Homework # 8

MATH 3160 – Complex Variables
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Problem 1

Find the Taylor series expansion at $z_0 = 0$ of the function

$$f(z) = \frac{z}{z^4 + 9} = \frac{z}{9} \cdot \frac{1}{1 + \left(\frac{z^4}{9}\right)}$$

Problem 2

Find the Taylor series expansion of

$$f(z) = \frac{1}{1 - z}$$

at the following points, if it exists (or say it doesn't exist if the series diverges).

(a) $z = 0$

(b) $z = 1$

(c) $z = 2$

Problem 3

Derive the Laurent series representation of

$$f(z) = \frac{e^z}{(z+1)^2}$$

about the point $z_0 = -1$.

Problem 4

Represent the function

$$f(z) = \frac{z+1}{z-1}$$

(a) By a Taylor series at $z_0 = 0$ and state its domain of convergence.

(b) By a Laurent series about $z_0 = 1$.

Problem 5

On the domain $0 < |z - 1| < 2$, find the Laurent series about $z_0 = 1$ of:

$$f(z) = \frac{z}{(z-1)(z-3)}$$