

Worksheet # 4

MATH 3160 – Complex Variables
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Problem 1:

Write the following functions $f(z)$ in the form $f(z) = u(x, y) + iv(x, y)$

(a) $f(z) = z^3 + z + 1$

writing out each z as $x + iy$

$$\begin{aligned} f(z) &= z^3 + z + 1 = (x + iy)^3 + (x + iy) + 1 \\ (x + iy)^3 &= (x + iy)(x + iy)(x + iy) = (x^2 + 2ixy + i^2y^2)(x + iy) = \\ (x^2 + 2ixy - y^2)(x + iy) &= x^3 + ix^2y + 2ix^2y + 2i^2xy^2 - y^2x - iy^3 = \\ &= x^3 - 3xy^2 + i(3x^2y - y^3) \\ f(z) &= x^3 - 3xy^2 + i(3x^2y - y^3) + (x + iy) + 1 = \\ &= \boxed{(x^3 - 3xy^2 + x + 1) + i(3x^2y - y^3 + y)} \end{aligned}$$

(b) $f(z) = \frac{\bar{z}^2}{z}$ for $z \neq 0$

$$\begin{aligned} f(z) &= \frac{\bar{z}^2}{z} = \frac{(\overline{x + iy})^2}{(x + iy)} = \frac{(x - iy)^2}{(x + iy)} = \\ &= \frac{(x^2 - 2ixy + i^2y^2)}{(x + iy)} = \frac{(x^2 - 2ixy - y^2)(x - iy)}{(x + iy)(x - iy)} = \\ &= \frac{(x^3 - x^2iy - 2ix^3y + 2i^2xy^2 - y^2x + iy^3)}{(x^2 + iyx - iyx - i^2y^2)} = \\ &= \frac{(x^3 - x^2iy - 2ix^3y - 2xy^2 - y^2x + iy^3)}{(x^2 + y^2)} = \\ &= \boxed{\frac{x^3 - 3xy^2}{x^2 + y^2} + i\frac{-3x^2y + y^3}{x^2 + y^2}} \end{aligned}$$

Problem 2:

Consider the mapping $z \rightarrow z^2$.

- (a) What is the image of the line $z = x + i$?

$$\begin{aligned} z \rightarrow z^2 &= (x + iy) \rightarrow (x + iy)^2 \\ (x^2 + 2ixy - y^2) &= (x^2 + 2ix(1) - (1)^2) = x^2 + 2xi - 1 \\ &= (x^2 - 1) + i(2x) \end{aligned}$$

- (a) What is the image of the square bounded by the four lines $z = \pm 1 + iy$ and $z = x \pm i$?
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Problem 3:

Compute the following limits (or state that they do not exist)

(a) $\lim_{z \rightarrow i} \frac{iz^3 - 1}{z + i}$

(b) $\lim_{z \rightarrow i} \left(z + \frac{1}{z} \right)$

(c) $\lim_{z \rightarrow 0} \frac{1}{z^2}$

Problem 4:

Does the following limit exist?

(a) $\lim_{z \rightarrow 0} \left(\frac{\bar{z}}{z} \right)^2$

no, b/c diff paths give diff result
