Worksheet # 4

MATH 3160 – Complex Variables
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Problem 1:

Write the following functions $f(z)$ in the form $f(z) = u(x, y) + iv(x, y)$

(a) $f(z) = z^3 + z + 1$

(b) $f(z) = \frac{\bar{z}^2}{z}$ for $z \neq 0$

Problem 2:

Consider the mapping $z \to z^2$.

(a) What is the image of the line $z = x + i$?

(a) What is the image of the square bounded by the four lines $z = \pm 1 + iy$ and $z = x \pm i$?

Problem 3:

Compute the following limits (or state that they do not exist)

(a) $\lim_{z \to i} \frac{iz^3 - 1}{z + i}$

(b) $\lim_{z \to i} \left(z + \frac{1}{z} \right)$

(c) $\lim_{z \to 0} \frac{1}{z^2}$

Problem 4:

Does the following limit exist?

(a) $\lim_{z \to 0} \left(\frac{\bar{z}}{z}\right)^2$
