Worksheet

MATH 3160 – Complex Variables
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Problem 1

Show that $\mathrm{Re}(z) = \frac{z+\bar{z}}{2}$ and $\mathrm{Im}(z) * i = \frac{z-\bar{z}}{2}$ for any complex number $z = a + bi$

Problem 2

Find the fourth roots of $-8 - 8\sqrt{3}i$. express the roots in rectangular coordinates, exhibit them as the vertices of a certain square, and point out the principal root.

Problem 3

Find the four zeros of the polynomial $z^4 + 4$, given that one of them is:

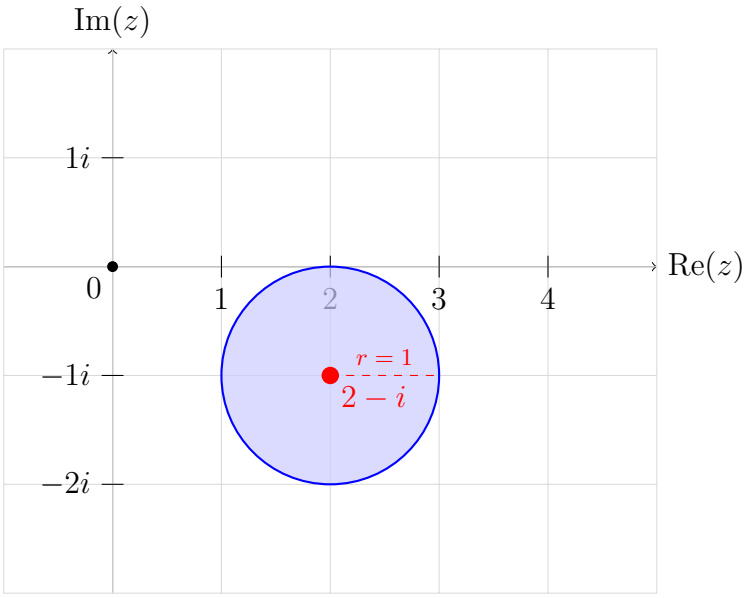
$$z_0 = \sqrt{2}e^{i\frac{\pi}{4}} = 1 + i$$

Use these zeros to factor $z^4 + 4$ into quadratic factors with real coefficients.

Problem 4

Sketch the following sets and state whether each set is open, connected, a domain, and whether it is bounded.

(a) $|z - 2 + i| \leq 1$



- **Open:** No, because the boundary is included ($\leq$ condition)
- **Connected:** Yes, it's a disk which is connected
- **Domain:** No, because it's not open
- **Bounded:** Yes, all points are within distance 1 from center $(2, -1)$

(b) $|2z + 3| > 4$

(c) $\text{Im}(z) > 1$

(d) $\text{Im}(z) = 1$

(e) $0 \leq \arg(z) \leq \frac{\pi}{4}$, where $z \neq 0$

(f) $|z - 4| \geq |z|$