HW5 problems in Mathematica

#2 - 7.2 a

In[175]:=

```
x1 = {1, 1, 1, 1, 0, 0, 0, 0}
len = Length[x1];
k = Range[0, len - 1];
X1 = Table[Sum[x1[[m + 1]] * Exp[I * (-2 * Pi * k[[n + 1]] * m / len)], {m, 0, len - 1}], {n, 0, len - 1}];
Simplify[X1]
N[Simplify[X1]]
ListPlot[(Tooltip[{Re[#1], Im[#1]}] &) /@ N[Simplify[X1]], AspectRatio → 1,
 AxesLabel → {"Real", "Imaginary"}, AxesStyle → Gray, ImageSize → Medium]
```

Out[175]=

{1, 1, 1, 1, 0, 0, 0, 0}

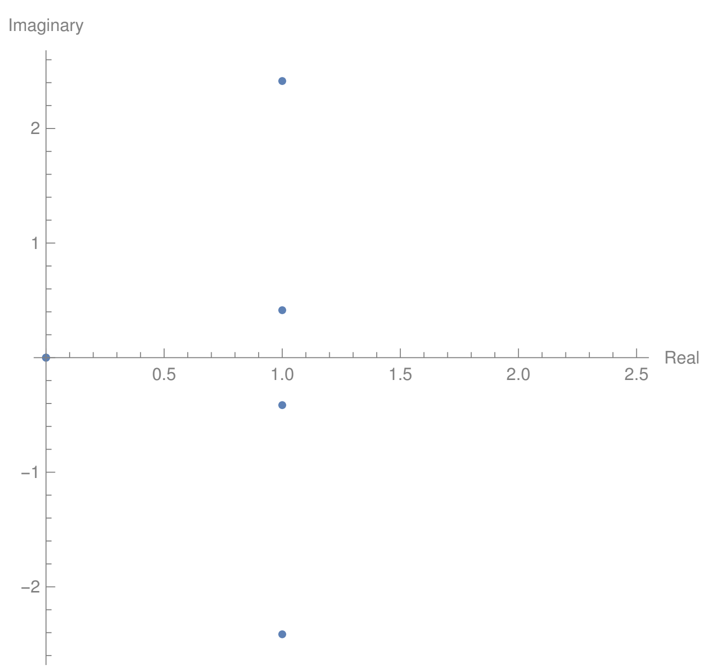
Out[179]=

$$\left\{4,\ -i\left((1+i)+\sqrt{2}\right),\ 0,\ (1+i)-i\sqrt{2},\ 0,\ (1-i)+i\sqrt{2},\ 0,\ (1+i)+i\sqrt{2}\right\}$$

Out[180]=

$$\{4.,\ 1.-2.41421\,i,\ 0.,\ 1.-0.414214\,i,\ 0.,\ 1.+0.414214\,i,\ 0.,\ 1.+2.41421\,i\}$$

Out[181]=



In[182]:=

```
len = 8
k = Range[0, len - 1]
n = Range[0, len - 1];
x2 = Sin[3 * Pi * n / len]
X2 = Table[Sum[x2[[m + 1]] * Exp[I * (-2 * Pi * k[[j + 1]] * m / len)], {m, 0, len - 1}], {j, 0, len - 1}];
Simplify[X2]
N[Simplify[X2]]
ListPlot[(Tooltip[{Re[#1], Im[#1]}] &) /@ N[Simplify[X2]], AspectRatio → 1,
  AxesLabel → {"Real", "Imaginary"}, AxesStyle → Gray, ImageSize → Medium]
```

Out[182]=

8

Out[183]=

{0, 1, 2, 3, 4, 5, 6, 7}

Out[185]=

$$\left\{0,\ \mathrm{Cos}\left[\frac{\pi}{8}\right],\ \frac{1}{\sqrt{2}},\ -\mathrm{Sin}\left[\frac{\pi}{8}\right],\ -1,\ -\mathrm{Sin}\left[\frac{\pi}{8}\right],\ \frac{1}{\sqrt{2}},\ \mathrm{Cos}\left[\frac{\pi}{8}\right]\right\}$$

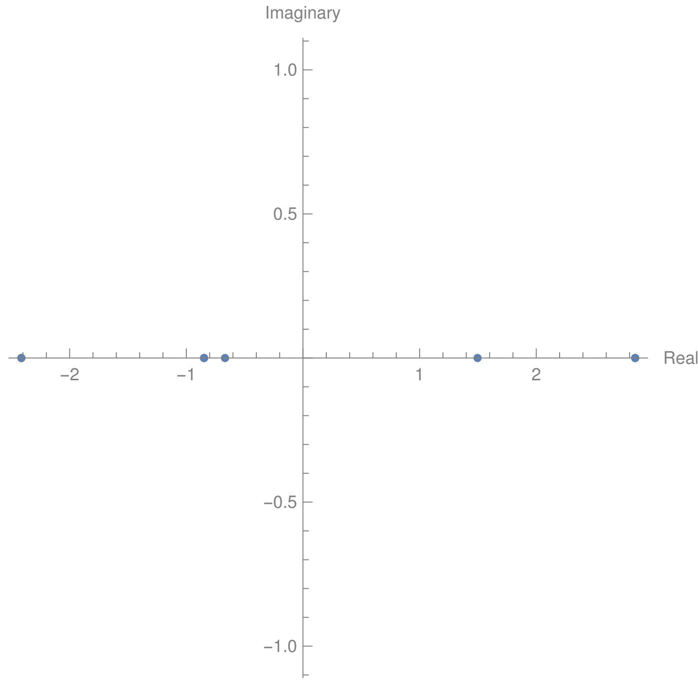
Out[187]=

$$\left\{-1+\sqrt{2}+2\,\mathrm{Cos}\left[\frac{\pi}{8}\right]-2\,\mathrm{Sin}\left[\frac{\pi}{8}\right],\ 1+\frac{1}{2}\,\mathrm{Csc}\left[\frac{\pi}{8}\right]+\sqrt{2}\,\mathrm{Sin}\left[\frac{\pi}{8}\right],\ -1-\sqrt{2},\ 1-\frac{\mathrm{Csc}\left[\frac{\pi}{8}\right]}{\sqrt{2}},\right.$$
$$\left.-1+\sqrt{2}-2\,\mathrm{Cos}\left[\frac{\pi}{8}\right]+2\,\mathrm{Sin}\left[\frac{\pi}{8}\right],\ 1-\frac{\mathrm{Csc}\left[\frac{\pi}{8}\right]}{\sqrt{2}},\ -1-\sqrt{2},\ 1+\frac{1}{2}\,\mathrm{Csc}\left[\frac{\pi}{8}\right]+\sqrt{2}\,\mathrm{Sin}\left[\frac{\pi}{8}\right]\right\}$$

Out[188]=

{1.49661, 2.84776, -2.41421, -0.847759, -0.668179, -0.847759, -2.41421, 2.84776}

Out[189]=



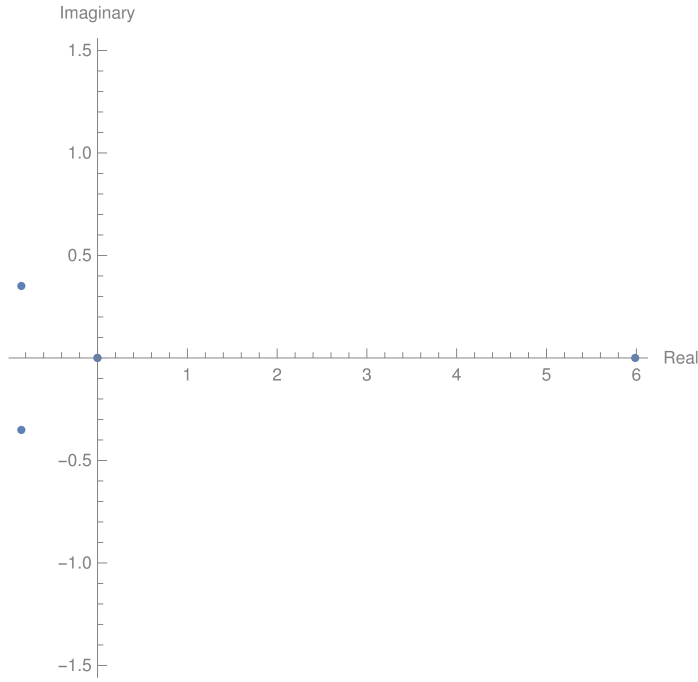
In[190]:= **N[Simplify[X1 * X2]]**

Out[190]=

{5.98642, 2.84776 - 6.8751 *i*, 0., -0.847759 + 0.351153 *i*, 0., -0.847759 - 0.351153 *i*, 0., 2.84776 + 6.8751 *i*}

In[192]:=
```
ListPlot[(Tooltip[{Re[#1], Im[#1]}] &) /@ N[Simplify[X1 * X2]], AspectRatio → 1,
  AxesLabel → {"Real", "Imaginary"}, AxesStyle → Gray, ImageSize → Medium]
```

Out[192]=



Same thing as above but using the fourier version in Mathematica to verify

In[205]:=
```
len = 8;
n = Range[0, len - 1];
x1 = {1, 1, 1, 1, 0, 0, 0, 0};
x2 = Sin[3 * Pi * n / len];

X11 = Fourier[x1, FourierParameters → {1, -1}];
X22 = Fourier[x2, FourierParameters → {1, -1}];
X1 == X11 (*This is to check against mine*)
X2 == X22
Y = X11 * X22
y = InverseFourier[Y, FourierParameters → {1, -1}];
y = Simplify[y]
Show[Show[ListPlot[(Tooltip[{Re[#1], Im[#1]}] &) /@ N[Simplify[X11 * X22]], AspectRatio → 1],
  AxesStyle → Gray], ImageSize → Medium]
```

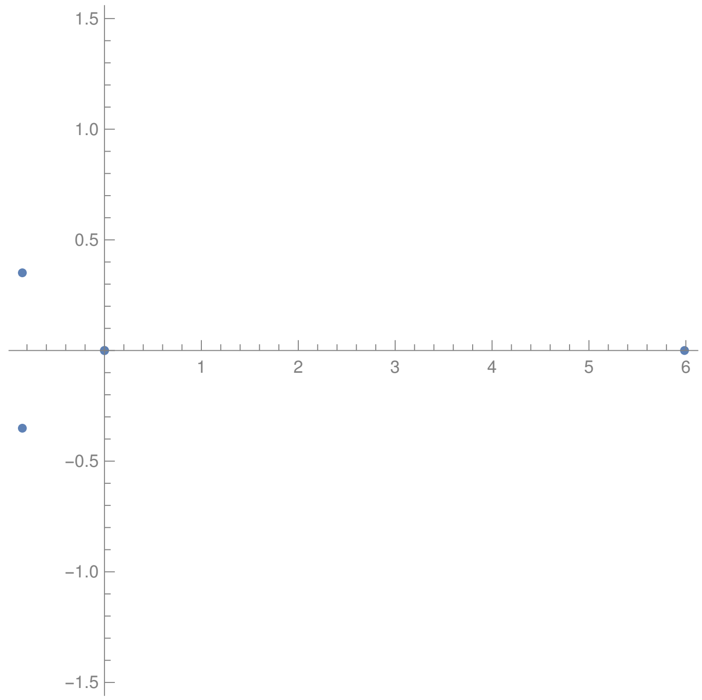
Out[211]=
```
True
```

Out[212]=
```
True
```

Out[213]=
```
{5.98642 + 0. i, 2.84776 - 6.8751 i, 0. + 0. i, -0.847759 + 0.351153 i,
 0. + 0. i, -0.847759 - 0.351153 i, 0. + 0. i, 2.84776 + 6.8751 i}
```

Out[215]=
```
{1.2483, 2.55487, 2.55487, 1.2483, 0.248303, -1.05826, -1.05826, 0.248303}
```

Out[216]=



#3 - 7.4 a and b

a)

In[217]:=

```
len = 8
k = Range[0, len - 1]
n = Range[0, len - 1];
x1 = Cos[2 * Pi * n / len]
x2 = Sin[2 * Pi * n / len]
X1 = Table[Sum[x1[[m + 1]] * Exp[I * (-2 * Pi * k[[j + 1]] * m / len)], {m, 0, len - 1}], {j, 0, len - 1}];
Simplify[X1]
N[Simplify[X1]]

X2 = Table[Sum[x2[[m + 1]] * Exp[I * (-2 * Pi * k[[j + 1]] * m / len)], {m, 0, len - 1}], {j, 0, len - 1}];
Simplify[X2]
N[Simplify[X2]]
ListPlot[(Tooltip[{Re[#1], Im[#1]}] &) /@ N[Simplify[X1 * X2]], AspectRatio → 1,
  AxesLabel → {"Real", "Imaginary"}, AxesStyle → Gray, ImageSize → Medium]
```

Out[217]=

8

Out[218]=

{0, 1, 2, 3, 4, 5, 6, 7}

Out[220]=

$$\left\{1, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, -1, -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right\}$$

Out[221]=

$$\left\{0, \frac{1}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, -1, -\frac{1}{\sqrt{2}}\right\}$$

Out[223]=

{0, 4, 0, 0, 0, 0, 0, 4}

Out[224]=

{0., 4., 0., 0., 0., 0., 0., 4.}

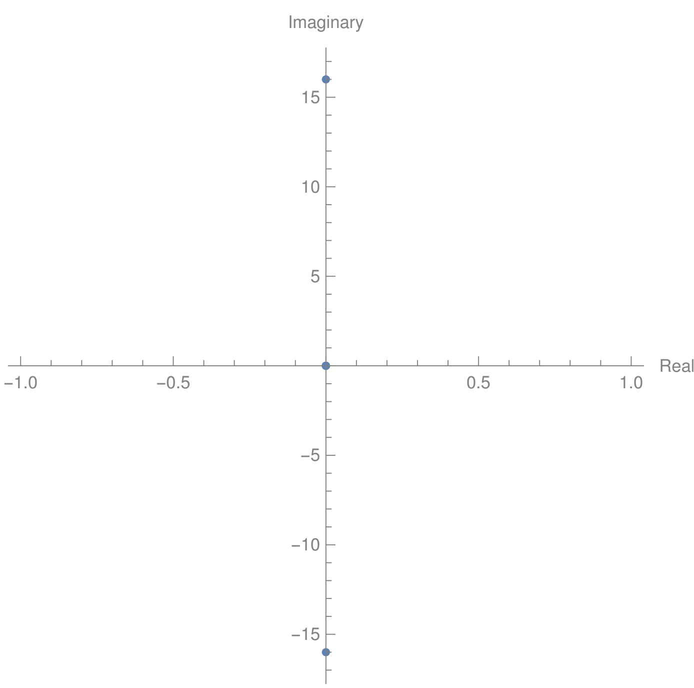
Out[226]=

$\{0, -4\,i, 0, 0, 0, 0, 0, 4\,i\}$

Out[227]=

$\{0., 0. - 4.\,i, 0., 0., 0., 0., 0., 0. + 4.\,i\}$

Out[228]=



b)

In[229]:=
```
len = 8
k = Range[0, len - 1]
n = Range[0, len - 1];
x1 = Cos[2 * Pi * n / len]
x2 = Sin[2 * Pi * n / len]
X1 = Table[Sum[x1[[m + 1]] * Exp[I * (-2 * Pi * k[[j + 1]] * m / len)], {m, 0, len - 1}], {j, 0, len - 1}];
Simplify[X1];
N[Simplify[X1]];

X2 = Table[Sum[x2[[m + 1]] * Exp[I * (-2 * Pi * k[[j + 1]] * m / len)], {m, 0, len - 1}], {j, 0, len - 1}];
X2 = Simplify[X2]
X2 = Conjugate[X2]
X3 = Simplify[X1 * X2]
convRes =
 1 / len * Table[Sum[X3[[m + 1]] * Exp[I * (2 * Pi * k[[j + 1]] * m / len)], {m, 0, len - 1}], {j, 0, len - 1}]
 (*inverse is just the same with pos j and div / len*)
ListPlot[(Tooltip[{Re[#1], Im[#1]}] &) /@ N[convRes], AspectRatio → 1,
 AxesLabel → {"Real", "Imaginary"}, AxesStyle → Gray, ImageSize → Medium]
```

Out[229]=

8

Out[230]=

{0, 1, 2, 3, 4, 5, 6, 7}

Out[232]=

$$\left\{1, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, -1, -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right\}$$

Out[233]=

$$\left\{0, \frac{1}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, -1, -\frac{1}{\sqrt{2}}\right\}$$

Out[238]=

$$\{0, -4\,i, 0, 0, 0, 0, 0, 4\,i\}$$

Out[239]=

$$\{0, 4\,i, 0, 0, 0, 0, 0, -4\,i\}$$

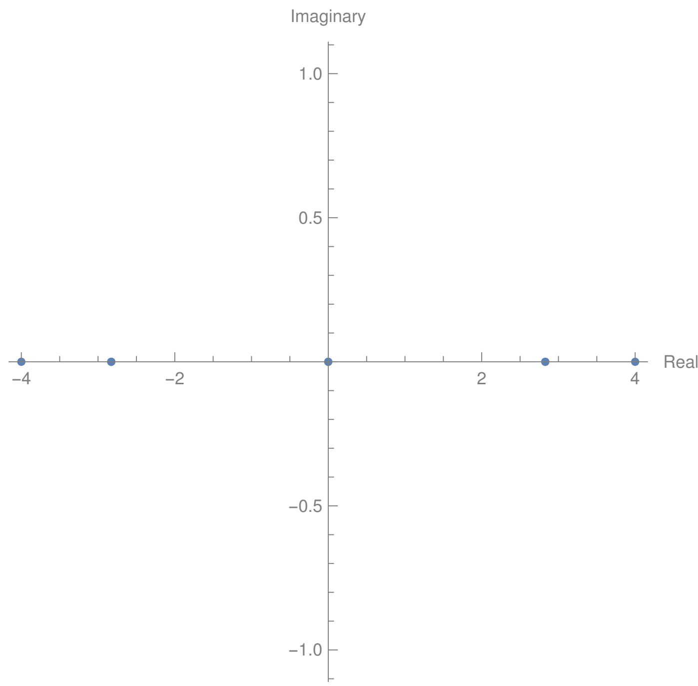
Out[240]=

$$\{0, 16\,i, 0, 0, 0, 0, 0, -16\,i\}$$

Out[241]=

$$\left\{0,\ \frac{1}{8}\left(-16\,i\,e^{-\frac{i\pi}{4}}+16\,i\,e^{\frac{i\pi}{4}}\right),\ -4,\ \frac{1}{8}\left(-16\,i\,e^{-\frac{3i\pi}{4}}+16\,i\,e^{\frac{3i\pi}{4}}\right),\ 0,\ \frac{1}{8}\left(16\,i\,e^{-\frac{3i\pi}{4}}-16\,i\,e^{\frac{3i\pi}{4}}\right),\ 4,\ \frac{1}{8}\left(16\,i\,e^{-\frac{i\pi}{4}}-16\,i\,e^{\frac{i\pi}{4}}\right)\right\}$$

Out[242]=



Same thing as above but using the fourier version in Mathematica to verify

In[243]:=
```
(*verification*)
len = 8; (*This should be the length of x1 and x2*)
n = Range[0, len - 1];
x1 = Sin[2 * Pi * n / len]; (*Your first signal*)
x2 = Sin[2 * Pi * n / len]; (*Your second signal,could be any other function*)

X1 = Fourier[x1, FourierParameters → {1, -1}]
X2 = Conjugate[Fourier[x2, FourierParameters → {1, -1}]]
R = X1 * X2 (*Pointwise multiplication*)
r = InverseFourier[R, FourierParameters → {1, -1}]
(*Inverse DFT to get the circular correlation*)

r = Simplify[r]
ListPlot[(Tooltip[{Re[#1], Im[#1]}] &) /@ N[r], AspectRatio → 1,
 AxesLabel → {"Real", "Imaginary"}, AxesStyle → Gray, ImageSize → Medium]
```

Out[247]=
```
{0. + 0. i, 0. - 4. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 4. i}
```

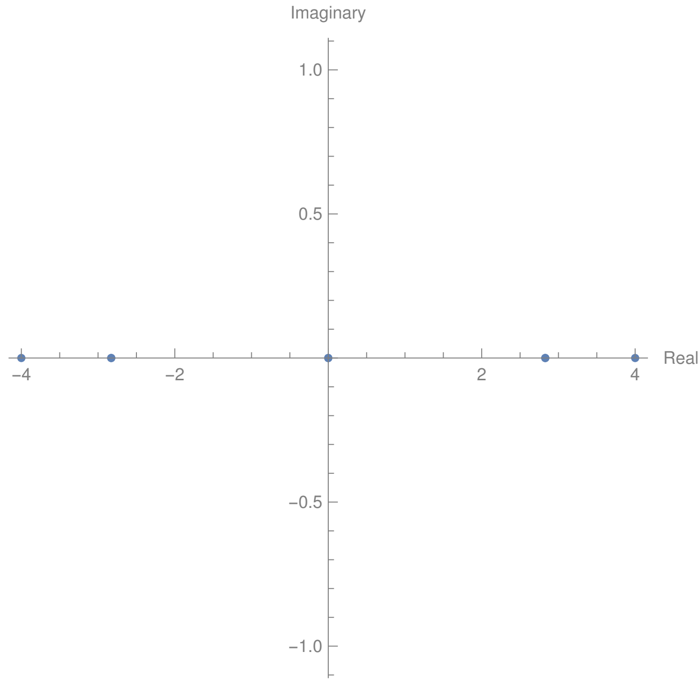
Out[248]=
```
{0. + 0. i, 0. + 4. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. - 4. i}
```

Out[249]=
```
{0. + 0. i, 16. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 0. + 0. i, 16. + 0. i}
```

Out[250]=
```
{4., 2.82843, 0., -2.82843, -4., -2.82843, 0., 2.82843}
```

Out[251]=
```
{4., 2.82843, 0., -2.82843, -4., -2.82843, 0., 2.82843}
```

Out[252]=



#5 - 7.8

In[253]:=
```
x1 = {1, 2, 3, 1}
x2 = {4, 3, 2, 2}
```

Out[253]=
```
{1, 2, 3, 1}
```

Out[254]=
```
{4, 3, 2, 2}
```

In[255]:=
```
len = Length[x1]
k = Range[0, len - 1]
n = Range[0, len - 1]
```

Out[255]=
```
4
```

Out[256]=
```
{0, 1, 2, 3}
```

Out[257]=
```
{0, 1, 2, 3}
```

In[258]:=

```
X1 = Table[Sum[x1[[m + 1]] * Exp[I * (-2 * Pi * k[[j + 1]] * m / len)], {m, 0, len - 1}], {j, 0, len - 1}]
X2 = Table[Sum[x2[[m + 1]] * Exp[I * (-2 * Pi * k[[j + 1]] * m / len)], {m, 0, len - 1}], {j, 0, len - 1}]
X3 = X2 * X1
```

Out[258]=

```
{7, -2 - i, 1, -2 + i}
```

Out[259]=

```
{11, 2 - i, 1, 2 + i}
```

Out[260]=

```
{77, -5, 1, -5}
```

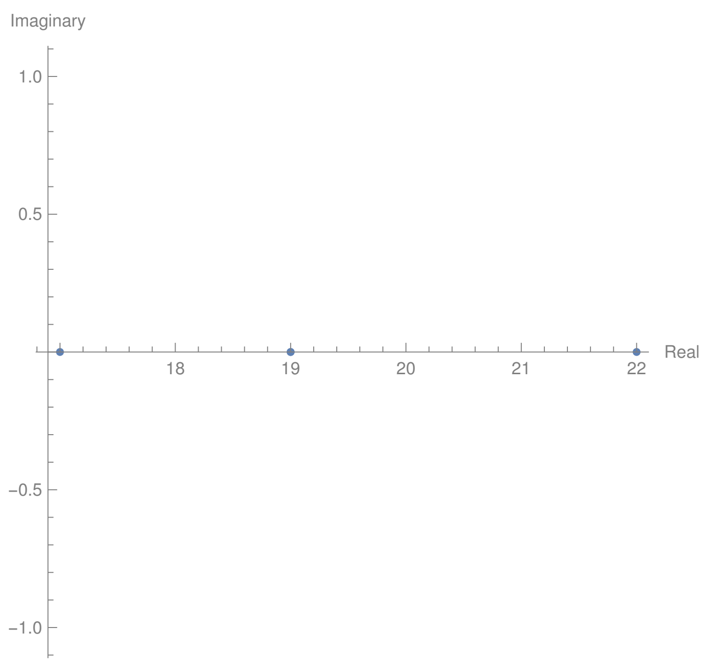
In[261]:=

```
circConv =
 1 / len * Table[Sum[X3[[m + 1]] * Exp[I * (2 * Pi * k[[j + 1]] * m / len)], {m, 0, len - 1}], {j, 0, len - 1}]
 (*inverse is just the same with pos j and div / len*)
ListPlot[(Tooltip[{Re[#1], Im[#1]}] &) /@ N[circConv], AspectRatio → 1,
 AxesLabel → {"Real", "Imaginary"}, AxesStyle → Gray, ImageSize → Medium]
```

Out[261]=

```
{17, 19, 22, 19}
```

Out[262]=



Same thing as above but using the fourier version in Mathematica to verify

In[273]:=

```
(*verification*)
x1 = {1, 2, 3, 1}
x2 = {4, 3, 2, 2}
len = Length[x1]; (*This should be the length of x1 and x2*)
n = Range[0, len - 1];

X1 = Fourier[x1, FourierParameters → {1, -1}]
X2 = Fourier[x2, FourierParameters → {1, -1}]
R = X1 * X2 (*Pointwise multiplication*)
r = InverseFourier[R, FourierParameters → {1, -1}]
(*Inverse DFT to get the circular correlation*)

r = Simplify[r]
ListPlot[(Tooltip[{Re[#1], Im[#1]}] &) /@ N[r], AspectRatio → 1,
 AxesLabel → {"Real", "Imaginary"}, AxesStyle → Gray, ImageSize → Medium]
```

Out[273]=

{1, 2, 3, 1}

Out[274]=

{4, 3, 2, 2}

Out[277]=

{7. + 0. i, -2. - 1. i, 1. + 0. i, -2. + 1. i}

Out[278]=

{11. + 0. i, 2. - 1. i, 1. + 0. i, 2. + 1. i}

Out[279]=

{77. + 0. i, -5. + 0. i, 1. + 0. i, -5. + 0. i}

Out[280]=

{17., 19., 22., 19.}

Out[281]=

{17., 19., 22., 19.}

Out[282]=