ECE 6310 - Advanced Electromagnetic Fields: Homework Set #1

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8 days - 7 hours - 44 min until deadline

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1 Preliminaries

In this document, we use standard notation for electromagnetic theory. Key equations and concepts are summarized below:

Vector Notation

- $\mathbf{E}$: Electric field intensity
- $\mathbf{H}$: Magnetic field intensity
- $\mathbf{D}$: Electric flux density
- $\mathbf{B}$: Magnetic flux density
- $\mathbf{J}$: Current density
- ρ_v: Volume charge density

Differential Operators

- $\nabla\cdot$: Divergence of a vector field
- $\nabla\times$: Curl of a vector field

- ∇ : Gradient of a scalar field
- ∂_i : Partial derivative with respect to the independent basis element i

Maxwell's Equations

In integral form, Maxwell's equations are given by:

$$\oint_{\partial V} \boldsymbol{\mathscr{E}} \cdot d\boldsymbol{\ell} = -\frac{d}{dt}\int_V \boldsymbol{\mathscr{B}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Faraday's Law of Induction)} \tag{1}$$

$$\oint_{\partial V} \boldsymbol{\mathscr{H}} \cdot d\boldsymbol{\ell} = \int_V \boldsymbol{\mathscr{J}} \cdot d\boldsymbol{\mathcal{S}} + \frac{d}{dt}\int_V \boldsymbol{\mathscr{D}} \cdot d\boldsymbol{\mathcal{S}} \qquad \text{(Ampère's Circuital Law)} \tag{2}$$

$$\oiint_{\partial V} \boldsymbol{\mathscr{D}} \cdot d\boldsymbol{\mathcal{S}} = \int_V \rho_v dV \qquad \text{(Gauss's Law for Electricity)} \tag{3}$$

$$\oiint_{\partial V} \boldsymbol{\mathscr{B}} \cdot d\boldsymbol{\mathcal{S}} = 0 \qquad \text{(Gauss's Law for Magnetism)} \tag{4}$$

Other Relevant Equations

- Continuity Equation: $\nabla \cdot \mathbf{J} + \partial_t \rho_v = 0$
- Relationship between $\mathbf{E}$, $\mathbf{D}$: $\mathbf{D} = \epsilon \mathbf{E}$
- Relationship between $\mathbf{H}$, $\mathbf{B}$: $\mathbf{B} = \mu \mathbf{H}$

Boundary Conditions

Discuss the boundary conditions for $\mathbf{E}$, $\mathbf{H}$, $\mathbf{D}$, and $\mathbf{B}$ at interfaces between different media.

Problem 1.3

The electric flux density inside a cube is given by:

(a) $\vec{D} = \hat{a}_x(3 + x)$

(b) $\vec{D} = \hat{a}_y(4 + y^2)$

Find the total electric charge enclosed inside the cubical volume when the cube is in the first octant with three edges coincident with the x, y, z axes and one corner at the origin. Each side of the cube is 1 m long.
Ok, this one we can do by using the following expressions from the text [1]:

$$\oiint_s \boldsymbol{\mathscr{D}} ds = \iiint_v \mathscr{q}_{ev} dv = \mathscr{Q}_e \tag{5}$$

$$\mathscr{q}_{ev} = \Delta \cdot \boldsymbol{\mathscr{D}} \tag{6}$$

We apply expression 6 then plug into expression 5 to get the total electric charge.

$$\iiint_V \mathscr{q}_{ev} = \iiint_V \Delta \cdot (\hat{a}_x(3+x))dV = \iiint_V \partial_x(\hat{a}_x(3+x))dV = \iiint_0^1 1dV = 1$$

$$\iiint_V \mathscr{q}_{ev} = \iiint_V \Delta \cdot (\hat{a}_y(4+y^2))dV = \iiint_V \partial_y(\hat{a}_y(4+y^2))dV = \iiint_0^1 2y\ dV$$

$$\iiint_0^1 2y\ dV = \iint_0^1 \int_0^1 2ydy\ ds = \iint_0^1 y^2|_0^1 ds = \iint_0^1 (1-0)\ ds = 1$$

Problem 1.4

An infinite planar interface between media, as shown in the figure, is formed by having air (medium #1) on the left of the interface and lossless polystyrene (medium #2) to the right of the interface. An electric surface charge density $\sigma_{es} = 0.2\,\mathrm{C/m^2}$ exists along the entire interface. The static electric flux density inside the polystyrene is given by $\vec{D}_2 = 6\hat{a}_x + 3\hat{a}_z\,\mathrm{C/m^2}$. Determine the corresponding vector:

(a) Electric field intensity inside the polystyrene.

(b) Electric polarization vector inside the polystyrene.

(c) Electric flux density inside the air medium.

(d) Electric field intensity inside the air medium.

(e) Electric polarization vector inside the air medium.

Leave your answers in terms of ε_0, μ_0.

Problem 1.13

The instantaneous magnetic flux density in free space is given by:

$$\vec{\mathscr{B}} = \hat{a}_x B_x \cos(2y)\sin(\omega t - \pi z) + \hat{a}_y B_y \cos(2x)\cos(\omega t - \pi z)$$

where B_x and B_y are constants. Assuming there are no sources at the observation points x, y, determine the electric displacement current density.

Problem 1.20

The instantaneous electric field inside a conducting rectangular pipe (waveguide) of width a is given by:

$$\vec{\mathscr{E}} = \hat{a}_y E_0 \sin\left(\frac{\pi x}{a}\right)\cos(\omega t - \beta_z z)$$

where β_z is the waveguide's phase constant. Assuming there are no sources within the free-space-filled pipe determine the:

(a) Corresponding instantaneous magnetic field components inside the conducting pipe.

(b) Phase constant β_z.

(c) The height of the waveguide is b.

Problem 2.18

The time-varying electric field inside a lossless dielectric material of polystyrene, of infinite dimensions and with a relative permittivity (dielectric constant) of 2.56, is given by:

$$\vec{\mathscr{E}} = \hat{a}_x 10^{-3}\sin(2\pi \times 10^7 t)\ \mathrm{V/m}$$

Determine the corresponding:

(a) Electric susceptibility of the dielectric material.

(b) Time-harmonic electric flux density vector.

(c) Time-harmonic electric polarization vector.

(d) Time-harmonic displacement current density vector.

(e) Time-harmonic polarization current density vector defined as the partial derivative of the corresponding electric polarization vector.

Leave your answers in terms of ε_0, μ_0.

Problem 2.25

Aluminum has a static conductivity of about $\sigma = 3.96 \times 10^7\,\mathrm{S/m}$ and an electron mobility of $\mu_e = 2.2 \times 10^{-3}\,\mathrm{m^2/(V\text{-}s)}$. Assuming that an electric field of $\vec{E} = \hat{a}_x 2\,\mathrm{V/m}$ is applied perpendicularly to the square area of an aluminum wafer with cross-sectional area of about $10\,\mathrm{cm}^2$, find the:

(a) Electron charge density q_{ve}.

(b) Electron drift velocity v_e.

(c) Electric current density J.

(d) Electric current flowing through the square cross section of the wafer.

(e) Electron density N_e.

Leave your answers in terms of ε_0, μ_0.

References

[1] Constantine A. Balanis. *advanced engineering electromagnetics*, chapter 1, page 2–3. John Wiley & Sons, 2024.