

# ECE 6310 - Advanced Electromagnetic Fields: Homework Set #1

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8 days - 9 hours - 33 min until deadline

*A B C D E F G H I J K L M N O P Q R S T U V W X Y Z*  
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## Problem 1.3

The electric flux density inside a cube is given by:

(a)  $\vec{D} = \hat{a}_x(3 + x)$

(b)  $\vec{D} = \hat{a}_y(4 + y^2)$

Find the total electric charge enclosed inside the cubical volume when the cube is in the first octant with three edges coincident with the x, y, z axes and one corner at the origin. Each side of the cube is 1 m long.

Ok, this one we can do by using the following expressions from the text [1]:

$$\oiint_s \mathcal{D} ds = \iiint_v \mathcal{q}_{ev} dv = \mathcal{Q}_e \quad (1)$$

$$\mathcal{q}_{ev} = \Delta \cdot \mathcal{D} \quad (2)$$

We apply expression 2 then plug into expression 1 to get the total electric charge.

$$\iiint_V \mathcal{q}_{ev} = \iiint_V \Delta \cdot (\hat{a}_x(3 + x)) dV = \iiint_V \partial_x(\hat{a}_x(3 + x)) dV = \iiint_0^1 1 dV = 1$$

$$\begin{aligned}\iiint_V \mathcal{Q}_{ev} &= \iiint_V \Delta \cdot (\hat{a}_y(4 + y^2))dV = \iiint_V \partial_y(\hat{a}_y(4 + y^2))dV = \iiint_0^1 2y \, dV \\ \iiint_0^1 2y \, dV &= \iint_0^1 \int_0^1 2y dy \, ds = \iint_0^1 y^2|_0^1 ds = \iint_0^1 (1 - 0) \, ds = 1\end{aligned}$$

## Problem 1.4

An infinite planar interface between media, as shown in the figure, is formed by having air (medium #1) on the left of the interface and lossless polystyrene (medium #2) to the right of the interface. An electric surface charge density  $\sigma_{es} = 0.2 \text{ C/m}^2$  exists along the entire interface. The static electric flux density inside the polystyrene is given by  $\vec{D}_2 = 6\hat{a}_x + 3\hat{a}_z \text{ C/m}^2$ . Determine the corresponding vector:

- (a) Electric field intensity inside the polystyrene.
- (b) Electric polarization vector inside the polystyrene.
- (c) Electric flux density inside the air medium.
- (d) Electric field intensity inside the air medium.
- (e) Electric polarization vector inside the air medium.

Leave your answers in terms of  $\varepsilon_0, \mu_0$ .

## Problem 1.13

The instantaneous magnetic flux density in free space is given by:

$$\vec{\mathcal{B}} = \hat{a}_x B_x \cos(2y) \sin(\omega t - \pi z) + \hat{a}_y B_y \cos(2x) \cos(\omega t - \pi z)$$

where  $B_x$  and  $B_y$  are constants. Assuming there are no sources at the observation points  $x, y$ , determine the electric displacement current density.

## Problem 1.20

The instantaneous electric field inside a conducting rectangular pipe (waveguide) of width  $a$  is given by:

$$\vec{\mathcal{E}} = \hat{a}_y E_0 \sin\left(\frac{\pi x}{a}\right) \cos(\omega t - \beta_z z)$$

where  $\beta_z$  is the waveguide's phase constant. Assuming there are no sources within the free-space-filled pipe determine the:

- (a) Corresponding instantaneous magnetic field components inside the conducting pipe.
- (b) Phase constant  $\beta_z$ .
- (c) The height of the waveguide is  $b$ .

## Problem 2.18

The time-varying electric field inside a lossless dielectric material of polystyrene, of infinite dimensions and with a relative permittivity (dielectric constant) of 2.56, is given by:

$$\vec{\mathcal{E}} = \hat{a}_x 10^{-3} \sin(2\pi \times 10^7 t) \text{ V/m}$$

Determine the corresponding:

- (a) Electric susceptibility of the dielectric material.
- (b) Time-harmonic electric flux density vector.
- (c) Time-harmonic electric polarization vector.
- (d) Time-harmonic displacement current density vector.
- (e) Time-harmonic polarization current density vector defined as the partial derivative of the corresponding electric polarization vector.

Leave your answers in terms of  $\varepsilon_0, \mu_0$ .

## Problem 2.25

Aluminum has a static conductivity of about  $\sigma = 3.96 \times 10^7 \text{ S/m}$  and an electron mobility of  $\mu_e = 2.2 \times 10^{-3} \text{ m}^2/(\text{V}\cdot\text{s})$ . Assuming that an electric field of  $\vec{E} = \hat{a}_x 2 \text{ V/m}$  is applied perpendicularly to the square area of an aluminum wafer with cross-sectional area of about  $10 \text{ cm}^2$ , find the:

- (a) Electron charge density  $q_{ve}$ .
- (b) Electron drift velocity  $v_e$ .
- (c) Electric current density  $J$ .
- (d) Electric current flowing through the square cross section of the wafer.
- (e) Electron density  $N_e$ .

Leave your answers in terms of  $\varepsilon_0, \mu_0$ .

## References

- [1] Constantine A. Balanis. *advanced engineering electromagnetics*, chapter 1, page 2–3. John Wiley & Sons, 2024.