ECE 5530/6530 Digital Signal Processing
Assignment 5

Submission guidelines (READ CAREFULLY)

- All submissions (pdf or doc) must be done using Canvas.
- Scanned hand written submissions will be accepted; however, it is the student's responsibility to ensure that the answers are easily legible. Otherwise, you may lose points.
- Make sure you are using the correct edition (4th) of the textbook to ensure you are solving the correct problems.
- For late policy refer to syllabus.

Questions from the textbook

1. Problem 7.1 *(10 points)*
2. Problem 7.2 part (a) *(10 points)* → work shown in mathematica
3. Problem 7.4 parts (a) and (b) *(10 points)* → work done in mathematica
4. Problem 7.7 solve only for $x_c(n)$ *(10 points)*
5. Problem 7.8 *(10 points)* * more work shown in Mathematica
6. Problem 7.9 *(10 points)*

   You may use MATLAB for your computations and only write down the intermediate transform results and the final transform and final sequence.
7. Problem 7.11 part (a) *(10 points)*

   Clarification let $X(k)$ be the eight point DFT of $x(n)$. Find the DFTs of $x_1(n)$ and $x_2(n)$ in terms of $X(k)$.
8. Problem 7.23 parts (b) and (d) *(10 points)*
9. Problem 8.4 *(10 points)*
10. Problem 8.6 *(10 points)*

Bonus
6
7
9
10

7.1 The first five points of the eight-point DFT of a real-valued sequence are $\{0.25, 0.125 - j0.3018, 0, 0.125 - j0.0518, 0\}$. Determine the remaining three points.

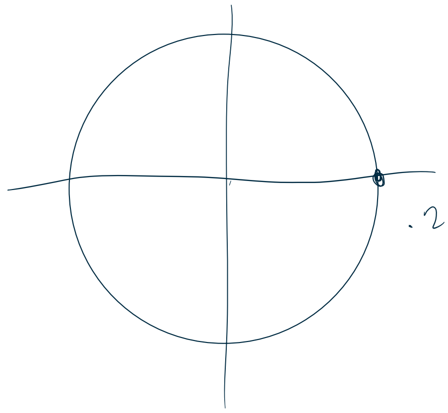
$\{0.25, [.125, -.3018], [0.125, -.0518], 0\}$. - - - - -

3 4 5 6 7

$N = 8$

$x(n) = ?$

$$C_k = \sum_{n=0}^{7} x(n) e^{-j\frac{2\pi k n}{N}}$$



Cyclic w/ periodicity 8

1	2	3	4	5	6	7	8
∴ {0.25,	b,	0,	c,	?, c^*,	?, 0,	?, b^*,	?, .25 }

$0 = 8 = 16 = 24 \ldots$ mod $8 = 0 \rightarrow x(0)$

$1 = 9 = 17 = 25 \ldots$

$2 = 10 = 18 = 26 \ldots$

$3 = 11 = 19 = 27 \ldots$

$4 = 12 = 20 = 28 \ldots$

⋮

$7 = 15 = 23 = 31 \ldots$

8 + 1	2	3	4	5	6	7	8
∴ {0.25,	−b,	−c,	0,	?,	?,	?,	? }
9	10	11	12	13	14	15	16

Real-valued by, 7.2.24 $|X(N-k)| = |X^*(k)|$

by $-k \% N = N-k \% N$

$|X(8-8)| = |X^*(8)|$

$|X(0)| = |X^*(8)| = 0.25$

$|X(8-7)| = |X^*(7)| = |(0.125 - j.3018)|$

$|X(1)| =$ & by *, $\angle X^*(k) = -\angle X^*(k)$

$\therefore\ X(7) = 0.125 + j.3018$

$X(6) = 0$

$X(5) = 0.125 + j.0518$

$$\{\overset{1}{0.25}, \overset{2}{0.125 - j0.318}, \overset{3}{0}, \overset{4}{0.125 - j0.0518}, \overset{5}{0.125 + .0518}, \overset{6}{0}, \overset{7}{0.125 + j0.3018}, \overset{8}{0.25}\}$$

7.2 Compute the eight-point circular convolution for the following sequences.

(a) $x_1(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

$x_2(n) = \sin \frac{3\pi}{8} n, \qquad 0 \le n \le 7$

$X_1 \circledN X_2 = X_1 \cdot X_2$

$x_2(n) = \left\{0, \frac{3\pi}{8}, \frac{3\pi}{4}, \frac{9\pi}{8}, \frac{3\pi}{2}, \frac{15\pi}{8}, \frac{18\pi}{8}, \frac{21\pi}{8}\right\}$

$\sin(x_2(n)) = \{ \ldots \}$

circular conv $= X_1(k) \cdot X_2(k)$

$X_1 = \sum_{n=0}^{N-1} x_1(n) e^{-j\frac{2\pi kn}{N}} \qquad k = 0 : N-1 \qquad w_k = \frac{2\pi}{N}$

$X_2 = \sum_{n=0}^{N-1} x_2(n) e^{-j\frac{2\pi kn}{N}} \qquad k = 0 : N-1$

$X_1 X_2 = \sum_{n=0}^{N-1} x_1(n) e^{-jw_1 km} \sum_{m=0}^{N-1} x_2(m) e^{-jw_2 km}$

$X_1 = 0$ for $n \ge 4$

$= 1 \cdot 1 + 1 \cdot e^{-jw_1 k} + 1 \cdot e^{-jw_1 \cdot 2k} + 1 \cdot e^{-jw_1 \cdot 3k}$

$N = 8, \quad w_1 = \frac{2\pi}{8} = \frac{\pi}{4}$

$1 + e^{-j\frac{\pi k}{4}} + e^{-j\frac{\pi}{2}k} + e^{-j\frac{3\pi}{4}k} + 0 + 0 + 0 + 0$

0 1 2 3 4 5 6 7

$k = 0$

$X_1(0) = 4$

$X_1(1) = 1 + e^{-j\frac{\pi}{4}} + e^{-j\frac{2\pi}{4}} + e^{-j3\frac{\pi}{4}} + 0 + \ldots$

$X_1(2) = 1 + e^{-j\frac{2\pi}{4}} + e^{-j\frac{4}{4}\pi} + e^{-j3\frac{\pi}{2}} + 0 + \ldots$

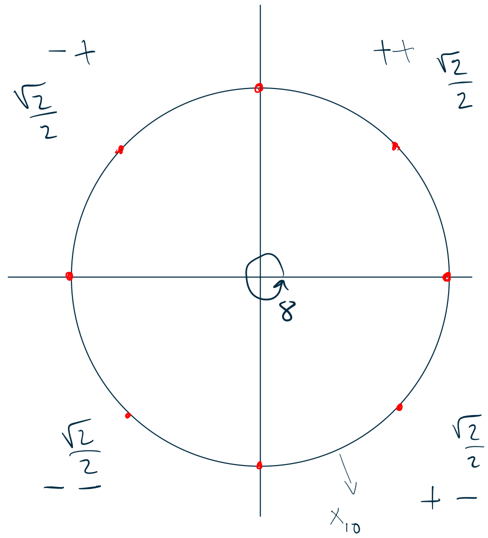
$X_1(3) = 1 + e^{-j\frac{3\pi}{4}} + e^{-j\frac{6\pi}{4}} + e^{-j\frac{9\pi}{4}} + 0 + \ldots$

$X_1(4) = 1 + e^{-j\frac{4\pi}{4}} + e^{-j\frac{8\pi}{4}} + e^{-j\frac{12}{4}\pi} + 0 + \ldots$

$X_1(5) = 1 + e^{-j\frac{5\pi}{4}} + e^{-j\frac{10\pi}{4}} + e^{-j\frac{15\pi}{4}} + 0 + \ldots$

$X_1(6) = 1 + e^{-j\frac{6\pi}{4}} + e^{-j\frac{12\pi}{4}} + e^{-j\frac{9\pi}{4}} + 0 + \ldots$

$X_1(7) = 1 + e^{-j\frac{7\pi}{4}} + e^{-j\frac{14\pi}{4}} + e^{-j\frac{21\pi}{4}} + 0 + \ldots$



$X_1 = \{ 1 + \frac{\sqrt{2}}{2}(1-j) - j - \frac{\sqrt{2}}{2}(1+j)$

$(1-j) + \frac{\sqrt{2}}{2}\left[(1-j) - (1+j)\right]$

$1 - j - 1 - j$

$1 - j + \frac{\sqrt{2}}{\cancel{2}}\left[\cancel{-2}j\right]$

$1 - j(1 + \sqrt{2})$

Rest in mathematica to prevent mistakes

3. Problem 7.4 parts (a) and (b) *(10 points)*

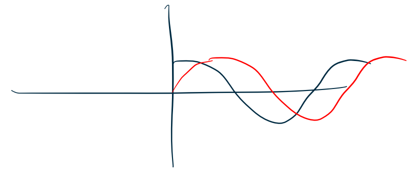
7.4 For the sequences

$$x_1(n) = \cos\frac{2\pi}{N}n, \qquad x_2(n) = \sin\frac{2\pi}{N}n, \qquad 0 \le n \le N-1$$

determine the N-point:

(a) Circular convolution $x_1(n)$ Ⓝ $x_2(n)$

(b) Circular correlation of $x_1(n)$ and $x_2(n)$



$\cos(\theta) = \sin(\theta + \frac{\pi}{2})$

$$\cos\left(\frac{2\pi}{N}n\right) \rightarrow X = \frac{1}{2}\sum_{k=-\infty}^{\infty} \delta(\omega \pm \omega_0 \mp 2\pi k) \quad \text{or} \quad \frac{N}{2}\delta((k \pm k_0))_N$$

$$\sin\left(\frac{2\pi}{N}n\right) = \cos\left(\frac{2\pi}{N}n - \frac{\pi}{2}\right)$$

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \rightarrow \theta = \frac{2\pi n}{N} - \frac{\pi}{2}$$

$$\sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\frac{2\pi n}{2} = \omega$$

$$\frac{1}{2}\left(e^{j\left(\frac{2\pi n}{N} - \frac{\pi}{2}\right)} + e^{-j\left(\frac{2\pi n}{N} - \frac{\pi}{2}\right)}\right)$$

$$\frac{1}{2}\left(e^{j\omega - j\frac{\pi}{2}} + e^{-j\omega + j\frac{\pi}{2}}\right.$$

$$\frac{1}{2}\left(e^{-j\frac{\pi}{2}} \cdot e^{j\omega} + e^{j\frac{\pi}{2}} e^{-j\omega}\right)$$

$$\frac{e^{j\omega}}{e^{j\omega}} \cdot e^{-j\omega}$$

$$e^{j\omega} \cdot e^{-2j\omega}$$

$$\frac{1}{2}\left(\left(e^{-j\pi}\right)e^{j\frac{\pi}{2}} e^{j\omega} + e^{j\frac{\pi}{2}} e^{-j\omega}\right)$$

$$\frac{e^{j\frac{\pi}{2}}}{2}\left(e^{j(\omega - \pi)} + e^{-j\omega}\right)$$

$$\omega = \frac{2\pi n}{N} \qquad \omega - \pi = \left(\frac{2\pi n}{N} - \pi\right)$$

$$\omega - \pi = \left(\frac{2n}{N} - 1\right)\pi$$

$$\sum_{n=0}^{N-1} x_1(n)\, e^{-j\frac{2\pi k n}{N}} \qquad k = 0 : N-1$$

$$e^{-j\frac{2\pi k n}{N}} \cdot \frac{1}{2}\left(e^{j\frac{2\pi n}{N}} + e^{-j\frac{2\pi n}{N}}\right)$$

$$\frac{1}{2}\left(e^{j\left(\frac{2\pi n}{N}(1-k)\right)} + e^{-j\left(\frac{2\pi k n}{N} + \frac{2\pi n}{N}\right)}\right)$$

$$\frac{1}{2}\left(e^{-j\frac{2\pi n}{N}(k-1)} + e^{-j\frac{2\pi n}{N}(k+1)}\right) \qquad \omega = \frac{2\pi n}{N}$$

$$\frac{1}{2}\left(e^{-j\omega(k-1)} + e^{-j\omega(k+1)}\right)$$

$$\sum_{n=0}^{N} \frac{1}{2}\,\delta[\omega(k-1)] + \delta[\omega(k+1)] \rightarrow \frac{N}{2}\delta((k \pm 1))_N$$

$$\therefore \cos \cdot \sin = \delta_c \cdot \delta_s = \sum_{n=0}^{N-1}$$

Rest in mathematica

7.7 If $X(k)$ is the DFT of the sequence $x(n)$, determine the N-point DFTs of the sequences

$$x_c(n) = x(n)\cos\frac{2\pi k_0 n}{N}, \qquad 0 \le n \le N-1$$

$$x(n) \cdot \cos\left(\frac{2\pi k_0 n}{N}\right)$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi k n}{N}} \rightarrow x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi k n}{N}}$$

$$x(n)\cdot\cos(wn) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi k n}{N}} \cdot \cos(wn)$$

$$\ldots \quad e^{j\underbrace{\frac{2\pi k n}{N}}} \cdot \left(\frac{1}{2}\left(e^{jwn} + e^{-jwn}\right)\right)$$

$$\ldots$$

$$w_0 n$$

$$x(n)\cos(wn) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\,\frac{1}{2}\left(e^{jw_0 n}\cdot e^{jwn} + e^{-jwn}\cdot e^{jw_0 n}\right)$$

$$\left(e^{j(w+w_0)n} + e^{-j(w-w_0)n}\right)$$

$$\frac{1}{N}\sum_{k=0}^{N-1} X(k) \cdot$$

becomes a freq shift in both directions

which is then $\boxed{\frac{1}{2}\left(\delta((k-k_0))_N + \delta((k+k_0))_N\right)}$

by $w = \frac{2\pi k}{N}$ & $w_0 = \frac{2\pi k_0}{N}$

$$\therefore w - w_0 = \frac{2\pi}{N}\underbrace{(k-k_0)}_{\text{arg for } \delta \text{ circular}}$$

7.8 Determine the circular convolution of the sequences

$$x_1(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{4}, 3, 2, 2\}$$

$x_1 \, Ⓝ \, x_2 = X_1(k) \cdot X_2(k)$ or $\sum_{m=0}^{N-1} x_1(m)_N \cdot x_2(n-m)_N$

$N = 4$ so we do mod 4

$x_1 = \{1, 2, 3, 1\}$

	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	4	6
3	0	3	6	9

$= k \cdot n$

$$X_1 = \sum_{n=0}^{N-1} x_1(n) \cdot e^{-j\frac{2\pi kn}{N}}$$

$$= \{1e^0 + 2e^0 + 3e^0 + 1e^0,$$
$$1e^0 + 2e^{\frac{-1\pi}{4}} + 3e^{\frac{-2\pi}{4}} + 1e^{\frac{-3\pi}{4}},$$
$$1e^0 + 2e^{\frac{-2\pi}{4}} + 3e^{\frac{-4\pi}{4}} + 1e^{\frac{-6\pi}{4}},$$
$$1e^0 + 2e^{\frac{-3\pi}{4}} + 3e^{\frac{-6\pi}{4}} + 1e^{\frac{-9\pi}{4}}\}$$

$= \{7, -2-j, 1, -2+i\}$

$$X_2 = \{4e^0 + 3e^0 + 2e^0 + 2e^0,$$
$$4e^0 + 3e^{\frac{-1\pi}{4}} + 2e^{\frac{-2\pi}{4}} + 2e^{\frac{-3\pi}{4}},$$
$$4e^0 + 3e^{\frac{-2\pi}{4}} + 2e^{\frac{-4\pi}{4}} + 2e^{\frac{-6\pi}{4}},$$
$$4e^0 + 3e^{\frac{-3\pi}{4}} + 2e^{\frac{-6\pi}{4}} + 2e^{\frac{-9\pi}{4}}\}$$

$= \{11, 2-j, 1, 2+i\}$

multiplication done in mathematica to prevent errors

7.23 Compute the N-point DFTs of the signals

[redacted]

(b) $x(n) = \delta(n - n_0), \quad 0 < n_0 < N$

[redacted]

(d) $x(n) = \begin{cases} 1, & 0 \le n \le N/2 - 1 \quad (N \text{ even}) \\ 0, & N/2 \le n \le N - 1 \end{cases}$

u(n) up to N/2

0 elsewhere

$$X = \sum_{n=0}^{N-1} x(n) e^{-j\omega n} \qquad \omega = \frac{2\pi k}{N}$$

$\delta(k) = 1$ @ k, 0 elsewhere

b) $$X = \sum_{n=0}^{N-1} \delta(n - n_0) e^{-j\omega n}$$

$\therefore$ sum is 1 term, at $n = n_0$ $\boxed{e^{-j\omega n_0}}$

d) 1 from $0 \to N/2 - 1$ 0 elsewhere

$$X = \sum_{n=0}^{N/2-1} x(n) e^{-j\omega n} = \sum_{n=0}^{N/2-1} e^{-j\omega n} \rightarrow a = (e^{-j\omega}) \qquad N/2 = M$$

$$= \sum_{n=0}^{M-1} a^n = \frac{1 - a^M}{1 - a}$$

$$X = \frac{1 - e^{-j\omega M}}{1 - e^{-j\omega}} \qquad e^{-j\omega M} = e^{-j\frac{2\pi k}{N}\frac{N}{2}} = e^{-j\pi k} = (e^{-j\pi})^k = (-1)^k$$

$$\boxed{X = \frac{1 - (-1)^k}{1 - e^{-j\omega}}}$$

$X_1 \cdot X_2 = \{17, 19, 22, 19\}$

Bonus!!!

7.9 Use the four-point DFT and IDFT to determine the sequence

$$x_3(n) = x_1(n) \circledN x_2(n)$$

where $x_1(n)$ and $x_2(n)$ are the sequence given in Problem 7.8.

Results of 7.8 used in mathematica

$x_3(n) = x_1(n) \circledN x_2(n)$

$X_3(k) = X_1(k) \cdot X_2(k)$ → This is the same as method I used in prob 7.8 .... maybe I am misunderstanding

$X_3(k) = \{77, -5, 1, -5\}$

$$x_3(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_3(k) e^{\frac{j2\pi kn}{N}}$$

$= \{17, 19, 22, 19\}$

7.11 Given the eight-point DFT of the sequence

$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & 4 \le n \le 7 \end{cases}$$

compute the DFT of the sequences

$\{1, 1, 1, 1, 0, 0, 0, 0\}$

(a) $x_1(n) = \begin{cases} 1, & n = 0 \\ 0, & 1 \le n \le 4 \\ 1, & 5 \le n \le 7 \end{cases}$

(b) $x_2(n) = \begin{cases} 0, & 0 \le n \le 1 \\ 1, & 2 \le n \le 5 \\ 0, & 6 \le n \le 7 \end{cases}$

a) $\{1, 0, 0, 0, 0, 1, 1, 1\} = x((n - n_0))_N$ $\quad N = 8$

$n_0 = 5$

$x_1 = x((n-5))_8$

$\therefore X_1(k) = X(k)\, e^{-j\frac{2\pi 5 kn}{8}}$

$$\boxed{= X(k)\, e^{-j\frac{5}{4}\pi kn}}$$

b) $\{0, 0, 1, 1, 1, 1, 0, 0\}$ $\quad n_0 = 2$

$N = 8$

Same as above, shift by 2 here

$$X_2(k) = X(k)\, e^{-j\frac{2\pi 2kn}{8}} = \boxed{X(k) \cdot e^{-j\frac{\pi kn}{2}}}$$