# Homework # 7

MATH 3160 – Complex Variables
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## Problem 1

Compute the integral

$$\int_{C_i} z^{1/2} dz$$

where $z^{1/2}$ is taken on the branch $0 < arg(z) < 2\pi$ and along the contours $C_1, C_2$, where $C_1$ is any contour from $-3$ to 3 lying in the domain except the point 3 that approaches 3 from above the real axis, while $C_2$ approaches 3 from below the real axis.

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## Problem 2

Apply the Cauchy-Goursat theorem to show that

$$\int_C f(z)dz = 0$$

for the contour $C$ being the unit circle about the origin, or determine that Cauchy-Goursat theorem does not apply.

(a) $f(z) = \frac{z^2}{z-3}$

(b) $f(z) = ze^{-z}$

(c) $f(z) = \frac{z}{2z-i}$

(d) $f(z) = \tan(z)$

(e) $f(z) = Log(z+2)$

(f) $f(z) = \frac{1}{z^2+3z+2}$

(g) $f(z) = log(z)$, any branch.

## Problem 3

Let C denote the positively oriented boundary of the square whose sides lie along $x \pm 2$ and $y \pm 2$. Evaluate the following integrals.

(a) $\int_C \frac{e^{-z}}{z - \frac{\pi i}{2}} dz$

(b) $\int_C \frac{z}{2z+1} dz$

(c) $\int_C \frac{\cosh(z)}{z}$

## Problem 4

Use the Cauchy integral formula to integrate $\int_C f(z)dz$ for $f(z) = \frac{1}{(z-i)(z-1)}$ over a contour $C$ being a positive-oriented circle at the origin with radius 2.

## Problem 5

Using the extended Cauchy integral formula, compute

$$\int_C \frac{e^{2iz}}{(z-3i)^4}dz$$

on the curve $C$ being a positive-oriented circle with radius 4 centered at the origin.

## Problem 6

Using the extended Cauchy integral formula, compute

$$\int_C \frac{\cosh(z)}{(z-i)^2} dz$$

on the contour $C$ being the positive-oriented circle with radius 2 centered at the origin. Recall that $\cosh(z) := \frac{e^z + e^{-z}}{2}$.