Homework # 3:

MATH 3160 – Complex Variables
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Problem 1:

(a) Write the function
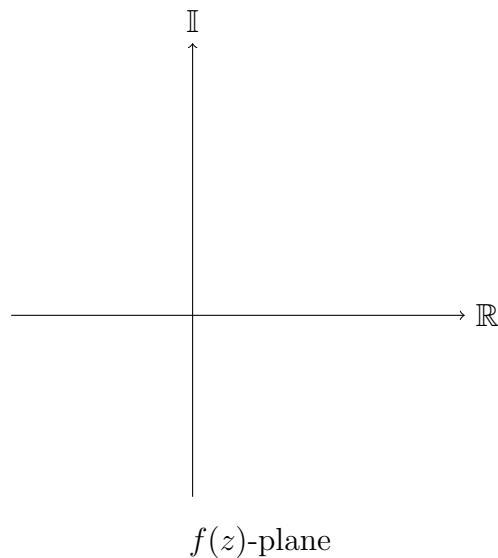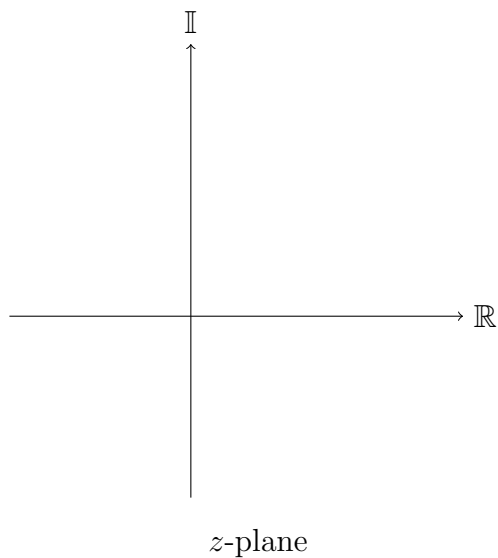
$$f(z) = z + \frac{1}{z} \qquad (z \neq 0)$$

in the form $f(z) = u(r, \theta) + iv(r, \theta)$.

(b) Show that the image of the points in the upper half plane $(y > 0)$ that are exterior to the circle $|z| = 1$ are mapped under f to the entire upper half plane $v > 0$.

(a)

(b)

z-plane
$f(z)$-plane

Problem 2:

Use the rectangular forms or exponential forms for the following functions to prove that

(a) $\lim\limits_{z \to z_0} Re(z) = Re(z_0)$

(b) $\lim\limits_{z \to z_0} \bar{z} = \bar{z}_0$

(c) $\lim\limits_{z \to 0} \frac{\bar{z}^2}{z} = 0$

Problem 3:

Show that the limit of the function
$$f(z) = \left(\frac{z}{\bar{z}}\right)^3$$
as z tends to zero does not exist. Do so by examining several test paths going to zero.

Problem 4:

Does $f(x + iy) = \dfrac{x + iy}{x + 2iy}$ have a limit as $x + iy \to 0$?