

A Basic PID Algorithm

Definition of terms (memory elements in *Italics*):

Counting indices:

j = represents the index of the current cycle
 $j-1$ = represents the index of the previous cycle

Temperature measurement:

T_j = current temperature of this cycle [°C], also can be written as just T
 T_{j-1} = temperature of the last cycle [°C]
 $\delta T = T_j - T_{j-1}$ temperature change since the last cycle

Time measurement:

t_j = current time of this cycle (sec), also can be written as just t
 t_{j-1} = time of the last cycle (sec)
 $\delta t = t_j - t_{j-1}$ elapsed time since the last cycle

Control Parameters:

T_s = Setpoint (desired temperature) [°C]

allowing us to define the error signal :

$\epsilon_j = T_j - T_s$ Error Signal [°C], also can be written as just $\epsilon = T - T_s$
 $\epsilon_{j-1} = T_{j-1} - T_s$ Error Signal for the previous cycle [°C]
 $\delta \epsilon = \epsilon_j - \epsilon_{j-1} = T_j - T_{j-1}$ error change since the last cycle [°C],
equal to the temperature change since the last cycle

Δ = Control bandwidth [°C]
 S = Settle bandwidth [°C]
 τ_i = Integral time (sec)
 I_{on} = Integral control on, Boolean, True = on, False = off
 τ_d = Derivative time (sec)
 D_{on} = Derivative control on, Boolean, True = on, False = off
 t_s = Settle time [sec]
 $T_H = T_s + \Delta/2$ High limit of control band
 $T_L = T_s - \Delta/2$ Low limit of control band
 $T_{SH} = T_s + S/2$ High limit of settle band
 $T_{SL} = T_s - S/2$ Low limit of settle band

Power terms:

P_p = Proportional power [normalized]
 k_j = current count of the number of cycles outside the control band
 k_{j-1} = count of the number of cycles outside the control band on the last cycle
 P_{Ij} = current integral power [normalized], also can be written as just P_I
 P_{Ij-1} = integral power of last cycle [normalized]
 P_d = Derivative power [normalized]
 $P_r = P_p + P_{Ij} + P_d$ Raw total power [normalized]
 P_t = Final total power [normalized 0-1]

Settle terms:

G_o = Temperature is settled, Boolean, True = on, False = off

t_{ej} = current elapsed running time in the settle band [sec]

t_{ej-1} = elapsed running time in the settle band of the previous cycle [sec]

Power Algorithm:

Proportional power:

$$\begin{aligned} P_p &= 1/2 - \varepsilon/\Delta & \text{if } T_L \leq T_j \leq T_H & \text{or} \\ &= 1 & \text{if } T_j < T_L & \text{or} \\ &= 0 & \text{if } T_H < T_j & \end{aligned}$$

Derivative power:

$$\begin{aligned} P_d &= -[\tau_d/\Delta][\delta\varepsilon/\delta t] & \text{if } D_{on} = \text{True} & \text{or} \\ &= 0 & \text{if } D_{on} = \text{False} & \end{aligned}$$

Out counter:

$$\begin{aligned} k_j &= k_{j-1} + 1 & \text{if } T_j < T_L \text{ or } T_H < T_j \\ &= 0 \text{ and } k_{j-1} = 0 & \text{if } T_L \leq T_j \leq T_H \end{aligned}$$

Integral power:

$$\begin{aligned} P_{ij} &= P_{ij-1} - [(\varepsilon * \delta t)/[\Delta * \tau_i]] & \text{if } T_L \leq T_j \leq T_H \text{ and } I_{on} = \text{True} \\ &= 0 \text{ and } P_{ij-1} = 0 & \text{if } k_j > 1 \text{ or } I_{on} = \text{False} \end{aligned}$$

Raw power:

$$P_r = P_p + P_{ij} + P_d$$

Final total power:

$$\begin{aligned} P_t &= P_r & \text{if } 0 < P_r < 1 \\ P_t &= 1 & \text{if } 1 < P_r \\ P_t &= 0 & \text{if } P_r < 0 \end{aligned}$$

Settle Algorithm:

Settle time tracking:

$$\begin{aligned} t_{ej} &= t_{ej-1} + \delta t & \text{if } T_{SL} < T_j < T_{SH} \\ &= 0 & \text{if } T_j < T_{SL} \\ &= 0 & \text{if } T_{SH} < T_j \end{aligned}$$

Determination of settled condition:

$$G_o = \text{True} \quad \text{if } t_s < t_{ej}$$

Initialization:

$$T_{j-1} = T_j$$

$$t_{j-1} = t_j$$

$$k_{j-1} = 0$$

$$P_{ij-1} = 0$$

$$t_{ej-1} = 0$$

$$P_d = 0$$

$$P_{lj} = 0$$

Reasonable Starting Values for the “bug”:

$$\Delta = 4 \quad \text{Control bandwidth [}^{\circ}\text{C]}$$

$$S = 1 \quad \text{Settle bandwidth [}^{\circ}\text{C]}$$

$$\tau_i = 30 \quad \text{Integral time (sec)}$$

$$\tau_d = 0.5 \quad \text{Derivative time (sec)}$$

$$t_s = 60 \quad \text{Settle time [sec]}$$