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11:00

Scribe Notes 4/7

<u>PMT (Probability Massed Table)</u>

- x= price of iPhone
- y= number of iPhones sold by AT&T store
- y = f(x) = 2,500,000 * $x^{-1.7}$
- E(x) = expected value of x
- var(x)= E(f(x))

Possible price of outcome	Weight	Intermediate step (weight times value)	function		function	
x_k	w_k	w_k*x_k	(x_k-mean)^2	w_k*{(x_k-mean)^2}	f(x_k)	w_k*(f(x_k))
350	0.18	63	2704	486.72	118.3107843	21.29594117
400	0.6	240	4	2.4	94.28400526	56.57040316
450	0.22	99	2304	506.88	77.17537266	16.97858199
	Mean Weight	402				
	Variance	996				
	# of iPhones We Expect to Sell	94.84492631				

<u>PMF</u>

PMF: probability mass function

$W_k = P(x=x_k) = h(x_k)$

Before:	outcome (x_k)	prob (w_k)
	X_1	w_1
	X_2	w_2
	Etc….	Etc…
	x_D	w_D

Example: x = number of no shows on AA2937 from DFW → AUS
MD80 w/ 190 seats

PMF: function that <u>could</u> build the lookup table
Built from standard families of distributions

Ex: binomial distribution
N: customers
P: probability of one person not showing up

$$P(X=x_k) = h(x_k) = \binom{N}{x_k} p^{x_k} (1-p)^{N-x_k}$$

all of this gives a modeling language to describe variables

PDF

PDF: probability density function for continuous random variables

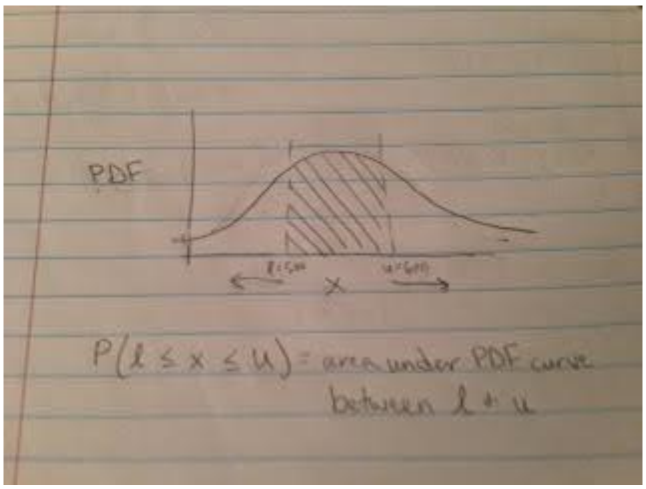
What is the possible price of apple stock tomorrow? Could be anywhere between 0 and GDP of globe, so it is mathematically impossible to list the numbers because there is an infinite amount.

So we draw a probability density function.

L= lower (500)

U = upper (600)

What is the probability that x is between L and U? The answer is the area under the curve.



We have now met three ways of formally describing random variables

1. The table: the brute force approach
2. PMF: function that explicitly can allow you to construct that table
3. PDF: take the area under the curve to get the probability that the random variables will fall within the bounds

There are other ways of describing random variables but we will use these three and montecarlo simulation for our purposes.

Joint Distributions

x= % of return on APPL this week

y= % of return on GOOG this week

P(x,y) joint distribution

X_k= possible APPL price	Y_k= possible GOOG price	w_k : this column is joint probability; $P(x= x_k , y=y_k)$
-1	-1	.3
1	-1	.2
-1	1	.2
1	1	.3

<u>Functions</u>

May be interested in the probability distribution of f(x)
Consider some function f(x,y)

Example: Hold 70% of my wealth in APPL and 30% in GOOG

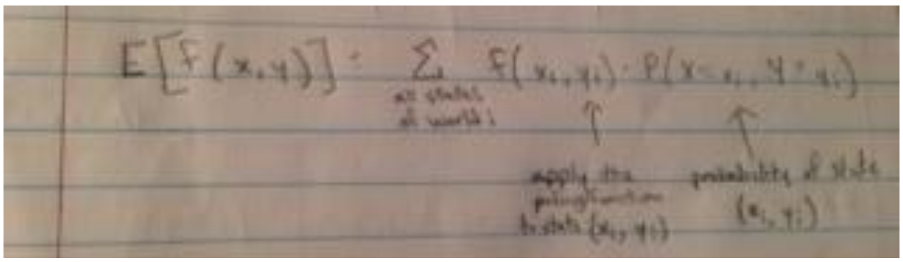
$F(x)= 0.7x + 0.3y$

*Functions take possible states of the world and give you a number.

P(X,Y): description of uncertainty about the future for x & y
F(X,Y): policy of decision or "happiness function"

Now we want to compute expected value for multiple functions

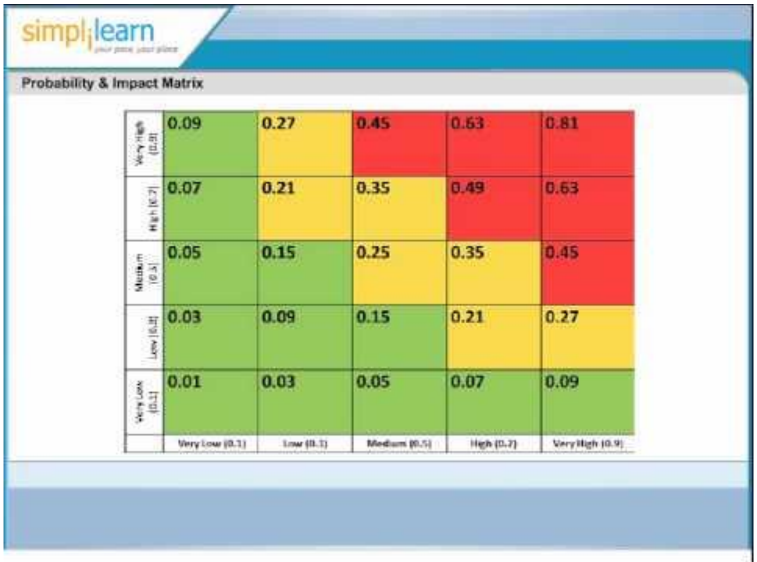
Equation: $E[f(x,y)] = \sum f(x_{i.}, y_i) * P(X= x_{i.}, Y= y_i)$



Every row is a different state of the world.

	A	B	C	D	E	F	G	H
1	x	y	JointProb	w_k*x_k	w_k*y_k	x-difference	y-difference	covariance
2	-2.5	-2.5	0.001588	-0.00397	-0.00397	-3.45318407	-2.97669621	0.016318
3	-2.5	-1.5	0.007115	-0.01779	-0.01067	-3.45318407	-1.97669621	0.048565
4	-2.5	-0.5	0.004315	-0.01079	-0.00216	-3.45318407	-0.97669621	0.014555
5	-2.5	0.5	0.000354	-0.00089	0.000177	-3.45318407	0.02330379	-2.9E-05
6	-2.5	1.5	3.94E-06	-9.9E-06	5.91E-06	-3.45318407	1.02330379	-1.4E-05
7	-2.5	2.5	1.00E-08	-2.5E-08	2.5E-08	-3.45318407	2.02330379	-7E-08
8	-2.5	3.5	0	0	0	-3.45318407	3.02330379	0
9	-1.5	-2.5	0.001588	-0.00238	-0.00397	-2.45318407	-2.97669621	0.011593
10	-1.5	-1.5	0.01934	-0.02901	-0.02901	-2.45318407	-1.97669621	0.093785

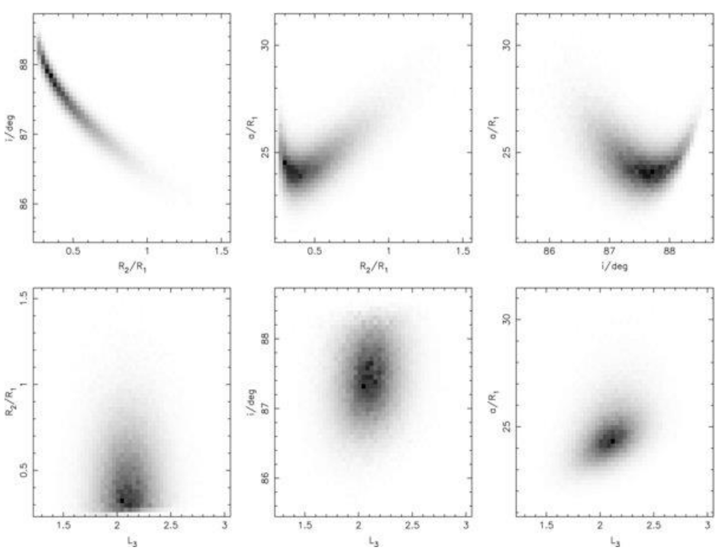
Joint Distribution can be shown in a matrix table. Below is an example of probability versus impact.



Very High (0.9)	0.09	0.27	0.45	0.63	0.81
High (0.7)	0.07	0.21	0.35	0.49	0.63
Medium (0.5)	0.05	0.15	0.25	0.35	0.45
Low (0.3)	0.03	0.09	0.15	0.21	0.27
Very Low (0.1)	0.01	0.03	0.05	0.07	0.09
	Very Low (0.1)	Low (0.3)	Medium (0.5)	High (0.7)	Very High (0.9)

The probability matrices can sometimes be hard to visualize with just numbers. So, we can map the numbers to a grayscale in which the higher the number the darker the color. So, the darker colors mean a more likely joint event and the lighter areas mean a less likely joint event.

Below are examples of grayscale cloud images.



In the APPL and GOOG example, both up events and both down events are more likely than the GOOG up and APPL down or APPL up and GOOG down events. Therefore, these events are a correlated joint distribution.

CoVariance

Covariance of x & y: " how coupled are x & y"

Equation: cov(x,y) = E[[X – E(X)]*[Y-E(Y)]

$$= \sum_{\text{all states } i} [x_i - E(x)][y_i - E(y)] * P(x = x_i, y = y_i)$$

*what happens if on average x_i and y_i are above the mean? Then both differences are positive, so when multiplied the covariance is positive

*what happens if on average x_i and y_i are below the mean? Then both differences are negative, so when multiplied the covariance is positive

*what happens if on average x_i and y_i are on opposite sides of the mean? Then one difference is positive and the other is negative, so when multiplied the covariance is negative.

Covariance of the excel sheet from above.

45	3.5	-1.5	8.80E-07	3.08E-06	-1.3E-06	2.54681593	-1.97669621	-4.4E-06
46	3.5	-0.5	0.00021485	0.000752	-0.00011	2.54681593	-0.97669621	-0.00053
47	3.5	0.5	0.00711487	0.024902	0.003557	2.54681593	0.02330379	0.000422
48	3.5	1.5	0.03188664	0.111603	0.04783	2.54681593	1.02330379	0.083102
49	3.5	2.5	0.01934023	0.067691	0.048351	2.54681593	2.02330379	0.09966
50	3.5	3.5	0.00158754	0.005556	0.005556	2.54681593	3.02330379	0.012224
51								
52			Sum w_k*x_k:	0.953184			Total Covariance:	0.891451
53			Sum w_k*y_k:	0.476696				