Homework # 7

MATH 3160 – Complex Variables
Miguel Gomez

Completed: October 27, 2025

Problem 1

Compute the integral

$$\int_{C_i} z^{1/2} dz$$

where $z^{1/2}$ is taken on the branch $0 < arg(z) < 2\pi$ and along the contours C_1, C_2, where C_1 is any contour from -3 to 3 lying in the domain except the point 3 that approaches 3 from above the real axis, while C_2 approaches 3 from below the real axis.

Problem 2

Apply the Cauchy-Goursat theorem to show that

$$\int_C f(z)dz = 0$$

for the contour C being the unit circle about the origin, or determine that Cauchy-Goursat theorem does not apply.

(a) $f(z) = \frac{z^2}{z-3}$

(b) $f(z) = ze^{-z}$

(c) $f(z) = \frac{z}{2z-i}$

(d) $f(z) = \tan(z)$

(e) $f(z) = Log(z+2)$

(f) $f(z) = \frac{1}{z^2+3z+2}$

(g) $f(z) = log(z)$, any branch.

(a)

$f(z) = \frac{z^2}{z-3}$

(b)

$f(z) = ze^{-z}$

(c)

$f(z) = \frac{z}{2z-i}$

(d)

$f(z) = \tan(z)$

(e)

$f(z) = Log(z+2)$

(f)

$f(z) = \frac{1}{z^2+3z+2}$

(g)

$f(z) = log(z)$, any branch.

Problem 3

Let C denote the positively oriented boundary of the square whose sides lie along $x \pm 2$ and $y \pm 2$. Evaluate the following integrals.

(a) $\int_C \frac{e^{-z}}{z - \frac{\pi i}{2}} dz$

(b) $\int_C \frac{z}{2z+1} dz$

(c) $\int_C \frac{\cosh(z)}{z}$

(a)

$\int_C \frac{e^{-z}}{z - \frac{\pi i}{2}} dz$

(b)

$\int_C \frac{z}{2z+1} dz$

(c)

$\int_C \frac{\cosh(z)}{z}$

Problem 4

Use the Cauchy integral formula to integrate $\int_C f(z)dz$ for $f(z) = \frac{1}{(z-i)(z-1)}$ over a contour C being a positive-oriented circle at the origin with radius 2.

Problem 5

Using the extended Cauchy integral formula, compute

$$\int_C \frac{e^{2iz}}{(z - 3i)^4} dz$$

on the curve C being a positive-oriented circle with radius 4 centered at the origin.

Problem 6

Using the extended Cauchy integral formula, compute

$$\int_C \frac{\cosh{(z)}}{(z-i)^2}dz$$

on the contour C being the positive-oriented circle with radius 2 centered at the origin. Recall that $\cosh{(z)} := \frac{e^z + e^{-z}}{2}$.