Worksheet # 4

MATH 3160 – Complex Variables
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Problem 1

Recall that we have defined the complex exponential function e^z by the formula $e^z = e^x e^{iy} = e^x(\cos(y) + i\sin(y))$, where $x = Re(z)$ and $y = Im(z)$.

Calculate $f'(z)$ for each of the following functions:

(a) $f(z) = (z+2)^5$

$$f(z) = (z+2)^5$$
$$f'(z) = \frac{df}{dz}$$
$$f'(g(z)) = \frac{df}{dg}\frac{dg}{dz} = 5(z+2)^4(1)$$

(b) $f(z) = e^{z^3+z+1}$

$$f(z) = e^{z^3+z+1} = e^{z^3}e^z e^1 = e(e^{z^3}e^z)$$
$$\frac{d(ab)}{dz} = a'b + ab'$$
$$a = e^{z^3} \to a' = e^{z^3}(3z^2)$$
$$b = e^z \to b' = e^z(1)$$
$$a'b + ab' = e^{z^3}(3z^2)e^z + e^{z^3}e^z = e^{z^3}e^z(3z^2+1)$$
$$\therefore f'(z) = e^{z^3}e^z e(3z^2+1) = e^{z^3+z+1}(3z^2+1)$$

(c) $f(z) = e^{1/z}$

$$f'(z) = \frac{d}{dz}e^{1/z} = \frac{d}{dz}e^{z^{-1}} = e^{z^{-1}}(-1)z^{-2}$$
$$= -e^{1/z}\frac{1}{z^2}$$

Problem 2

Use the Cauchy-Riemann equations to show that $f'(z)$ does not exist at any point for the following:

(a) $f(z) = z - \bar{z}$

(b) $f(z) = e^x e^{-iy}$

The Cauchy-Riemann equations are the following:

$$\frac{du}{dx} = \frac{dv}{dy}$$
$$\frac{du}{dy} = -\frac{dv}{dx}$$

$f'(z_0) \iff$ these expressions above hold when evaluated at (x_0, y_0).

(a)

$$f(z) = z - \bar{z} = (x + iy) - (x - iy) = (x - x) + i(y + y) = i2y$$
$$u(x,y) = 0 \qquad v(x,y) = 2y$$
$$\frac{du}{dx} = 0 \qquad \frac{dv}{dy} = 2 \qquad \frac{du}{dy} = 0 \qquad \frac{dv}{dx} = 0$$
$$\frac{du}{dx} \neq \frac{dv}{dy}$$
$$\therefore f'(z)\ DNE$$

(b)

$$f(z) = e^x e^{-iy} = e^x(\cos(-y) + i\sin(-y)) = e^x(\cos(y) - i\sin(y))$$
$$u(x,y) = e^x\cos(y) \qquad v(x,y) = -e^x\sin(y)$$
$$\frac{du}{dx} = e^x\cos(y) \qquad \frac{dv}{dy} = -e^x\cos(y) \qquad \frac{du}{dy} = -e^x\sin(y) \qquad \frac{dv}{dx} = -e^x\sin(y)$$
$$\frac{du}{dx} = e^x\cos(y)$$
$$\frac{dv}{dy} = -e^x\cos(y)$$
$$e^x\cos(y) = -e^x\cos(y) \to 2e^x\cos(y) = 0$$

Only possible if $\cos(y) = 0$ and similar situation for the other equation

$$\frac{du}{dy} = -\frac{dv}{dx}$$
$$-e^x\sin(y) = e^x\sin(y) \to 2e^x\sin(y) = 0$$

$\sin(\theta)$ and $\cos(\theta)$ cannot both be 0 simultaneously

$\therefore f'(z)\ DNE$