Worksheet # 9

MATH 3160 – Complex Variables
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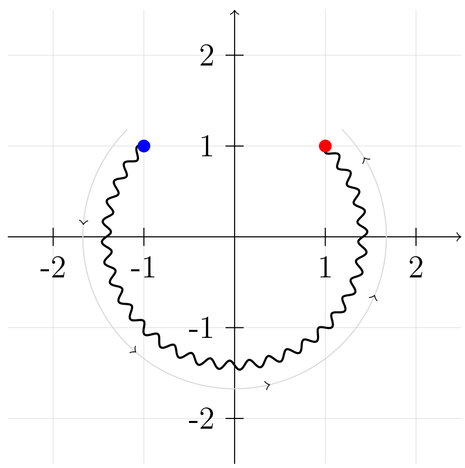
Problem 1

let C be the contour shown below, traversed counter-clockwise from the blue point to the red:



find $\int_C \frac{1}{z} dz$ (Hint: Consider a new branch of the logarithm function by $\log(re^{i\theta}) = \ln(r) + i\theta$, where $\pi/2 < \theta \leq \pi/2$, and check that this is an anti-derivative of $1/z$.)

Problem 2

Show that $\int_C f(z)dz = 0$ for C the unit circle and :

(i) $f(z) = \frac{z^2}{z+3}$

(ii) $f(z) = \frac{1}{z^2+2z+2}$

(i)

Since we are evaluating with C within the unit circle, any point which lies outside of the unit circle does not matter for our evaluation as we only need the curve and its interior to be a simply connected domain D. in the denominator, we see that we have $z+3$, meaning that it only becomes 0 if $\text{Re}(z)$ is -3. So the vertical line $x = -3$ in the complex plane will give a divide by zero issue. The unit circle only contains points where the magnitude of z is less than or equal to 1.

$\therefore$ the C-G equation holds and we have a path with a simply connected interior region with the same starting and ending point whose integral evaluates to 0.

Problem 3

Let C_1 denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 1$, and $y = \pm i$, and let C_2 denote the positively oriented circle $|z| = 4$. Explain why:

$$\int_{C_1} f(z)dz = \int_{C_2} f(z)dz$$

when

(a) $f(z) = \frac{1}{2x^2+1}$

(b) $f(z) = \frac{z+2}{sin(z/2)}$

(c) $f(z) = \frac{z}{1-e^z}$