ECE 5530/6530 Digital Signal Processing
Assignment 6

Submission guidelines (READ CAREFULLY)

- All submissions (pdf or doc) must be done using Canvas.
- Scanned hand written submissions will be accepted; however, it is the student's responsibility to ensure that the answers are easily legible. Otherwise, you may lose points.
- Make sure you are using the correct edition (4th) of the textbook to ensure you are solving the correct problems.
- For late policy refer to syllabus.

Questions from the textbook

1. Problem 10.1 a to c. Use MATLAB *(30 points)*.
2. Problem 10.6. Use MATLAB *(30 points)*.
3. Problem 9.6 *(30 points)*.
4. Problem 9.7 parts a and b *(30 points)*.
5. Problem 9.9 part a *(30 points)*. Note: you don't have to answer the question about stability, I am asking for only the diagrams of the structures.

9.6 Determine a_1, a_2 and c_1, and c_0 in terms of b_1 and b_2 so that the two systems in Fig. P9.6 are equivalent.

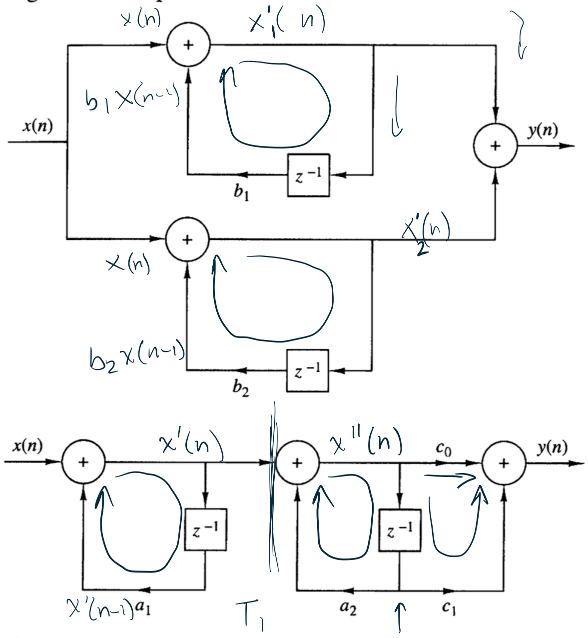


Figure P9.6

$x_1'(n) = x(n) + b_1 x(n-1)$

$x_2'(n) = x(n) + b_2 x(n-1)$

$y(n) = x_1'(n) + x_2'(n)$

$= x(n) + b_1 x(n-1) + b_2 x(n-1)$

$x(n) + (b_1+b_2) x(n-1)$

$y(n) = C_0 x''(n) + C_1 x''(n-1)$

$x''(n-1)$

$a_2 x''(n-1) + x'(n)$

$x''(n) = a_2 x''(n-1) + x'(n)$

$X_1'(n) = x(n) + b_1 x'(n-1)$

$X_2'(n) = x(n) + b_2 x'(n-1)$

$X_1'(w) = X(w) + b_1 z^{-1} X_1'(w)$

$X_2'(w) = X(w) + b_2 z^{-1} X_2'(w)$

$X_1'(w)(1-b_1z^{-1}) = X(w)$

$X_1'(w) = (1-b_1z^{-1})^{-1} X(w)$

$X_2'(w) = (1-b_2z^{-1})^{-1} X(w)$

$X_1' + X_2' = (1-b_1z^{-1})^{-1} X(w) + (1-b_2z^{-1})^{-1} X(w)$

$= X(w)\left[(1-b_1z^{-1})^{-1} + (1-b_2z^{-1})^{-1}\right]$

#2 at T_1

$x'(n) = x(n) + a_1 x'(n-1)$

$X'(w) = X(w) + a_1 X'(w) z^{-1}$

$X'(w)(1-a_1z^{-1}) = X(w)$

$X'(w) = (1-a_1z^{-1})^{-1} X(w)$

$X''(w) = a_2 z^{-1} X''(w) + X'(w)$

$X''(w)(1-a_2z^{-1}) = X'(w)$

$X''(w) = (1-a_2z^{-1}) X'(w)$

$Y(w) = C_0 X''(w) + C_1 z^{-1} X''(w)$

$(C_0 + C_1 z^{-1}) X''(w)$

$(C_0 + C_1 z^{-1})\left[(1-a_2z^{-1})^{-1}\left[(1-a_1z^{-1})^{-1} X(w)\right]\right]$

$$Y(w) = \frac{C_0 + C_1 z^{-1}}{(1-a_2 z^{-1})(1-a_1 z^{-1})} X(w)$$

$$\frac{Y(w)}{X(w)} = \frac{C_0 + C_1 z^{-1}}{(1-a_2 z^{-1})(1-a_1 z^{-1})}$$

$$\frac{Y}{X} = \frac{1}{1-b_1 z^{-1}} + \frac{1}{1-b_2 z^{-1}}$$

$$\frac{1-b_2 z^{1} + 1 - b_1 z^{1}}{(1-b_1 z^{-1})(1-b_2 z^{-1})} = \frac{C_0 + C_1 z^{-1}}{(1-a_2 z^{-1})(1-a_1 z^{-1})}$$

$$2 - (b_1 + b_2) z^{-1}$$

$$\therefore \boxed{\begin{aligned} C_1 &= -(b_1 + b_2) \\ C_0 &= 2 \end{aligned}}$$

9.7 Consider the filter shown in Fig. P9.7.

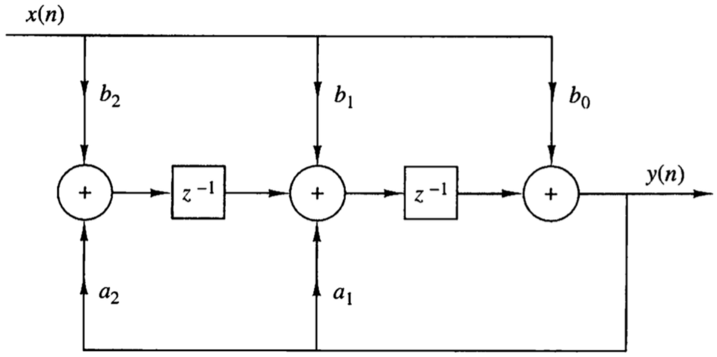


Figure P9.7

(a) Determine its system function.

(b) Sketch the pole–zero plot and check for stability if

1. $b_0 = b_2 = 1, \quad b_1 = 2, \quad a_1 = 1.5, a_2 = -0.9$

2. $b_0 = b_2 = 1, \quad b_1 = 2, \quad a_1 = 1, a_2 = -2$

$$\left[\left(b_2 x(n) + a_2 y(n)\right)^{-1} + \left(b_1 x(n) + a_1 y(n)\right)\right]^{-1} + b_0 x(n) = y(n)$$

$$\left[\left(b_2 x(n-1) + a_2 y(n-1)\right) + b_1 x(n) + a_1 y(n)\right]^{-1} + b_0 x(n) = y(n)$$

$$\underline{b_2 x(n-2)} + a_2 y(n-2) + \underline{b_1 x(n-1)} + a_1 y(n-1) + \underline{b_0 x(n)} = y(n)$$

$$b_2 x(n-2) + b_1 x(n-1) + b_0 x(n) = y(n) - a_1 y(n-1) - a_2 y(n-2)$$

$$b_2 X(z) z^{-2} + b_1 X(z) z^{-1} + b_0 X(z) = Y(z) - a_1 Y(z) z^{-1} - a_2 Y(z) z^{-2}$$

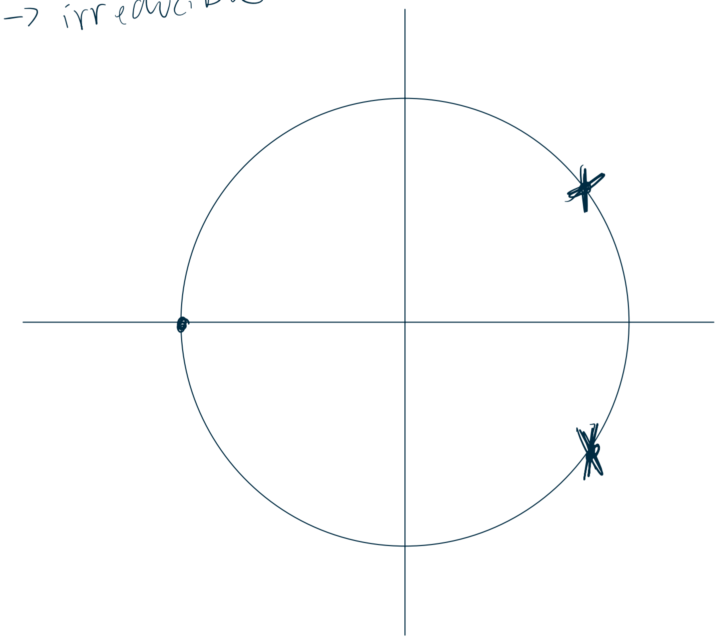
$$Y(z)\left(1 - a_1 z^{-1} - a_2 z^{-2}\right) = X(z)\left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right)$$

(a) $$\boxed{H = \frac{Y}{X} = \frac{\left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right)}{\left(1 - a_1 z^{-1} - a_2 z^{-2}\right)}} = \boxed{\frac{b_0 z^2 + b_1 z + b_2}{z^2 - a_1 z - a_2}}$$

(b) Sketch the pole–zero plot and check for stability if

1. $b_0 = b_2 = 1, \quad b_1 = 2, \quad a_1 = 1.5, a_2 = -0.9$
2. $b_0 = b_2 = 1, \quad b_1 = 2, \quad a_1 = 1, a_2 = -2$

b.1) $\dfrac{z^2 + 2z + 1}{z^2 - \frac{3}{2}z + \frac{9}{10}} = \dfrac{(z+1)^2}{z^2 - \frac{3}{2}z + \frac{9}{10}}$ -> irreducible



done in mathematica

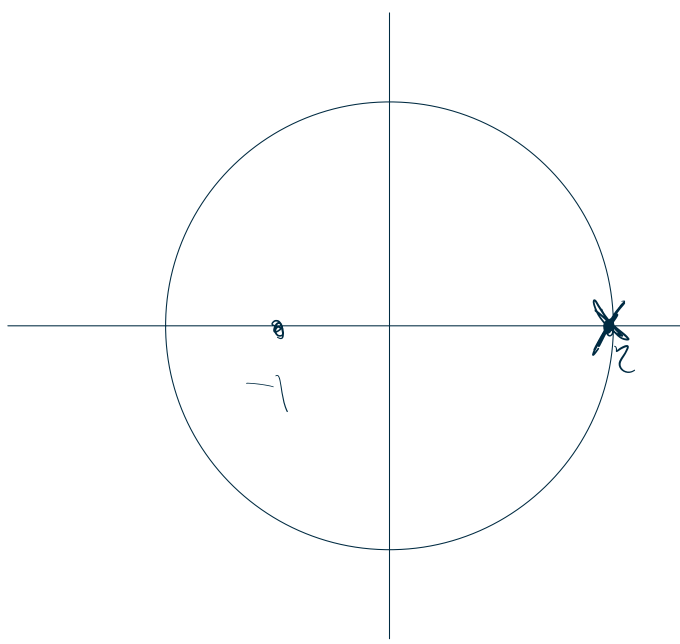
file included w/ HW

b.2 -> $H_1 = \dfrac{z^2 + 2z + 1}{z^2 - z + 2} = \dfrac{(z+1)^2}{(z+1)(z-2)} = \dfrac{z+1}{z-2} = \dfrac{1 + z^{-1}}{1 - 2z^{-1}}$

$a_1 = 1$ $\quad$ $ROC_1 = z, 1$

$a_2 = -2$ $\quad$ $ROC_2 = z, -2$

between 1 & 2?

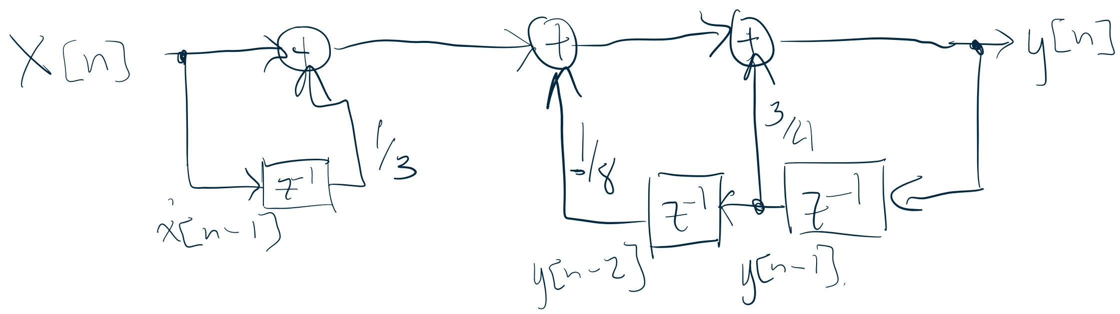


9.9 Obtain the direct form I, direct form II, cascade, and parallel structures for the following systems.

(a) $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$

Direct I

a)



Parallel ?