Worksheet # 8

MATH 3160 – Complex Variables
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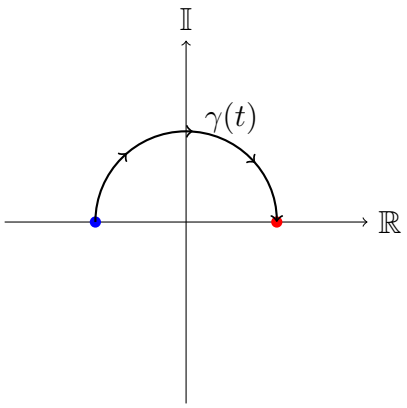
Problem 1

A contour C is parametrized by $\gamma(t) = e^{i(\pi - t)} \quad (0 \leq t \leq \pi)$. Draw the contour C, carefully indicating its starting point and ending point.

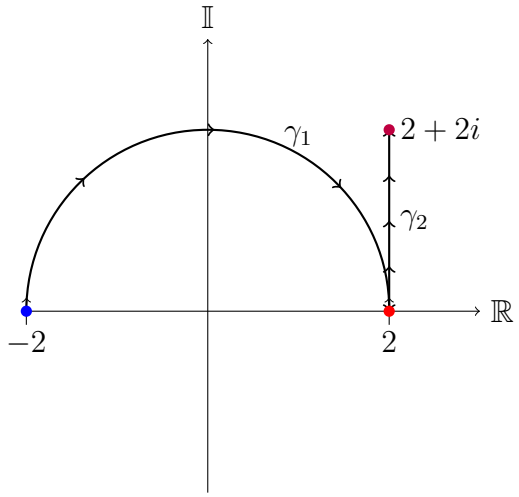
$$e^{i(\pi-t)} = e^{i\pi}e^{-it}$$

This can be understood as taking the path for e^{-it} which from 0 to π should sweep around clockwise from 0 to $-\pi$. Every point in this path is then rotated in a counter-clockwise rotation by π. So the path is the following:



Problem 2

Write down the parametrization of the following contour:



Starting from the blue point above, we move in a circular path along the arc, landing us at the red point. The following is the parametrization of that arc:

$$\gamma_1(t): \ [0,1] \to 2e^{-i\pi(t+1)} = 2e^{-i(\pi t)}e^{-i\pi} \quad 0 \le t \le 1$$

We start gamma at π by including the factor of $e^{-i\pi}$. Then as t sweeps from 0 to 1, we end at $e^{-i2\pi}$, effectively rotating the semicircular path on the bottom of the circle around the origin. Then for γ_2 we will then do the following:

$$\gamma_2 : [1,2] \to 2 + 2i(t-1) \quad 1 \le t \le 2$$
