

ECE 6310 - Advanced Electromagnetic Fields: Homework Set #4

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4 days - 10 hours - 42 min until deadline

Preliminaries

In this document, we use standard notation for electromagnetic theory. Key equations and concepts are summarized below:

Vector Notation

- \mathcal{E} : Electric field intensity
- \mathcal{H} : Magnetic field intensity
- \mathcal{D} : Electric flux density
- \mathcal{B} : Magnetic flux density
- \mathcal{J} : Current density
- ρ_v : Volume charge density

Differential Operators

- $\nabla \cdot$: Divergence of a vector field
- $\nabla \times$: Curl of a vector field
- ∇ : Gradient of a scalar field
- ∂_i : Partial derivative with respect to the independent basis element i

Maxwell's Equations

In integral form, Maxwell's equations are given by:

$$\oint_{\partial V} \mathcal{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt} \int_V \mathcal{B} \cdot d\boldsymbol{\mathcal{S}} \quad (\text{Faraday's Law of Induction}) \quad (1)$$

$$\oint_{\partial V} \mathcal{H} \cdot d\boldsymbol{\ell} = \int_V \mathcal{J} \cdot d\boldsymbol{\mathcal{S}} + \frac{d}{dt} \int_V \mathcal{D} \cdot d\boldsymbol{\mathcal{S}} \quad (\text{Ampère's Circuital Law}) \quad (2)$$

$$\oiint_{\partial V} \mathcal{D} \cdot d\boldsymbol{\mathcal{S}} = \int_V \rho_v dV \quad (\text{Gauss's Law for Electricity}) \quad (3)$$

$$\oiint_{\partial V} \mathcal{B} \cdot d\boldsymbol{\mathcal{S}} = 0 \quad (\text{Gauss's Law for Magnetism}) \quad (4)$$

Other Relevant Equations

- Continuity Equation: $\nabla \cdot \mathcal{J} + \partial_t \rho_v = 0$
- Relationship between \mathcal{E} , \mathcal{D} : $\mathcal{D} = \epsilon \mathcal{E}$
- Relationship between \mathcal{H} , \mathcal{B} : $\mathcal{B} = \mu \mathcal{H}$

Boundary Conditions

Discuss the boundary conditions for \mathcal{E} , \mathcal{H} , \mathcal{D} , and \mathcal{B} at interfaces between different media.

1 - 8.2

A standard X-band (8.2–12.4GHz) rectangular waveguide with inner dimensions of 0.9 in. (2.286 cm) by 0.4 in. (1.016 cm) is filled with lossless polystyrene ($\epsilon_r = 2.56$). For the lowest-order mode of the waveguide, determine at 10GHz the following values.

(a) Cutoff Freq (f_c) in GHz

We can find this with expression (8 – 16) from [1] with $a = 2.286$ cm.

$$\begin{aligned}(f_c)_{mn} &= \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \\(f_c)_{10} &= \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{1\pi}{a}\right)^2 + \left(\frac{0\pi}{b}\right)^2} \\&= \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2} \\&= \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{\pi}{2.86 * 10^{-2}}\right)^2} = 4.09832\text{GHz}\end{aligned}$$

(b) Guide wavelength (λ_g) in cm

This one can be done with a combination of expressions from chapter 8 in [1], (8 – 21a, b, c). we can do this with a because the frequency in question is 10GHz which is above the cutoff found in part a.

$$\begin{aligned}(\lambda_g)_{mn} &= \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \\&= \frac{\lambda}{\sqrt{1 - \left(\frac{4.09832e^9}{1e^{10}}\right)^2}} \\&= \frac{0.0187375}{\sqrt{1 - (.409832)^2}} \\&= 0.0205419 [m] = 2.05419 [cm]\end{aligned}$$

(c) Wave impedance (η)

This can be found with $(8 - 20a)$ in [1].

$$\begin{aligned}
Z_w^+ &= \frac{\sqrt{\frac{\mu}{\epsilon}}}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \\
Z_w^+ &= \frac{\sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}}}{\sqrt{1 - (.409832)^2}} \\
\eta_0 &= 120\pi \\
&= \frac{\frac{\eta_0}{\sqrt{\epsilon_r}}}{\sqrt{1 - (.409832)^2}} = \frac{120\pi}{\sqrt{2.56 \cdot 0.912161}} \\
&= 258.309 \Omega
\end{aligned}$$

(d) Phase velocity (v_p) in $\frac{m}{s}$

(e) Group velocity in $\frac{m}{s}$

This velocity, as well as the group velocity, can be found using expression $(8.49a - c)$ from [1]. These reduce to a simple expression in $(8 - 52)$

$$\begin{aligned}
v_p &= \frac{v}{\cos(\Psi)} \\
v_g &= v \cos(\Psi) \\
\Psi &= \cos^{-1} \left(\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \right)
\end{aligned}$$

Since inverse cos returns an angle, taking the cos of the inverse is the argument itself.

$$\begin{aligned}
v_p &= \frac{v}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \\
v_g &= v \sqrt{1 - \left(\frac{f_c}{f}\right)^2}
\end{aligned}$$

2 - 9.14

A circular waveguide with radius of $3cm$ is made of copper ($\sigma = 5.76107 \frac{S}{m}$). For the dominant TE_{11} and low-loss TE_{01} modes, determine their corresponding cut-off frequencies and attenuation constants (in $\frac{Np}{m}$ and $\frac{dB}{m}$ at a frequency of $7GHz$. Assume that the waveguide is filled with air.

3 - 11.2

A magnetic line source of infinite length and constant magnetic current I_m is placed parallel to the z axis at a height h above a PEC ground plane of infinite extent, as shown in Figure 11-2 except that we now have a magnetic line source.

- (a) Determine the total magnetic field at ρ, ϕ for $0 \leq \phi \leq 180^\circ$.
- (b) Simplify the expressions when the observations are made at very large distances (far field).
- (c) Determine the smallest height h (in λ) that will introduce a null in the far field amplitude pattern at:
 - $\phi = 30^\circ$
 - $\phi = 90^\circ$

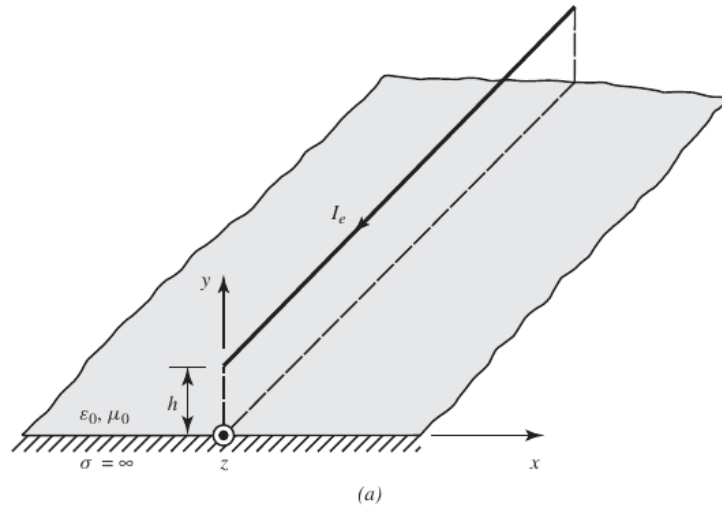


Figure 1: fig. 11-2

4 - 11.20

For a strip of width $w = 2\lambda$, plot the $\frac{RCS}{\lambda_o^2}$ (in dB) when the length of the strip is $l = 5\lambda$, 10λ and 20λ (plot all three graphs on the same figure). Use the approximate relation between the 2D SW and the 3D RCS. Assume normal incidence.

References

- [1] Constantine A. Balanis. *advanced engineering electromagnetics*, chapter 1, page 2–3. John Wiley & Sons, 2024.