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We first went over the most recent homework on probability, expected value, and decision modeling. This is a review of the homework answers.

Question 1: Bayes Rule Calculations

Part A- Asked what is the probability that a patient has SOS, given that he or she tests positive for the disease?

D=has the disease
T= tests positive

P (P|T)= P(D) * P(T|D) / P(T) =~ 0.086

- The probability of D occurring, P(D), =1/1000
- The probability of testing positive given that they have the disease, P(T|D),= 0.95
- P(T)= the marginal propensity that you will test positive

Addition Rule

P(T and D) + P(T and ~D)
"The probability of T and D" plus "the probability of T and not D"

- Due to the addition rule these must add up to P(T)

Joint Probability

P(D)*P(T|D) + P(~D)*P(T|~D)
=[1/1000]*[0.95] + [999/1000]*[0.01]

- This outcome of roughly 8% seems like a low number, however this relates back to the jury trial example that claimed 95% accuracy but in fact was considerably lower. Out of all the people who test positive for the disease, only approximately 8% of them will have the disease.

Part B- Is the proposed policy of free, universal SOS testing a smart financial move for your company?

In order to solve this problem you need to compare the costs with the corresponding benefits.

Costs= $100,000,000 + $5,000,000= $105,000,000
= "cost of testing everyone" + "follow up costs for false positives"

Benefits= $10,000 * 10,000[0.95] =~ $95,000,000
"0.95"= probability of detecting given that you have the disease

- Costs outweigh the benefits, so it is not a good decision.

Part C- What does x have to be to make this a breakeven trade?
=~ roughly 11,050

There are a few ways to do this; we can either plot the benefits vs. costs and see where the intersection point is or solve for x.

<u>Question 2: Dice Game</u>
Part A- Does the game have a positive expected value?

P(Win) + P(Lose) = 1
P(Lose) = P(not rolling 1's in 4 games)
= P(not 1)^4
= (5/6)^4= 0.48
P[win]= 1-0.48=0.52

Part B- Suppose you agree to play the game from Part A ten times in a row—that is, ten distinct games of four rolls each. What is the probability that you will be left with a net profit of at least $1 at the end of all 10 games?

To complete this problem you needed to find the number of games where you will make a positive profit.

If you win 6 and lose 4 it will result in at least a dollar with of profit.
P(6 wins or more)
= P(6 wins) + P(7 wins) + … + P (10 wins) = 0.422 which is found through the binomial

- P[6 wins]
- [0.518}^6 * [0.482]^4

Combinations example- number of wins* number of loses
= WWWWWW*LLLL
* ALL have p=0.518 *
- There are many ways to rearrange wins and losses and to solve this, you need to know the # of possible arrangements.

Binomial Distribution

$\binom{10}{6}$ (0.518)^6 * (0.482)^4 = 0.422

<u>Question 3: Decision Trees</u>
1. Reduce every stochastic node to its expected value.
2. At a decision node, the best value of that node becomes the value of that decision.
3. Start from the right of a tree and work towards the left in time.

E [F]= → $500,000
→ $ -250,000

E (F)= P2* 500,000 + (1-P2)(-250,000)
= 750,000P2 - 250,000

E(G) = 510,000P1 + (1-P1)- 240,000
= 750,000P1- 240,000

* Applied rule #1 to F and G by determining their expected values.

- Take the option with the higher expected values.

E(F) = 750,000*P2 - 250,000 > -200,000
and E(F) > E(G)
= 750,000*P1 -240,000

- Go with F if it is greater than 200,000. Otherwise, go with G.

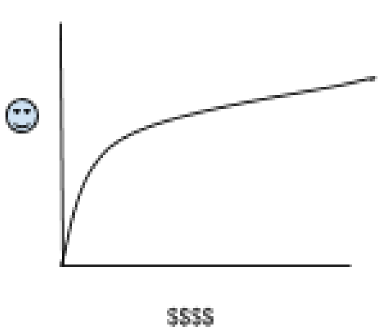
Reduce node E to its expected value and choose the best option of all the stochastic nodes. Keep pruning until you are back down at the beginning of the decision tree on the left most side.

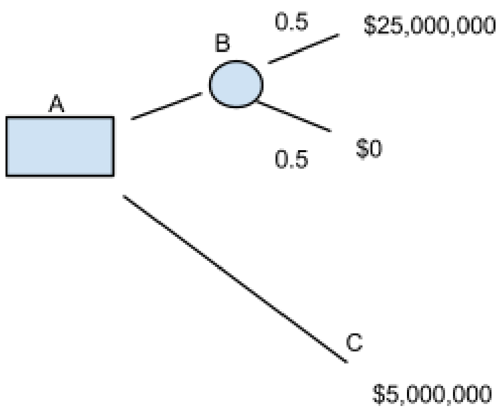
Propose F when
750,000*P2 -250,000 > 750,000*P1 - 240,000
750,000*P2 >750,000*P1 + 10,000
P2>P1 + (10,000/750,000)
P2> P1 + (1/75)

<u>Utility Function</u>- shows that as you get more money the utility or happiness may begin to diminish marginally.

Ex: A $10 bill is likely to make you happier if you have no money than if you are a millionaire.

1. Maps wealth to happiness (or utility)
2. Subjective- describes your own risk preferences
3. Not completely arbitrary
4. Provides a looser set of requirements for rational action than the principle of expected value.





- Terminal wealth of $5,000,000 , $25,000,000 , or $0

Expected value of point B=
E[B] = 0.5 * $25,000,000 + 0.5 * $0
= $12,500,000 which is clearly the best option compared to the expected value, E[C] of option C of $5,000,000.

- This is an illustration of how the Utility Function introduces subjectivity into decision-making. When following the principle of expected value, Option C is the clear choice. However, the decision depends more on an individual's personal Utility Function and their level of risk aversion.

<u>Calculating decisions based on Utility Functions:</u>

Our random variable, wealth, is designated by a W.
The expected utility of this random variable is shown by E(U(W)).
We must find U(W) and maximize this new variable of utility rather than the expected value.

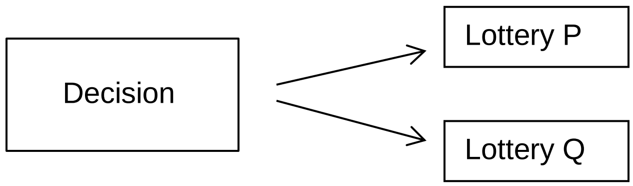
Going back to the previous example, we would map the two options' utilities with the following:

E(U(B)) = 0.5(U($25,000,00)) + 0.5(U(0))
E(U(C)) = U($5,000,000)

Therefore, we would choose Option C when the expected utility of $5,000,000 is greater than half (0.5) that of $25,000,000. This is arbitrarily decided by our individual Utility Functions.

- All Utility Functions are risk-averse. They are concave functions that plateau eventually. This is simply a natural reaction to the effect of marginal gains.

Example of a Decision (Lottery):



Lottery: a simple (binary) random variable with possibly asymmetric outcomes and possibly asymmetric possibilities. P1 and P2 (probabilities of outcomes) do not have to be equal to each other. X1 and X2 (values of outcomes) do not have to be equal either.

Criteria for Coherent Utility Functions: the following four rules are for self-consistency. If all four of the rules are met in any given circumstance, we can create a valid Utility Function.

1. Completeness Rule- given lotteries P and Q, either P > Q, Q > P, or P~Q.
2. Transitivity Rule- given lotteries P, Q and R, if P > Q and Q > R, then P > R.
3. "Split the difference" Rule
4. "Irrelevant Option" Rule

Rules 3 and 4 are illustrated in pages 4-5 of the Utility notes in the course packet.