Worksheet 2

MATH 3160
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Problem 1

Reduce each of these to a real number

(a) $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$

$$\frac{1+2i}{3-4i} + \frac{2-i}{5i} =$$
$$\frac{(1+2i)(3+4i)}{(3-4i)(3+4i)} + \frac{(2-i)(-5i)}{(5i)(-5i)} =$$
$$\frac{(1+2i)(3+4i)}{9-16i^2} + \frac{(2-i)(-5i)}{-25i^2} =$$
$$\frac{(3+4i+6i+8\cancel{i^2})}{9-16\cancel{i^2}}^{-1} + \frac{(-10i+5\cancel{i^2})}{-25\cancel{i^2}}^{-1} =$$
$$\frac{(3+4i+6i-8)}{25} + \frac{(-10i-5)}{25} =$$
$$\frac{(3+4i+6i-8)}{25} + \frac{(-10i-5)}{25} =$$
$$\frac{(-5+10i)}{25} + \frac{(-10i-5)}{25} =$$
$$\frac{(-5-5+10i-10i)}{25} = -\frac{2}{5}$$

(b) $\frac{5i}{(1-i)(2-i)(3-i)}$

$$\frac{5i}{(1-i)(2-i)(3-i)} = \frac{5i}{(2-i-2i+i^2)(3-i)} =$$
$$\frac{5i}{(1-3i)(3-i)} = \frac{5i}{(1-3i)(3-i)} =$$
$$\frac{5i}{-10i} = -\frac{1}{2}$$

Problem 2

Find the principal argument $\operatorname{Arg} z$ when..

preliminary necessary expressions:

$$\begin{aligned} \arg(z) &= \operatorname{Arg}(z) + 2 \cdot pi \cdot k \quad ; k \in \mathbb{Z} \\ e^{i\theta} &= \cos\theta + i\sin\theta \\ -\pi &< \theta \leq \pi \end{aligned}$$

(a) $z = \frac{-2}{1+\sqrt{3}i}$

(b) $z = \frac{2i}{i-1}$

(c) $z = (\sqrt{3} - i)^6$

Problem 3

For the next few questions write the individual factors on the left in exponential form, perform the needed operations on complex numbers, and finally change back to rectangular coordinates *Show that*:

(a) $i(1 - \sqrt{3}i)(\sqrt{3} + i) = 2(1 + \sqrt{3}i)$

(b) $(\sqrt{3}+i)^6 = -64$

(c) $(1+\sqrt{3}i)^{-10} = 2^{-11}(-1+\sqrt{3}i)$

Problem 4

Use exponential form to find $(1-i)^5$