Homework #2

MATH 3160 – complex variables
Your Name Here

Completed: August 27, 2025

Problem 1

By writing the individual factors on the left in exponential form, performing the needed operations, and finally changing back to rectangular coordinates, show that

(a) $i(1-\sqrt{3}i)(\sqrt{3}+i) = 2(1+\sqrt{3}i)$

(b) $\frac{5i}{2+i} = 1+2i$

(c) $(-1+i)^7 = -8(1+i)$

(d) $(1+\sqrt{3}i)^{-10} = 2^{-11}(-1+\sqrt{3}i)$

Problem 2

Find the square roots of (a) $2i$ and (b) $(1 - \sqrt{3}i)$ express them in rectangular coordinates

Problem 3

Find all roots and indicate in rectangular coordinates

(a) $(-16)^{\frac{1}{4}}$

(b) $(-8-8\sqrt{3}i)^{\frac{1}{4}}$

Problem 4

Find the four zeros of $z^4 + 4$

Problem 5

Show that if c is an n^{th} root of 1 other than 1 itself, then:

$$1 + c + c^2 + ... + c^{n-1} = 0$$

Hint: multiply above by $(c - 1)$

multiplying the above by $(c - 1)$ gives the following

$$(c-1) \cdot (1 + c + c^2 + ... + c^{n-1}) = (c-1) \cdot 0$$

$$c + c^2 + c^3 + ... + c^{n-1} + c^n + \quad \text{expanding } c$$

$$-1 - c - c^2 - ... - c^{n-1} \quad \text{expanding } -1$$

$$-1 + (c - c) + (c^2 - c^2) + (c^3 - c^3) + ... + (c^{n-1} - c^{n-1}) + c^n = 0$$

$$-1 + \cancelto{0}{(c - c)} + \cancelto{0}{(c^2 - c^2)} + \cancelto{0}{(c^3 - c^3)} + ... + \cancelto{0}{(c^{n-1} - c^{n-1})} + c^n = 0$$

$$-1 + c^n = 0$$

$$c^n = 1$$

$$\sqrt[n]{c^n} = \sqrt[n]{1}$$

$$c = 1$$

However, if this is the case and $c = 1$, then the lhs should equal 0 when we plug in 1 for c:

$$1 + (1) + (1)^2 + ... + (1)^{n-1} = 0$$

$$1 + 1 + 1 + ... + 1 = n \neq 0$$

This appears to be a contradiction. Now using something other than 1, i.e. $c \neq 1$, then the sum cannot be 0.

$$1 + c + c^2 + ... + c^{n-1} = S$$

same steps as before

$$-1 + c^n = S(c-1) = Sc - S$$

$$c^n - Sc + S - 1 = 0$$

Problem 6

For each of the below, indicate the domain of definition.

(a) $f(z) = \frac{1}{z^2+1}$

(b) $f(z) = \text{Arg}\left(\frac{1}{z}\right)$

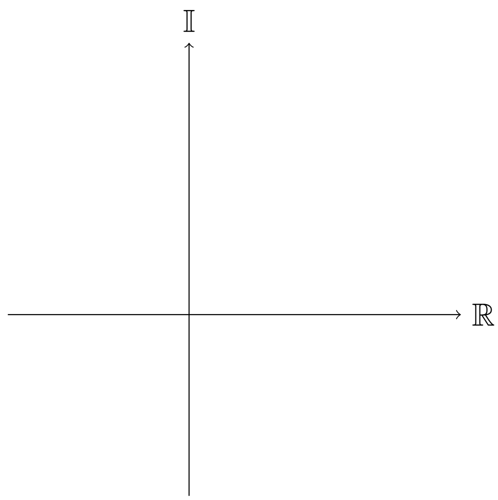
(c) $f(z) = \frac{z}{z+\bar{z}}$

(d) $f(z) = \frac{1}{(1-|z|^2)}$

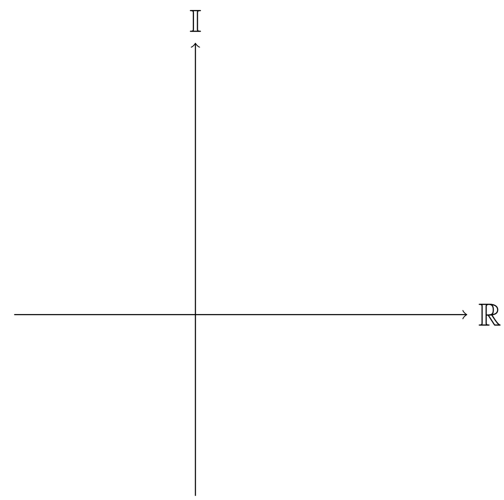
1 Problem 7

Sketch the region onto which the sector $r \leq 1;\ 0 \leq \theta \leq \frac{\pi}{4}$ in the z-plane is mapped to the $w = f(z)$-plane by the transformations

(a) $w = z^2$

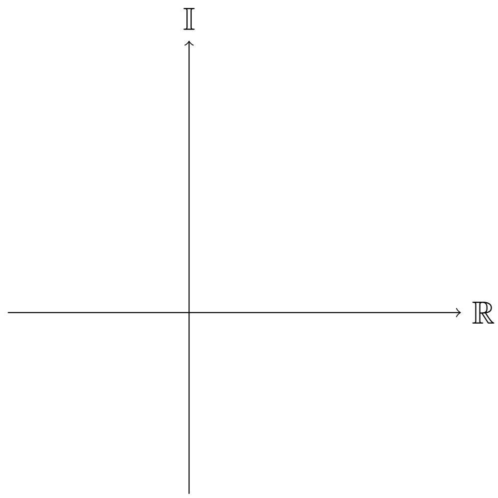


z-plane



w-plane

(b) $w = z^3$



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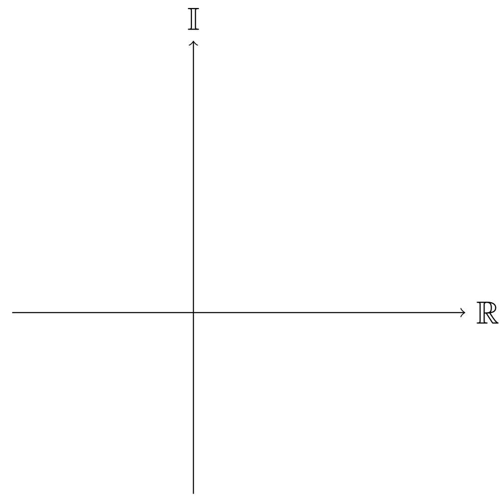
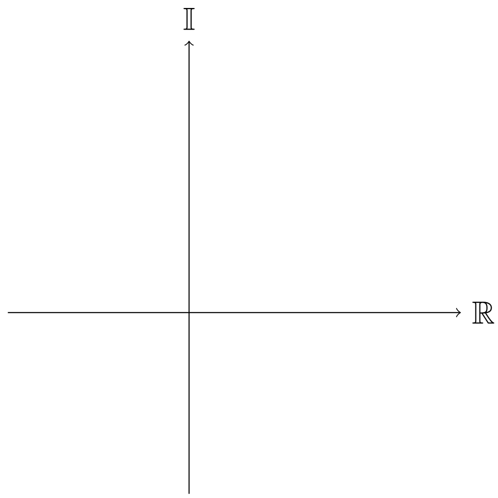


w-plane

(c) $w = z^4$



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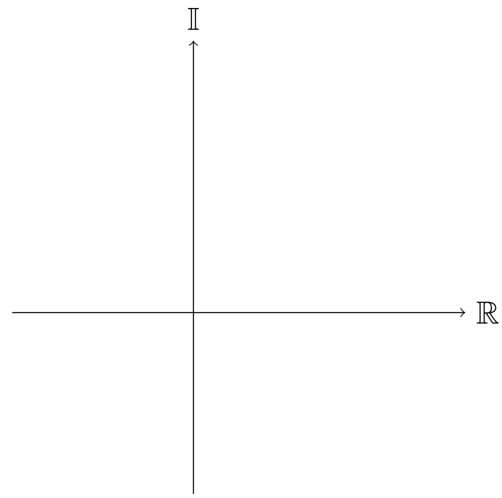

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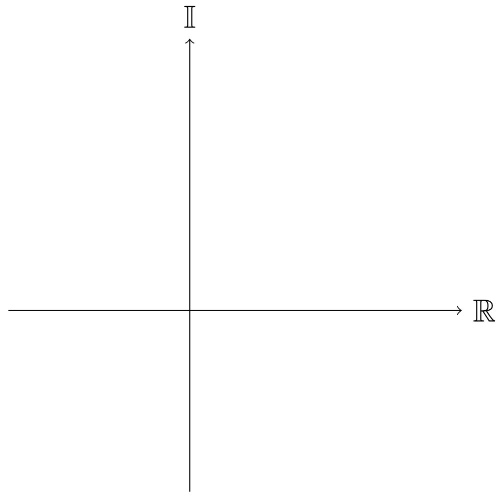
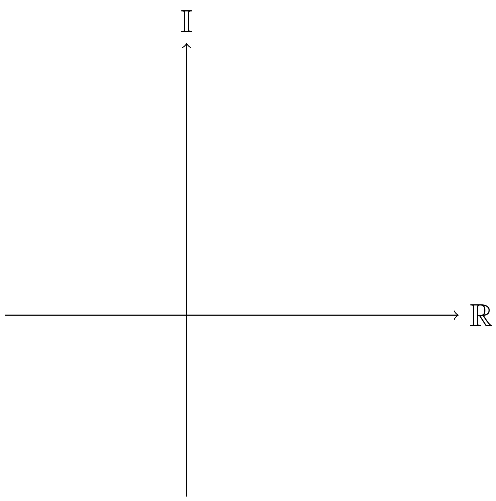
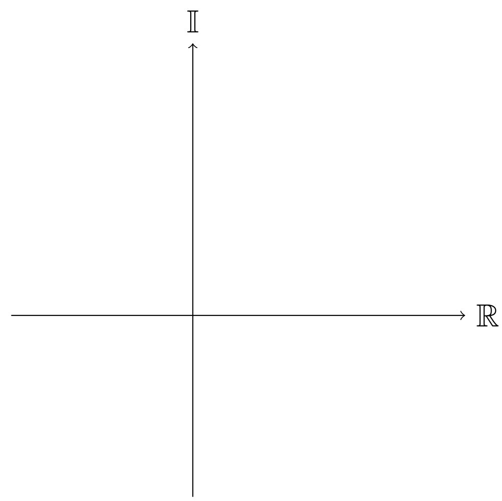




z-plane



w-plane