 47Hz but we only need to worry about the subsequent ones that we were asked for. Aligned at their centers is a vertical line and a tick mark indicating the center frequency. this bank of filters is what we will use to pass the time data created in the equation above. Filters one and two do not pass anything since they are both too low to capture the first freq of 220Hz , but the third has a center of 189Hz , meaning it does capture the first frequency. In all cases, we can see that the transition points have the characteristic peaks associated with instant changes in frequency. Any instant change results in

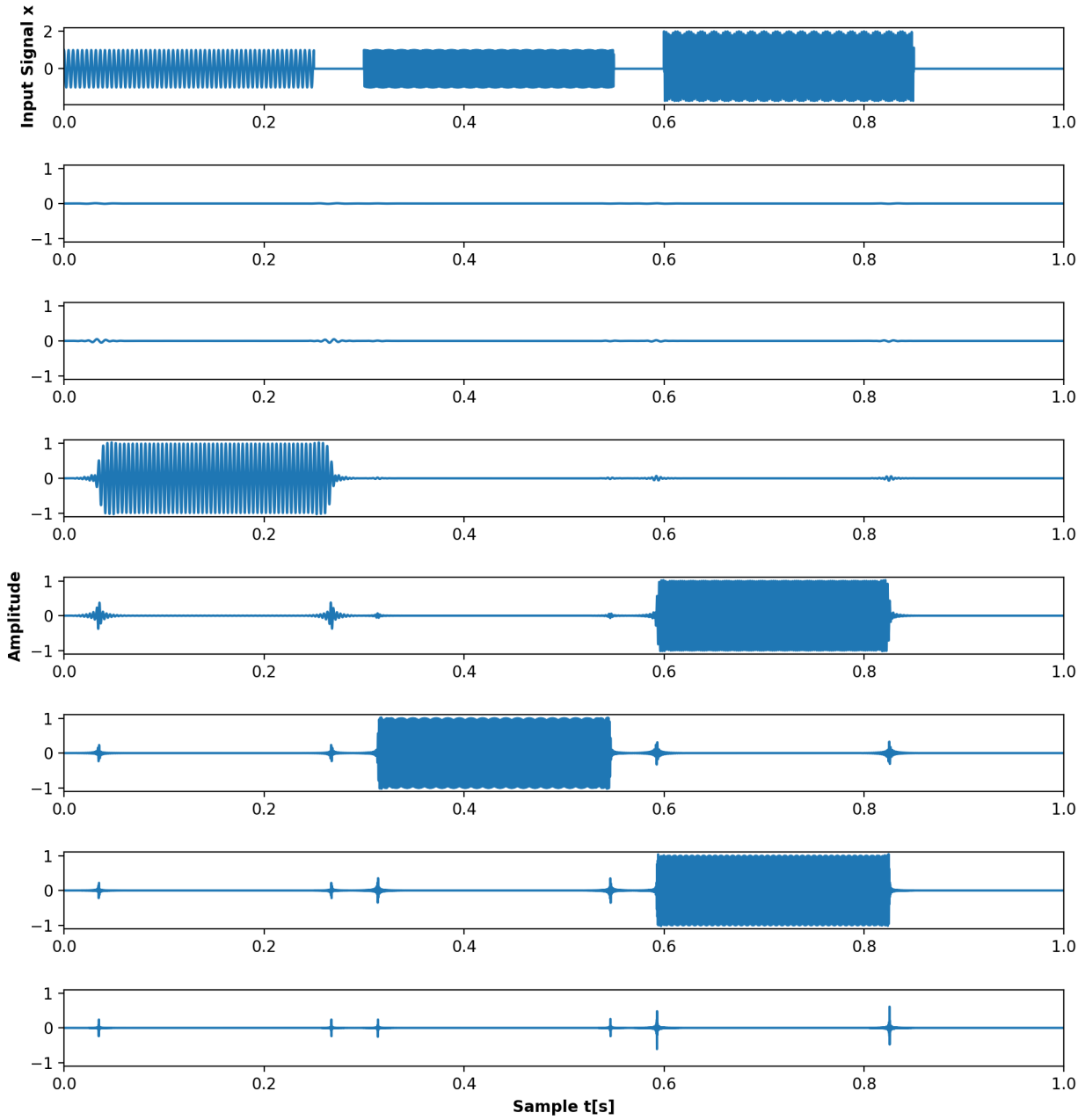


Figure 2: plot for all BPF's processing the time data at the top

a sharp change which corresponds to an infinite number of frequencies. the third filter is capable of capturing the $880Hz$, the fourth captures the $1760Hz$ in superposition with the $880Hz$, and no signal is passed through the last except the transients that appear in all filters.

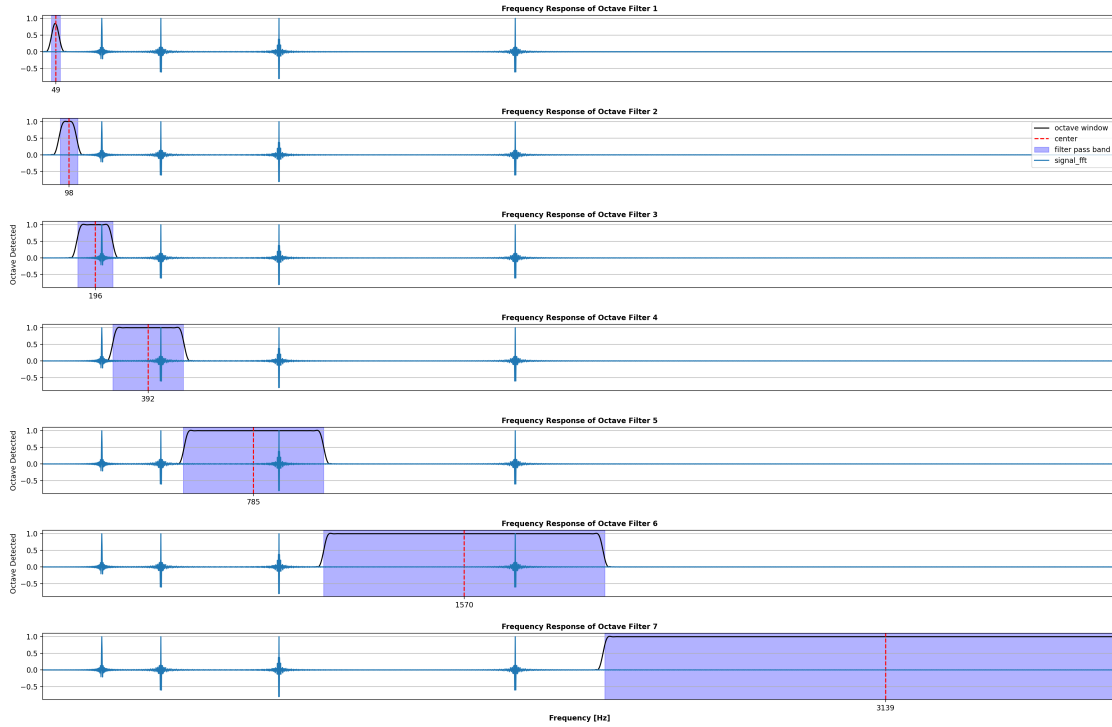


Figure 3: plot for all BPF's bandpass regions

In this figure, it is easier to see that the band-pass filters correctly capture the frequencies that we see in the time plot of fig. 2. In the figure above fig. 3, we can see that the filters that do not capture any signal do not have an overlapping note peak in the shaded region. However, the ones that we see with a signal passed through them do have a corresponding note in the shaded band-pass region.

6.1 Creating the FIR Filters

To create the FIR filters, we used a python class shown in subsection 7.2. The class works by first defining N , the number of points to be used in the FIR filter. We then declare the pass band region, the frequencies we wish for the FIR filter to pass through while the others are rejected. This is shown in fig. 4. Effectively, we are specifying the desired frequency response of the FIR filter but only using N number of points to define the filter in the frequency domain.

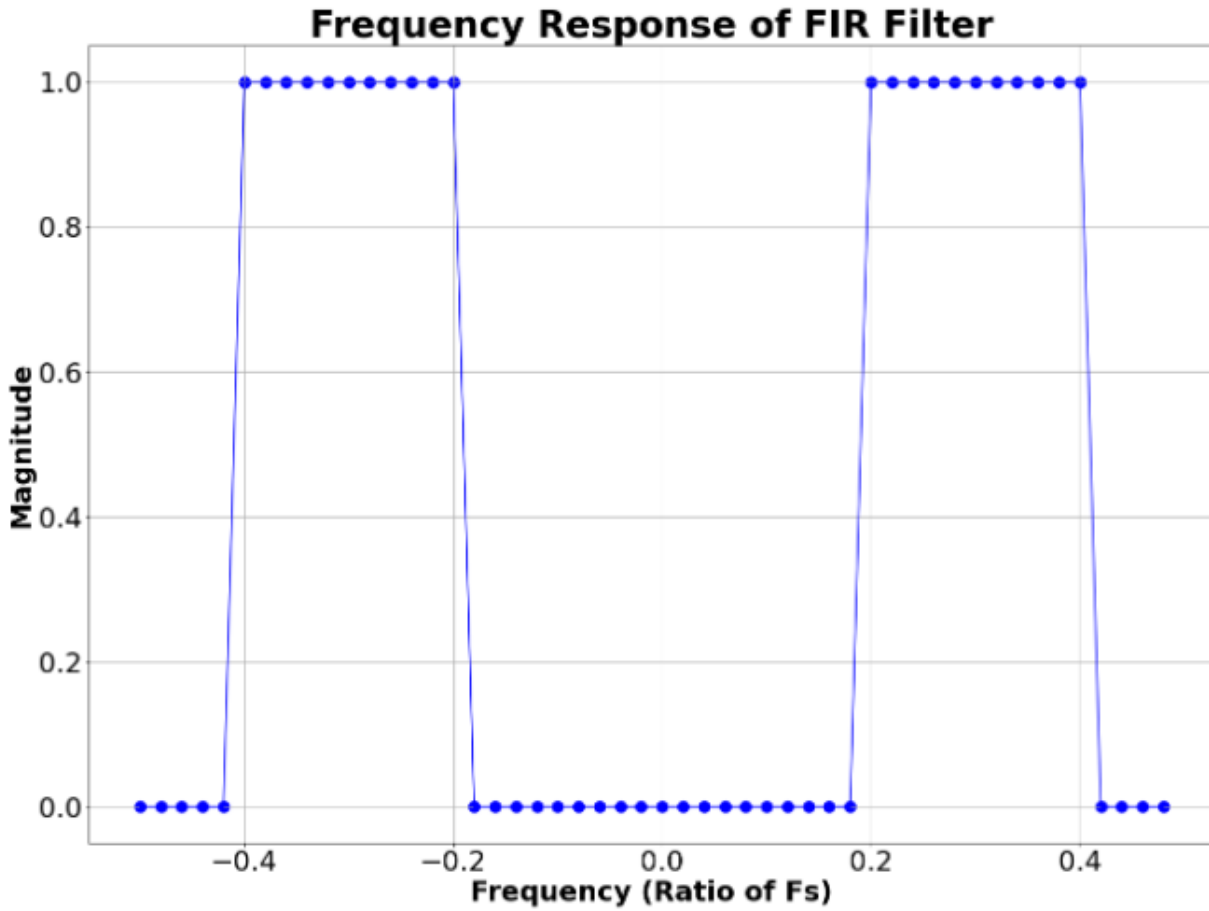


Figure 4: Band-pass region of the FIR filter being created. The filter response is specified by N number of points where N is the length of the FIR filter being created.

After defining the frequency response with N number of points, we need to also ensure that the frequency response is symmetric about the y-axis. This ensures that our FIR filter will be purely real. To obtain the time domain FIR filter, we then take the inverse fourier transform of the N number of points specifying the frequency response. The FIR filter corresponding to the frequency response shown in 4 above.

7 Appendix A

7.1 Matlab Code

```
1 function print_octaves(n,fs)
2 % n should be a range relative to A4. ex: -1:1 gives A3,A4,A5
3 A4 = 440;
4 C4 = A4.*2.^(-9./12);
5 B4 = A4.*2.^(2./12);
6 Octaves = 2.^n;
7 Cs = C4.*Octaves;
8 Bs = B4.*Octaves;
9 n_range = 0:length(n)-1;
10 Centers = (Cs + Bs)./2;
11 cell(size(Centers));
12 octave_array = arrayfun(@(x) sprintf('Octave %d', x), n_range, 'UniformOutput',
    false);
13 w_Cs = Cs.*2.*pi./fs;
14 w_Bs = Bs.*2.*pi./fs;
15 w_Centers = Centers.*2.*pi./fs;
16 rows = {'Lower (Hz)', 'Lower (Rad)', 'Upper (Hz)', 'Upper (Rad)', 'Center (Hz)', 'Center
    (Rad)'};
17 % Summarize data in a table
18 T = array2table([Cs; w_Cs; Bs; w_Bs; Centers; w_Centers], 'VariableNames',
    octave_array, 'RowName', rows);
19 disp(T)
20 disp('Hz are not normalized.')
21 disp('Radians are normalized by sampling frequency.')
22 end
```

7.2 Python FIR Filter Class

```
1 import numpy as np
2 from numpy import zeros, append
3 from numpy.fft import fftshift, fft
4 import matplotlib.pyplot as plt
5
6
7 class FIRFilter:
8     def __init__(self, N=10000, fmin=3, fmax=7, padding_factor=9, fs=8000):
9         self.N = N
10        self.padding_factor = padding_factor
11        self.fs = fs # Sampling rate
12        self.H = zeros(N)
13        self.w = zeros(N)
14        self.pos = np.arange(N)
15        self.fmin = fmin*self.N/self.fs
16        self.fmax = fmax*self.N/self.fs
17
18        self.h = None
19        self.h_pad = None
20        self.H_pad = None
21        self.w_pad = None
22        self.h_ham = None
23        self.H_ham_pad = None
24
25        self.create_filter()
26        self.apply_padding()
27        self.apply_hamming_window()
28
29
30    def create_filter(self):
31        k = np.arange(-int(self.N/2), int(self.N/2))
32        self.w = k * self.fs / self.N # Adjusted to use actual frequency values
33        self.H = np.where((np.abs(k) >= self.fmin) & (np.abs(k) <= self.fmax), 1,
34    0)
35        self.h = fftshift(fft(fftshift(self.H)))
36
37    def apply_padding(self):
38        NP = self.N + self.padding_factor * self.N
39        self.h_pad = append(self.h, zeros(self.padding_factor * self.N))
40        self.H_pad = fftshift(fft(self.h_pad)) / self.N
41        k = np.arange(-NP/2, NP/2)
42        self.w_pad = k * self.fs / NP
43
44    def apply_hamming_window(self):
45        self.h_ham = self.h * 0.5 * (1 + np.cos(2 * np.pi * (self.pos - self.N / 2)
46    / self.N))
47        self.h_ham_pad = append(self.h_ham, zeros(self.padding_factor * self.N))
48        self.H_ham_pad = fftshift(fft(self.h_ham_pad)) / self.N
49
50    def process(self, input_data):
51        return np.convolve(input_data, self.h_ham)/self.N
52
53    def plot_filter(self, fig, row, col, pos):
54        # Frequency Response Plot
55        ax1 = fig.add_subplot(row, col, pos)
56        ax1.scatter(self.w, self.H.real, c='b', s=150)
57        # ax1.plot(self.w_pad, abs(self.H_pad), 'r')
58        ax1.plot(self.w_pad, abs(self.H_ham_pad), 'black')
59        ax1.set_xlim(0, .5)
60        ax1.set_title(f'Frequency Response of Octave Filter {pos}', fontsize=5,
```



```

fontweight='bold')
59     if pos == row-1:
60         ax1.legend(['Hamming'], prop={'size': 5})
61         ax1.set_xlabel('Frequency (Ratio of Fs)', fontsize=5, fontweight='bold'
)
62         ax1.set_ylabel('Magnitude', fontsize=5, fontweight='bold')
63         ax1.grid(True)
64
65     def plot_filter1(self):
66         # Matplotlib plotting
67         fig = plt.figure(figsize=(22, 16))
68
69         # Frequency Response Plot
70         ax1 = fig.add_subplot(211)
71         ax1.scatter(self.w, self.H.real, c='b', s=150)
72         ax1.plot(self.w_pad, abs(self.H_pad), 'r')
73         ax1.plot(self.w_pad, abs(self.H_ham_pad), 'black')
74         ax1.set_xlabel('Frequency (Hz)', fontsize=15, fontweight='bold')
75         ax1.set_ylabel('Magnitude', fontsize=15, fontweight='bold')
76         ax1.set_title('Frequency Response of FIR Filter', fontsize=15, fontweight='
bold')
77         ax1.legend(['Ideal', 'Actual', 'Hamming'], prop={'size': 15})
78         ax1.tick_params(axis='both', labelsize=15)
79         ax1.grid(True)
80
81         # Time Domain Plot
82         ax2 = fig.add_subplot(212)
83         ax2.vlines(self.pos, 0, self.h.real, 'b')
84         # ax2.vlines(self.pos, 0, self.h.imag, 'r')
85         ax2.scatter(self.pos, self.h.real, c='b', s=150)
86         # ax2.scatter(self.pos, self.h.imag, c='r', s=150)
87         ax2.set_xlabel('Position', fontsize=15, fontweight='bold')
88         ax2.set_ylabel('Value (Unscaled)', fontsize=15, fontweight='bold')
89         ax2.set_title('Time Domain FIR Filter', fontsize=15, fontweight='bold')
90         # ax2.legend(['Real', 'Imag'], prop={'size': 15})
91         ax2.tick_params(axis='both', labelsize=15)
92         ax2.grid(True)
93
94         plt.show(block=False)

```

7.3 Python Code - Generate Plots

```
1 from util.OctaveBandFilt import OctaveBandFilter, ourOctaveBandFilter
2 from util.FIR_filter import FIRFilter
3 import numpy as np
4 import matplotlib.pyplot as plt
5 from rich import print
6 import math
7
8 fontsize = 10
9
10 def calculate_grid_dimensions(n):
11     columns = round(math.sqrt(n))
12     rows = math.ceil(n / columns)
13     return rows, columns
14
15 def calculate_octave_ranges(base_freq, num_octaves, sample_rate, N):
16     octave_ranges = []
17     for i in range(num_octaves+1):
18         fmin = base_freq * (2 ** i) # Minimum frequency for the octave
19         if N < 500:
20             fmax = base_freq * (2 ** (i+(11/12)))
21         else:
22             fmax = base_freq * (2 ** (i+1)) # Maximum frequency for the octave
23
24         # Normalize the frequencies
25         fmin_normalized = fmin
26         fmax_normalized = fmax
27
28         octave_ranges.append((fmin_normalized, fmax_normalized))
29     return octave_ranges
30
31 # Create a set of filters
32 N = 600
33 sampling_frequency = 8000 # Sampling frequency in Hz
34 # Sample rate and base frequency
35 base_freq = 32.703 # Starting frequency of the first octave
36 # Calculate octave ranges
37 num_octaves = 6
38 # for the filtering
39 octave_ranges = calculate_octave_ranges(base_freq, num_octaves, sampling_frequency,
40                                         2*N)
41 # for the midpoints
42 octave_ranges1 = calculate_octave_ranges(base_freq, num_octaves, sampling_frequency
43                                         , N/2)
44
45 filters = [FIRFilter(N, fmin=fmin, fmax=fmax, padding_factor=9, fs=
46                 sampling_frequency) for fmin, fmax in octave_ranges]
47 num_filters = len(filters)
48 #filters = [FIRFilter(N, fmin=fmin*N, fmax=fmax*N, padding_factor=9) for fmin, fmax
49             in octave_ranges]
50 print(f"octave ranges: {octave_ranges}")
51 # Calculate grid size
52 total_subplots = num_filters
53 rows, cols = calculate_grid_dimensions(total_subplots)
54 print(f"rows: {rows}")
55 print(f"cols: {cols}")
56 # Create a figure for plotting
57 fig = plt.figure(figsize=(10, 10))
```

```

57 mids = [(b+a)/2 for a, b in octave_ranges]
58 mids1 = [(b+a)/2 for a, b in octave_ranges1]
59 print(f"midpoints_overlap: {mids}")
60 print(f"midpoints_no_overlap: {mids1}")
61 # Plot each filter's response
62 for i, filter in enumerate(filters):
63     # Calculate position
64     position = i * 2 + 1 # Position for frequency response plot
65     x = mids1[i]
66     filter.plot_filter(fig, num_filters+1, 1, i+1)
67     plt.axvline(x=x, ymin=0, ymax=1, color='r', linestyle='--')
68     if i == 0:
69         plt.legend(['octave window', 'center'])
70     plt.xticks([x])
71     plt.yticks([0, .5, 1])
72     ax = plt.gca()
73     ax.set_xlim([0, filter.fs/2])
74
75 plt.tight_layout()
76 plt.show(block=False)
77 plt.savefig("images/post_lock_freq_data.png", dpi=200, transparent=False)
78 plt.close()
79
80 num_octaves = 6
81 octave_ranges = calculate_octave_ranges(base_freq, num_octaves, sampling_frequency,
82                                         N)
83 filters = [FIRFilter(N, fmin=fmin, fmax=fmax, padding_factor=9, fs=
84                 sampling_frequency) for fmin, fmax in octave_ranges]
85 num_filters = len(filters)
86 # Generate the signal 5.2
87 fs = sampling_frequency
88 t1 = np.linspace(0, 0.25, int(fs*0.25), endpoint=False)
89 t2 = np.linspace(0.3, 0.55, int(fs*0.25), endpoint=False)
90 t3 = np.linspace(0.6, 0.85, int(fs*0.25), endpoint=False)
91 t_end = np.linspace(0.85, 1, int(fs*0.15), endpoint=False)
92
93 x1 = np.cos(2*np.pi*220*t1)
94 x2 = np.cos(2*np.pi*880*t2)
95 x3 = np.cos(2*np.pi*440*t3) + np.cos(2*np.pi*1760*t3)
96
97 zero_padding = np.zeros(int(fs*0.05))
98 zeros_end = np.zeros(int(fs*0.15))
99 x = np.concatenate((x1, zero_padding, x2, zero_padding, x3, zeros_end))
100 t = np.linspace(0, 1, len(x), endpoint=False)
101 # Filter and plot the signal
102 fig = plt.figure(figsize=(10, 10))
103 plt.title('Filtered Signal with Filters', fontsize=fontsize+10, fontweight='bold')
104 plt.subplot(len(filters)+1, 1, 1)
105 plt.plot(t, x)
106 plt.ylabel('Input Signal x', fontsize=fontsize, fontweight='bold')
107 ax = plt.gca()
108 ax.set_xlim([0, 1])
109
110 for i, filter in enumerate(filters):
111     filtered_x = filter.process(x)
112     t = np.linspace(0, 1, len(filtered_x), endpoint=False)
113     filtered_sig = filtered_x
114     plt.subplot(len(filters)+1, 1, i+2)
115     plt.plot(t, np.real(filtered_sig))
116     ax = plt.gca()
117     ax.set_ylim([-1.100, 1.100])
118     ax.set_xlim([0, 1])

```

```

117     plt.tight_layout(pad=2.0)
118     plt.ylabel('Amplitude', fontsize=fontsize, fontweight='bold') if i == (len(
filters)//2) else None
119
120
121 plt.xlabel('Sample t[s]', fontsize=fontsize, fontweight='bold')
122 plt.show(block=False)
123 plt.savefig("images/post_lock_time_data.png", dpi=200, transparent=False)
124 plt.close()
125
126
127 fig = plt.figure(figsize=(22, 16))
128 x_fftd = np.fft.fft(x)
129
130 # Plot each filter's response
131 for i, filter in enumerate(filters):
132     # Calculate position
133     position = i * 2 + 1 # Position for frequency response plot
134     x = mids[i]
135     # Determine the width of the rectangle from the octave range
136     fmin, fmax = octave_ranges[i]
137     rect_width = fmax - fmin
138     filter.plot_filter(fig, num_filters+1, 1, i+1)
139     plt.axvline(x=x, ymin=0, ymax=1, color='r', linestyle='--')
140
141     # Adding the shaded rectangle
142     plt.axvspan(x - rect_width/2, x + rect_width/2, ymin=0, ymax=1, alpha=0.3,
color='blue')
143
144     plt.plot(x_fftd/max(x_fftd))
145     plt.legend(['octave window', 'center', 'filter pass band', 'signal_fft'],
fontsize=10, loc='upper right', bbox_to_anchor=(1, 1)) if i == 1 else None
146     plt.xticks([x])
147     ax = plt.gca()
148     ax.set_xlim([0, filter.fs/2])
149     ax.set_ylabel("Octave Detected", fontsize=10) if i >= 2 and i < 6 and i != 3
else ax.set_ylabel(" ", fontsize=10)
150     plt.tight_layout(pad=2.0)
151
152
153 plt.show(block=False)
154 plt.savefig("images/post_lock_fft_freqdata.png", dpi=200, transparent=False)
155 plt.close()

```