

# Worksheet # 4

MATH 3160 – Complex Variables  
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## Problem 1:

Write the following functions  $f(z)$  in the form  $f(z) = u(x, y) + iv(x, y)$

(a)  $f(z) = z^3 + z + 1$

(b)  $f(z) = \frac{\bar{z}^2}{z}$  for  $z \neq 0$

**Problem 2:**

Consider the mapping  $z \rightarrow z^2$ .

(a) What is the image of the line  $z = x + i$ ?

(a) What is the image of the square bounded by the four lines  $z = \pm 1 + iy$  and  $z = x \pm i$ ?

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**Problem 3:**

Compute the following limits (or state that they do not exist)

(a)  $\lim_{z \rightarrow i} \frac{iz^3 - 1}{z + i}$

(b)  $\lim_{z \rightarrow i} \left( z + \frac{1}{z} \right)$

(c)  $\lim_{z \rightarrow 0} \frac{1}{z^2}$

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**Problem 4:**

Does the following limit exist?

(a)  $\lim_{z \rightarrow 0} \left( \frac{\bar{z}}{z} \right)^2$

no, b/c diff paths give diff result

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