

Worksheet # 5

MATH 3160 – Complex Variables
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Problem 1

Recall that we have defined the complex exponential function e^z by the formula $e^z = e^x e^{iy} = e^x(\cos(y) + i \sin(y))$, where $x = \operatorname{Re}(z)$ and $y = \operatorname{Im}(z)$.

Calculate $f'(z)$ for each of the following functions:

(a) $f(z) = (z + 2)^5$

$$\begin{aligned} f(z) &= (z + 2)^5 \\ f'(z) &= \frac{df}{dz} \\ f'(g(z)) &= \frac{df}{dg} \frac{dg}{dz} = 5(z + 2)^4(1) \end{aligned}$$

(b) $f(z) = e^{z^3+z+1}$

$$\begin{aligned} f(z) &= e^{z^3+z+1} = e^{z^3} e^z e^1 = e(e^{z^3} e^z) \\ \frac{d(ab)}{dz} &= a'b + ab' \\ a &= e^{z^3} \rightarrow a' = e^{z^3} (3z^2) \\ b &= e^z \rightarrow b' = e^z (1) \\ a'b + ab' &= e^{z^3} (3z^2) e^z + e^{z^3} e^z = e^{z^3} e^z (3z^2 + 1) \\ \therefore f'(z) &= e^{z^3} e^z e (3z^2 + 1) = e^{z^3+z+1} (3z^2 + 1) \end{aligned}$$

(c) $f(z) = e^{1/z}$

$$\begin{aligned} f'(z) &= \frac{d}{dz} e^{1/z} = \frac{d}{dz} e^{z^{-1}} = e^{z^{-1}} (-1) z^{-2} \\ &= -e^{1/z} \frac{1}{z^2} \end{aligned}$$

Problem 2

Use the Cauchy-Riemann equations to show that $f'(z)$ does not exist at any point for the following:

(a) $f(z) = z - \bar{z}$

(b) $f(z) = e^x e^{-iy}$

The Cauchy-Riemann equations are the following:

$$\frac{du}{dx} = \frac{dv}{dy}$$

$$\frac{du}{dy} = -\frac{dv}{dx}$$

$f'(z_0) \iff$ these expressions above hold when evaluated at (x_0, y_0) .

(a)

$$f(z) = z - \bar{z} = (x + iy) - (x - iy) = (x - x) + i(y + y) = i2y$$

$$u(x, y) = 0 \quad v(x, y) = 2y$$

$$\frac{du}{dx} = 0 \quad \frac{dv}{dy} = 2 \quad \frac{du}{dy} = 0 \quad \frac{dv}{dx} = 0$$

$$\frac{du}{dx} \neq \frac{dv}{dy}$$

$$\therefore f'(z) \text{ DNE}$$

(b)

$$f(z) = e^x e^{-iy} = e^x (\cos(-y) + i \sin(-y)) = e^x (\cos(y) - i \sin(y))$$

$$u(x, y) = e^x \cos(y) \quad v(x, y) = -e^x \sin(y)$$

$$\frac{du}{dx} = e^x \cos(y) \quad \frac{dv}{dy} = -e^x \cos(y) \quad \frac{du}{dy} = -e^x \sin(y) \quad \frac{dv}{dx} = -e^x \sin(y)$$

$$\frac{du}{dx} = e^x \cos(y)$$

$$\frac{dv}{dy} = -e^x \cos(y)$$

$$e^x \cos(y) = -e^x \cos(y) \rightarrow 2e^x \cos(y) = 0$$

Only possible if $\cos(y) = 0$ and similar situation for the other equation

$$\frac{du}{dy} = -\frac{dv}{dx}$$

$$-e^x \sin(y) = e^x \sin(y) \rightarrow 2e^x \sin(y) = 0$$

$\sin(\theta)$ and $\cos(\theta)$ cannot both be 0 simultaneously

$$\therefore f'(z) \text{ DNE}$$