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4/9/14 9:30 Class Notes

Joint Distributions

- Continue with joint distributions
  - Go to the page for the week and download the SimpleJointDistribtuion CSV file
  - Going through expected value of a joint distribution
- Review (for two random variables)
  - X and Y are two related RV's
    - X: Apple Stock
    - Y: Google Stock
  - A joint distribution P(x,y) describes the joint variation of these two together.
    - Simplest way to do this is to make a probability mass table (PMT)
    - Think of f(x,y) as a policy or a decision/allocation
      - Ex: f(x,y) = aX + bY (portfolio)
    - To calculate the expected value of all possible events:
      - E[f(x,y) = the sum of f(xi, yj)*P(X=xi, Y=yj) as joint events
    - Strategy
      - Apply the function to every possible outcome
      - Take the weighted sum of the function values
  - Covariance
    - Cov(x,y) = E[[X-E[x]*[Y-E[y]] = E[f(x,y)]
      - F(x,y) = [X-E[x]]*[Y-E[y]]
- Back to the CSV file
  - The function f(x,y) is equal to our return of both Apple and Google stats
    - Column A – how much you would gain or lose on a share of Apple stock
    - Column B – how much you would gain or lose on a share of Google stock
    -

x	y	JointProb
-2.5	-2.5	0.001588
-2.5	-1.5	0.007115
-2.5	-0.5	0.004315
-2.5	0.5	0.000354
-2.5	1.5	3.94E-06
-2.5	2.5	1.00E-08
-2.5	3.5	0

  - Our f(x,y) tells us the outcome of the joint distribution of both of the stocks, given that we have 10 shares of both
  - Multiplying Column C, by our joint probability, we return our expected value, and all of those summed up = 14.298

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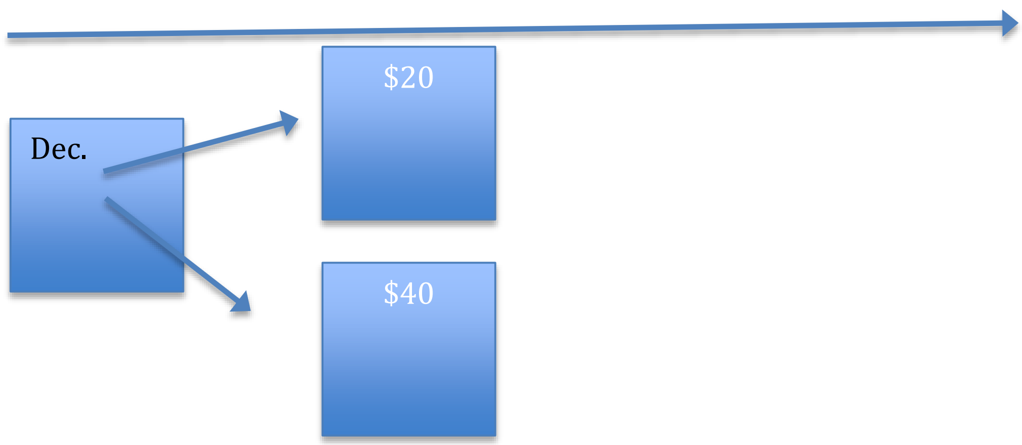
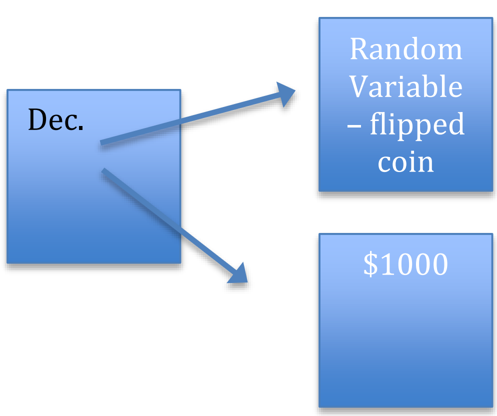
- Let's go back to probability
  - It makes no sense to say you would never do something that included certain death as a possibility – there are probabilities of us dying crossing the street!
  - Let's look at the stats
    - Cause of death and number
      - Botulism – 2
      - Flood – 200
      - Heart disease – 800,000
      - Homicide – 11,000
      - Motor vehicle accidents – 40,000
      - Pregnancy – 450
      - Stomach cancer – 90,000
      - Tornado – 90
  - We're trying to get to a point where we can think systematically about risk.
- Decision Problems



- Imagine the same scenario, except that the $20, is now a coin flip, for $40 or $10, and the other option is $10
- Applying our principle:
  - Reduce the stochastic (random) nodes to expected values, and proceed as before, searching for an expected value
  - Prune the tree
  - Time runs left to right, but we want to work left to right



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- Here, payoff is equal to $2^k, for as many heads are flipped until a tails is flipped.

Xk	Wk
$1	.5
$2	.25
$4	.125

Wk = P(k hears first, then tails) = (1/2)^k+1

So, naively apply the expected value, and our expected value is infinity! So why would we turn it down? When is enough, enough? We're essentially looking at how we can affect our utility – how happy are we with taking a probability, given the risk?

Now, we're looking to compare utilities – rather than straight cash – because the world runs on happiness.