ECE 6310 - Advanced Electromagnetic Fields: Homework Set #4

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0 days - 11 hours - 51 min until deadline!!

Preliminaries

In this document, we use standard notation for electromagnetic theory. Key equations and concepts are summarized below:

Vector Notation

- $\mathscr{E}$: Electric field intensity
- $\mathscr{H}$: Magnetic field intensity
- $\mathscr{D}$: Electric flux density
- $\mathscr{B}$: Magnetic flux density
- $\mathscr{J}$: Current density
- ρ_v: Volume charge density

Differential Operators

- $\nabla \cdot$: Divergence of a vector field
- $\nabla \times$: Curl of a vector field
- ∇ : Gradient of a scalar field
- ∂_i : Partial derivative with respect to the independent basis element i

Maxwell's Equations

In integral form, Maxwell's equations are given by:

$$\oint_{\partial V} \boldsymbol{\mathscr{E}} \cdot d\boldsymbol{\ell} = -\frac{d}{dt}\int_V \boldsymbol{\mathscr{B}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Faraday's Law of Induction)} \tag{1}$$

$$\oint_{\partial V} \boldsymbol{\mathscr{H}} \cdot d\boldsymbol{\ell} = \int_V \boldsymbol{\mathscr{J}} \cdot d\boldsymbol{\mathcal{S}} + \frac{d}{dt}\int_V \boldsymbol{\mathscr{D}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Ampère's Circuital Law)} \tag{2}$$

$$\oiint_{\partial V} \boldsymbol{\mathscr{D}} \cdot d\boldsymbol{\mathcal{S}} = \int_V \rho_v dV \qquad \text{(Gauss's Law for Electricity)} \tag{3}$$

$$\oiint_{\partial V} \boldsymbol{\mathscr{B}} \cdot d\boldsymbol{\mathcal{S}} = 0 \qquad \text{(Gauss's Law for Magnetism)} \tag{4}$$

Other Relevant Equations

- Continuity Equation: $\nabla \cdot \boldsymbol{\mathscr{J}} + \partial_t \rho_v = 0$
- Relationship between $\boldsymbol{\mathscr{E}}$, $\boldsymbol{\mathscr{D}}$: $\boldsymbol{\mathscr{D}} = \epsilon \boldsymbol{\mathscr{E}}$
- Relationship between $\boldsymbol{\mathscr{H}}$, $\boldsymbol{\mathscr{B}}$: $\boldsymbol{\mathscr{B}} = \mu \boldsymbol{\mathscr{H}}$

Boundary Conditions

Discuss the boundary conditions for $\boldsymbol{\mathscr{E}}$, $\boldsymbol{\mathscr{H}}$, $\boldsymbol{\mathscr{D}}$, and $\boldsymbol{\mathscr{B}}$ at interfaces between different media.

1 - 8.2

A standard X-band (8.2–12.4GHz) rectangular waveguide with inner dimensions of 0.9 in. (2.286 cm) by 0.4 in. (1.016 cm) is filled with lossless polystyrene ($\epsilon_r = 2.56$). For the lowest-order mode of the waveguide, determine at 10GHz the following values.

(a) Cutoff Freq (f_c) in GHz
We can find this with expression $(8-16)$ from [1] with $a = 2.286$ cm.

$$\begin{aligned}
(f_c)_{mn} &= \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \\
(f_c)_{10} &= \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{1\pi}{a}\right)^2 + \left(\frac{0\pi}{b}\right)^2} \\
&= \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2} \\
&= \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{\pi}{2.86*10^{-2}}\right)^2} = 4.09832GHz
\end{aligned}$$

(b) Guide wavelength (λ_g) in cm
This one can be done with a combination of expressions from chapter 8 in [1], $(8-21a, b, c)$. we can do this with a because the frequency in question is $10GHz$ which us above the cutoff found in part a.

$$\begin{aligned}
(\lambda_g)_{mn} &= \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \\
&= \frac{\lambda}{\sqrt{1-\left(\frac{4.09832e^9}{1e^{10}}\right)^2}} \\
&= \frac{0.0187375}{\sqrt{1-(.409832)^2}} \\
&= 0.0205419\ [m] = 2.05419\ [cm]
\end{aligned}$$

(c) Wave impedance (η)

This can be found with $(8-20a)$ in [1].

$$
\begin{aligned}
Z_w^+ &= \frac{\sqrt{\frac{\mu}{\epsilon}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \\
Z_w^+ &= \frac{\sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}}}{\sqrt{1-(.409832)^2}} \\
\eta_0 &= 120\pi \\
&= \frac{\frac{\eta_0}{\sqrt{\epsilon_r}}}{\sqrt{1-(.409832)^2}} = \frac{120\pi}{\sqrt{2.56}\cdot 0.912161} \\
&= 258.309\ \Omega
\end{aligned}
$$

(d) Phase velocity (v_p) in $\frac{m}{s}$

(e) Group velocity in $\frac{m}{s}$
This velocity, as well as the group velocity, can be found using expression $(8.49a-c)$ from [1]. These reduce to a simple expression in $(8-52)$

$$
\begin{aligned}
v_p &= \frac{v}{\cos(\Psi)} \\
v_g &= v\cos(\Psi) \\
\Psi &= \cos^{-1}\left(\sqrt{1-\left(\frac{f_c}{f}\right)^2}\right)
\end{aligned}
$$

Since inverse cos returns an angle, taking the cos of the inverse is the argument to the inverse itself.

$$
\begin{aligned}
v_p &= \frac{v}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \\
v_g &= v\sqrt{1-\left(\frac{f_c}{f}\right)^2}
\end{aligned}
$$

Since v is the expression $\frac{c}{\sqrt{\epsilon_r}}$, we can input those into the expression and solve for the speeds.

$$v_p = \frac{c}{\sqrt{\epsilon_r}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} =$$

$$v_g = \frac{c}{\sqrt{\epsilon_r}}\sqrt{1 - \left(\frac{f_c}{f}\right)^2} =$$

2 - 9.14

A circular waveguide with radius of $3cm$ is made of copper ($\sigma = 5.76107\frac{S}{m}$). For the dominant TE_{11} and low-loss TE_{01} modes, determine their corresponding cut-off frequencies and attenuation constants (in $\frac{Np}{m}$ and $\frac{dB}{m}$ at a frequency of $7GHz$. Assume that the waveguide is filled with air.

3 - 11.2

A magnetic line source of infinite length and constant magnetic current Im is placed parallel to the z axis at a height h above a PEC ground plane of infinite extent, as shown in Figure 11-2 except that we now have a magnetic line source.

(a) Determine the total magnetic field at ρ, ϕ for $0 \leq \phi \leq 180°$.

(b) Simplify the expressions when the observations are made at very large distances (far field).

(c) Determine the smallest height h (in λ) that will introduce a null in the far field amplitude pattern at:

- $\phi = 30°$
- $\phi = 90°$

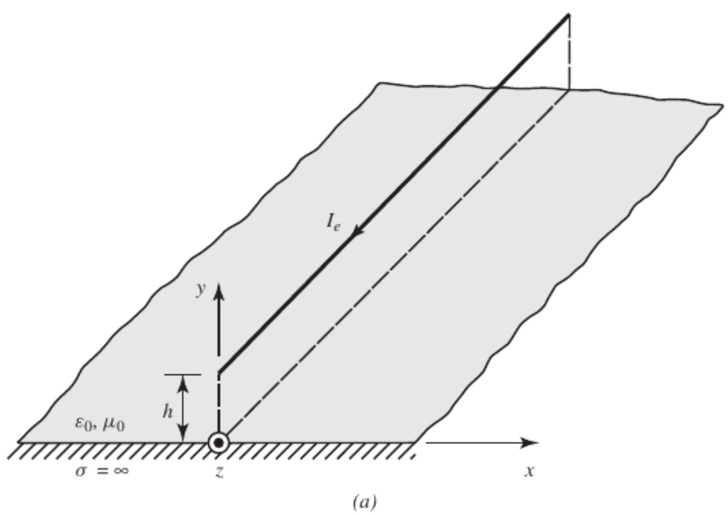

Figure 1: fig. 11-2

4 - 11.20

For a strip of width $w = 2\lambda$, plot the $\frac{RCS}{\lambda_o^2}$ (in dB) when the length of the strip is $l = 5\lambda$, 10λ and 20λ (plot all three graphs on the same figure). Use the approximate relation between the 2D SW and the 3D RCS. Assume normal incidence.

References

[1] Constantine A. Balanis. *advanced engineering electromagnetics*, chapter 1, page 2–3. John Wiley & Sons, 2024.