Homework # 4

MATH 3160 – Complex Variables
Your Name Here

Completed: September 11, 2025

Problem 1

Find $f'(z)$ using differentiation rules.

(a) $f(z) = 3z^2 - 2z + 4$

(b) $f(z) = (1 - 4z)^3$

(c) $f(z) = \frac{z-1}{2z+1}$, assume $z \neq -1/2$

(d) $f(z) = \frac{(z^2+1)^4}{z^2}$, assume $z \neq 0$

(e) $f(z) = z\, e^{z^2+3}$.

Problem 2

Show that $f'(z_0)$ does not exist at any point z_0 in two ways: using the limit definition and using the Cauchy-Riemann equations. Here, $z = x + iy$ and $x, y \in \mathbb{R}$.

(a) $f(z) = 2x + ixy^2$

(b) $f(z) = e^x e^{-iy}$

Problem 3

Using the exponential function e^z, we can now define the complex cosine and sine function for any $z \in \mathbb{C}$ as follows:

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2}$$

and

$$\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}.$$

Using these formulas,

(a) express $\cos(z)$ and $\sin(z)$ in rectangular coordinates $u(x, y) + iv(x, y)$ where $z = x + iy$.

(b) show that the complex cosine and sine functions are analytic over $\mathbb{C}$ and calculate their derivatives.
