# ECE 6310 - Advanced Electromagnetic Fields: Homework Set #3

Miguel Gomez

2024-02-15

## Preliminaries

In this document, we use standard notation for electromagnetic theory. Key equations and concepts are summarized below:

### Vector Notation

- $\boldsymbol{\mathscr{E}}$: Electric field intensity
- $\boldsymbol{\mathscr{H}}$: Magnetic field intensity
- $\boldsymbol{\mathscr{D}}$: Electric flux density
- $\boldsymbol{\mathscr{B}}$: Magnetic flux density
- $\boldsymbol{\mathscr{J}}$: Current density
- $\rho_v$: Volume charge density

### Differential Operators

- $\nabla\cdot$ : Divergence of a vector field
- $\nabla\times$ : Curl of a vector field
- $\nabla$ : Gradient of a scalar field
- $\partial_i$ : Partial derivative with respect to the independent basis element $i$

## Maxwell's Equations

In integral form, Maxwell's equations are given by:

$$\oint_{\partial V} \boldsymbol{\mathscr{E}} \cdot d\boldsymbol{\mathscr{l}} = -\frac{d}{dt}\int_V \boldsymbol{\mathscr{B}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Faraday's Law of Induction)} \tag{1}$$

$$\oint_{\partial V} \boldsymbol{\mathscr{H}} \cdot d\boldsymbol{\mathscr{l}} = \int_V \boldsymbol{\mathscr{J}} \cdot d\boldsymbol{\mathcal{S}} + \frac{d}{dt}\int_V \boldsymbol{\mathscr{D}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Ampère's Circuital Law)} \tag{2}$$

$$\oiint_{\partial V} \boldsymbol{\mathscr{D}} \cdot d\boldsymbol{\mathcal{S}} = \int_V \rho_v dV \qquad \text{(Gauss's Law for Electricity)} \tag{3}$$

$$\oiint_{\partial V} \boldsymbol{\mathscr{B}} \cdot d\boldsymbol{\mathcal{S}} = 0 \qquad \text{(Gauss's Law for Magnetism)} \tag{4}$$

## Other Relevant Equations

- Continuity Equation: $\nabla \cdot \boldsymbol{\mathscr{J}} + \partial_t \rho_v = 0$
- Relationship between $\boldsymbol{\mathscr{E}}$, $\boldsymbol{\mathscr{D}}$: $\boldsymbol{\mathscr{D}} = \epsilon \boldsymbol{\mathscr{E}}$
- Relationship between $\boldsymbol{\mathscr{H}}$, $\boldsymbol{\mathscr{B}}$: $\boldsymbol{\mathscr{B}} = \mu \boldsymbol{\mathscr{H}}$

## Boundary Conditions

Discuss the boundary conditions for $\boldsymbol{\mathscr{E}}$, $\boldsymbol{\mathscr{H}}$, $\boldsymbol{\mathscr{D}}$, and $\boldsymbol{\mathscr{B}}$ at interfaces between different media.

# 1 - 6.19

Show that for observations made at very large distances ($\beta r \gg 1$) the electric and magnetic fields of Example 6-3 reduce to the following:

$$
\begin{aligned}
E_\theta &= \\
H_\phi &\approx \\
E_r &\approx 0 \\
E_\phi = H_r = H_\theta &= 0
\end{aligned}
$$

# 2 - 6.20

# 3 - 6.25

# 4 - 6.26

# 5 - 6.38

# 6 - 7.3

# 7 - 7.15

# 8 - 7.37

# References

[1] Constantine A. Balanis. *advanced engineering electromagnetics*, chapter 1, page 2–3. John Wiley & Sons, 2024.