Worksheet # 7

MATH 3160 – Complex Variables
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Completed: October 14, 2025

Problem 1

suppose $l_1(z)$ and $l_2(z)$ denote two different branches of log. What can you say about the function $l_1(z) - l_2(z)$? How many different values can the difference take? Use this to conclude that $\frac{d}{dz}(l(z)) = \frac{1}{z}$ for any branch of log (away from its branch cut).

This seems similar to the argument made in class about the arg of a complex number and the value it could take in a branch. Some similar reasoning will need to be made here to show that the result must be $\frac{1}{z}$.

Problem 2

Check that for any $c_1\ c_2 \in \mathbb{C}$, and any $0 \neq z \in \mathbb{C}$, we have $z^{(c_1+c_2)} = z^{c_1} z^{c_2}$ using the similar rule for the exponential function.

$$
\begin{aligned}
c_1\ ,\ c_2 &= x_1 + iy_1 \quad , \quad x_2 + iy_2 \\
w = c_1 + c_2 &= x_1 + iy_1 + x_2 + iy_2 = x_1 + x_2 + iy_1 + iy_2 \\
z^w &= z^{(c_1+c_2)} = z^{x_1+iy_1+x_2+iy_2} = z^{x_1+x_2+iy_1+iy_2} \\
&= z^{x_1} z^{x_2} z^{iy_1} z^{iy_2} = z^{x_1} z^{iy_1} z^{x_2} z^{iy_2} \\
&= z^{x_1+iy_1} z^{x_2+iy_2} = z^{c_1} z^{c_2} \\
\therefore z^{(c_1+c_2)} &= z^{c_1} z^{c_2} \quad \forall x,y \in \mathbb{R} | z \neq 0
\end{aligned}
$$

Problem 3

Using the chain rule, show that $\frac{d}{dz} z^c = cz^{c1}$.