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ECE 6310 - Advanced Electromagnetic Fields: Homework Set #2

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1 Preliminaries

In this document, we use standard notation for electromagnetic theory. Key equations and concepts are summarized below:

Vector Notation

- $\mathcal{E}$: Electric field intensity
- $\mathcal{H}$: Magnetic field intensity
- $\mathcal{D}$: Electric flux density
- $\mathcal{B}$: Magnetic flux density
- $\mathcal{J}$: Current density
- ρ_v: Volume charge density

Differential Operators

- $\nabla\cdot$: Divergence of a vector field
- $\nabla\times$: Curl of a vector field
- ∇ : Gradient of a scalar field
- ∂_i : Partial derivative with respect to the independent basis element i

Maxwell's Equations

In integral form, Maxwell's equations are given by:

$$\oint_{\partial V} \mathcal{E} \cdot d\updownarrow = -\frac{d}{dt} \int_V \mathcal{B} \cdot d\mathcal{S} \qquad \text{(Faraday's Law of Induction)} \tag{1}$$

$$\oint_{\partial V} \mathcal{H} \cdot d\updownarrow = \int_V \mathcal{J} \cdot d\mathcal{S} + \frac{d}{dt} \int_V \mathcal{D} \cdot d\mathcal{S} \qquad \text{(Ampère's Circuital Law)} \tag{2}$$

$${}_{\partial V}\mathcal{D} \cdot d\mathcal{S} = \int_V \rho_v dV \qquad \text{(Gauss's Law for Electricity)} \tag{3}$$

$${}_{\partial V}\mathcal{B} \cdot d\mathcal{S} = 0 \qquad \text{(Gauss's Law for Magnetism)} \tag{4}$$

Other Relevant Equations

- Continuity Equation: $\nabla \cdot \mathcal{J} + \partial_t \rho_v = 0$
- Relationship between $\mathcal{E}$, $\mathcal{D}$: $\mathcal{D} = \epsilon \mathcal{E}$
- Relationship between $\mathcal{H}$, $\mathcal{B}$: $\mathcal{B} = \mu \mathcal{H}$

Boundary Conditions

Discuss the boundary conditions for $\mathcal{E}$, $\mathcal{H}$, $\mathcal{D}$, and $\mathcal{B}$ at interfaces between different media.

Problem 3.2

Verify that (3-28a) and (3-28b) are solutions to (3-26a)
To solve this problem, we can first set up the expressions we need

(3-26a)

$$\frac{d^2 f}{dx^2} = -\beta_x^2 f \tag{5}$$

(3-28a)

$$\begin{aligned}
f(x) &= A_1 e^{-j\beta_x x} + B_1 e^{j\beta_x x} \\
\frac{d}{dx} f(x) &= -j\beta_x A_1 e^{-j\beta_x x} + j\beta_x B_1 e^{j\beta_x x} \\
\frac{d^2}{dx^2} f(x) &= (-j\beta_x)^2 A_1 e^{-j\beta_x x} + (j\beta_x)^2 B_1 e^{j\beta_x x} \\
\frac{d^2}{dx^2} f(x) &= -\beta_x^2 A_1 e^{-j\beta_x x} - \beta_x^2 B_1 e^{j\beta_x x} \\
&= -\beta_x^2 (A_1 e^{-j\beta_x x} + B_1 e^{j\beta_x x}) \\
&= -\beta_x^2 f(x)
\end{aligned}$$

(3-28b)

$$\begin{aligned}
f(x) &= C_1 \cos(\beta_x x) + D_1 \sin(\beta_x x) \\
\frac{d}{dx} f(x) &= -\beta_x C_1 \sin(\beta_x x) + \beta_x D_1 \cos(\beta_x x) \\
\frac{d^2}{dx^2} f(x) &= -\beta_x^2 C_1 \cos(\beta_x x) - \beta_x^2 D_1 \sin(\beta_x x) \\
&= -\beta_x^2 (C_1 \cos(\beta_x x) + D_1 \sin(\beta_x x)) \\
&= -\beta_x^2 f(x)
\end{aligned}$$

Problem 4.2

Using Maxwell's equations, find the magnetic field components for the wave whose electric field is given in Example 4-1. Compare your answer with that obtained in the solution of Example 4-1.

Problem 4.22

Sea water is an important medium in communication between submerged submarines or between submerged submarines and receiving and transmitting stations located above the surface of the sea. Assuming the constitutive electrical parameters of the sea are $\sigma = 4\frac{S}{m}$, $\epsilon_r = 81$, $\mu_r = 1$, and $f = 104Hz$, find the:

(a) Complex propagation constant $\gamma\frac{1}{m}$.

(b) Phase velocity $v\frac{m}{s}$.

(c) Wavelength λ $[m]$.

(d) Attenuation constant $\alpha\frac{N}{m}$.

(e) Skin depth δ $[m]$.

Problem 4.26

In a source-free, free-space region, the complex magnetic field of a time-harmonic field is represented by:

$$H = \left[\hat{a}_x(1+j) + \hat{a}_z\left(\sqrt{2}e^{-j\frac{\pi}{4}}\right)\right]\frac{E_0}{\eta_0}e^{-j\beta_0 y}$$

where E_0 is a constant and η_0 is the intrinsic impedance of free space. Determine the:

(a) Polarization of the wave (linear, circular, or elliptical). Justify your answer.

(b) Sense of rotation, if any.

(c) Corresponding electric field.

Problem 5.4

A vertical interface is formed by having free space to its left and a lossless dielectric medium to its right with $\epsilon = 4\epsilon_0$ and $\mu = \mu_0$, as shown in Figure P5-4. The incident electric field of a uniform plane wave traveling in the free-space medium and incident normally upon the interface has a value of $2 \cdot 10^3\frac{V}{m}$ right before it strikes the boundary. At a frequency of $3GHz$, find the:

[scale=1.5, axis/.style=->, >=stealth']
[gray!40] (0,-1.5) – (1.5,-1.5) – (1.5,1.5) – (0,1.5) – cycle; [axis] (0,0) – (2,0) node(xline)[right] z; [axis] (0,0) – (0,2) node(yline)[above] x;
(0,0) circle (3.5pt); [black] (0,0) circle (1.75pt);
at (-0.5,1) ε_0, μ_0; at (1,1) $4\varepsilon_0, \mu_0$;
[axis] (0,0) – (0,-1.5) node(yline)[below] y;
at (0.5,-2.5) **Figure P5-4**;

(a) Reflection coefficient.

(b) SWR in the free-space medium.

(c) Positions (in meters) in the free-space medium where the electric field maxima and minima occur.

Problem 5.9

A uniform plane wave traveling in air is incident normally on a half space occupied by a lossless dielectric medium of relative permittivity of 4. The reflections can be eliminated by placing another dielectric slab, $\frac{\lambda}{4}$ thick, between the air and the original dielectric medium, as shown in Figure P5-9. To accomplish this, the intrinsic impedance η_1 of the slab must be equal to $\sqrt{\eta_0\eta_2}$ where η_0 and η_2 are, respectively, the intrinsic impedances of air and the original dielectric medium. Assuming that the relative permeabilities of all the media are unity, what should the relative permittivity of the dielectric slab be to accomplish this?

[scale=1.5, axis/.style=->, >=stealth']
[gray!40] (0,-1.5) – (1.5,-1.5) – (1.5,1.5) – (0,1.5) – cycle; [gray!60] (1,-1.5) – (2,-1.5) – (2,1.5) – (1,1.5) – cycle; [axis] (-.5,0) – (2.75,0) node(xline)[right] ; [axis] (0,0) – (0,2) node(yline)[above] ; [axis] (0,.5) – (1,.5) node(yline)[] ;
at (.5,.7) $\frac{\lambda_1}{4}$;
(0,0) circle (3.5pt); [black] (0,0) circle (1.75pt);
at (-0.5,1.25) η_0; at (.5,1.25) η_1; at (.5,-.5) $\epsilon_{r1} =?$; at (1.5,1.25) η_2; at (1.5,-.5) $\epsilon_{r2} = 4$;
[axis] (0,0) – (0,-2) node(yline)[below] ;
at (0.5,-2.5) **Figure P5-9**;

This problem is a simple one. First we set the expressions for each of the impedances:

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$
$$\eta_1 = \sqrt{\frac{\mu_{r1}\mu_0}{\epsilon_{r1}\epsilon_0}} = \sqrt{\frac{\mu_{r1}}{\epsilon_{r1}}}\eta_0$$
$$\eta_2 = \sqrt{\frac{\mu_{r2}\mu_0}{\epsilon_{r2}\epsilon_0}} = \sqrt{\frac{\mu_{r2}}{\epsilon_{r2}}}\eta_0$$

Plugging in the values for what we know:

$$\eta_2 = \sqrt{\frac{1}{4}}\eta_0 = \frac{1}{2}\eta_0$$
$$\eta_1 = \sqrt{\frac{\mu_{r1}}{\epsilon_{r1}}}\eta_0 = \sqrt{\eta_0\eta_2} = \sqrt{\frac{1}{2}\eta_0^2} = \sqrt{\frac{1}{2}}\ \eta_0$$

Now we set the expression for η_1 equal to the expression we have, and solve for the permittivity:

$$\eta_1 = \sqrt{\frac{\mu_{r1}}{\epsilon_{r1}}}\eta_0 = \sqrt{\frac{1}{2}}\ \eta_0$$
$$\therefore \quad \sqrt{\frac{\mu_{r1}}{\epsilon_{r1}}} = \sqrt{\frac{1}{2}}$$

If we have $\mu_{r1} = 1$ then we can say that the relative permittivity of the middle slab should be 2.

Problem 5.21

A uniform plane wave traveling in a lossless dielectric is incident normally on a flat interface formed by the presence of air. For ϵ_r 's of 2.56, 4, 9, 16, 25, and 81:

(a) Determine the critical angles.

(b) Find the Brewster angles if the wave is of parallel polarization.

(c) Compare the critical and Brewster angles found in parts (a) and (b).

(d) Plot the magnitudes of the reflection coefficients for both perpendicular, $|\Gamma_\perp|$, and parallel, $|\Gamma_\parallel|$, polarizations versus incidence angle.

(e) Plot the phase (in degrees) of the reflection coefficients for both perpendicular and parallel polarizations versus incidence angle.