Worksheet # 4

MATH 3160 – Complex Variables
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Problem 1

Recall that we have defined the complex exponential function e^z by the formula $e^z = e^x e^{iy} = e^x(\cos(y) + i\sin(y))$, where $x = Re(z)$ and $y = Im(z)$.

Calculate $f'(z)$ for each of the following functions:

(a) $f(z) = (z+2)^5$

$$\begin{aligned} f(z) &= (z+2)^5 \\ f'(z) &= \frac{df}{dz} \\ f'(g(z)) &= \frac{df}{dg}\frac{dg}{dz} = 5(z+2)^4(1) \end{aligned}$$

(b) $f(z) = e^{z^3+z+1}$

$$\begin{aligned} f(z) &= e^{z^3+z+1} = e^{z^3}e^z e^1 = e(e^{z^3}e^z) \\ \frac{d(ab)}{dz} &= a'b + ab' \\ a &= e^{z^3} \to a' = e^{z^3}(3z^2) \\ b &= e^z \to b' = e^z(1) \\ a'b + ab' &= e^{z^3}(3z^2)e^z + e^{z^3}e^z = e^{z^3}e^z(3z^2+1) \\ \therefore f'(z) &= e^{z^3}e^z e(3z^2+1) = e^{z^3+z+1}(3z^2+1) \end{aligned}$$

(c) $f(z) = e^{1/z}$

$$\begin{aligned} f'(z) &= \frac{d}{dz}e^{1/z} = \frac{d}{dz}e^{z^{-1}} = e^{z^{-1}}(-1)z^{-2} \\ &= -e^{1/z}\frac{1}{z^2} \end{aligned}$$

Problem 2

Use the Cauchy-Riemann equations to show that $f'(z)$ does not exist at any point for the following:

(a) $f(z) = z - \bar{z}$

(b) $f(z) = e^x e^{-iy}$