Homework Assignment #3

ECE 6530: Digital Signal Processing
September 25, 2023

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Homework set #3

Due Date: Sep 29, 2023
(75 points)

1 Problem 3.2 parts a, b, d, f, and h

Determine the z-transform of the following signals and sketch the ROC of the following:

a) $x(n) = (1+n)u(n)$

b) $x(n) = (a^n + a^{-n})u(n)$ real a

c) $x(n) = (na^n \sin \omega_0 n)u(n)$

d) $x(n) = Ar^n \cos(\omega_0 n + \phi)u(n)$

e) $x(n) = \left[\frac{1}{2}\right]^n [u(n) - u(n-10)]$

Problem a can be split into two parts:

$$x(n) = (1+n)u(n) = u(n) + nu(n)$$

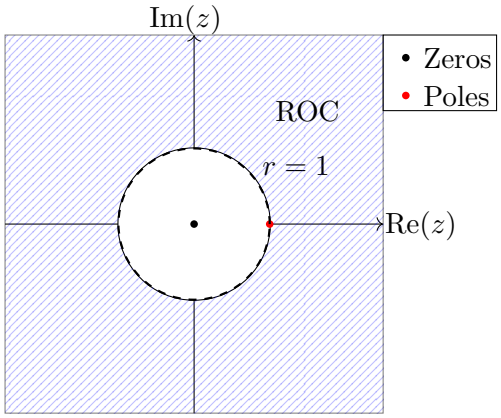
The first is a simple one that we can solve by geometric sum. But we have a table in the book that has these simple cases so we can skip ahead a bit:

$$\begin{aligned}
X_{tot}(z) &= X_1(z) + X_2(z) \\
X_{tot}(z) &= \left[\frac{1}{1-z^{-1}}\right] - z\frac{dX(z)}{dz} \\
X_{tot}(z) &= \left[\frac{1}{1-z^{-1}}\right] - z\left[\frac{-1}{(1-z^{-1})^2}\right](z^{-2}) \\
X_{tot}(z) &= \left[\frac{1}{1-z^{-1}}\right] + \left[\frac{z^{-1}}{(1-z^{-1})^2}\right] \\
X_{tot}(z) &= \left[\frac{1-z^{-1}}{(1-z^{-1})^2}\right] + \left[\frac{z^{-1}}{(1-z^{-1})^2}\right] \\
X_{tot}(z) &= \left[\frac{1}{(1-z^{-1})^2}\right]
\end{aligned}$$

The poles are clearly at 1 since a value of 1 for z would cause the denominator to go to 0. The zeros would need us to multiply top and bottom by z^2.

$$X_{tot}(z) = \left[\frac{z^2}{z^2(1-z^{-1})^2}\right] = \left[\frac{z^2}{(z-1)^2}\right]$$

This shows the zeros as well as the poles. both with multiplicity 2.



2 Problem 3.3 a-d

a)

$$x_1(n) = \begin{cases} \left(\frac{1}{3}\right)^n & \text{if } n \geq 0 \\ \left(\frac{1}{2}\right)^{-n} & \text{if } n < 0 \end{cases}$$

b)

$$x_2(n) = \begin{cases} \left(\frac{1}{3}\right)^n - 2^n & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$$

c) $x_3(n) = x_1(n+4)$

d) $x_4(n) = x_1(-n)$

3 Problem 3.7