Homework Set #3

ECE 6310 - Advanced Electromagnetic Fields

Balanis (2nd Edition): 6.19, 6.20, 6.25, 6.26, 6.38
Balanis (2nd Edition): 7.3, 7.15, 7.37

Verify your solutions for the far field E_θ for a half-wavelength dipole (6.26) in HFSS; design the antenna to be resonant at 1GHz.

6.19. Show that for observations made at very large distance ($\beta r \gg 1$) the electric and magnetic fields of Example 6-3 reduce to

$$E_\theta = j\eta \frac{\beta I_e \ell e^{-j\beta r}}{4\pi r} \sin\theta$$

$$H_\phi \simeq \frac{E_\theta}{\eta}$$

$$E_r \simeq 0$$

$$E_\phi = H_r = H_\theta = 0$$

6.20. For Problem 6.19, show that the:

- Time-average power density is

$$\mathbf{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^*\right] = \hat{\mathbf{a}}_r W_{av} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r \frac{\eta}{8}\left|\frac{I_o \ell}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2}$$

- Radiation intensity is

$$U = r^2 S_{av} = \frac{\eta}{8}\left|\frac{I_o \ell}{\lambda}\right|^2 \sin^2\theta$$

- Radiated power is

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi) \times \sin\theta\, d\theta\, d\phi = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 \ell}{\lambda}\right|^2$$

- Directivity is $D_o = \dfrac{4\pi U_{\max}(\theta,\phi)}{P_{rad}} = \dfrac{3}{2}$ (dimensionless) $= 1.761$ dB
- Radiation resistance is

$$R_r = \frac{2P_{rad}}{|I_o|^2} = 80\pi^2\left(\frac{\ell}{\lambda}\right)^2$$

6.25. The current distribution on a very thin wire dipole antenna of overall length ℓ is given by

$$\mathbf{I}_e = \begin{cases} \hat{\mathbf{a}}_z I_0 \sin\left[\beta\left(\dfrac{\ell}{2} - z'\right)\right] & 0 \le z' \le \dfrac{\ell}{2} \\ \hat{\mathbf{a}}_z I_0 \sin\left[\beta\left(\dfrac{\ell}{2} + z'\right)\right] & -\dfrac{\ell}{2} \le z' \le 0 \end{cases}$$

where I_0 is a constant. Representing the distance R of (6-112) by the far-field approximations of (6-112a) through (6-112b), derive the far-zone electric and magnetic fields radiated by the dipole using (6-97a) and the far-field formulations of Section 6.7.

6.26. Show that the radiated far-zone electric and magnetic fields derived in Problem 6.25 reduce for a half-wavelength dipole ($\ell = \lambda/2$) to

$$E_\theta \simeq j\eta \frac{I_0 e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

$$H_\phi \simeq \frac{E_\theta}{\eta}$$

$$E_r \simeq E_\phi \simeq H_r \simeq H_\theta \simeq 0$$

6.38. A coaxial line of inner and outer radii a and b, respectively, is mounted on an infinite conducting ground plane. Assuming that the electric field over the aperture of the coax is

$$\mathbf{E}_a = -\hat{\mathbf{a}}_\rho \frac{V}{\varepsilon \ln(b/a)} \frac{1}{\rho'}, \qquad a \le \rho' \le b$$

where V is the applied voltage and ε is the permittivity of medium in the coax, find the far-zone spherical electric and magnetic field components radiated by the aperture.

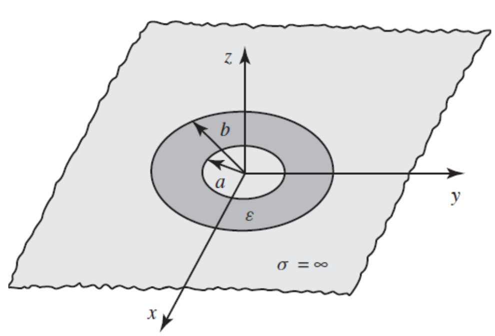

Figure P6-38

7.3. An infinitesimal vertical magnetic dipole of length l and constant current I_m is placed symmetrically about the origin and it is directed along the z axis, as shown in Figure 6-2a. Derive expressions valid everywhere, near and far field, for the:

- Electric vector potential components (F_r, F_θ, F_ϕ).
- Electric field components (E_r, E_θ, E_ϕ).
- Magnetic field components (H_r, H_θ, H_ϕ).
- Time-average power density, defined as $\mathbf{S}_{av} = \frac{1}{2}\text{Re}\left[\mathbf{E} \times \mathbf{H}^*\right]$.
- Radiation intensity, defined in the far field as $U \approx r^2 S_{av}$.
- Power radiated, defined as $P_{rad} = \int_0^{2\pi}\int_0^{\pi} U\,(\theta, \phi)\sin\theta d\theta d\phi$.
- Maximum directivity, defined as $D_0 = \dfrac{4\pi U_{\max}\,(\theta, \phi)}{P_{rad}}$
- Radiation resistance, defined as $R_r = \dfrac{2P_{rad}}{|I_m|^2}$

You can minimize the derivations as long as you justify the procedure.

7.15. An infinitesimal electric dipole is placed at an angle of 30° at a height h above a perfectly conducting electric ground plane. Determine the location and orientation of its image. Do this by sketching the image.

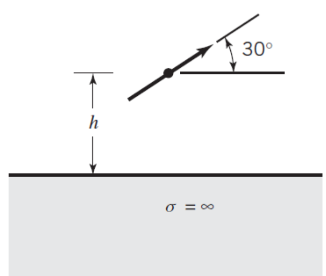

Figure P7-15

7.37. For the aperture shown in Figure 6-4c and assuming it is mounted on an infinite PEC ground plane:

(a) Form the most practical, exact or approximate (when necessary to solve the problem), equivalent currents $\mathbf{J}_s$ and $\mathbf{M}_s$.

(b) Find the far-zone electric and magnetic fields. The electric field distribution at the aperture is given by (E_o is a constant)

$$\mathbf{E}_a = \hat{\mathbf{a}}_y E_o$$

$$-a/2 \le x' \le a/2; -b/2 \le y' \le b/2$$