ECE 6310 - Advanced Electromagnetic Fields: Homework Set #2

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10 days - 0 hours - 5 min until deadline

$\mathscr{A}\mathscr{B}\mathscr{C}\mathscr{D}\mathscr{E}\mathscr{F}\mathscr{G}\mathscr{H}\mathscr{I}\mathscr{J}\mathscr{K}\mathscr{L}\mathscr{M}\mathscr{N}\mathscr{O}\mathscr{P}\mathscr{Q}\mathscr{R}\mathscr{S}\mathscr{T}\mathscr{U}\mathscr{V}\mathscr{W}\mathscr{X}\mathscr{Y}\mathscr{Z}$
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1 Preliminaries

In this document, we use standard notation for electromagnetic theory. Key equations and concepts are summarized below:

Vector Notation

- $\boldsymbol{\mathscr{E}}$: Electric field intensity
- $\boldsymbol{\mathscr{H}}$: Magnetic field intensity
- $\boldsymbol{\mathscr{D}}$: Electric flux density
- $\boldsymbol{\mathscr{B}}$: Magnetic flux density
- $\boldsymbol{\mathscr{J}}$: Current density
- ρ_v: Volume charge density

Differential Operators

- $\nabla\cdot$: Divergence of a vector field
- $\nabla\times$: Curl of a vector field

- ∇ : Gradient of a scalar field
- ∂_i : Partial derivative with respect to the independent basis element i

Maxwell's Equations

In integral form, Maxwell's equations are given by:

$$\oint_{\partial V} \boldsymbol{\mathscr{E}} \cdot d\boldsymbol{\ell} = -\frac{d}{dt}\int_V \boldsymbol{\mathscr{B}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Faraday's Law of Induction)} \tag{1}$$

$$\oint_{\partial V} \boldsymbol{\mathscr{H}} \cdot d\boldsymbol{\ell} = \int_V \boldsymbol{\mathscr{J}} \cdot d\boldsymbol{\mathcal{S}} + \frac{d}{dt}\int_V \boldsymbol{\mathscr{D}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Ampère's Circuital Law)} \tag{2}$$

$$\oiint_{\partial V} \boldsymbol{\mathscr{D}} \cdot d\boldsymbol{\mathcal{S}} = \int_V \rho_v dV \qquad \text{(Gauss's Law for Electricity)} \tag{3}$$

$$\oiint_{\partial V} \boldsymbol{\mathscr{B}} \cdot d\boldsymbol{\mathcal{S}} = 0 \qquad \text{(Gauss's Law for Magnetism)} \tag{4}$$

Other Relevant Equations

- Continuity Equation: $\nabla \cdot \boldsymbol{\mathscr{J}} + \partial_t \rho_v = 0$
- Relationship between $\boldsymbol{\mathscr{E}}$, $\boldsymbol{\mathscr{D}}$: $\boldsymbol{\mathscr{D}} = \epsilon \boldsymbol{\mathscr{E}}$
- Relationship between $\boldsymbol{\mathscr{H}}$, $\boldsymbol{\mathscr{B}}$: $\boldsymbol{\mathscr{B}} = \mu \boldsymbol{\mathscr{H}}$

Boundary Conditions

Discuss the boundary conditions for $\boldsymbol{\mathscr{E}}$, $\boldsymbol{\mathscr{H}}$, $\boldsymbol{\mathscr{D}}$, and $\boldsymbol{\mathscr{B}}$ at interfaces between different media.

Problem 1.3

(a) Electron charge density q_{ve}.

(b) Electron drift velocity v_e.

(c) Electric current density J.

(d) Electric current flowing through the square cross section of the wafer.

(e) Electron density N_e.

Leave your answers in terms of ϵ_0, μ_0.

References

[1] Constantine A. Balanis. *advanced engineering electromagnetics*, chapter 1, page 2–3. John Wiley & Sons, 2024.