# Homework Assignment #2

ECE 6530: Digital Signal Processing
September 8, 2023

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**Homework set #2**

Due Date: Sep 14, 2023
(100 points)

## 1

2.7 - Answer subquestions (1) through (5) for parts (a) through (e). (10 points)

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a) $y(n) = \cos(x(n))$

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

c) $y(n) = x(n)\cos(\omega_0 n)$

d) $y(n) = x(-n+2)$

e) $y(n) =$Trun$[x(n)]$ where Trun$[x(n)]$ denotes the integer part of $x(n)$, obtained by truncation.

## 2

2.11 - Problem 2.11 (10 points)

The following inputoutput pairs have been observed during the operation of a linear system:

$$x_1(n) = \{-1, \underset{\uparrow}{2}, 1\} \quad \xleftrightarrow{\mathcal{T}} \quad y_1(n) = \{1, \underset{\uparrow}{2}, -1, 0, 1\}$$

$$x_2(n) = \{1, \underset{\uparrow}{-1}, -1\} \quad \xleftrightarrow{\mathcal{T}} \quad y_2(n) = \{-1, \underset{\uparrow}{1}, 0, 2\}$$

$$x_3(n) = \{0, \underset{\uparrow}{1}, 1\} \quad \xleftrightarrow{\mathcal{T}} \quad y_3(n) = \{\underset{\uparrow}{1}, 2, 1\}$$

Can you draw any conclusions about the time invariance of this system?

$$x_1(n) + x_2(n) = \{-1, \underset{\uparrow}{2}, 1\} + \{1, \underset{\uparrow}{-1}, -1\}$$

$$y_1(n) + y_2(n) = \{1, \underset{\uparrow}{2}, -1, 0, 1\} + \{-1, \underset{\uparrow}{1}, 0, 2\}$$

Since the red match, we would expect the blue to match as well if they are time invariant. Since they do not, it tells us there is some relationship other than the impulse going on.

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## 3

2.16 - Problem 2.16 part (b3) and (b11) (10 points)

a) If $y(n) = x(n) * h(n)$, show that $\sum_y = \sum_x \sum_h$, where $\sum_x = \sum_{-\infty}^{\infty} x(n)$.

b) Compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the correctness of the results by using the test in $(a)$.

b3 ) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \left\{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\right\}$

b11) $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

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b3) Convolution of discrete systems can be done by matrix multiplication:

$$\begin{bmatrix} x[0] & 0 & 0 & 0 \\ x[1] & x[0] & 0 & 0 \\ x[2] & x[1] & x[0] & 0 \\ x[3] & x[2] & x[1] & x[0] \\ x[4] & x[3] & x[2] & x[1] \\ 0 & x[4] & x[3] & x[2] \\ 0 & 0 & x[4] & x[3] \\ 0 & 0 & 0 & x[4] \end{bmatrix} \cdot \begin{bmatrix} h[0] \\ h[1] \\ h[2] \\ h[3] \end{bmatrix} = \begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ y[3] \\ y[4] \\ y[5] \\ y[6] \\ y[7] \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -2 & 1 & 0 \\ -4 & 3 & -2 & 1 \\ 0 & -4 & 3 & -2 \\ 0 & 0 & -4 & 3 \\ 0 & 0 & 0 & -4 \end{bmatrix} \cdot \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 \\ 1 \\ -2+1 \\ 3-2+2 \\ -4+3-4+1 \\ -4+6-2 \\ -8+3 \\ -4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 \\ 1 \\ -1 \\ 3 \\ -4 \\ 0 \\ -5 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{3}{2} \\ -2 \\ 0 \\ -\frac{5}{2} \\ -2 \end{bmatrix}$$

$$y = \{0, \tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{3}{2}, -2, 0, -\tfrac{5}{2}, -2\}$$

b11) can be done by expressing it as a sum:

$$x(n) = \left(\frac{1}{2}\right)^n u(n),\ h(n) = \left(\frac{1}{4}\right)^n u(n)$$

$$x * h = \sum_{k=-\infty}^{\infty} x(n)h(n-k) \ = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

Swapping the order of the convolution should not make a difference

$$= \sum_{k=-\infty}^{\infty} h(k)x(n-k) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{4}\right)^k u(k) \left(\frac{1}{2}\right)^{(n-k)} u(n-k)$$

Since the sum goes from $k$ and is bounded by $n$, we can absorb those into the sum:

$$= \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k \left(\frac{1}{2}\right)^{(n-k)} = \sum_{k=0}^{n} \left(\left(\frac{1}{2}\right)^2\right)^k \left(\frac{1}{2}\right)^{(n-k)}$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{2k} \left(\frac{1}{2}\right)^{-k} \left(\frac{1}{2}\right)^{n} = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n+k}$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k$$

By the expression for a finite sum:

$$= \sum_{k=1}^{n} = \frac{1-r^n}{1-r} \quad \& \quad r = \frac{1}{2}$$

$$= \left(\frac{1}{2}\right)^n \frac{1-\left(\frac{1}{2}\right)^n}{1-\left(\frac{1}{2}\right)}$$

$$= \frac{\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^n}{\left(\frac{1}{2}\right)}$$

$$= 2\left(\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^{2n}\right)$$

$$= \left(\left(\frac{1}{2}\right)^{n+1} - \left(\frac{1}{2}\right)^{2n+1}\right)$$

Need to verify this. ask about my solution next week

Verifying using $\sum_y = \sum_x \sum_h$, where $\sum_x = \sum_{-\infty}^{\infty} x(n)$

$$\sum_y = \sum_x \sum_h \quad \text{, where} \quad \sum_x = \sum_{-\infty}^{\infty} x(n)$$

## 4

2.24 - Problem 2.24. (10 points)

The discrete-time system:

$$y(n) = ny(n-1) + x(n), \qquad n \geq 0$$

:is at rest [i.e., $y(1) = 0$]. Check if the system is linear time invariant and BIBO stable.

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## 5

2.27 - Problem 2.27. Determine the homogeneous, particular and total solutions. (15 points)
Determine the particular solution of the difference equation:

$$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$$

when the forcing function is $x(n) = 2u(n)$.

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## 6

2.31 - Problem 2.31. (10 points)
Determine the impulse response of the following causal system:

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

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## 7

2.46 - Problem 2.46. (10 points)
Determine the direct form II realization for each of the following LTI systems:

a) $2y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-5)$
b) $y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$

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## 8

2.49 - Problem 2.49 part (a). Assume that the system is relaxed. (10 points)
A discrete-time system is realized by the structure shown in Fig. P49.

a) Determine the impulse response.

b) Determine a realization for its inverse system, that is, the system which produces $x(n)$ as an output when $y(n)$ is used as an input.

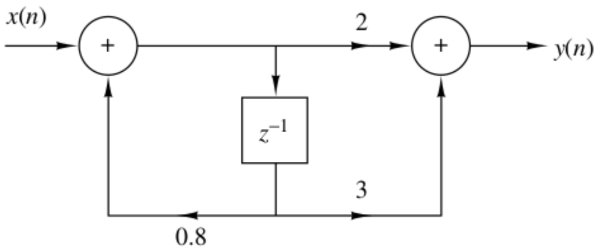


Figure 1: Figure P49 from textbook

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## 9

2.52 - Problem 2.52 part (a). (15 points)
Consider the systems shown in Fig. P52.

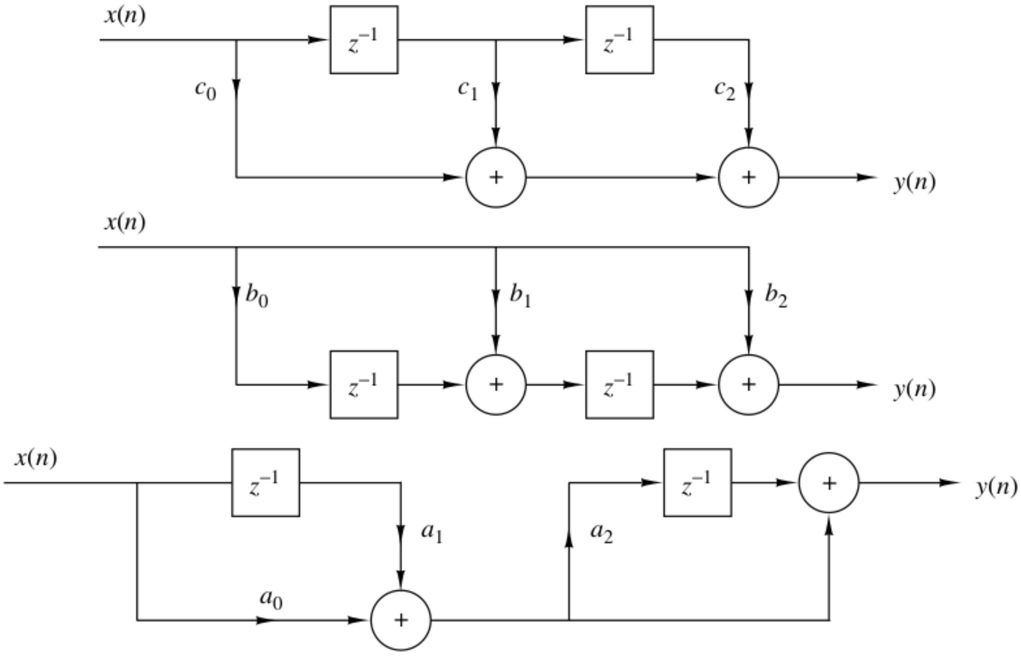


Figure 2: Figure P52 from textbook

a) Determine and sketch their impulse responses $h1(n)$, $h2(n)$, and $h3(n)$.

b) Is it possible to choose the coefficients of these systems in such a way that:

$$h_1(n) = h_2(n) = h_3(n)$$