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4/2 SCRIBE NOTES

Started by reminding us what we went over last class, We started probablty and begean Baye's Rule.

Continuing on Bayes' Rule: You have 2 events: A & B

Then: $P(A|B) = P(A) \cdot \frac{P(B|A)}{P(B)}$

Posterior Probability — Prior Probability — Updates Term

(Bayes' Formula updates your P(A) given new information

Bayes' Rule Example 1:

A: You Like Shrek B: You like Finding Nemo

$P(A) = .6$
$P(B|A) = .9$
$P(B|\sim A) = .3$
Given Values

* $\sim A$ = "Not A"

$$P(A|B) = \frac{P(A)\,P(B|A)}{P(B)}$$

We know P(A) & P(B|A), but what is P(B)?

We Know that
$$P(B) = P(A)\,P(B|A) + P(\sim A)\,P(B|\sim A)$$
$$= .6 \times .9 + .4 \times .3$$
$$= .66$$

Now we can simply plug into the formula:
$$P(A|B) = \frac{P(A)\,P(B|A)}{P(B)} = \frac{.6 \times .9}{.66} \approx .82$$

Bayes' Rule Example 2:

Two Events - G & D

G: Guilty

D: DNA Evidence matched DNA at crimescene

$P(G) = \frac{1}{10{,}000{,}000}$ => Assumption of innocence

$P(D|\sim G) = \frac{1}{1{,}000{,}000}$ => Probability of false match

$P(D|G) \approx 1$, Almost 1 so we shall estimate it as 1

What is the probability the defendant is guilty given the new evidence of the DNA matching?

$$P(G|D) = \frac{P(G)\,P(D|G)}{P(D)}$$

Find $P(D) = P(G)\,P(D|G) + P(\sim G)\,P(D|\sim G)$

$$= \frac{1}{10{,}000{,}000} \times 1 + \frac{9{,}999{,}999}{10{,}000{,}000} \times \frac{1}{1{,}000{,}000}$$

$$\approx \frac{1}{10{,}000{,}000} + \frac{1}{1{,}000{,}000}$$

$$= \frac{11}{10{,}000{,}000}$$

$$P(G|D) = \frac{P(G)\,P(D|G)}{P(D)} = \frac{\frac{1}{10{,}000{,}000} \times 1}{\frac{11}{10{,}000{,}000}} = \frac{1}{11}$$

Summary of Bayes' Rule:

Formula uses new evidence to update probabilities in a methodical way.

Be aware of the fallacy of confusing ~~P(B|A)~~ P(A|B) with P(A|B).

Often called the Base Rate or Prosecutor's Fallacy.

Next, we covered Odds:

Odds $\stackrel{Def.}{=}$ Just a different way of expressing probabilities

Example: A = Event that "Robert Apprentice wins Kentucky Derby"

$P(A) = .2$

$$Odds(A) = \frac{P(A)}{1-P(A)} = \frac{\text{Prob of } A}{\text{Prob of } \sim A} = \frac{.2}{.8} = \frac{1}{4}$$

Odds are often expressed as "odds against".

$$\text{Odds Against}(A) = \frac{1}{Odds(A)} = \frac{1-P(A)}{P(A)} = \frac{4}{1}$$

Read as the odds agaist A are 4 to 1.

Then we moved onto RANDOM VARIABLES:

X is a random variable => uncertain quantity

A probability distribution is a description of the possible outcomes for X with their probabilites.

Distribution as: $P(X = x_k) = w_k$

Value(x_k)	Probability(w_k)
x_1	w_1
x_2	w_2
$\vdots$	$\vdots$
x_k	w_k

Example: the probability distribution where X: Coin Flip

Outcomes(x_k)	Probabilities(w_k)
Heads	.5
Tails	.5

* These are discrete distributions.

Next, we went over the concept of EXPECTED VALUE.

Expected Value = $E(x)$ = average outcome of a random variable

$$E(X) = \text{weighted average} = \sum_{k=1}^{D} w_k x_k$$

Example: $E(X)$ where X = Price of Apple Stock

	Value (x_k)	Probability (w_k)	
x_1	400	.2	w_1
x_2	500	.6	w_2
x_3	600	.2	w_3

$$E(X) = 400 \times .2 + 500 \times .6 \times 600 \times .2$$

* WEIGHTED AVERAGE

Expected Value can also be applied to functions of Variables: $E[f(x)]$

* Quick Note: $E(f(x)) \neq f(E(x))$ => Jensen's Inequality
in general

Example: $E[f(x)]$ where $f(X)$ is the money you get from sellin Apple Stock = $.75(X)$

X = Price of Apple Stock

$Y = f(x) = .75(x)$

x_k EVENT	w_k Probability	$f(x_k)$ Function
400	.2	$.75 \times 400 = 300$
500	.6	$.75 \times 500 = 375$
600	.2	$.75 \times 600 = 450$

$$E[f(x)] = \sum_{k=1}^{D} w_k f(x_k)$$
$$= .2 \times 300 + .6 \times 375 + .2 \times 450$$

Example 2 of $E[f(x)]$: X = Price of Copper
$Y = f(x)$ = Copper Demanded
$Y = Kx^{\beta}$ where $K = 1000$ & $\beta = -2$

EVENT x_k	PROBABILITY w_k	$f(x)$
2	.25	1000×2^{-2}
3	.5	1000×3^{-2}
4	.25	1000×4^{-2}

$$E(Y) = E[f(x)] = \sum_{k=1}^{D} w_k f(x_k)$$

$$= .25 \times (1000 \times 2^{-2}) + .5 \times (1000 \times 3^{-2}) + .25 \times (1000 \times 4^{-2})$$

Notice that ~~EB~~ expected values are missing a notion of dispersion. So we must consider Variance.

Ideally you want $E(x) \uparrow$, Variance $\downarrow$, CoVariance $\downarrow$

VARIANCE

- Not the same concept as sample variance
- Talking about the spread of a distribution

X: Random variable with distribution P

$$Var(x) = E\{[X - E(X)]^2\}$$

$$\mu = E(X)$$

$$Var(X) = E((X-\mu)^2) = E[f(x)] \text{ where } f(x) = (x-\mu)^2$$

Identity: $var(x) = E(x^2) - [E(x)]^2$
$= E(x^2) - \mu^2$

$$sd(x) = \sqrt{var(x)}$$

* $var(x)$ = variance
$sd(x)$ = standard deviation

SUMMARY:

We started class by completing Bayes' Rule. Then, we covered Odds which are just a different way of stating probabilities. Next, we covered the concept of random variables which are any variables with uncertain quantities. We looked at the probability distributions of these random variables.

Last we covered the idea of Expected Value which is the average outcome of a certain thing whether it is a random variable of a function of a random variable. We covered how expected values by themselves are lacking and we must look at the Variance of the distributions as well.