ECE 6310 - Advanced Electromagnetic Fields: Homework Set #3

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1 Preliminaries

In this document, we use standard notation for electromagnetic theory. Key equations and concepts are summarized below:

Vector Notation

- $\mathscr{E}$: Electric field intensity

- $\mathscr{H}$: Magnetic field intensity

- $\mathscr{D}$: Electric flux density

- $\mathscr{B}$: Magnetic flux density

- $\mathscr{J}$: Current density

- ρ_v: Volume charge density

Differential Operators

- $\nabla \cdot$: Divergence of a vector field

- $\nabla \times$: Curl of a vector field

- ∇ : Gradient of a scalar field

- ∂_i : Partial derivative with respect to the independent basis element i

Maxwell's Equations

In integral form, Maxwell's equations are given by:

$$\oint_{\partial V} \boldsymbol{\mathcal{E}} \cdot d\boldsymbol{\ell} = -\frac{d}{dt} \int_V \boldsymbol{\mathcal{B}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Faraday's Law of Induction)}$$

$$(1)$$

$$\oint_{\partial V} \boldsymbol{\mathcal{H}} \cdot d\boldsymbol{\ell} = \int_V \boldsymbol{\mathcal{J}} \cdot d\boldsymbol{\mathcal{S}} + \frac{d}{dt} \int_V \boldsymbol{\mathcal{D}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Ampère's Circuital Law)}$$

$$(2)$$

$$\oiint_{\partial V} \boldsymbol{\mathcal{D}} \cdot d\boldsymbol{\mathcal{S}} = \int_V \rho_v dV \qquad \text{(Gauss's Law for Electricity)}$$

$$(3)$$

$$\oiint_{\partial V} \boldsymbol{\mathcal{B}} \cdot d\boldsymbol{\mathcal{S}} = 0 \qquad \text{(Gauss's Law for Magnetism)}$$

$$(4)$$

Other Relevant Equations

- Continuity Equation: $\nabla \cdot \boldsymbol{\mathcal{J}} + \partial_t \rho_v = 0$

- Relationship between $\boldsymbol{\mathcal{E}}$, $\boldsymbol{\mathcal{D}}$: $\boldsymbol{\mathcal{D}} = \epsilon \boldsymbol{\mathcal{E}}$

- Relationship between $\boldsymbol{\mathcal{H}}$, $\boldsymbol{\mathcal{B}}$: $\boldsymbol{\mathcal{B}} = \mu \boldsymbol{\mathcal{H}}$

Boundary Conditions

Discuss the boundary conditions for $\boldsymbol{\mathcal{E}}$, $\boldsymbol{\mathcal{H}}$, $\boldsymbol{\mathcal{D}}$, and $\boldsymbol{\mathcal{B}}$ at interfaces between different media.

References

[1] Constantine A. Balanis. *advanced engineering electromagnetics*, chapter 1, page 2–3. John Wiley & Sons, 2024.