Project 2 IEOR 4004: NBA Rescheduling Problem

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December 2025

1 Introduction

Throughout NBA history, countless superstars such as Kobe Bryant, Kyrie Irving, Kevin Durant, and James Harden have suffered major injuries at critical moments, some of which have had lifelong consequences and fundamentally altered the trajectories of their careers. For professional athletes, the relationship between scheduling, fatigue, and injury risk is therefore not an abstract concept but a core factor that shapes both the length and quality of their playing careers.

The National Basketball Association (NBA) is one of the most widely watched and highest-grossing sports organizations of the last few decades. Part of their success comes from organizing fair play and schedules for all their teams. All teams should play evenly at home and away with an adequate amount of time to prepare for each game. The purpose of this project is to propose alternatives to the current schedule that more effectively emphasize and ensure fairness.

1.1 Project Goals and Core Challenges

Building upon the context above, this project aims to use Integer Programming to systematically analyze and optimize the preliminary schedule for the NBA's 2025/2026 season. The core challenge we face is to find a feasible, or even improved, schedule that satisfies a complex set of interconnected constraints related to competitive fairness. Specifically, our work entails:

1.Extract the fixed schedule framework for each team from the given preliminary schedule. This includes each team's set of home-game dates, the number of times each team played another team at home, the number of times each team played another team away (at the opponent's arena), and the full set of dates on which each team played away. We printed all of this information in our code and organized each team's data as shown in Figure 2. The computed data form the "hard constraints" for the subsequent optimization model—the fundamental framework that any new schedule must preserve to ensure the competitive balance of the season is not disrupted.

2.Build an Integer Programming model with no objective function. The sole goal of this model is to find a feasible solution that satisfies all constraints extracted in the first phase. This means that, while keeping the game dates for each team and the number of games against each opponent completely unchanged, we only reassign the combinations of "who plays whom, and when." This serves as the foundation for all subsequent optimization work.

3.Introduce a critical optimization constraint to the base model: limiting

```
TEAM: Atlanta Hawks
(a) Home dates:
    2025-11-03, 2025-11-07, 2025-11-15, 2025-11-17, 2025-11-19, 2025-11-23, 2025-11-27, 2025-11-28, 2025-11-29, 2025-12-25
(b) # times HOSTED each opponent:
    vs Boston Celtics at HOME: 1
    vs Chicago Bulls at HOME: 1
    vs Dallas Mavericks at HOME: 1
    vs Golden State Warriors at HOME: 1
    vs Los Angeles Lakers at HOME: 1
    vs Miami Heat at HOME: 1
    vs Milwaukee Bucks at HOME: 1
    vs New York Knicks at HOME: 1
    vs Phoenix Suns at HOME: 1
    vs Toronto Raptors at HOME: 1
(c) # times VISITED each opponent:
    at Brooklyn Nets (AWAY): 1
    at Chicago Bulls (AWAY): 1
    at Cleveland Cavaliers (AWAY): 1
    at Denver Nuggets (AWAY): 1
    at Houston Rockets (AWAY): 1
    at Philadelphia 76ers (AWAY): 1
(d) Away dates:
    2025-11-01, 2025-11-05, 2025-11-11, 2025-11-13, 2025-11-21, 2025-12-01
```

Figure 2: Preliminary scheduling information for the Atlanta Hawks.

travel fatigue for teams playing three consecutive matches. Specifically, we first apply a hard constraint—no team may play three consecutive matches where the sum of the absolute time zone differences between any two adjacent match locations reaches or exceeds 4. If scheduling cannot meet this hard constraint, we relax the condition by imposing penalties on consecutive matches. Ultimately, our goal is to reduce injury risks caused by overly demanding schedules and enhance competitive fairness through optimization.

1.2 Model Assumptions

To build a tractable mathematical model, we make the following key assumptions:

1.Fixed Schedule Framework: We assume that the set of home and away dates for each team, as well as the total number of games against every other team (including the split between home and away), as provided in the league's preliminary schedule, are fixed league mandates that must be adhered to in the optimization. This ensures that the new schedule is consistent with the original plan in terms of the number of games and basic structure.

2.Opponent Interchangeability: We assume that, subject to constraints (e) to (h), any two games that meet the constraints can have their opponents swapped without affecting the fairness of the competition. For example, if Team A must play a home game on Date D and must play one home game each against Team B and Team C, then, as long as the total number of games is satisfied, the specific assignment of which opponent plays on Date D can be adjusted. This provides the degrees of freedom necessary for optimization.

Through the introduction and assumptions outlined above, we have clarified the practical problem this project aims to solve, the technical approach adopted, and the underlying rationale. The following sections will detail the specific components of the Integer Programming models established to achieve these goals.

2 Problem 2

To find a feasible solution such that a new schedule would fit the requirements, we formulated the following integer program:

1. Sets: **Sets**

   - I: set of NBA teams, index i, j
   - D: set of dates in the season, index d
   - $A_i \subseteq D$: set of dates on which team i must play at home (from part (a))
   - $B_i \subseteq D$: set of dates on which team i must play away (from part (d))

   Parameters

   - C_{ij}: number of times team i must play at home against team j (from part (b)), for all $i, j \in I$ with $i \neq j$
   - D_{ij}: number of times team i must play away at team j (from part (c)), for all $i, j \in I$ with $i \neq j$

   Decision Variable

   - $x_{i,j,d} \in \{0, 1\}$, equals 1 if team i hosts team j on date d, and 0 otherwise

   Constraints

$$
\begin{aligned}
\sum_{j \in I,\ j \neq i} x_{i,j,d} &= 1 && \forall i \in I,\ d \in A_i \\
\sum_{j \in I,\ j \neq i} x_{i,j,d} &= 0 && \forall i \in I,\ d \in D \setminus A_i \\
\sum_{j \in I,\ j \neq i} x_{j,i,d} &= 1 && \forall i \in I,\ d \in B_i \\
\sum_{j \in I,\ j \neq i} x_{j,i,d} &= 0 && \forall i \in I,\ d \in D \setminus B_i \\
\sum_{d \in A_i} x_{i,j,d} &= C_{ij} && \forall i, j \in I,\ i \neq j \\
\sum_{d \in B_i} x_{j,i,d} &= D_{ij} && \forall i, j \in I,\ i \neq j \\
x_{i,j,d} &\in \{0, 1\} && \forall i, j \in I,\ i \neq j,\ d \in D
\end{aligned}
$$

The feasible schedule is included as a csv file for the final submission.

3 Problem 3 Approach 1: Additional Travel Limits (hard constraint)

As a modification to the second problem, in the third problem the NBA wants to adjust the schedule to make the travel for each team less strenuous on the teams, meaning for any three consecutive games that a team plays, we want to make sure that the total number of time zones traveled is less than four. We need to compute a feasible schedule that satisfies this additional constraint. If we do not find such a feasible schedule, we need to conclude that no such schedule exists. The following is the set-up for problem 3, which takes parts of the set-ups in problem 2.

Sets and Parameters

$$
\begin{aligned}
I &= \text{set of teams}, \\
D &= \text{set of all dates in the scheduling horizon}, \\
G_i &= \text{ordered list of dates on which team } i \text{ plays (home or away)}, \\
n_i &= |G_i| \text{ total number of games played by team } i, \\
d_{i,k} &= k\text{-th game date in } G_i, \text{ for } k = 1, \dots, n_i, \\
H_{i,d} &= \begin{cases} 1, & \text{if team } i \text{ must be } \textbf{home} \text{ on date } d, \\ 0, & \text{otherwise}, \end{cases} \\
A_{i,d} &= \begin{cases} 1, & \text{if team } i \text{ must be } \textbf{away} \text{ on date } d, \\ 0, & \text{otherwise}, \end{cases} \\
H^{\text{pair}}_{i,j} &= \text{required number of home games where team } i \text{ hosts team } j, \\
A^{\text{pair}}_{i,j} &= \text{required number of away games where team } i \text{ plays at team } j, \\
z_i &= \text{time zone of team } i\text{'s home arena},
\end{aligned}
$$

Decision Variables

$$
\begin{aligned}
&x_{i,j,d} \in \{0,1\} && 1 \text{ if team } i \text{ hosts team } j \text{ on date } d, 0 \text{ otherwise}, \\
&\text{diff1}_{i,k} \geq 0 && \text{(continuous) time-zone jump between the 1st and 2nd games in triple } k \text{ for team } i, \\
&\text{diff2}_{i,k} \geq 0 && \text{(continuous) time-zone jump between the 2nd and 3rd games in triple } k \text{ for team } i,
\end{aligned}
$$

Derived Expressions

$$t_{i,k} = z_i \sum_{j \neq i} x_{i,j,d_{i,k}} \; + \; \sum_{j \neq i} z_j \, x_{j,i,d_{i,k}}, \qquad \forall i \in I, \; k = 1, \ldots, n_i - 2.$$

This expression gives the time zone of the arena in which team i plays on date $d_{i,k}$ (i.e., in its k-th game in G_i). It is not a decision variable; rather, it is a linear expression in the scheduling variables x that is used in the constraints defining the jump variables $\text{diff1}_{i,k}$ and $\text{diff2}_{i,k}$.

Objective Function

For this problem, our aim is to find a feasible solution that satisfies the constraints, or to conclude that the solutions are infeasible. Since we are not concerned about optimality here and there are no metrics to minimize or maximize, our objective function is simply

$$\text{Min } 0$$

which forces Gurobi to stop as soon as feasibility is reached, because once Gurobi finds a feasible solution, everything is already optimized, and so it stops immediately. Although Max 0 would give the same result, Min 0 aligns with Gurobi's default optimization direction and avoids unnecessary settings such as switching the solver to maximization mode.

Constraints

1. **Home schedule constraints**

$$\sum_{j \neq i} x_{i,j,d} = H_{i,d}, \qquad \forall i \in I, \; d \in D.$$

2. **Away schedule constraints**

$$\sum_{j \neq i} x_{j,i,d} = A_{i,d}, \qquad \forall i \in I, \; d \in D.$$

3. **Total home pair requirements**

$$\sum_{d \in D} x_{i,j,d} = H^{\text{pair}}_{i,j}, \qquad \forall i, j \in I, \; i \neq j.$$

4. **Total away pair requirements**

$$\sum_{d \in D} x_{j,i,d} = A^{\text{pair}}_{i,j}, \qquad \forall i, j \in I, \; i \neq j.$$

5. **Linearization of time-zone jumps for each triple**

   For each team i and each triple index $k = 1, \ldots, n_i - 2$, let $t_{i,k}, t_{i,k+1}, t_{i,k+2}$ be the time-zone expressions defined above. We enforce

$$\mathrm{diff1}_{i,k} \geq t_{i,k+1} - t_{i,k}, \qquad \mathrm{diff1}_{i,k} \geq t_{i,k} - t_{i,k+1},$$
$$\mathrm{diff2}_{i,k} \geq t_{i,k+2} - t_{i,k+1}, \qquad \mathrm{diff2}_{i,k} \geq t_{i,k+1} - t_{i,k+2},$$

   so that $\mathrm{diff1}_{i,k}$ and $\mathrm{diff2}_{i,k}$ represent the absolute time-zone jumps between the first and second, and between the second and third games of the triple, respectively.

6. **Hard three-consecutive-game travel constraint**

   For each team i and triple index k, we require

$$\mathrm{diff1}_{i,k} + \mathrm{diff2}_{i,k} \leq 3, \qquad \forall i \in I,\ k = 1, \ldots, n_i - 2.$$

   This enforces the project rule that

$$|t_{i,k} - t_{i,k+1}| + |t_{i,k+1} - t_{i,k+2}| \leq 3,$$

   i.e., the total time-zone change over any three consecutive games is less than 4.

7. **Domain of the jump variables**

$$\mathrm{diff1}_{i,k} \geq 0, \quad \mathrm{diff2}_{i,k} \geq 0, \qquad \forall i \in I,\ k = 1, \ldots, n_i - 2.$$

8. **Domain of the scheduling variables**

$$x_{i,j,d} \in \{0, 1\}, \qquad \forall i, j \in I,\ i \neq j,\ d \in D.$$

Results

We could not find a feasible solution to enforce all constraints. After temporarily removing the hard triple constraint, a feasible solution was found. This indicates that it is impossible to create a schedule that strictly avoids the triple-jump pattern (three consecutive games with a cumulative time-zone difference of 4 or more).

To proceed, we decided to convert the hard triple constraint into a soft constraint. This approach allows us to find a solution that minimizes violations of the rule, getting us as close as possible to a fully compliant schedule.

4 Problem 3 Approach 2: Additional Travel Limits (soft constraint / penalty)

In the soft-penalty formulation, the triple-game travel requirement is not strictly enforced. Instead, we allow violations but penalize them in the objective function.

Sets and Parameters

$$
\begin{aligned}
I &= \text{set of teams,}\\
D &= \text{set of all dates in the scheduling horizon,}\\
G_i &= \text{ordered list of dates on which team } i \text{ plays (home or away),}\\
n_i &= |G_i| \text{ total number of games played by team } i,\\
d_{i,k} &= k\text{-th game date in } G_i, \text{ for } k = 1, \ldots, n_i,\\
H_{i,d} &= \begin{cases} 1, & \text{if team } i \text{ must be } \textbf{home} \text{ on date } d,\\ 0, & \text{otherwise,} \end{cases}\\
A_{i,d} &= \begin{cases} 1, & \text{if team } i \text{ must be } \textbf{away} \text{ on date } d,\\ 0, & \text{otherwise,} \end{cases}\\
H^{\text{pair}}_{i,j} &= \text{required number of home games where team } i \text{ hosts team } j,\\
A^{\text{pair}}_{i,j} &= \text{required number of away games where team } i \text{ plays at team } j,\\
z_i &= \text{time zone of team } i\text{'s home arena,}\\
M &= \text{big-}M \text{ parameter used in softening the travel constraint (in code, } M = 3).
\end{aligned}
$$

Decision Variables

$$
\begin{aligned}
&\text{diff1}_{i,k} \geq 0 && \text{time-zone jump between the 1st and 2nd games in triple } k \text{ for team } i,\\
&\text{diff2}_{i,k} \geq 0 && \text{time-zone jump between the 2nd and 3rd games in triple } k \text{ for team } i,\\
&x_{i,j,d} \in \{0,1\} && 1 \text{ if team } i \text{ hosts team } j \text{ on date } d, 0 \text{ otherwise,}\\
&\text{viol}_{i,k} \in \{0,1\} && \begin{cases} 1 & \text{if triple } k \text{ for team } i \text{ is allowed to exceed the travel limit,}\\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

Derived Expressions

$$
t_{i,k} = z_i \sum_{j \neq i} x_{i,j,d_{i,k}} + \sum_{j \neq i} z_j \, x_{j,i,d_{i,k}}, \qquad \forall i \in I, \; k = 1, \ldots, n_i - 2.
$$

This expression gives the time zone of the arena in which team i plays its k-th game (as determined by the scheduling variables x).

Objective Function

Since the original NBA constraint is "no team should play three consecutive games with a total time-zone jump of 4 or more," we incorporate this requirement by penalizing every violated triple. The objective is to minimize the number of violations:

$$\min \sum_{i \in I} \sum_{k=1}^{n_i - 2} \mathrm{viol}_{i,k}.$$

This objective encourages the solver to find a schedule with the fewest possible violations. If a schedule that satisfies all triple constraints exists, the minimum is 0; otherwise, we obtain the schedule with the least number of violations.

Constraints

1. **Home schedule constraints**

$$\sum_{j \neq i} x_{i,j,d} = H_{i,d}, \qquad \forall i \in I,\ d \in D.$$

2. **Away schedule constraints**

$$\sum_{j \neq i} x_{j,i,d} = A_{i,d}, \qquad \forall i \in I,\ d \in D.$$

3. **Total home pair requirements**

$$\sum_{d \in D} x_{i,j,d} = H_{i,j}^{\text{pair}}, \qquad \forall i, j \in I,\ i \neq j.$$

4. **Total away pair requirements**

$$\sum_{d \in D} x_{j,i,d} = A_{i,j}^{\text{pair}}, \qquad \forall i, j \in I,\ i \neq j.$$

5. **Linearization of time-zone jumps for each triple**

   For each team i and each triple index $k = 1, \ldots, n_i - 2$, let $t_{i,k}, t_{i,k+1}, t_{i,k+2}$ be the time-zone expressions defined above. Then we enforce

$$\mathrm{diff1}_{i,k} \geq t_{i,k+1} - t_{i,k}, \qquad \mathrm{diff1}_{i,k} \geq t_{i,k} - t_{i,k+1},$$

$$\mathrm{diff2}_{i,k} \geq t_{i,k+2} - t_{i,k+1}, \qquad \mathrm{diff2}_{i,k} \geq t_{i,k+1} - t_{i,k+2},$$

   so that $\mathrm{diff1}_{i,k}$ and $\mathrm{diff2}_{i,k}$ represent the absolute time-zone jumps between the first and second, and between the second and third games of the triple, respectively.

6. **Soft three-consecutive-game travel constraint**

   For each team i and triple index k, we allow the sum of the two jumps to exceed 3 only if $\mathrm{viol}_{i,k} = 1$:

   $$\mathrm{diff1}_{i,k} + \mathrm{diff2}_{i,k} \le 3 + M \cdot \mathrm{viol}_{i,k}, \qquad \forall i \in I,\ k = 1, \ldots, n_i - 2.$$

   When $\mathrm{viol}_{i,k} = 0$, this reduces to $\mathrm{diff1}_{i,k} + \mathrm{diff2}_{i,k} \le 3$, enforcing the original travel rule $|t_{i,k} - t_{i,k+1}| + |t_{i,k+1} - t_{i,k+2}| \le 3$. When $\mathrm{viol}_{i,k} = 1$, the RHS becomes $3 + M$, allowing a larger sum of jumps but penalizing this in the objective.

   This sets $t_{i,p}$ equal to the time zone of the arena in which team i plays at date d_{ip}.

7. **Domain of the jump variables**

   $$\mathrm{diff1}_{i,k} \ge 0, \quad \mathrm{diff2}_{i,k} \ge 0, \qquad \forall i \in I,\ k = 1, \ldots, n_i - 2.$$

8. **Domain of the scheduling variables**

   $$x_{i,j,d} \in \{0, 1\}, \qquad \forall i, j \in I,\ i \ne j,\ d \in D.$$

9. **Domain of violation indicators**

   $$\mathrm{viol}_{i,k} \in \{0, 1\}, \qquad \forall i \in I,\ k = 1, \ldots, n_i - 2.$$

Results

We were able to generate a schedule that minimizes the violations of constraint 3, with 16 violations, which is 6 less than the violations of the original schedule. The full csv file of this newly optimized schedule is included in the submission.

The optimized schedule shows a modest but clear improvement in travel efficiency. On average, teams experience slightly smaller time-zone jumps between consecutive games, with the average pair jump decreasing from about 0.83 to 0.80. Over three-game stretches, cumulative time-zone movement also drops, as the average triple sum declines from 1.58 to 1.53. Most importantly, the number of major three-game travel violations—instances where teams must cross multiple time zones in rapid succession—falls from 22 to 16, removing six of the most demanding travel sequences. Overall, the optimized schedule reduces travel strain in both routine transitions and the most disruptive multi-game stretches, which likely makes travel less exhausting for players. Moreover, because large time-zone jumps typically correspond to long-distance flights—the primary contributors to travel-related carbon emissions—these improvements strongly suggest that the optimized schedule would reduce total travel distance and therefore lower associated emissions, making it a more sustainable travel plan than the original schedule.

5 Conclusion

For problem 3, we were unable to find a feasible solution that satisfies all constraints in our integer programming model once the additional travel limits were imposed as a hard requirement. In particular, when we required that no team play three consecutive games with a cumulative time-zone jump of 4 or more, Gurobi did not return any feasible schedule. After temporarily removing this triple-game constraint, the solver was able to find a feasible schedule, which indicates that the current preliminary home/away structure and fixed game dates are not compatible with a schedule that completely avoids the triple-jump pattern.

To move forward, we converted the triple-game travel limit from a hard constraint into a soft constraint. We introduced violation indicators and a penalty term in the objective so that each triple that exceeds the allowed travel distance is counted and minimized. Under this soft-penalty formulation, the model successfully produced a feasible schedule. The result still contains several triple-jump violations, but the number is minimized given the fixed framework of dates, arenas, and opponent counts.

Overall, our results show that, under the existing league calendar and structural requirements, it is impossible to completely eliminate high-travel triples without relaxing at least one major assumption (such as game dates or home/away allocations). However, the soft-constraint approach provides a practical compromise: it identifies where violations are unavoidable and finds a schedule that reduces them as much as possible, offering the closest attainable approximation to the ideal travel policy within the current NBA scheduling framework.

From a practical standpoint, even a schedule that reduces—but does not fully eliminate—excessive travel sequences represents a meaningful step toward protecting player health and maintaining competitive integrity. As highlighted in the introduction, the physical and cognitive toll of long-distance, multi-time-zone travel is well documented in sports-science literature, and it has been implicated in several high-profile injury cases that altered the course of NBA seasons. While a "perfect" travel-restricted schedule may be mathematically infeasible under the current league framework, the optimized schedule produced by our soft-constraint model still yields tangible benefits: it lowers the frequency of the most demanding road stretches, modestly reduces average time-zone jumps, and likely decreases total travel distance. For league schedulers, such a model offers a transparent, optimization-driven method to trade off strict travel limits against other scheduling requirements, helping to balance athlete welfare, competitive fairness, and the logistical realities of an 82-game season. Future work could explore relaxing additional fixed elements (e.g., allowing limited changes to home/away patterns) or incorporating more nuanced fatigue metrics, but the present approach already demonstrates how integer programming can deliver a concrete, improved schedule that directly addresses the player-safety concerns that motivated this project.

References

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