Homework 1: Complex Numbers

MATH 3160
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Problem 1: Complex Number Reduction

Reduce each of these to a real number:

(a) $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$

$$
\begin{aligned}
\frac{1+2i}{3-4i} + \frac{2-i}{5i} &= \\
\frac{(1+2i)(3+4i)}{(3-4i)(3+4i)} + \frac{(2-i)(-5i)}{(5i)(-5i)} &= \\
\frac{(1+2i)(3+4i)}{9-16i^2} + \frac{(2-i)(-5i)}{-25i^2} &= \\
\frac{(3+4i+6i+8\cancelto{-1}{i^2})}{9-16\cancelto{-1}{i^2}} + \frac{(-10i+5\cancelto{-1}{i^2})}{-25\cancelto{-1}{i^2}} &= \\
\frac{(3+4i+6i-8)}{25} + \frac{(-10i-5)}{25} &= \\
\frac{(3+4i+6i-8)}{25} + \frac{(-10i-5)}{25} &= \\
\frac{(-5+10i)}{25} + \frac{(-10i-5)}{25} &= \\
\frac{(-5-5+10i-10i)}{25} = -\frac{2}{5}
\end{aligned}
$$

(b) $\frac{5i}{(1-i)(2-i)(3-i)}$

$$\frac{5i}{(1-i)(2-i)(3-i)} = \frac{5i}{(2-i-2i-i^2)(3-i)} =$$
$$\frac{5i}{(2-3i-\cancelto{-1}{i^2})(3-i)} = \frac{5i}{(2-3i+1)(3-i)} =$$
$$\frac{5i}{(3-3i)(3-i)} = \frac{5i}{(9-3i-9i-3i^2)} =$$
$$\frac{5i}{(9-12i-3\cancelto{-1}{i^2})} = \frac{5i}{(12-12i)} =$$
$$\frac{5i}{(12-12i)} \cdot \frac{(12+12i)}{(12+12i)} = \frac{5i}{(12-12i)} \cdot \frac{(12+12i)}{(12+12i)}$$

(c) $(1-i)^4$

Problem 2: Vector Addition and Subtraction

Locate the numbers $z_1 + z_2$ and $z_1 - z_2$ vectorially by drawing a graph when:

(a) $z_1 = 2i$, $z_2 = 2/3 - i$

(b) $z_1 = -\sqrt{3} + i$, $z_2 = \sqrt{3}$

(c) $z_1 = (3, 1)$, $z_2 = (1, 4)$

(d) $z_1 = x_1 + iy_1$, $z_2 = x_1 - iy_1$

Problem 3: Geometric Sets in the Complex Plane

In each case, sketch the set of points determined by the given condition:

(a) $|z - 1 + i| = 1$

(b) $|z + i| \leq 3$

(c) $|z - 4i| \geq 4$

Hint: Note that for any two complex numbers z_1, z_2, *the absolute value* $|z_1 - z_2|$ *is the distance between* z_1 *and* z_2 *in the complex plane.*

Problem 4: Principal Arguments

Find the principal argument $\mathrm{Arg}(z)$ when:

(a) $z = \frac{i}{-2-2i}$

(b) $z = (\sqrt{3} - i)^6$

Show your work.

Problem 5: Argument Properties

For any two non-zero complex numbers z_1, z_2, show that any angle θ in the set $\arg(z_1 z_2)$ can be written as

$$\theta = \theta_1 + \theta_2$$

where $\theta_1 \in \arg(z_1)$ and $\theta_2 \in \arg(z_2)$. Also find an example where the principal argument $\operatorname{Arg}(z_1 z_2)$ is not equal to $\operatorname{Arg}(z_1) + \operatorname{Arg}(z_2)$.

Problem 6: Principal Argument Addition

Show that if $\text{Re}(z_1) > 0$ and $\text{Re}(z_2) > 0$, then $\text{Arg}(z_1 z_2) = \text{Arg}(z_1) + \text{Arg}(z_2)$. Use polar form of z_1 and z_2 to do so.