4/9/14 STA371H Notes

Topics:

1. Joint Distribution Review
2. Decision Modeling
3. Utility Theory

Review from last class:

Joint distributions

- P(X, Y): notation for probability of both X and Y
- Joint outcome from two or more variables
- Describes uncertainty in both random variables (X,Y) at once because they're related to each other

PMT (Probability-Mass-Table)

- For bivariate joint distribution, use a PMT
- A PMT lists all the possible joint events (possible states of the world)

Example:

- X_i = % return on AAPL this week
- Y_j = % return on GOOG this week
- In a PMT, there are three columns and a row for every possible joint occurrence:

Event X_i	Event Y_j	$P(X=x_i, Y=y_j)$
-1	-1	P(-1,-1)
1	-1	P(1,-1)
-1	1	P(-1, 1)
1	1	P(1, 1)

Showed matrix of

- 49 possible joint events
- level of darkness tells you how likely that joint event is

FOR BIVARIATE JOINT DISTRIBUTION

- usually we care about functions of X, Y
- f(x,y) but it could be 10,000 variables (tomorrow's return for each individual stock traded on equity market)
- in general, joint distribution are on much larger scales "walk before we can run)

f(X,Y): function/policy/decision/allocation resulting from some future outcome

Ex. f(X,Y) = 10x + 10Y

- f(x,y) tells you how much you gain/lose in portfolio
- portfoilio is a joint distribution of all stocks

Ex 2: f(x,y) = [x – E(x)

- E(this f) = covariance (x,y)
    - Expected value of this function: = E[[x- E(x)] [Y – E(y)]]

Strategy to calculate E(f(x,y))

1. apply the function to al possible outcomes
2. take the weighted sum:
   a. E[f(x,y)] = sigma_all xi, yj of f(x_i, y_j) * P(x=xi, y=yj)

For covariance:

Function for covariance:

$E[[X - E(X)] [Y-E(Y)]$

The variance of a portfolio is intimately related to the covariance of each individual aspect.

IN real life, nobody is going to make you a table and hand it to you. In practice, we use Monte Carlo simulation. IN principle, strategy is correct. However, we apply principles to a different technique.

Play a game:

- 3 pennies and 1 dime in a jar
- Draw random coin.
- If it's a penny, you get 10 million.
- If it's a dime, God or Obama will strike you dead immediately.
- How many pennies have to be in the jar for you to be willing to play the game?

You make decisions all the time:

- Late for an interview and run yellow light→escalated risk of dying
- You came to stats class and crossed street
- We all go through life without trying to put spectrum of likelihood and how much we would like those outcomes
- Our life is really made up choices that have different levels of risks and rewards
- For big decisions, it's worthwhile to try to be more systematic/formal in everyday life

8 ways you can die:

Cause of Death	Annual Deaths (US)
Botulism	2
Flood	200
Heart disease	800,000
Homicide	11,000
Motor vehicle accident	40,000
Pregnancy	450
Stomach cancer	90,000
Tornado	90

We don't think in these terms:

- Don't consciously think about dying when we get in a car
- Don't think about risk of heart disease when we eat another burger

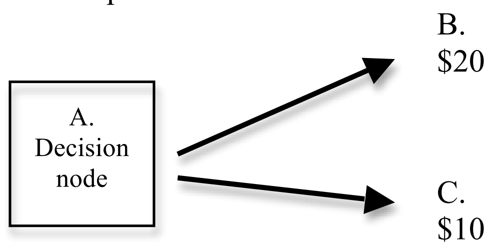
Maybe we should think more in these terms when making important decisions. Have to be more systematic so we drastically don't misestimate numbers.

Learning to Think Systematically About Decision-Making: Decision modeling and Utility Theory

The systematic framework we are building up to is expected value and expected utility.

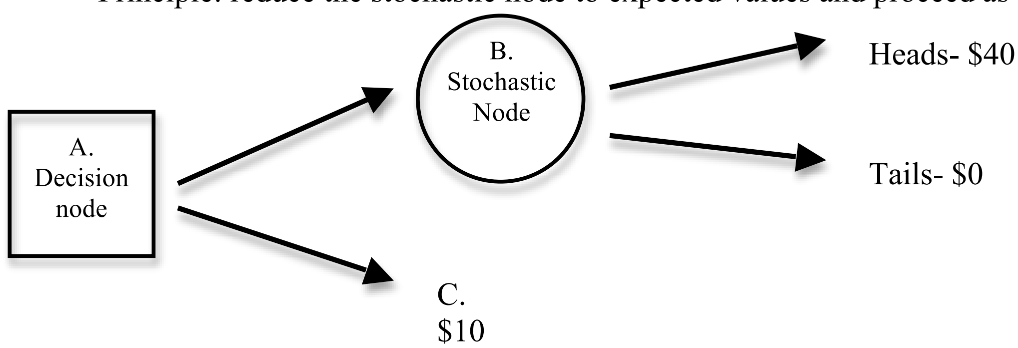
Decision Modeling (Decision Trees)

Decision Tree 1: Lets start simple. If you have to make a decision between two sure things, take the better option.



Decision Tree 2: addition of a 'stochastic node'. This node is a random even, such as a coin flip. If you flip heads, you get $40. If you flip tails, you get nothing.

*Principle: reduce the stochastic node to expected values and proceed as before.



St. Petersburg Paradox: Element of Paradox:

- Natural strategy with taking the expected value that is highest leads you to conclusion that it is always rational to take the higher value
- However, sometimes, taking the highest expected value can be very irrational
- Can be very rational to take the sure thing

Example of Paradox: Flipping Game

- Flip coin
- Keep flipping the coin until it stops coming up heads
    - K: flips
    - $2^k dollars

Calculate expected value of the flipping game

B: PMT for flipping game

Flips	Payoff for x_k = # of flips	W_k = probability	Payoff*probability
1	$2	0.5	1
2	$4	0.25	1
3	$8	1/8	1
4	$16	1/16	1

Flipping game has infinity payoff.
Expected value: infinity

- There is nobody who would pick the flipping game over the finite $10,000,000.
- Even though probabilities
- Vanishing small probability of astoomically large number
- At some point, you would be just as happy with $10 billion as you would be with $40 billion
- This is the paradox that gave birth utility theory.

Diminishing Return of Money Paradox

- Having twice as much money is not going to make you twice as happy
- E(X) → does not apply
- E(happiness(x))
- E(utility(x))
- E(u(x)): UTILITY FUNCTION
- Economists call happiness → "utility"
- Don't make decisions based on expected value, but based on expected utility

Utility Function: U(x)

- Aren't just functions of money
- Any possible intangible scenario that you construct can be put in utility function
- Mathematical function of the world
- How much utility would I get from a lovely afternoon on Enchanted Rock with sandwiches and ice cream?
- Some function that input can be any scenario and spit out a real number
- U(X): ANYTHING → Real Numbers
- What kind of tradeoffs should we make between clean air and economic development
- Defy easy characterization by money
- John von Neumann
    - Incredible contributions to physics, mathematics, economics
    - Invention of modern utility theory
    - Invention of game theory
    - Put all possible states of the world (all non-monetary things) put on common scale of value
    - Maps from all possible states of world to real numbers if this statement holds true:
        - Decision: $50 or afternoon at Enchanted Rock

As long as you can pick between choices along the decision tree, then there exists a utility function that maps events to real numbers

Allais Paradox

- Paradox in utility theory that shows that simple criterion for utility function cannot be decided willy-nilly
- Fundamental self consistency that has to be satisfied
- If it doesn't satisfy, then you're behaving incoherently