ECE 6310 - Advanced Electromagnetic Fields: Homework Set #3

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Preliminaries

In this document, we use standard notation for electromagnetic theory. Key equations and concepts are summarized below:

Vector Notation

- $\boldsymbol{\mathscr{E}}$: Electric field intensity
- $\boldsymbol{\mathscr{H}}$: Magnetic field intensity
- $\boldsymbol{\mathscr{D}}$: Electric flux density
- $\boldsymbol{\mathscr{B}}$: Magnetic flux density
- $\boldsymbol{\mathscr{J}}$: Current density
- ρ_v: Volume charge density

Differential Operators

- $\nabla\cdot$: Divergence of a vector field
- $\nabla\times$: Curl of a vector field
- ∇ : Gradient of a scalar field
- ∂_i : Partial derivative with respect to the independent basis element i

Maxwell's Equations

In integral form, Maxwell's equations are given by:

$$\oint_{\partial V} \boldsymbol{\mathscr{E}} \cdot d\boldsymbol{\ell} = -\frac{d}{dt}\int_V \boldsymbol{\mathscr{B}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Faraday's Law of Induction)} \tag{1}$$

$$\oint_{\partial V} \boldsymbol{\mathscr{H}} \cdot d\boldsymbol{\ell} = \int_V \boldsymbol{\mathscr{J}} \cdot d\boldsymbol{\mathcal{S}} + \frac{d}{dt}\int_V \boldsymbol{\mathscr{D}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Ampère's Circuital Law)} \tag{2}$$

$$\oiint_{\partial V} \boldsymbol{\mathscr{D}} \cdot d\boldsymbol{\mathcal{S}} = \int_V \rho_v dV \qquad \text{(Gauss's Law for Electricity)} \tag{3}$$

$$\oiint_{\partial V} \boldsymbol{\mathscr{B}} \cdot d\boldsymbol{\mathcal{S}} = 0 \qquad \text{(Gauss's Law for Magnetism)} \tag{4}$$

Other Relevant Equations

- Continuity Equation: $\nabla \cdot \boldsymbol{\mathscr{J}} + \partial_t \rho_v = 0$
- Relationship between $\boldsymbol{\mathscr{E}}$, $\boldsymbol{\mathscr{D}}$: $\boldsymbol{\mathscr{D}} = \epsilon \boldsymbol{\mathscr{E}}$
- Relationship between $\boldsymbol{\mathscr{H}}$, $\boldsymbol{\mathscr{B}}$: $\boldsymbol{\mathscr{B}} = \mu \boldsymbol{\mathscr{H}}$

Boundary Conditions

Discuss the boundary conditions for $\boldsymbol{\mathscr{E}}$, $\boldsymbol{\mathscr{H}}$, $\boldsymbol{\mathscr{D}}$, and $\boldsymbol{\mathscr{B}}$ at interfaces between different media.

1 - 6.19

Show that for observations made at very large distances ($\beta r \gg 1$) the electric and magnetic fields of Example 6-3 reduce to the following:

$$\begin{aligned} E_\theta &= \\ H_\phi &\approx \\ E_r &\approx 0 \\ E_\phi = H_r = H_\theta &= 0 \end{aligned}$$

2 - 6.20

For 6.19, show that

- the time average power density is:

$$\mathbf{S} = \frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*]$$

- Radiation Intensity
- Radiated Power
- Directivity
- Radiation Resistance

3 - 6.25

The current distribution on a very thin wire dipole antenna of overall length ℓ is given by:

$$I_e = \begin{cases} \hat{a}z I_0 \sin[\beta\left(\frac{\ell}{2} - z'\right)] & \text{if } 0 \leq z' \leq \frac{\ell}{2} \\ \hat{a}z I_0 \sin[\beta\left(\frac{\ell}{2} + z'\right)] & \text{if } -\frac{\ell}{2} \leq z' \leq 0 \end{cases}$$

where I0 is a constant. Representing the distance R of (6-112) by the far-field approximations of (6-112a) through (6-112b), derive the far-zone electric and magnetic fields radiated by the dipole using (6-97a) and the far-field formulations of Section 6.7

4 - 6.26

Show that the radiated far-zone electric and magnetic fields derived in Problem 6.25 reduce for a half-wavelength dipole ($\ell = \frac{\lambda}{2}$) to:

$$E_\theta \approx j\eta \frac{I_0 e^{-j\beta r}}{2\pi r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]$$

$$H_\phi \approx \frac{E_\theta}{\eta}$$

$$E_r \approx E_\phi \approx H_r \approx H_\theta \approx 0$$

5 - 6.38

A coaxial line of inner and outer radii a and b, respectively, is mounted on an infinite conducting ground plane. Assuming that the electric field over the aperture of the coax is:

$$E_a = -\hat{a}_\rho \frac{V}{\varepsilon \ln(b/a)\rho'}$$

$$\text{where } a \leq \rho' \leq b$$

where V is the applied voltage and is the permittivity of medium in the coax, find the far-zone spherical electric and magnetic field components radiated by the aperture.

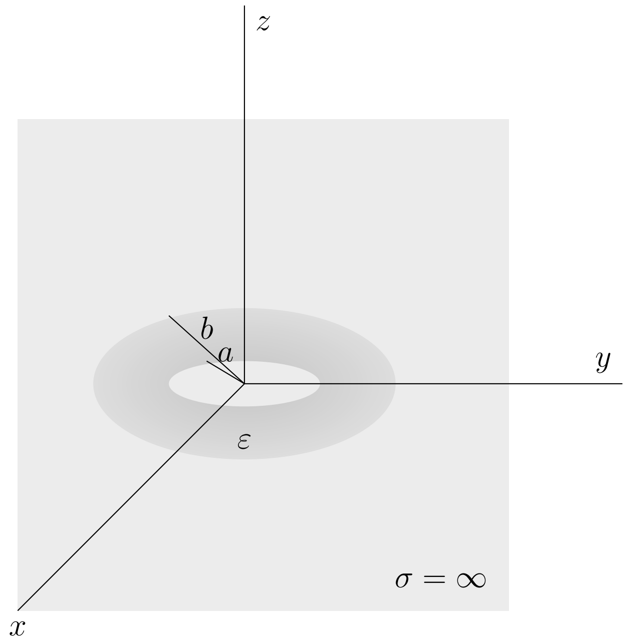

Figure P6-38

6 - 7.3

7 - 7.15

8 - 7.37

References

[1] Constantine A. Balanis. *advanced engineering electromagnetics*, chapter 1, page 2–3. John Wiley & Sons, 2024.