Statistical Analysis of Network Data

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How to follow this tutorial

Go to https://github.com/igraph/netuser15

You will need at least `igraph` version `1.0.0` and `igraphdata` version `1.0.0`. You will also need the `DiagrammeR` package. To install them from within R, type:

```
install.packages("igraph")
install.packages("igraphdata")
install.packages("DiagrammeR")
```

Outline

- Introduction
- Manipulate network data
- Questions

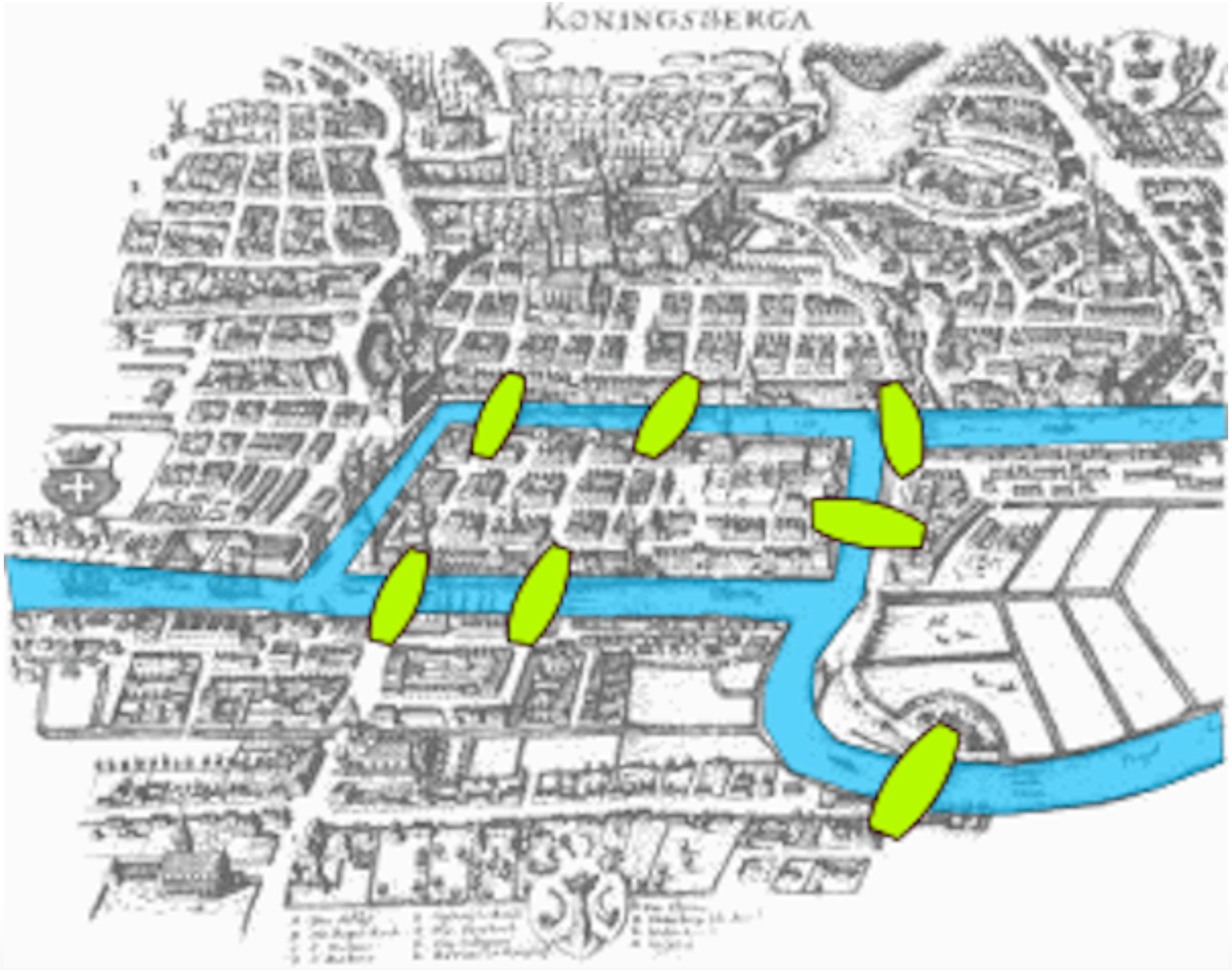
BREAK

- Classic graph theory: paths
- Social network concepts: centrality, groups
- Visualization
- Questions

Why networks?

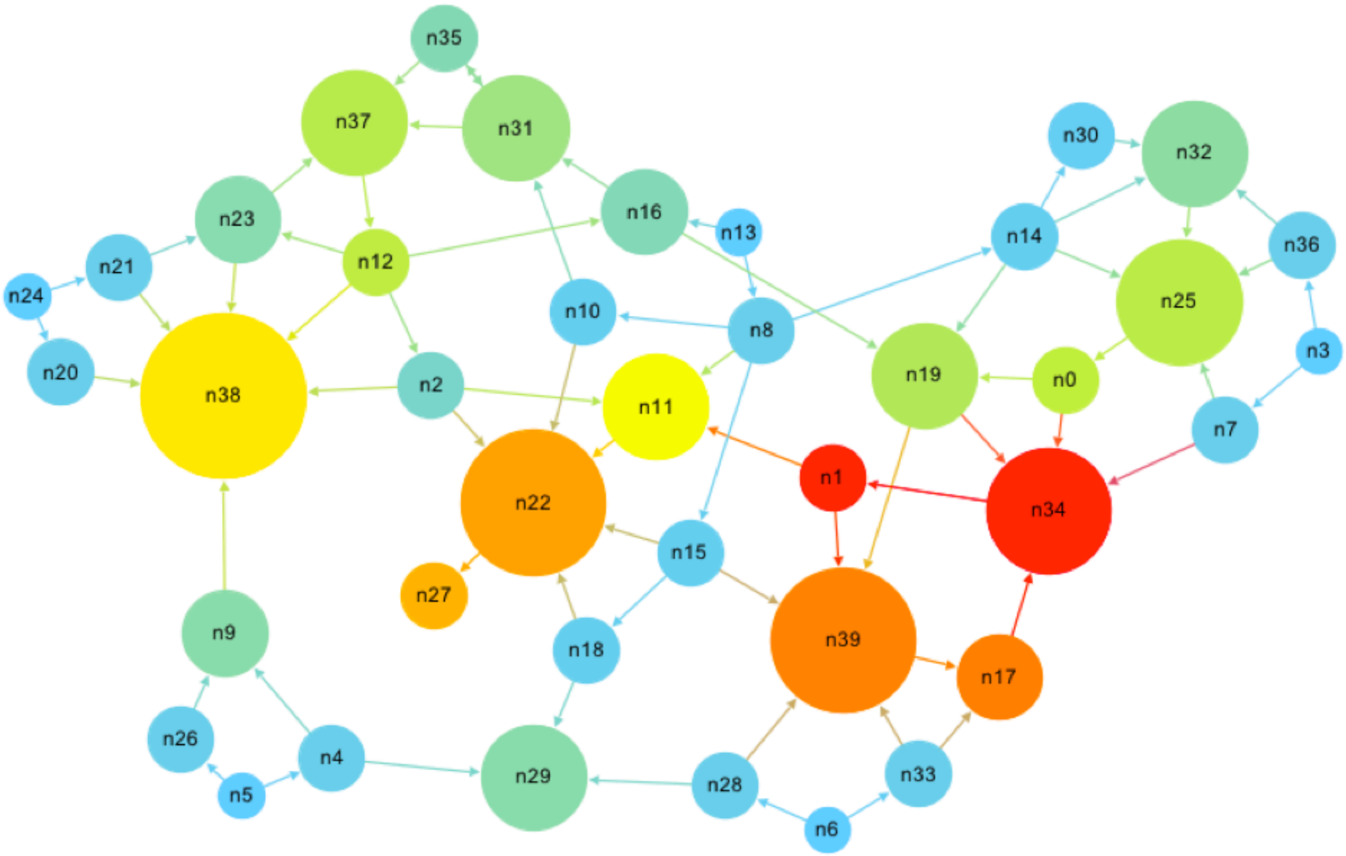
Sometimes connections are important, even more important than (the properties of) the things they connect.

Example 1: Königsberg Bridges



– Bogdan Giuşcă, CC BY-SA 3.0, Wikipedia

Example 2: Page Rank

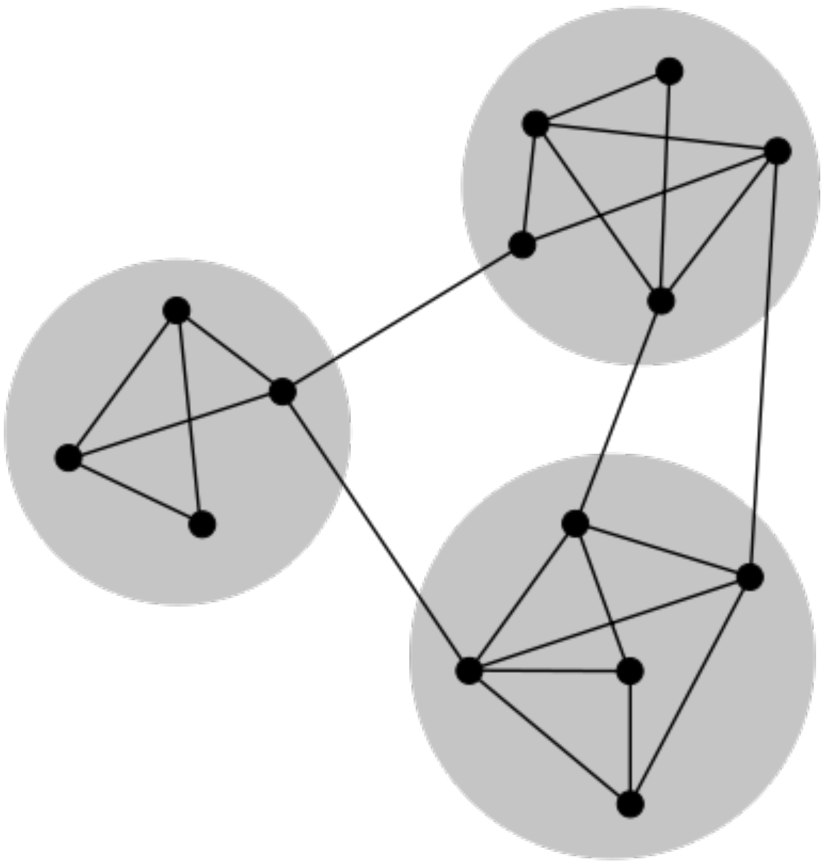


http://computationalculture.net/article/what_is_in_pagerank

Example 3: Matching Twitter to Facebook

http://morganlinton.com/wp-content/uploads/2013/12/twitter-facebook-branding2.png

Example 4: Detection of groups



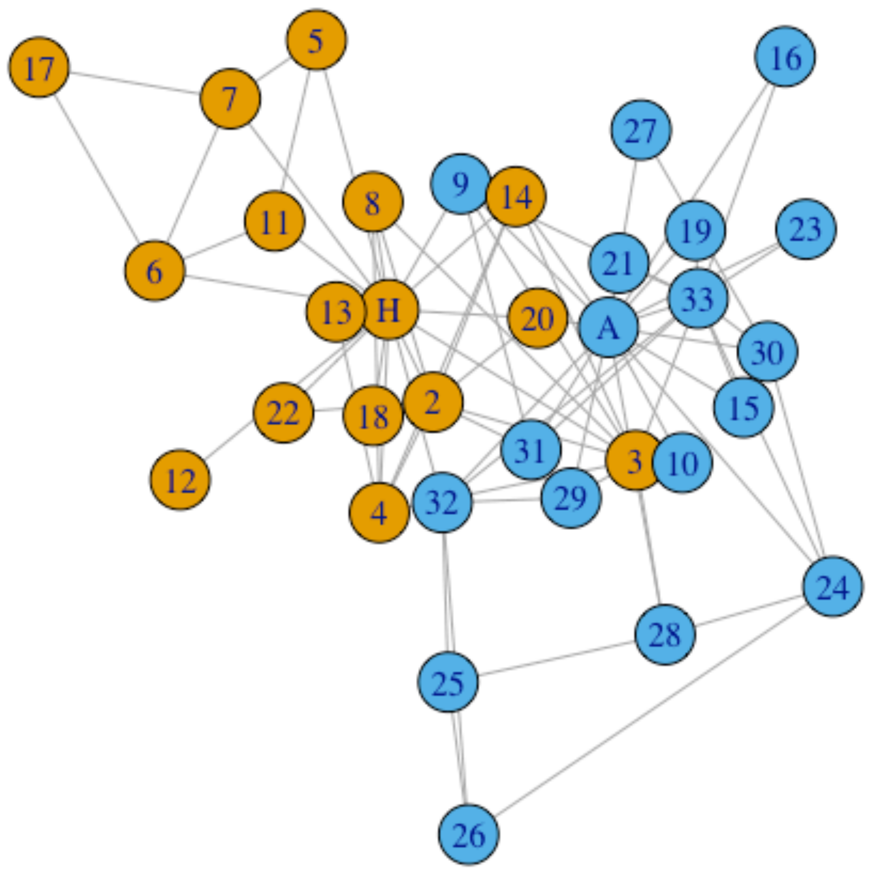
https://en.wikipedia.org/wiki/Community_structure#/media/File:Network_Community_Structure.svg

About igraph

- Network analysis library, written mostly in C/C++.
- Interface to R and Python
- https://github.com/igraph
- http://igraph.org
- Mailing list, stack overflow help.
- Open GitHub issues for bugs

Creating and manipulating networks in R/igraph.

What is a network or graph?



More formally:

- `V`: set of vertices
- `E`: subset of ordered or unordered pairs of vertices. Multiset, really.

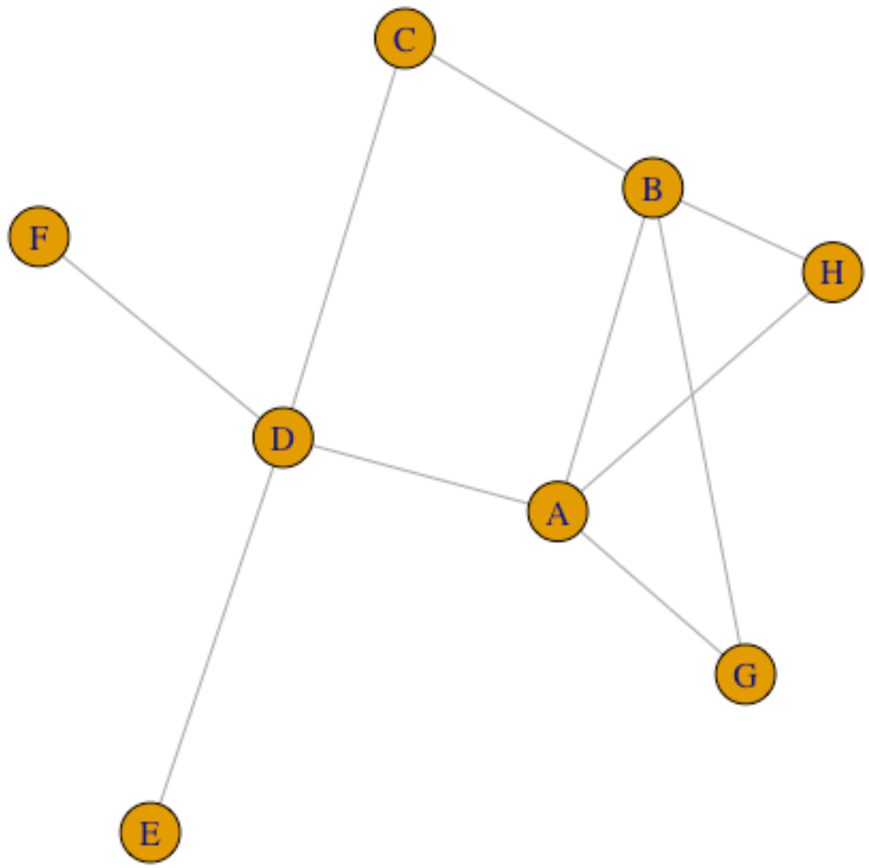
Creating toy networks with `make_graph`

```
library(igraph)
```

```
toy1 <- make_graph(~ A - B, B - C - D, D - E:F:A, A:B - G:H)
toy1
```

```
#> IGRAPH UN-- 8 10 --
#> + attr: name (v/c)
#> + edges (vertex names):
#>  [1] A--B A--D A--G A--H B--C B--G B--H C--D D--E D--F
```

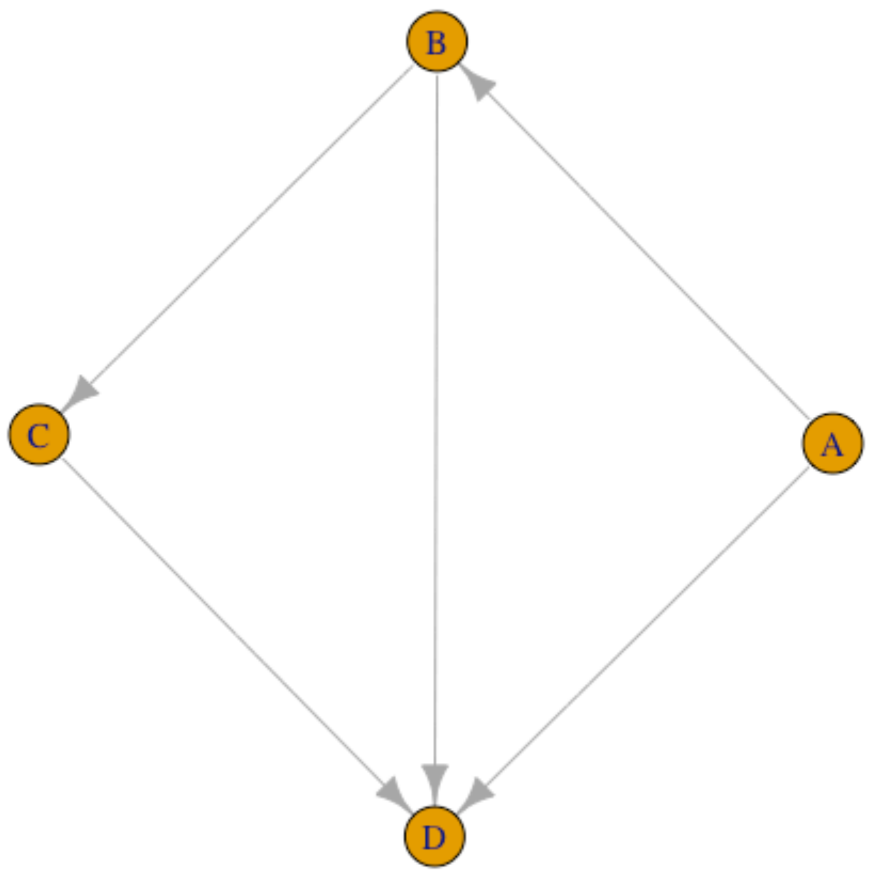
```
par(mar = c(0,0,0,0)); plot(toy1)
```



```r
toy2 <- make_graph(~ A -+ B, B -+ C -+ D +- A:B)
toy2
```

```
#> IGRAPH DN-- 4 5 --
#> + attr: name (v/c)
#> + edges (vertex names):
#> [1] A->B A->D B->C B->D C->D
```

```
par(mar = c(0,0,0,0)); plot(toy2)
```



Printout of a graph

```
toy2
```

```
#> IGRAPH DN-- 4 5 --
#> + attr: name (v/c)
#> + edges (vertex names):
#> [1] A->B A->D B->C B->D C->D
```

`IGRAPH` means this is a graph object. Next, comes a four letter code:

- `U` or `D` for undirected or directed
- `N` if the graph is named, always use named graphs for real data sets.
- `W` if the graph is weighted (has a `weight` edge attribute).
- `B` if the graph is bipartite (has a `type` vertex attribute).

Attributes

```
make_ring(5)
```

```
#> IGRAPH U--- 5 5 -- Ring graph
#> + attr: name (g/c), mutual (g/l), circular (g/l)
#> + edges:
#> [1] 1--2 2--3 3--4 4--5 1--5
```

- Some graphs have a name (`name` graph attribute), that comes after the two dashes.
- Then the various attributes are listed. Attributes are metadata that is attached to the vertices, edges, or the graph itself.
- `(v/c)` means that `name` is a vertex attribute, and it is character.
- `(e/.)` means an edge attribute, `(g/.)` means a graph attribute

```r
make_ring(5)
```

```
#> IGRAPH U--- 5 5 -- Ring graph
#> + attr: name (g/c), mutual (g/l), circular (g/l)
#> + edges:
#> [1] 1--2 2--3 3--4 4--5 1--5
```

- Attribute types: `c` for character, `n` for numeric, `l` for logical and `x` (complex) for anything else.
- igraph treats some attributes specially. Always start your non-special attributes with an uppercase letter.

Real network data

Adjacency matrices

```
A <- matrix(sample(0:1, 100, replace = TRUE), nrow = 10)
A
```

```
#>       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
#>  [1,]    1    0    1    1    0    0    1    0    1     1
#>  [2,]    1    1    0    1    0    0    1    0    0     0
#>  [3,]    0    1    1    0    0    0    1    0    0     0
#>  [4,]    1    0    1    1    1    1    1    0    1     1
#>  [5,]    1    0    0    0    0    0    1    0    1     1
#>  [6,]    1    1    1    1    1    1    0    1    1     1
#>  [7,]    1    1    0    0    1    1    0    0    0     0
#>  [8,]    0    0    1    0    1    0    1    0    0     1
#>  [9,]    1    0    0    1    1    0    1    1    0     1
#> [10,]    1    1    1    1    1    1    0    0    0     1
```

```r
graph_from_adjacency_matrix(A)
```

```
#> IGRAPH D--- 10 55 --
#> + edges:
#>  [1]  1-> 1  1-> 3  1-> 4  1-> 7  1-> 9  1->10  2-> 1  2-> 2  2-> 4
#> [10]  2-> 7  3-> 2  3-> 3  3-> 7  4-> 1  4-> 3  4-> 4  4-> 5  4-> 6
#> [19]  4-> 7  4-> 9  4->10  5-> 1  5-> 7  5-> 9  5->10  6-> 1  6-> 2
#> [28]  6-> 3  6-> 4  6-> 5  6-> 6  6-> 8  6-> 9  6->10  7-> 1  7-> 2
#> [37]  7-> 5  7-> 6  8-> 3  8-> 5  8-> 7  8->10  9-> 1  9-> 4  9-> 5
#> [46]  9-> 7  9-> 8  9->10 10-> 1 10-> 2 10-> 3 10-> 4 10-> 5 10-> 6
#> [55] 10->10
```

List of edges

```
L <- matrix(sample(1:10, 20, replace = TRUE), ncol = 2)
L
```

```
#>       [,1] [,2]
#>  [1,]    7    7
#>  [2,]    3    9
#>  [3,]    3    8
#>  [4,]    4    5
#>  [5,]   10    6
#>  [6,]   10    6
#>  [7,]    8    1
#>  [8,]    8    4
#>  [9,]    6    7
#> [10,]    1    9
```

```
graph_from_edgelist(L)
```

```
#> IGRAPH D--- 10 10 --
#> + edges:
#>  [1]  7->7  3->9  3->8  4->5 10->6 10->6  8->1  8->4  6->7  1->9
```

Two tables, one for vertices, one for edges

```
edges <- data.frame(
  stringsAsFactors = FALSE,
  from = c("BOS", "JFK", "LAX"),
  to   = c("JFK", "LAX", "JFK"),
  Carrier = c("United", "Jetblue", "Virgin America"),
  Departures = c(30, 60, 121)
)
vertices <- data.frame(
  stringsAsFactors = FALSE,
  name = c("BOS", "JFK", "LAX"),
  City = c("Boston, MA", "New York City, NY",
    "Los Angeles, CA")
)
```

```r
edges
```

```
#>   from  to        Carrier Departures
#> 1  BOS JFK         United         30
#> 2  JFK LAX        Jetblue         60
#> 3  LAX JFK Virgin America        121
```

```
vertices
```

```
#>   name               City
#> 1  BOS         Boston, MA
#> 2  JFK New York City, NY
#> 3  LAX   Los Angeles, CA
```

```r
toy_air <- graph_from_data_frame(edges, vertices = vertices)
toy_air
```

```
#> IGRAPH DN-- 3 3 --
#> + attr: name (v/c), City (v/c), Carrier (e/c), Departures (e/n)
#> + edges (vertex names):
#> [1] BOS->JFK JFK->LAX LAX->JFK
```

The real US airports data set is in the `igraphdata` package:

```
library(igraphdata)
data(USairports)
USairports
```

```
#> IGRAPH DN-- 755 23473 -- US airports
#> + attr: name (g/c), name (v/c), City (v/c), Position (v/c),
#> | Carrier (e/c), Departures (e/n), Seats (e/n), Passengers
#> | (e/n), Aircraft (e/n), Distance (e/n)
#> + edges (vertex names):
#>  [1] BGR->JFK BGR->JFK BOS->EWR ANC->JFK JFK->ANC LAS->LAX MIA->JFK
#>  [8] EWR->ANC BJC->MIA MIA->BJC TEB->ANC JFK->LAX LAX->JFK LAX->SFO
#> [15] AEX->LAS BFI->SBA ELM->PIT GEG->SUN ICT->PBI LAS->LAX LAS->PBI
#> [22] LAS->SFO LAX->LAS PBI->AEX PBI->ICT PIT->VCT SFO->LAX VCT->DWH
#> [29] IAD->JFK ABE->CLT ABE->HPN AGS->CLT AGS->CLT AVL->CLT AVL->CLT
#> [36] AVP->CLT AVP->PHL BDL->CLT BHM->CLT BHM->CLT BNA->CLT BNA->CLT
#> + ... omitted several edges
```

Converting it back to tables

```
as_data_frame(toy_air, what = "edges")
```

```
#>   from  to        Carrier Departures
#> 1  BOS JFK         United         30
#> 2  JFK LAX        Jetblue         60
#> 3  LAX JFK Virgin America        121
```

```r
as_data_frame(toy_air, what = "vertices")
```

```
#>     name                City
#> BOS  BOS          Boston, MA
#> JFK  JFK New York City, NY
#> LAX  LAX   Los Angeles, CA
```

Long data frames

```
as_long_data_frame(toy_air)
```

```
#>   from to        Carrier Departures from_name         from_City to_name
#> 1    1  2         United         30       BOS        Boston, MA     JFK
#> 2    2  3        Jetblue         60       JFK New York City, NY     LAX
#> 3    3  2 Virgin America        121       LAX   Los Angeles, CA     JFK
#>             to_City
#> 1 New York City, NY
#> 2   Los Angeles, CA
#> 3 New York City, NY
```

Quickly look at the metadata, without conversion:

```r
V(USairports)[[1:5]]
```

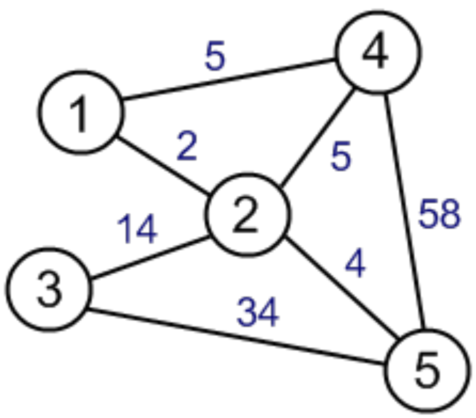
```
#> + 5/755 vertices, named:
#>   name          City         Position
#> 1  BGR    Bangor, ME N444827 W0684941
#> 2  BOS    Boston, MA N422152 W0710019
#> 3  ANC Anchorage, AK N611028 W1495947
#> 4  JFK  New York, NY N403823 W0734644
#> 5  LAS Las Vegas, NV N360449 W1150908
```

```r
E(USairports)[[1:5]]
```

```
#> + 5/23473 edges (vertex names):
#>   tail head tid hid              Carrier Departures Seats Passengers
#> 1  JFK  BGR   4   1 British Airways Plc          1   226        193
#> 2  JFK  BGR   4   1 British Airways Plc          1   299        253
#> 3  EWR  BOS   7   2 British Airways Plc          1   216        141
#> 4  JFK  ANC   4   3 China Airlines Ltd.         13  5161       3135
#> 5  ANC  JFK   3   4 China Airlines Ltd.         13  5161       4097
#>   Aircraft Distance
#> 1      627      382
#> 2      819      382
#> 3      627      200
#> 4      819     3386
#> 5      819     3386
```

Weighted graphs

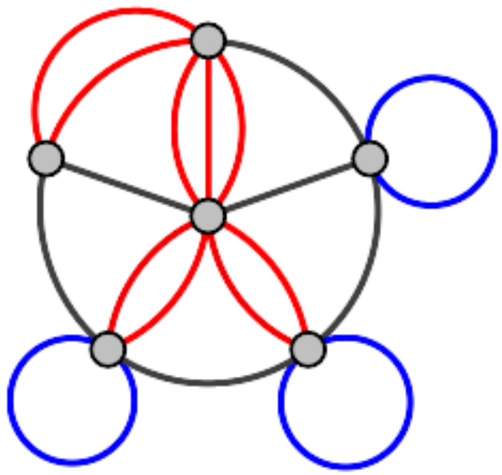
Numbers (usually real) assigned to edges. E.g. number of departures, or number of passengers.



http://web.cecs.pdx.edu/~sheard/course/Cs163/Doc/Graphs.html

Multigraphs

They have multiple (directed) edges between the same pair of vertices. A graph that has no multiple edges and no loop edges is a simple graph.



https://en.wikipedia.org/wiki/Multigraph

Multi-graphs are nasty. Always check if your graph is a multi-graph.

```r
is_simple(USairports)
```

```
#> [1] FALSE
```

```r
sum(which_multiple(USairports))
```

```
#> [1] 15208
```

```r
sum(which_loop(USairports))
```

```
#> [1] 53
```

`simplify()` creates a simple graph from a multigraph, in a flexible way: you can specify what it should do with the edge attributes.

```
air <- simplify(USairports, edge.attr.comb =
  list(Departures = "sum", Seats = "sum", Passengers = "sum", "ignore"))
is_simple(air)
```

```
#> [1] TRUE
```

```
summary(air)
```

```
#> IGRAPH DN-- 755 8228 -- US airports
#> + attr: name (g/c), name (v/c), City (v/c), Position (v/c),
#> | Departures (e/n), Seats (e/n), Passengers (e/n)
```

Querying and manipulating networks: the `[` and `[[` operators

The `[` operator treats the graph as an adjacency matrix.

```
    BOS JFK ANC EWR . . .
BOS   .   1   .   1
JFK   1   .   1   .
ANC   .   1   .   .
EWR   1   .   1   .
. . .
```

The `[[` operator treats the graph as an adjacency list.

```
BOS: JFK, LAX, EWR, MKE, PVD
JFK: BGR, BOS, SFO, BNA, BUF, SRQ, RIC RDU, MSP
LAX: DTW, MSY, LAS, FLL, STL,
. . .
```

Queries

Does an edge exist?

```
air["BOS", "JFK"]
```

```
#> [1] 1
```

```
air["BOS", "ANC"]
```

```
#> [1] 0
```

Convert the graph to an adjacency matrix, or just a part of it:

```
air[c("BOS", "JFK", "ANC"), c("BOS", "JFK", "ANC")]
```

```
#> 3 x 3 sparse Matrix of class "dgCMatrix"
#>     BOS JFK ANC
#> BOS   .   1   .
#> JFK   1   .   1
#> ANC   .   1   .
```

For weighted graphs, query the edge weight:

```
E(air)$weight <- E(air)$Passengers
air["BOS", "JFK"]
```

```
#> [1] 31426
```

All adjacent vertices of a vertex:

```
air[["BOS"]]
```

```
#> $BOS
#> + 79/755 vertices, named:
#>  [1] BGR JFK LAS MIA EWR LAX PBI PIT SFO IAD BDL BUF BWI CAK CLE CLT CMH
#> [18] CVG DCA DTW GSO IND LGA MDT MKE MSP MSY MYR ORF PHF PHL RDU RIC SRQ
#> [35] STL SYR ALB PVD ROC SCE FLL MCO TPA BHB IAH ORD PBG PQI MCI ATL AUS
#> [52] DEN DFW MDW PDX PHX RSW SAN SEA SLC ACY JAX MEM SJU STT SJC LGB FRG
#> [69] IAG ACK LEB MVY PVC BMG AUG HYA RKD RUT SLK
```

```r
air[[, "BOS"]]
```

```
#> $BOS
#> + 79/755 vertices, named:
#>  [1] BGR JFK LAS MIA EWR LAX PBI PIT SFO IAD BDL BUF BWI CAK CLE CLT CMH
#> [18] CVG DCA DTW IND LGA MDT MKE MSP MSY MYR PHF PHL RDU RIC SRQ STL SYR
#> [35] XNA ALB MHT PVD ROC SCE FLL MCO TPA BHB IAH ORD PBG PQI MCI ATL AUS
#> [52] DEN DFW MDW PDX PHX RSW SAN SEA SLC ACY JAX MEM SJU STT SJC LGB FRG
#> [69] PTK PGD ACK LEB MVY PVC AUG HYA RKD RUT SLK
```

Manipulation

Add an edge (and potentially set its weight):

```
air["BOS", "ANC"] <- TRUE
air["BOS", "ANC"]
```

```
#> [1] 1
```

Remove an edge:

```
air["BOS", "ANC"] <- FALSE
air["BOS", "ANC"]
```

```
#> [1] 0
```

Note that you can use all allowed indexing modes, e.g.

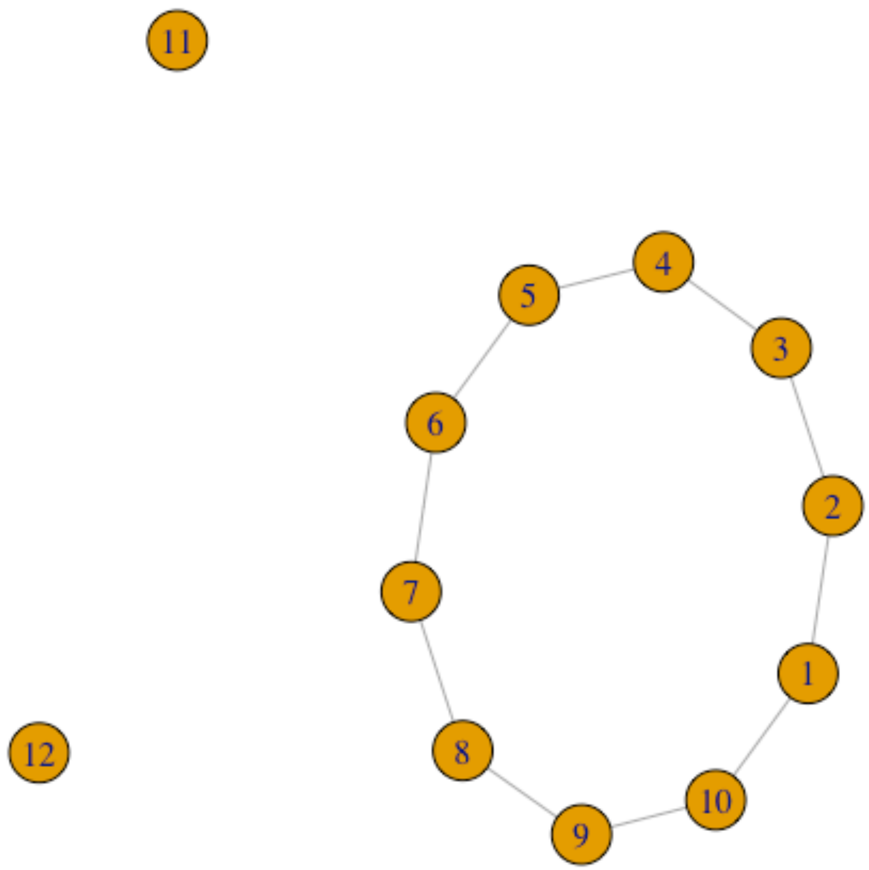
```
g <- make_empty_graph(10)
g[-1, 1] <- TRUE
g
```

```
#> IGRAPH D--- 10 9 --
#> + edges:
#> [1]  2->1  3->1  4->1  5->1  6->1  7->1  8->1  9->1 10->1
```

creates a star graph.

Add vertices to a graph:

```
g <- make_ring(10) + 2
par(mar = c(0,0,0,0)); plot(g)
```



Add vertices with attributes:

```
g <- make_(ring(10), with_vertex_(color = "grey")) +
  vertices(2, color = "red")
par(mar = c(0,0,0,0)); plot(g)
```

Add an edge

```
g <- make_(star(10), with_edge_(color = "grey")) +
  edge(5, 6, color = "red")
par(mar = c(0,0,0,0)); plot(g)
```

Add a chain of edges

```
g <- make_(empty_graph(5)) + path(1,2,3,4,5,1)
g2 <- make_(empty_graph(5)) + path(1:5, 1)
g
```

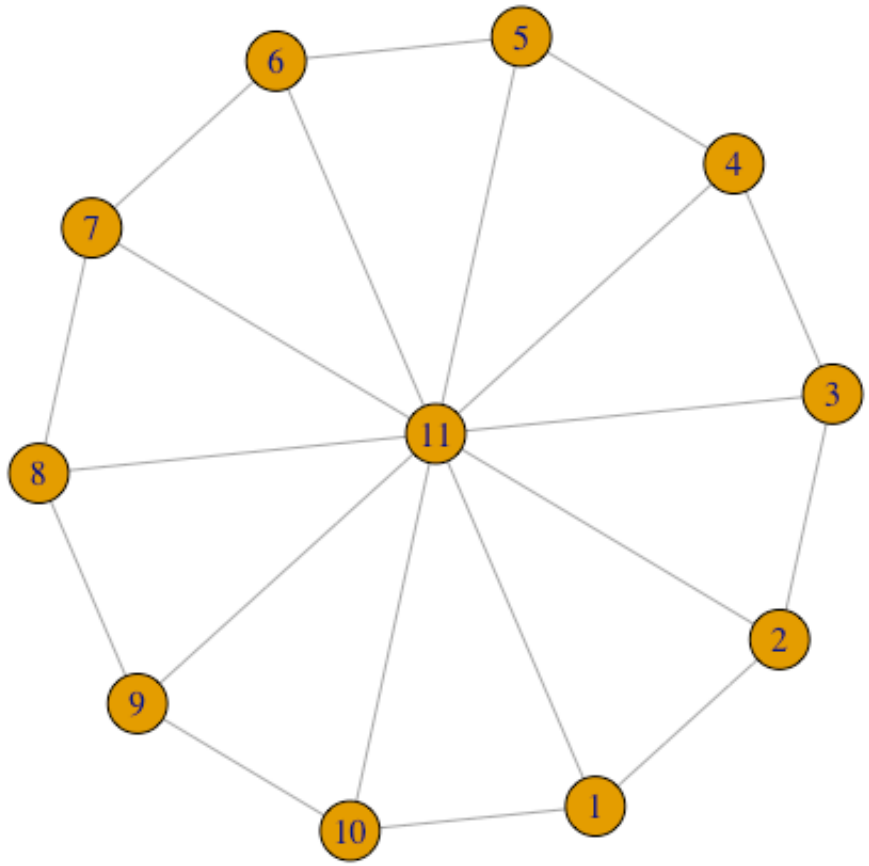
```
#> IGRAPH D--- 5 5 --
#> + edges:
#> [1] 1->2 2->3 3->4 4->5 5->1
```

```
g2
```

```
#> IGRAPH D--- 5 5 --
#> + edges:
#> [1] 1->2 2->3 3->4 4->5 5->1
```

Exercise

Create the wheel graph.



(A) solution

```
make_star(11, center = 11, mode = "undirected") + path(1:10, 1)
```

```
#> IGRAPH U--- 11 20 -- Star
#> + attr: name (g/c), mode (g/c), center (g/n)
#> + edges:
#>  [1]  1--11  2--11  3--11  4--11  5--11  6--11  7--11  8--11  9--11
#> [10] 10--11  1-- 2  2-- 3  3-- 4  4-- 5  5-- 6  6-- 7  7-- 8  8-- 9
#> [19]  9--10  1--10
```

Vertex sequences

They are the key objects to manipulate graphs. Vertex sequences can be created in various ways. Most frequently used ones:

expression	result
`V(air)`	All vertices.
`V(air)[1,2:5]`	Vertices in these positions
`V(air)[degree(air) < 2]`	Vertices satisfying condition
`V(air)[nei('BOS')]`	Neighbors of a vertex
`V(air)['BOS', 'JFK']`	Select given vertices

Edge sequences

The same for edges:

expresssion	result
`E(air)`	All edges.
`E(air)[FL %--% CA]`	Edges between two vertex sets
`E(air)[FL %->% CA]`	Edges between two vertex sets, directionally
`E(air, path = P)`	Edges along a path
`E(air)[to('BOS')]`	Incoming edges of a vertex
`E(air)[from('BOS')]`	Outgoing edges of a vertex

Manipulate attributes via vertex and edge sequences

```
FL <- V(air)[grepl("FL$", City)]
CA <- V(air)[grepl("CA$", City)]

V(air)$color <- "grey"
V(air)[FL]$color <- "blue"
V(air)[CA]$color <- "blue"
```

```r
E(air)[FL %--% CA]
```

```
#> + 21/8228 edges (vertex names):
#>  [1] MIA->LAX MIA->SFO MIA->SJC LAX->MIA LAX->FLL LAX->MCO LAX->TPA
#>  [8] SFO->MIA SFO->FLL SFO->MCO FLL->LAX FLL->SFO FLL->LGB MCO->LAX
#> [15] MCO->SFO TPA->LAX SMF->MIA JAX->OAK OAK->JAX LGB->FLL VNY->ORL
```

```r
E(air)$color <- "grey"
E(air)[FL %--% CA]$color <- "red"
```

Quick look at metadata

```r
V(air)[[1:5]]
```

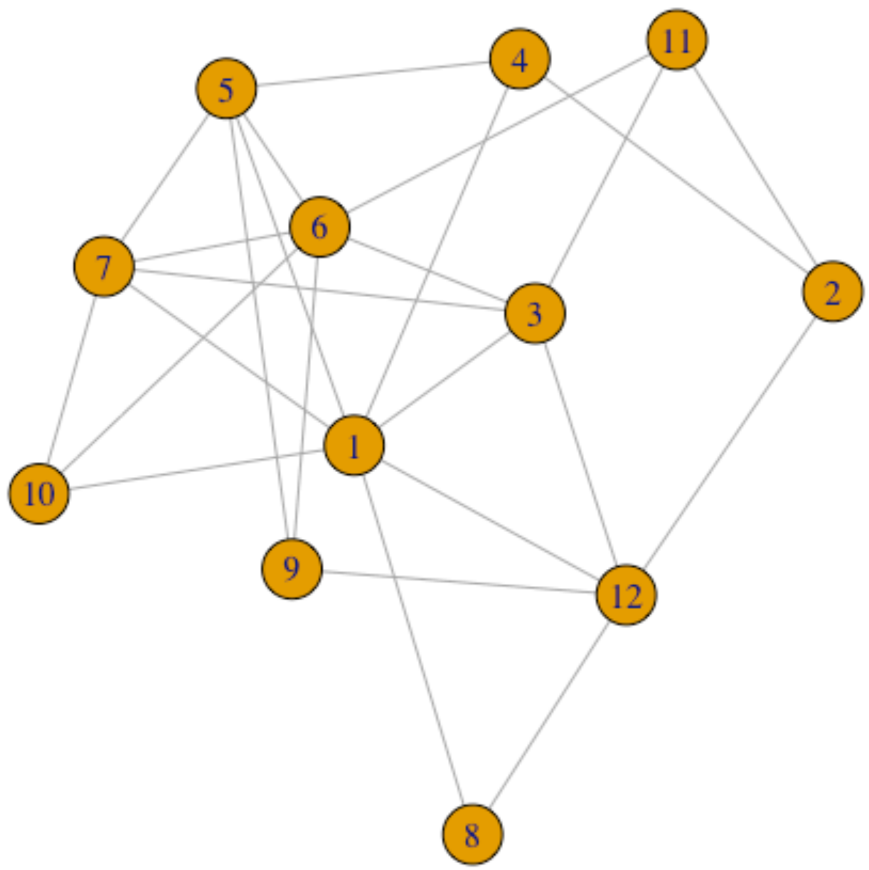
```
#> + 5/755 vertices, named:
#>   name            City          Position color
#> 1  BGR     Bangor, ME N444827 W0684941  grey
#> 2  BOS     Boston, MA N422152 W0710019  grey
#> 3  ANC Anchorage, AK N611028 W1495947  grey
#> 4  JFK   New York, NY N403823 W0734644  grey
#> 5  LAS Las Vegas, NV N360449 W1150908  grey
```

```r
E(air)[[1:5]]
```

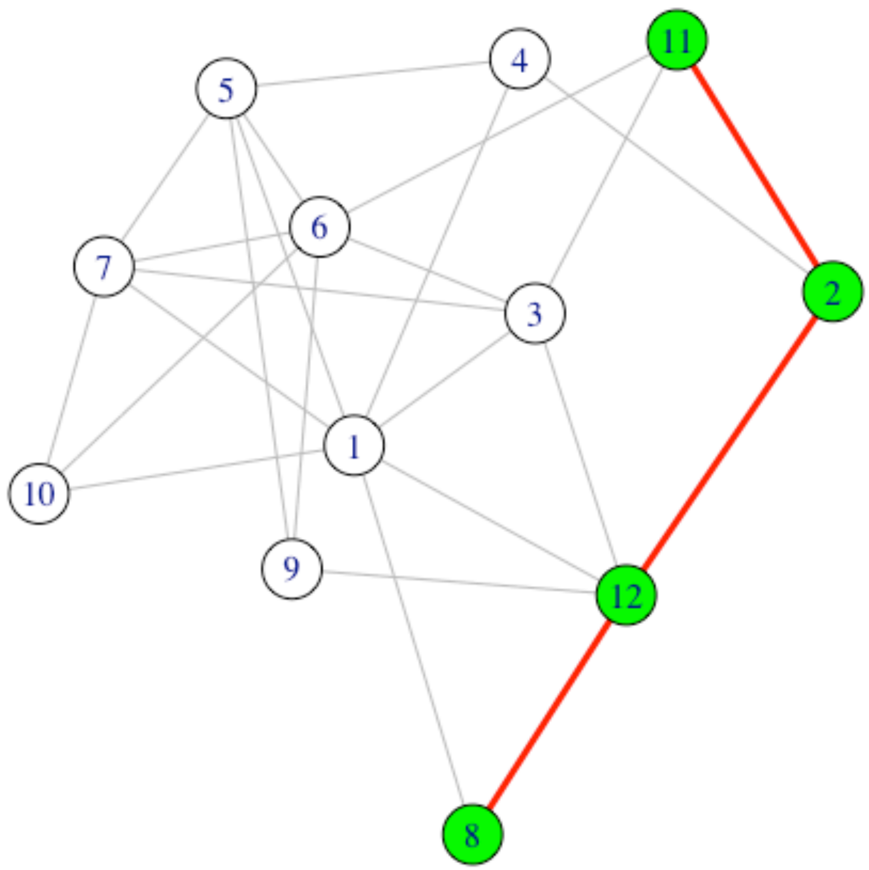
```
#> + 5/8228 edges (vertex names):
#>   tail head tid hid Departures Seats Passengers weight color
#> 1  BOS  BGR   2   1          1    34          6      6  grey
#> 2  JFK  BGR   4   1          2   525        446    446  grey
#> 3  MIA  BGR   6   1          1    12          4      4  grey
#> 4  EWR  BGR   7   1          4   758        680    680  grey
#> 5  DCA  BGR  43   1          4   200        116    116  grey
```

BREAK

Paths



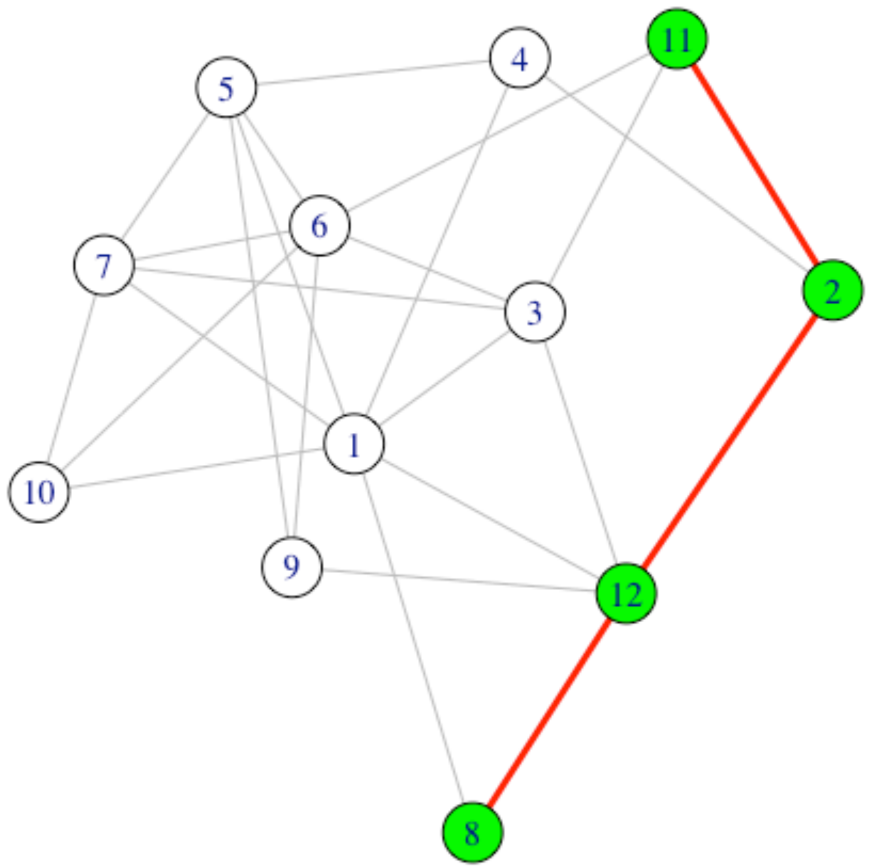
Paths



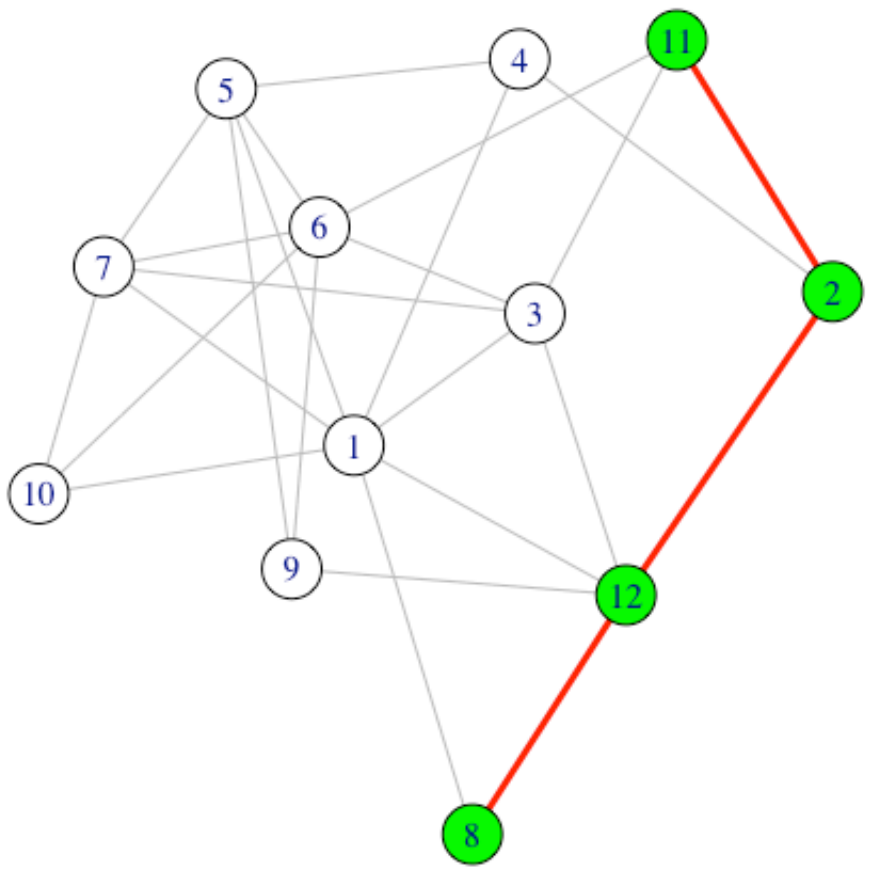
Define a path in igraph

```r
set.seed(42)
g <- sample_gnp(12, 0.25)

pa <- V(g)[11, 2, 12, 8]

V(g)[pa]$color <- 'green'
E(g)$color <- 'grey'
E(g, path = pa)$color <- 'red'
E(g, path = pa)$width <- 3
```

```r
par(mar=c(0,0,0,0))
plot(g, margin = 0, layout = layout_nicely)
```



Shortest paths



Length of the shortest path: distance. How many planes to get from `PBI` to `BDL`?

```
air <- delete_edge_attr(air, "weight")
distances(air, 'PBI', 'ANC')
```

```
#>     ANC
#> PBI   2
```

```r
sp <- shortest_paths(air, 'PBI', 'ANC', output = "both")
sp
```

```
#> $vpath
#> $vpath[[1]]
#> + 3/755 vertices, named:
#> [1] PBI JFK ANC
#>
#>
#> $epath
#> $epath[[1]]
#> + 2/8228 edges (vertex names):
#> [1] PBI->JFK JFK->ANC
#>
#>
#> $predecessors
#> NULL
#>
#> $inbound_edges
#> NULL
```

```r
all_shortest_paths(air, 'PBI', 'ANC')$res
```

```
#> [[1]]
#> + 3/755 vertices, named:
#> [1] PBI ORD ANC
#>
#> [[2]]
#> + 3/755 vertices, named:
#> [1] PBI EWR ANC
#>
#> [[3]]
#> + 3/755 vertices, named:
#> [1] PBI JFK ANC
```

Weighted paths

```
wair <- simplify(USairports, edge.attr.comb =
    list(Departures = "sum", Seats = "sum", Passangers = "sum",
         Distance = "first", "ignore"))
E(wair)$weight <- E(wair)$Distance
```

Weighted (shortest) paths

```
distances(wair, c('BOS', 'JFK', 'PBI', 'AZO'),
                    c('BOS', 'JFK', 'PBI', 'AZO'))
```

```
#>       BOS  JFK  PBI  AZO
#> BOS     0  187 1197  745
#> JFK   187    0 1028  621
#> PBI  1197 1028    0 1116
#> AZO   745  621 1116    0
```

```r
shortest_paths(wair, from = 'BOS', to = 'AZO')$vpath
```

```
#> [[1]]
#> + 3/755 vertices, named:
#> [1] BOS DTW AZO
```

```r
all_shortest_paths(wair, from = 'BOS', to = 'AZO')$res
```

```
#> [[1]]
#> + 3/755 vertices, named:
#> [1] BOS DTW AZO
```

Mean path length

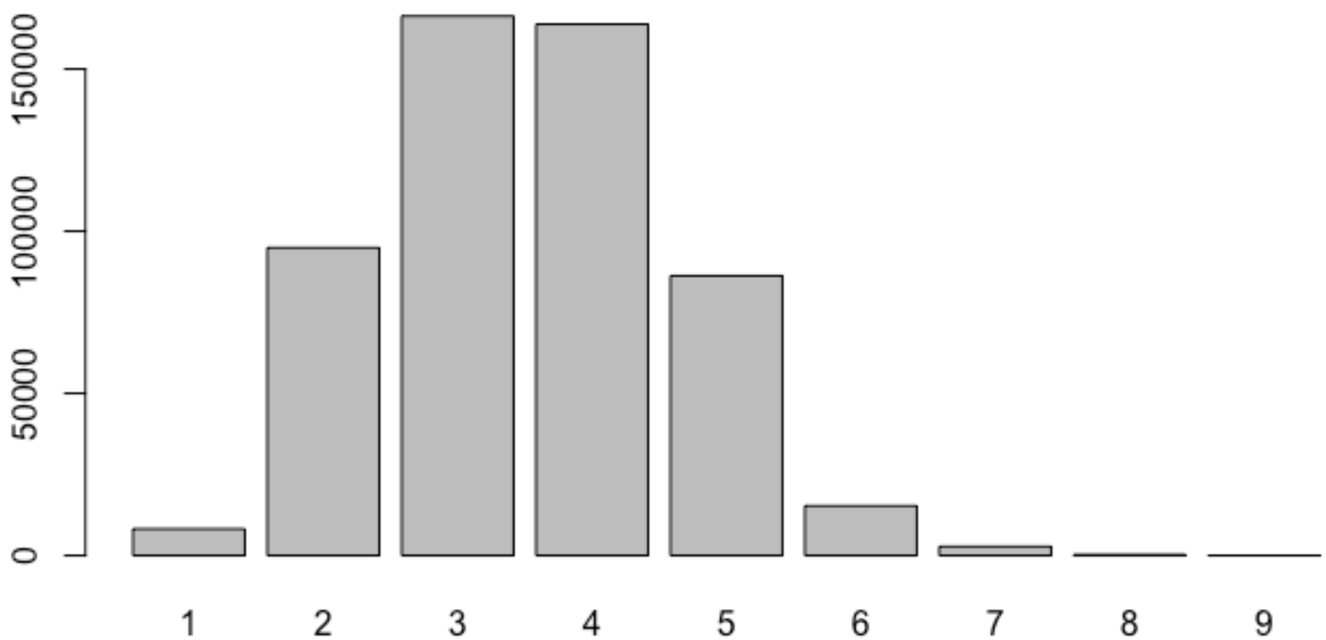
```
mean_distance(air)
```

```
#> [1] 3.52743
```

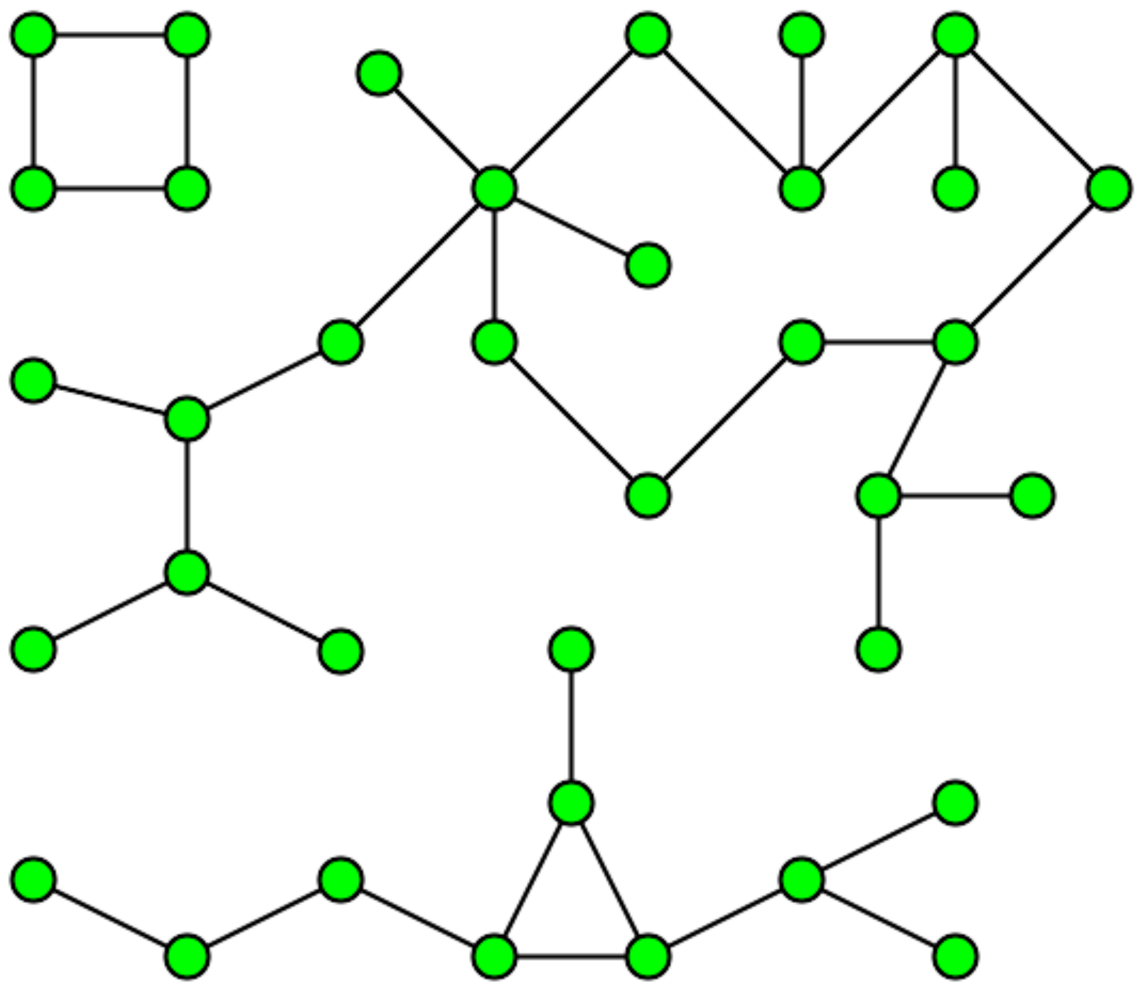
```
air_dist_hist <- distance_table(air)
air_dist_hist
```

```
#> $res
#> [1]   8228  94912 166335 163830  86263  15328   2793    291     27
#>
#> $unconnected
#> [1] 31263
```

```
barplot(air_dist_hist$res, names.arg = seq_along(air_dist_hist$res))
```

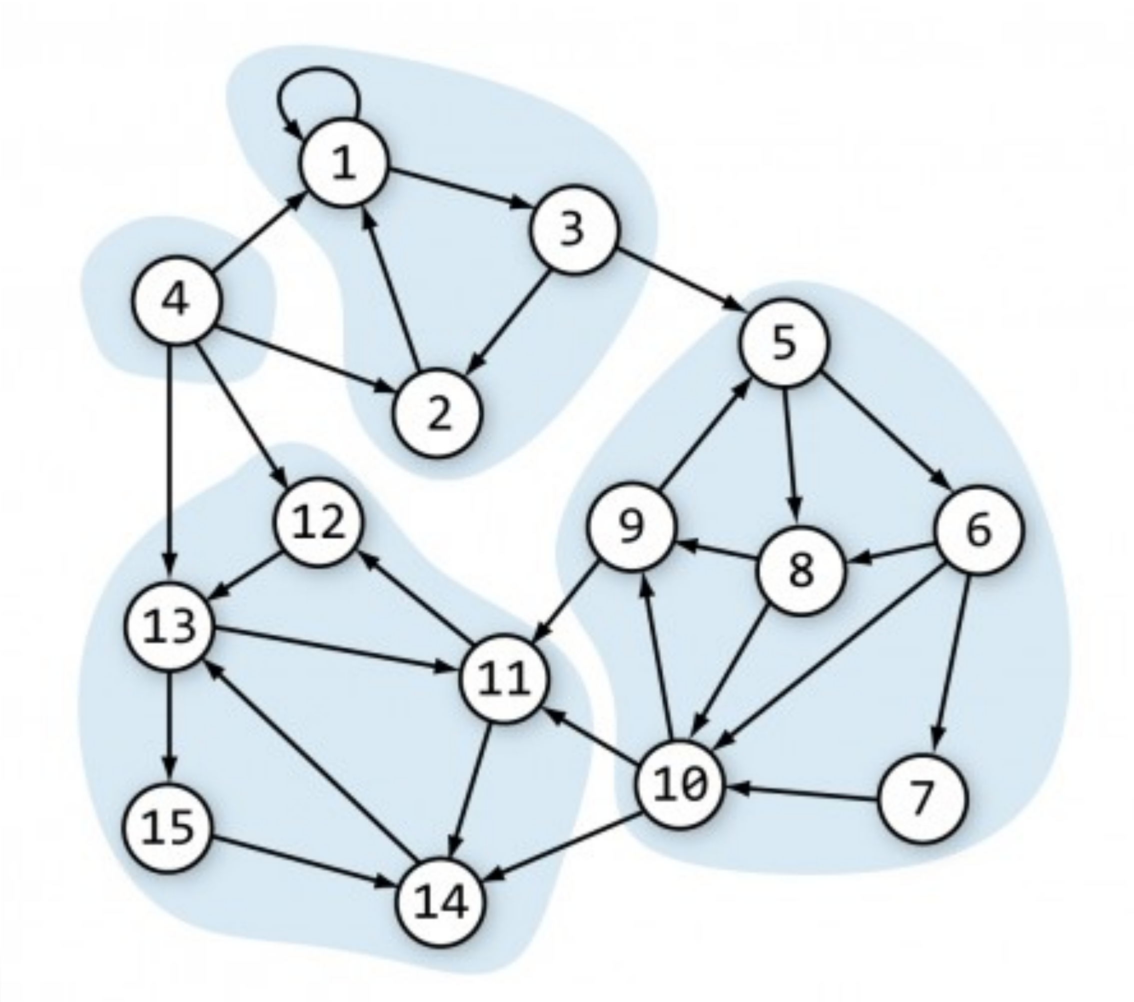


Components



David Eppstein, public domain

Strongly connected components



http://www.greatandlittle.com/studios/

```r
co <- components(air, mode = "weak")
co$csize
```

```
#> [1] 745   2   2   3   2   1
```

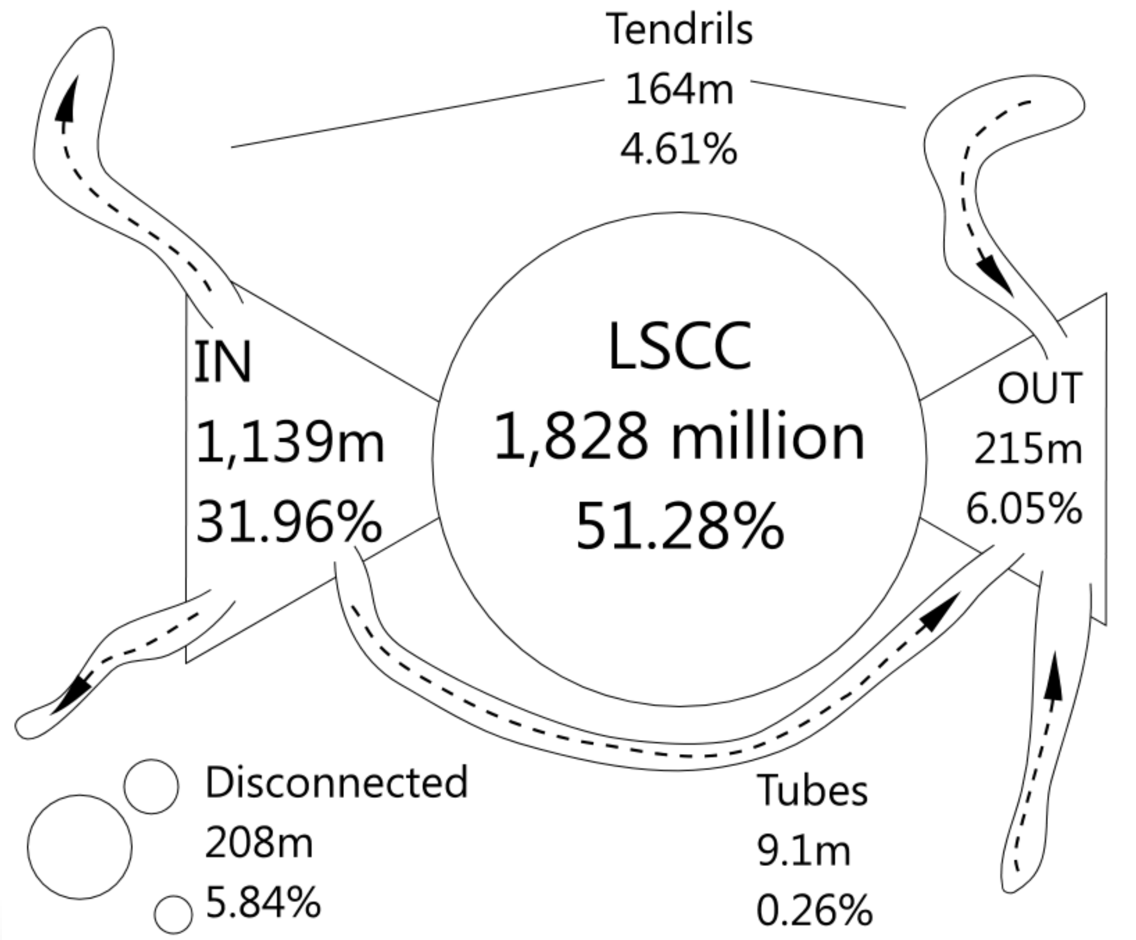
```r
groups(co)[[2]]
```

```
#> [1] "GKN" "MXY"
```

```r
co <- components(air, mode = "strong")
co$csize
```

```
#>  [1]   1   1   1   1   1   1   1   1   1   1   2   1   2   1   1   2   1
#> [18]   1   1   1   1   1   1   1 723   1   1   1   1   1
```

Bow-tie structure of a directed graph



http://webdatacommons.org/hyperlinkgraph/2012-

Exercise

1. Extract the large (strongly) connected component from the airport graph, as a separate graph. Hint: `components()`, `induced_subgraph()`. How many airports are not in this component?

2. In the large connected component, which airport is better connected, `LAX` or `BOS`? I.e. what is the mean number of plane changes that are required if traveling to a uniformly randomly picked airport?

3. Which airport is the best connected one? Which one is the worst (within the strongly connected component)?

Solution

```r
largest_component <- function(graph) {
  comps <- components(graph, mode = "strong")
  gr <- groups(comps)
  sizes <- vapply(gr, length, 1L)
  induced_subgraph(graph, gr[[ which.max(sizes) ]])
}
sc_air <- largest_component(air)
```

```r
table(distances(sc_air, "BOS"))
```

```
#>
#>   0   1   2   3   4   5
#>   1  83 355 135 147   2
```

```r
table(distances(sc_air, "LAX"))
```

```
#>
#>   0   1   2   3   4   5
#>   1 109 394 195  22   2
```

```r
mean(as.vector(distances(sc_air, "BOS")))
```

```
#> [1] 2.484094
```

```r
mean(as.vector(distances(sc_air, "LAX")))
```

```
#> [1] 2.185339
```

```r
D <- distances(sc_air)
sort(rowMeans(D))[1:10]
```

```
#>      ORD      MSP      SEA      DTW      LAX      PHX      EWR      ANC
#> 2.117566 2.146611 2.149378 2.170124 2.185339 2.218534 2.224066 2.230982
#>      SLC      JFK
#> 2.235131 2.275242
```

```r
sort(rowMeans(D), decreasing = TRUE)[1:10]
```

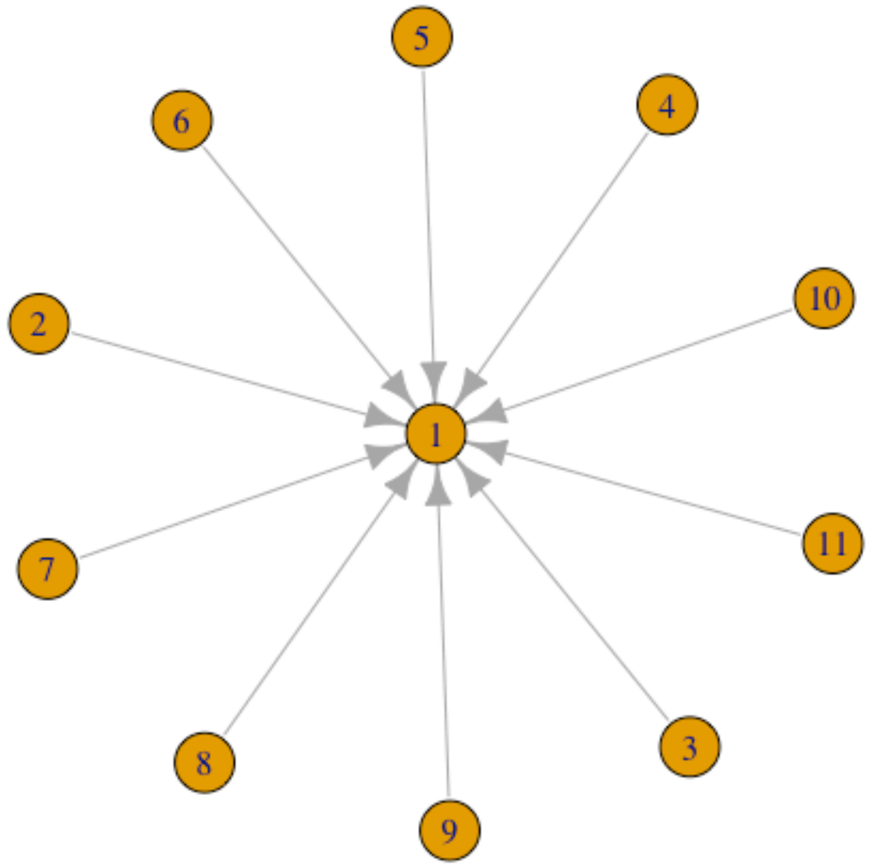
```
#>      DQR      SDX      BLD      TIQ      TCL      CPX      AFK      WHD
#> 6.147994 6.147994 5.150761 5.135546 4.889350 4.872752 4.820194 4.799447
#>      ZXH      DOF
#> 4.799447 4.798064
```

```r
V(sc_air)[[names(sort(rowMeans(D), decreasing = TRUE)[1:10])]]
```

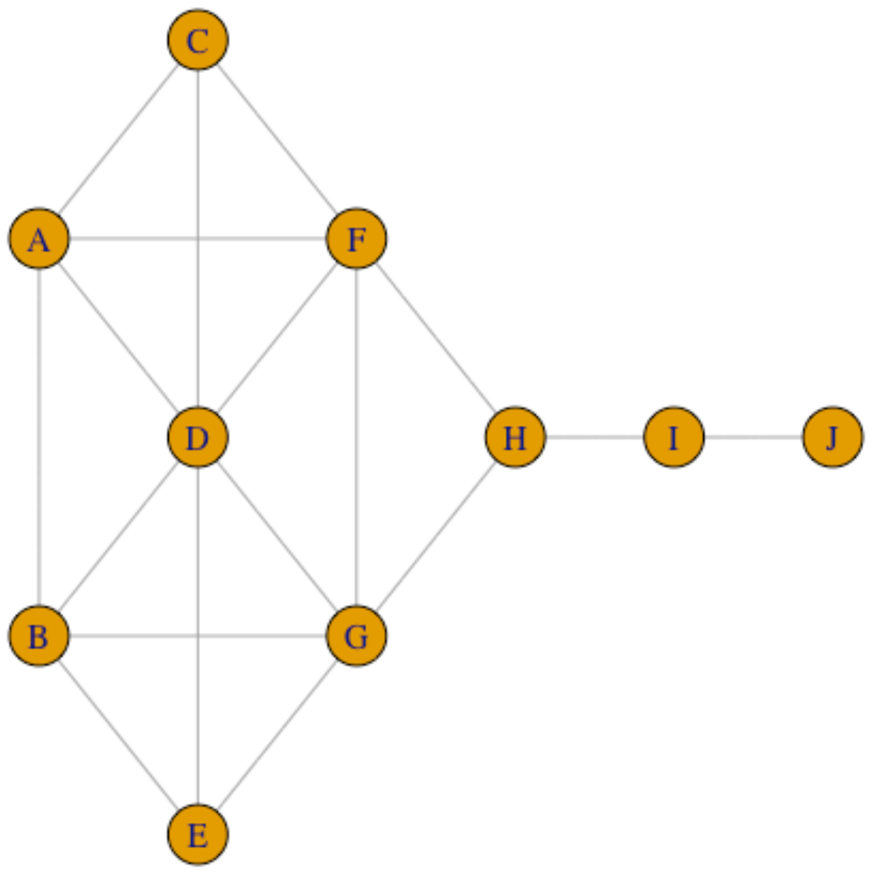
```
#> + 10/723 vertices, named:
#>      name                City          Position color
#> 567   DQR    Peach Springs, AZ N355919 W1134836  grey
#> 570   SDX           Sedona, AZ N345055 W1114718  grey
#> 566   BLD     Boulder City, NV N355651 W1145140  grey
#> 180   TIQ           Tinian, TT N145949 E1453705  grey
#> 688   TCL       Tuscaloosa, AL N331314 W0873641  grey
#> 722   CPX          Culebra, PR  N181848 W651816  grey
#> 670   AFK         Nebraska, NE  N403620 W955204  grey
#> 418   WHD            Hyder, AK N555412 W1300024  grey
#> 420   ZXH Chomondely Sound, AK N551421 W1320651  grey
#> 410   DOF         Dora Bay, AK N551400 W1321300  grey
```

Centrality

Finding important vertices in the network (family of concepts)



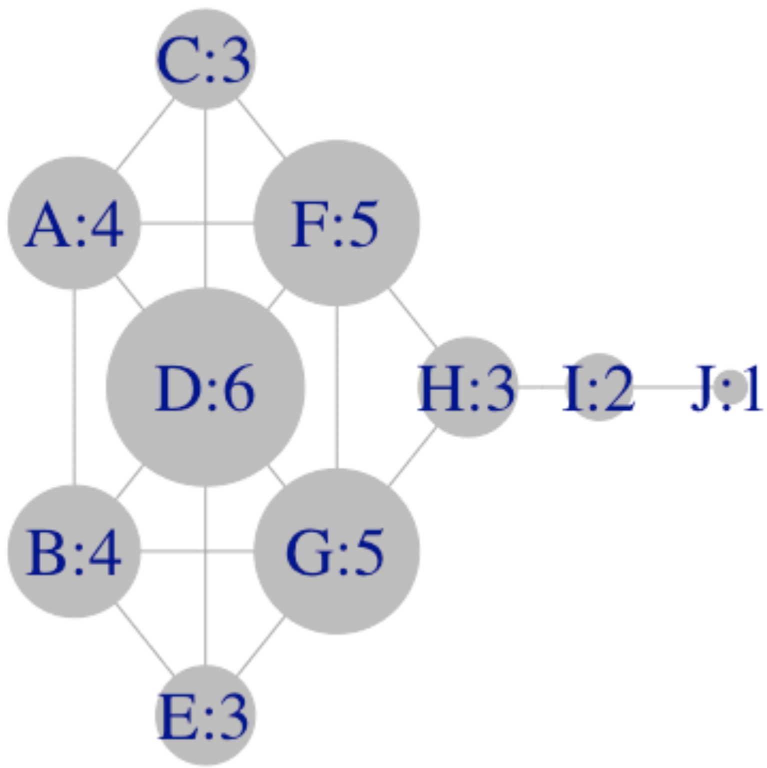
Centrality



Classic centrality measures: degree

```
V(kite)$label.cex <- 2
V(kite)$color <- V(kite)$frame.color <- "grey"
V(kite)$size <- 30
par(mar=c(0,0,0,0)) ; plot(kite)
```

```
d <- degree(kite)
par(mar = c(0,0,0,0))
plot(kite, vertex.size = 10 * d, vertex.label =
       paste0(V(kite)$name, ":", d))
```

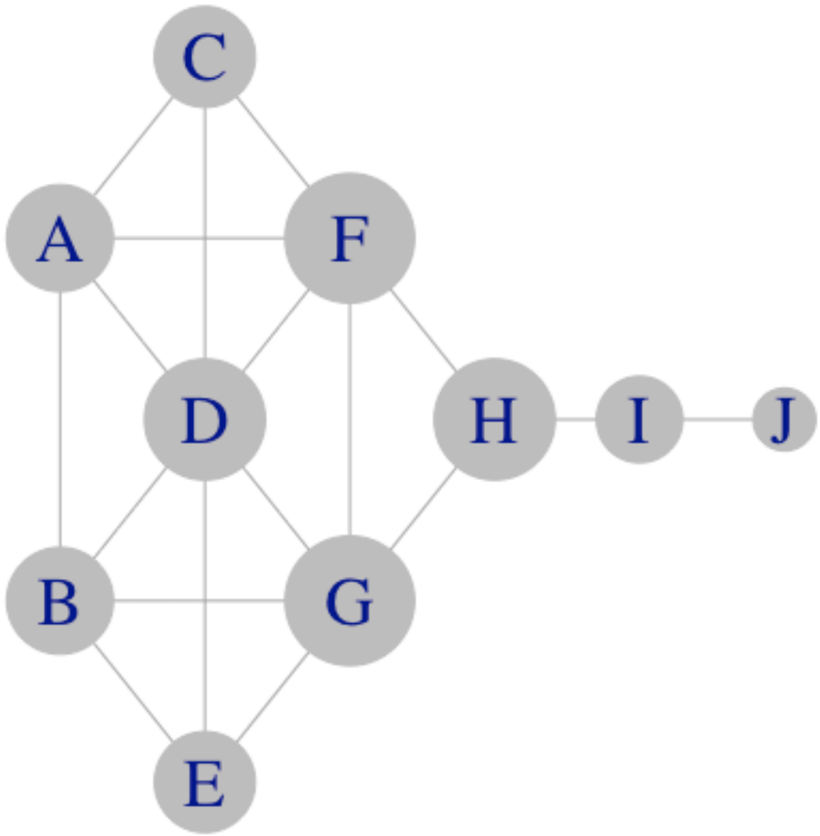


Classic centrality measures: closeness

1 / How many steps do you need to get there?

```
cl <- closeness(kite)
```

```r
par(mar=c(0,0,0,0)); plot(kite, vertex.size = 500 * cl)
```



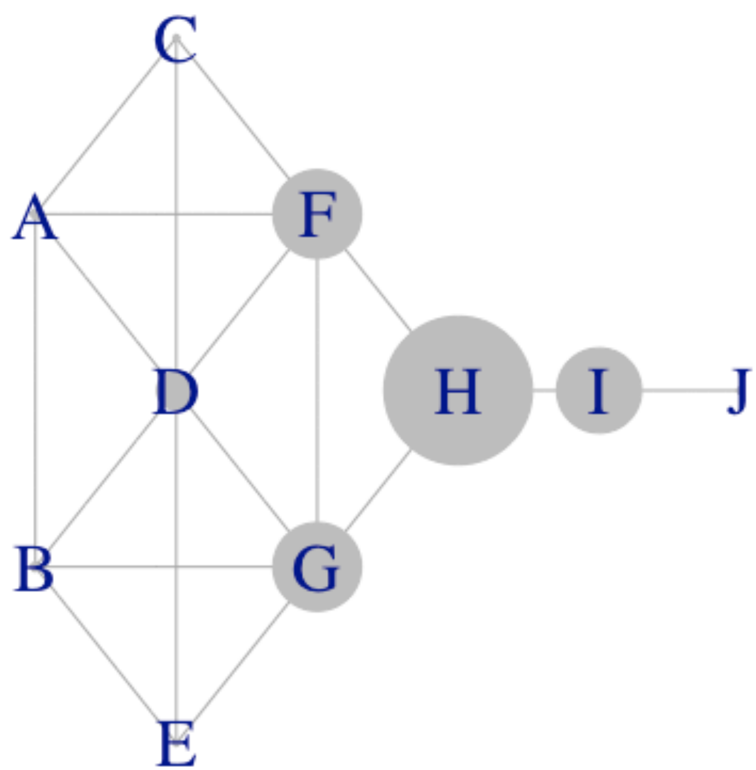
Classic centrality measures: betweenness

How many shortest paths goes through me

```
btw <- betweenness(kite)
btw
```

```
#>          A          B          C          D          E          F 
#>  0.8333333  0.8333333  0.0000000  3.6666667  0.0000000  8.3333333 
#>          G          H          I          J 
#>  8.3333333 14.0000000  8.0000000  0.0000000 
```

```
par(mar=c(0,0,0,0)); plot(kite, vertex.size = 3 * btw)
```



Eigenvector centrality

Typically for directed. Central vertex: it is cited by central vertices.

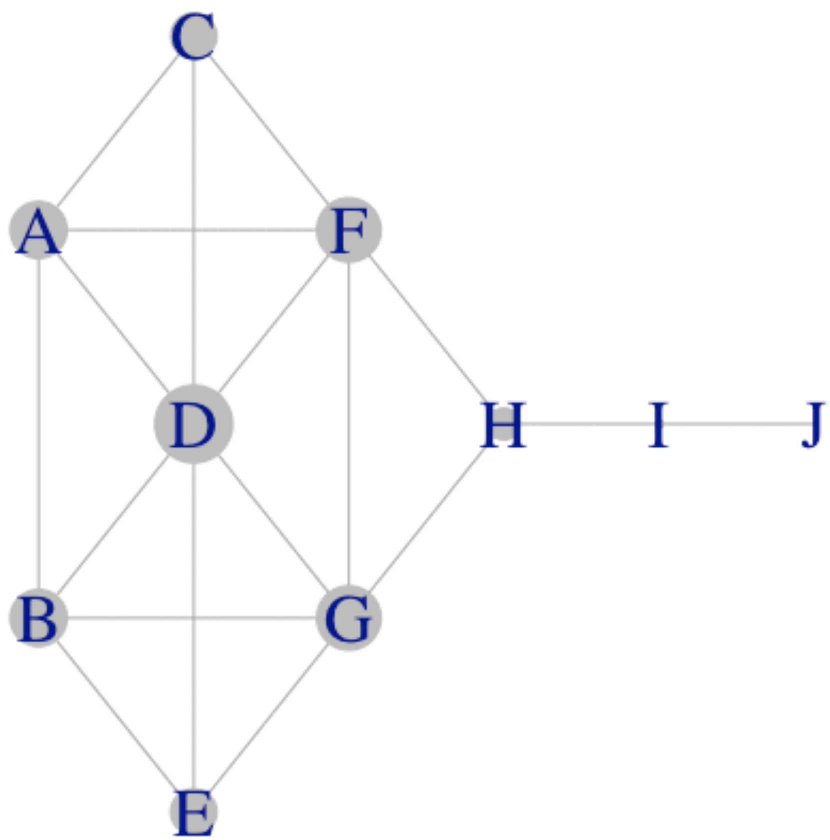
```
ec <- eigen_centrality(kite)$vector
ec
```

```
#>          A          B          C          D          E          F
#> 0.73221232 0.73221232 0.59422577 1.00000000 0.59422577 0.82676381
#>          G          H          I          J
#> 0.82676381 0.40717690 0.09994054 0.02320742
```

```
cor(ec, d)
```

```
#> [1] 0.9542561
```

```
par(mar=c(0,0,0,0)); plot(kite, vertex.size = 20 * ec)
```



Page Rank

Fixes the practical problems with eigenvector centrality

```
page_rank(kite)$vector
```

```
#>          A          B          C          D          E          F
#> 0.10191991 0.10191991 0.07941811 0.14714792 0.07941811 0.12890693
#>          G          H          I          J
#> 0.12890693 0.09524829 0.08569396 0.05141993
```

Exercise

Create a table that contains the top 10 most central airports according to all these centrality measures.

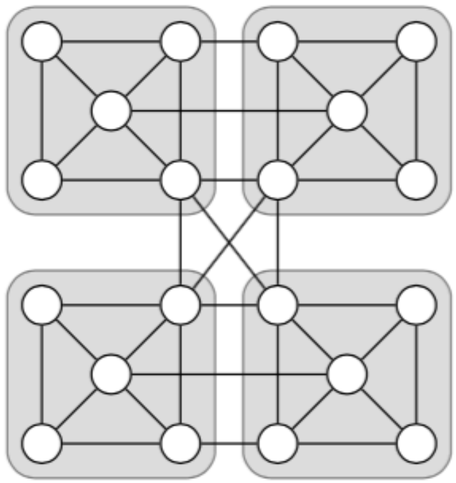
Clusters

Why finding groups

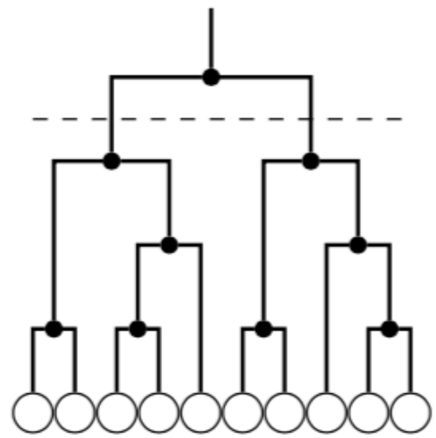
Finding groups in networks. Dimensionality reduction. Community detection.

We want to find dense groups.

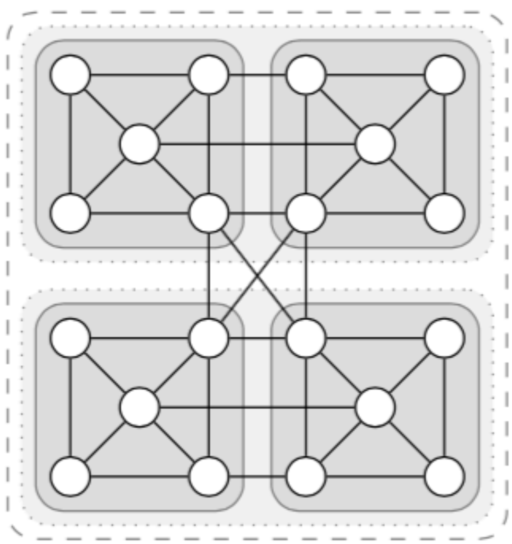
[redacted]



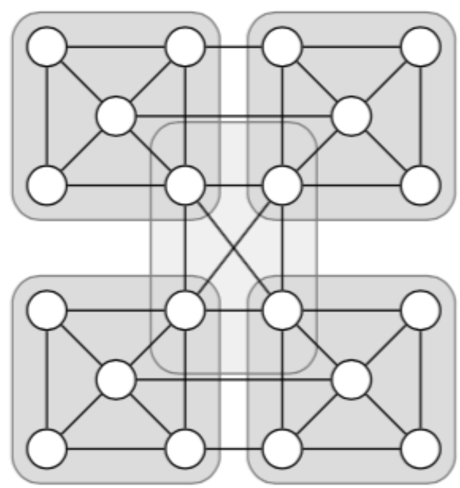
(a) Flat



(b) Hierarchical



(c) Multi-level

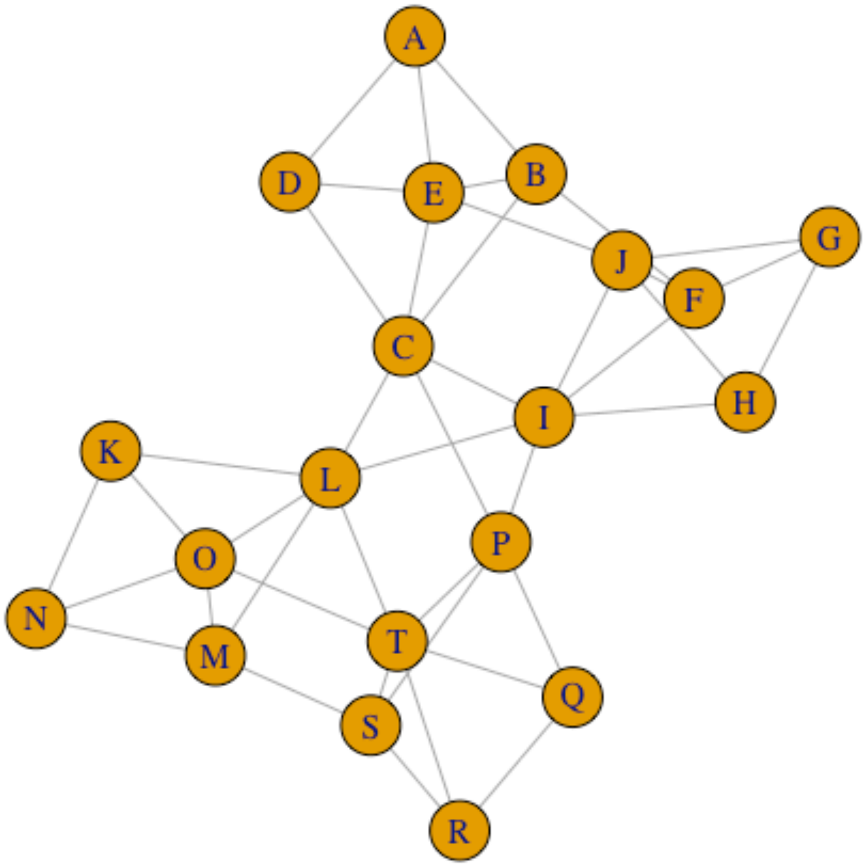


(d) Overlapping

Clusters by hand

```
graph <- make_graph( ~ A-B-C-D-A, E-A:B:C:D,
                       F-G-H-I-F, J-F:G:H:I,
                       K-L-M-N-K, O-K:L:M:N,
                       P-Q-R-S-P, T-P:Q:R:S,
                       B-F, E-J, C-I, L-T, O-T, M-S,
                       C-P, C-L, I-L, I-P)
```

```
par(mar=c(0,0,0,0)); plot(graph)
```



```
flat_clustering <- make_clusters(
    graph,
    c(1,1,1,1,1,2,2,2,2,2,3,3,3,3,3,4,4,4,4,4))
```

```r
flat_clustering
```

```
#> IGRAPH clustering unknown, groups: 4, mod: 0.51
#> + groups:
#>   $`1`
#>   [1] 1 2 3 4 5
#>
#>   $`2`
#>   [1]  6  7  8  9 10
#>
#>   $`3`
#>   [1] 11 12 13 14 15
#>
#>   $`4`
#>   + ... omitted several groups/vertices
```

```r
flat_clustering[[1]]
```

```
#> [1] 1 2 3 4 5
```

```r
length(flat_clustering)
```

```
#> [1] 4
```

```r
sizes(flat_clustering)
```

```
#> Community sizes
#> 1 2 3 4
#> 5 5 5 5
```

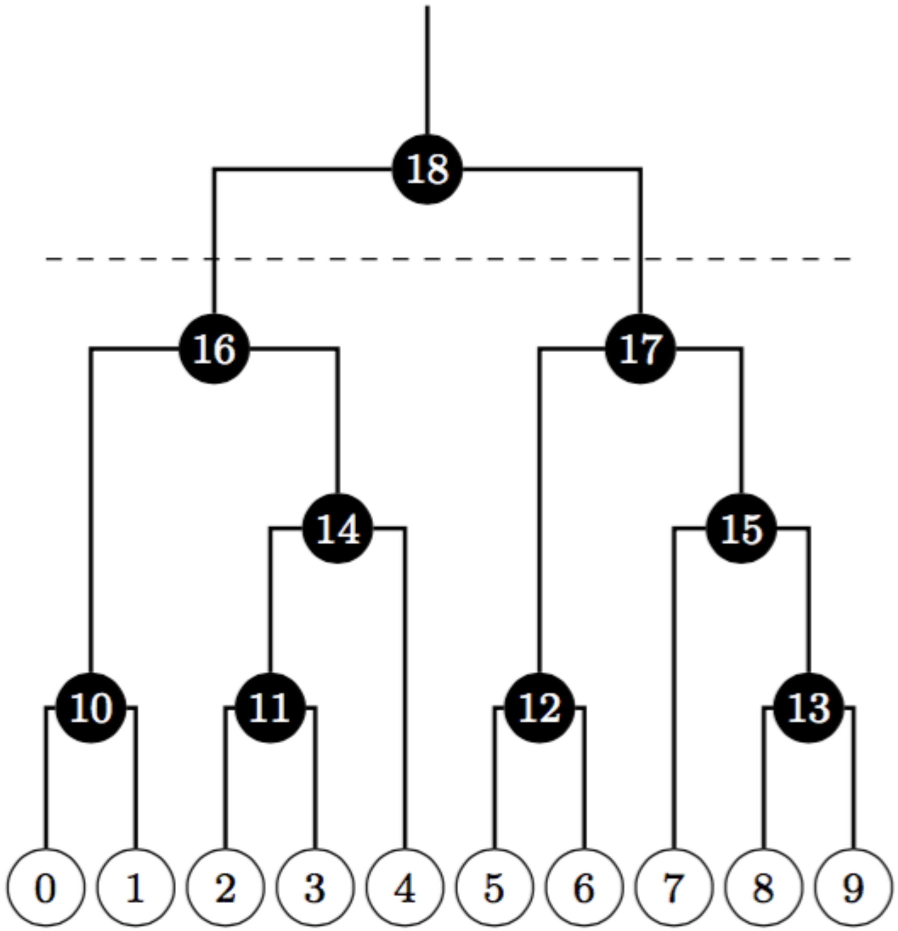
```r
induced_subgraph(graph, flat_clustering[[1]])
```

```
#> IGRAPH UN-- 5 8 --
#> + attr: name (v/c)
#> + edges (vertex names):
#> [1] A--B A--D A--E B--C B--E C--D C--E D--E
```

Hierarchical community structure

Typically produced by top-down or bottom-up clustering algorithms.

The outcome can be represented as a *dendrogram*, a tree-like diagram that illustrates the order in which the clusters are merged (in the bottom-up case) or split (in the top-down case).



$$
\left[\begin{array}{cc}
0 & 1 \\
2 & 3 \\
5 & 6 \\
8 & 9 \\
11 & 4 \\
7 & 13 \\
10 & 14 \\
12 & 15 \\
\hline
16 & 17
\end{array}\right]
\begin{array}{l}
\rightarrow 10 \\
\rightarrow 11 \\
\rightarrow 12 \\
\rightarrow 13 \\
\rightarrow 14 \\
\rightarrow 15 \\
\rightarrow 16 \\
\rightarrow 17 \\
\rightarrow 18
\end{array}
$$

Clustering quality measures

- External quality measures: require ground truth
- Internal quality measures: require assumption about *good* clusters.

External quality measures

Measure	Type	Range	igraph name
Rand index	similarity	0 to 1	`rand`
Adjusted Rand index	similarity	-0.5 to 1	`adjusted.rand`
Split-join distance	distance	0 to 2n	`split.join`
Variation of information	distance	0 to log n	`vi`
Normalized mutual information	similarity	0 to 1	`nmi`

External quality measures

```r
data(karate)
karate
```

```
#> IGRAPH UNW- 34 78 -- Zachary's karate club network
#> + attr: name (g/c), Citation (g/c), Author (g/c), Faction (v/n),
#> | name (v/c), label (v/c), color (v/n), weight (e/n)
#> + edges (vertex names):
#>  [1] Mr Hi  --Actor 2  Mr Hi  --Actor 3  Mr Hi  --Actor 4
#>  [4] Mr Hi  --Actor 5  Mr Hi  --Actor 6  Mr Hi  --Actor 7
#>  [7] Mr Hi  --Actor 8  Mr Hi  --Actor 9  Mr Hi  --Actor 11
#> [10] Mr Hi  --Actor 12 Mr Hi  --Actor 13 Mr Hi  --Actor 14
#> [13] Mr Hi  --Actor 18 Mr Hi  --Actor 20 Mr Hi  --Actor 22
#> [16] Mr Hi  --Actor 32 Actor 2--Actor 3  Actor 2--Actor 4
#> [19] Actor 2--Actor 8  Actor 2--Actor 14 Actor 2--Actor 18
#> + ... omitted several edges
```

```r
karate <- delete_edge_attr(karate, "weight")
```

```r
ground_truth <- make_clusters(karate, V(karate)$Faction)
length(ground_truth)
```

```
#> [1] 2
```

```r
ground_truth
```

```
#> IGRAPH clustering unknown, groups: 2, mod: 0.37
#> + groups:
#>   $`1`
#>    [1]  1  2  3  4  5  6  7  8 11 12 13 14 17 18 20 22
#>
#>   $`2`
#>    [1]  9 10 15 16 19 21 23 24 25 26 27 28 29 30 31 32 33 34
#>
```

Exercise

Write a naive clustering method that classifies vertices into two groups, based on two center vertices. Put the two centers in separate clusters, and other vertices in the cluster whose center is closer to it.

```
cluster_naive2 <- function(graph, center1, center2) {
  # ...
}
```

Solution

```
cluster_naive2 <- function(graph, center1, center2) {
  dist <- distances(graph, c(center1, center2))
  cl <- apply(dist, 2, which.min)
  make_clusters(graph, cl)
}
dist_memb <- cluster_naive2(karate, 'John A', 'Mr Hi')
```

```r
dist_memb
```

```
#> IGRAPH clustering unknown, groups: 2, mod: 0.31
#> + groups:
#>   $`1`
#>    [1] "Actor 9"  "Actor 10" "Actor 14" "Actor 15" "Actor 16" "Actor 19"
#>    [7] "Actor 20" "Actor 21" "Actor 23" "Actor 24" "Actor 25" "Actor 26"
#>   [13] "Actor 27" "Actor 28" "Actor 29" "Actor 30" "Actor 31" "Actor 32"
#>   [19] "Actor 33" "John A"
#>
#>   $`2`
#>    [1] "Mr Hi"    "Actor 2"  "Actor 3"  "Actor 4"  "Actor 5"  "Actor 6"
#>    [7] "Actor 7"  "Actor 8"  "Actor 11" "Actor 12" "Actor 13" "Actor 17"
#>   [13] "Actor 18" "Actor 22"
#>   + ... omitted several groups/vertices
```

Rand index

Check if pairs of vertices are classified correctly

```
rand_index <- compare(ground_truth, dist_memb, method = "rand")
rand_index
```

```
#> [1] 0.885918
```

Rand index

Random clusterings

```
random_partition <- function(n, k = 2) { sample(k, n, replace = TRUE) }
total <- numeric(100)
for (i in seq_len(100)) {
  c1 <- random_partition(100)
  c2 <- random_partition(100)
  total[i] <- compare(c1, c2, method = "rand")
}
mean(total)
```

```
#> [1] 0.5017414
```

Adjusted Rand index

```r
total <- numeric(100)
for (i in seq_len(100)) {
  c1 <- random_partition(100)
  c2 <- random_partition(100)
  total[i] <- compare(c1, c2, method = "adjusted.rand")
}
mean(total)
```

```
#> [1] 0.00168767
```

Adjusted rand index

```
compare(ground_truth, dist_memb, method = "adjusted.rand")
```

```
#> [1] 0.7718469
```

Internal quality metrics: density

```
edge_density(karate)
```

```
#> [1] 0.1390374
```

```
subgraph_density <- function(graph, vertices) {
  sg <- induced_subgraph(graph, vertices)
  edge_density(sg)
}
```

```
subgraph_density(karate, ground_truth[[1]])
```

```
#> [1] 0.275
```

```
subgraph_density(karate, ground_truth[[2]])
```

```
#> [1] 0.2287582
```

Internal quality metrics: modularity

Uses a null model

$$Q(G) = \frac{1}{2m} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(A_{ij} - p_{ij}\right) \delta_{ij}$$

A_{ij}: Adjacency matrix

δ_{ij}: i and j are in the same cluster

p_{ij} expected value for an (i, j) edge from the null model

Modularity

Common null model: degree-sequence (configuration) model

$$Q(G) = \frac{1}{2m} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta_{ij}$$

Modularity in igraph

```
modularity(ground_truth)
```

```
#> [1] 0.3714661
```

```
modularity(karate, membership(ground_truth))
```

```
#> [1] 0.3714661
```

Well behaving:

```r
modularity(karate, rep(1, gorder(karate)))
```

```
#> [1] 0
```

```r
modularity(karate, seq_len(gorder(karate)))
```

```
#> [1] -0.04980276
```

Heuristic algorithms

Edge-betweenness clustering

Exact modularity optimization

Greedy agglomerative algorithm to maximize modularity

Edge-betweenness clustering

```r
dendrogram <- cluster_edge_betweenness(karate)
dendrogram
```

```
#> IGRAPH clustering edge betweenness, groups: 5, mod: 0.4
#> + groups:
#>   $`1`
#>    [1] "Mr Hi"    "Actor 2"  "Actor 4"  "Actor 8"  "Actor 12" "Actor 13"
#>    [7] "Actor 14" "Actor 18" "Actor 20" "Actor 22"
#>
#>   $`2`
#>   [1] "Actor 3"  "Actor 25" "Actor 26" "Actor 28" "Actor 29" "Actor 32"
#>
#>   $`3`
#>   [1] "Actor 5"  "Actor 6"  "Actor 7"  "Actor 11" "Actor 17"
#>
#>   + ... omitted several groups/vertices
```

```r
membership(dendrogram)
```

```
#>    Mr Hi  Actor 2  Actor 3  Actor 4  Actor 5  Actor 6  Actor 7  Actor 8
#>        1        1        2        1        3        3        3        1
#>  Actor 9 Actor 10 Actor 11 Actor 12 Actor 13 Actor 14 Actor 15 Actor 16
#>        4        5        3        1        1        1        4        4
#> Actor 17 Actor 18 Actor 19 Actor 20 Actor 21 Actor 22 Actor 23 Actor 24
#>        3        1        4        1        4        1        4        4
#> Actor 25 Actor 26 Actor 27 Actor 28 Actor 29 Actor 30 Actor 31 Actor 32
#>        2        2        4        2        2        4        4        2
#> Actor 33   John A
#>        4        4
```

```r
compare_all <- function(cl1, cl2) {
  methods <- eval(as.list(args(compare))$method)
  vapply(methods, compare, 1.0, comm1 = cl1, comm2 = cl2)
}
compare_all(dendrogram, ground_truth)
```

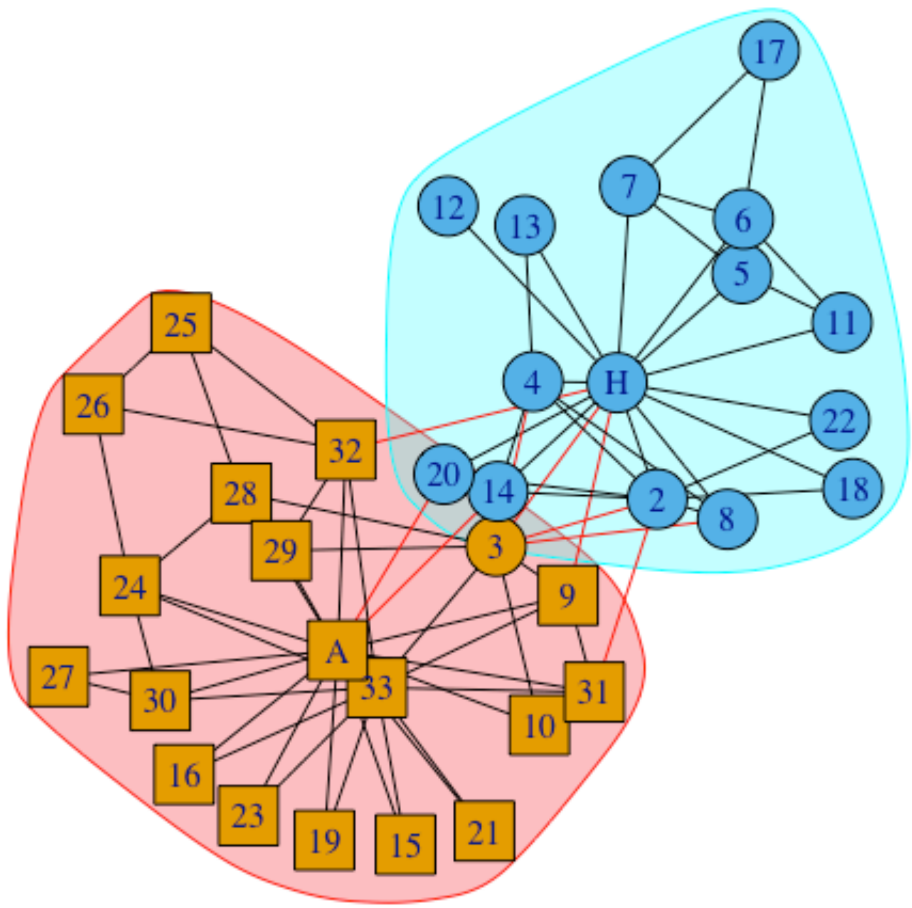
```
#>            vi           nmi    split.join          rand adjusted.rand
#>     0.8868344     0.5798278    13.0000000     0.7379679     0.4686165
```

```r
cluster_memb <- cut_at(dendrogram, no = 2)
compare_all(cluster_memb, ground_truth)
```

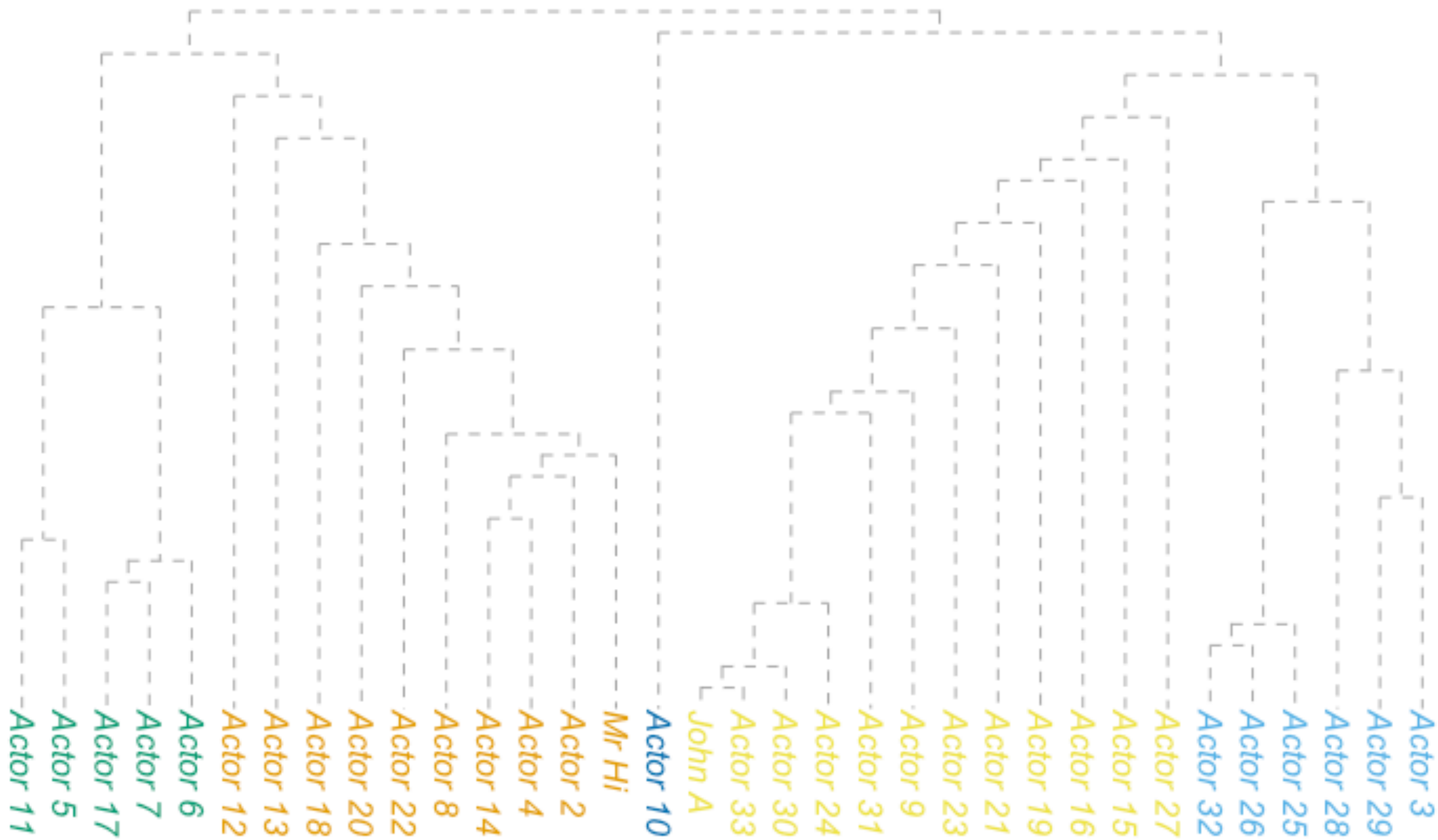
```
#>            vi           nmi    split.join          rand adjusted.rand
#>     0.2252446     0.8364981     2.0000000     0.9411765     0.8823025
```

```r
clustering <- make_clusters(karate, membership = cluster_memb)
```

```r
V(karate)[Faction == 1]$shape <- "circle"
V(karate)[Faction == 2]$shape <- "square"
par(mar=c(0,0,0,0)); plot(clustering, karate)
```



```r
par(mar=c(0,0,0,0)); plot_dendrogram(dendrogram, direction = "downwards")
```



Exact modularity maximization

```
optimal <- cluster_optimal(karate)
modularity(clustering)
```

```
#> [1] 0.3599606
```

```
modularity(optimal)
```

```
#> [1] 0.4197896
```

```
modularity(ground_truth)
```

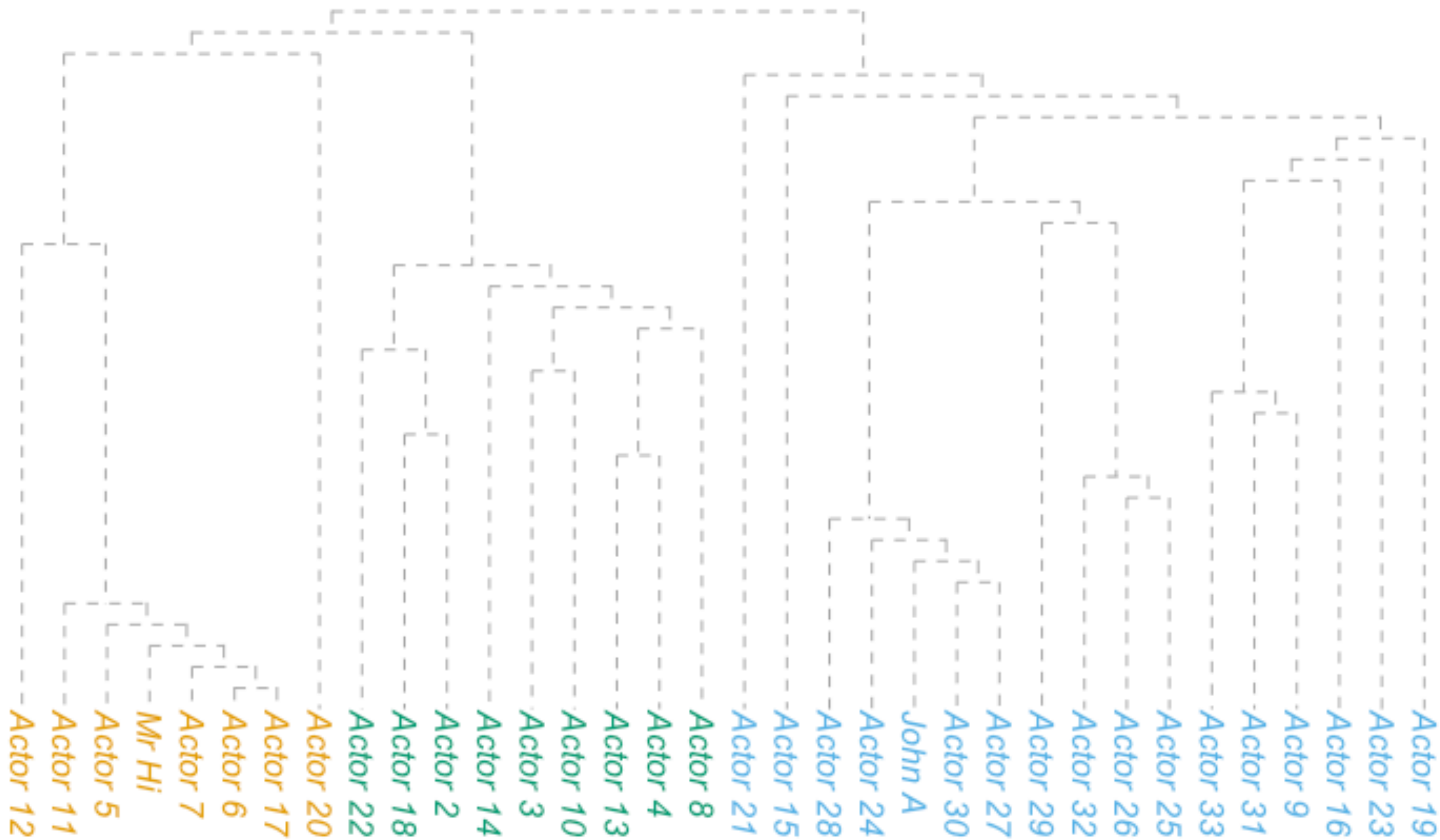
```
#> [1] 0.3714661
```

Heuristic modularity optimization

```
dend_fast <- cluster_fast_greedy(karate)
compare_all(dend_fast, ground_truth)
```

```
#>            vi           nmi    split.join          rand adjusted.rand
#>     0.5321150     0.6924673    10.0000000     0.8413547     0.6802559
```

```r
par(mar = c(0,0,0,0)); plot_dendrogram(dend_fast, direction = "downwards")
```

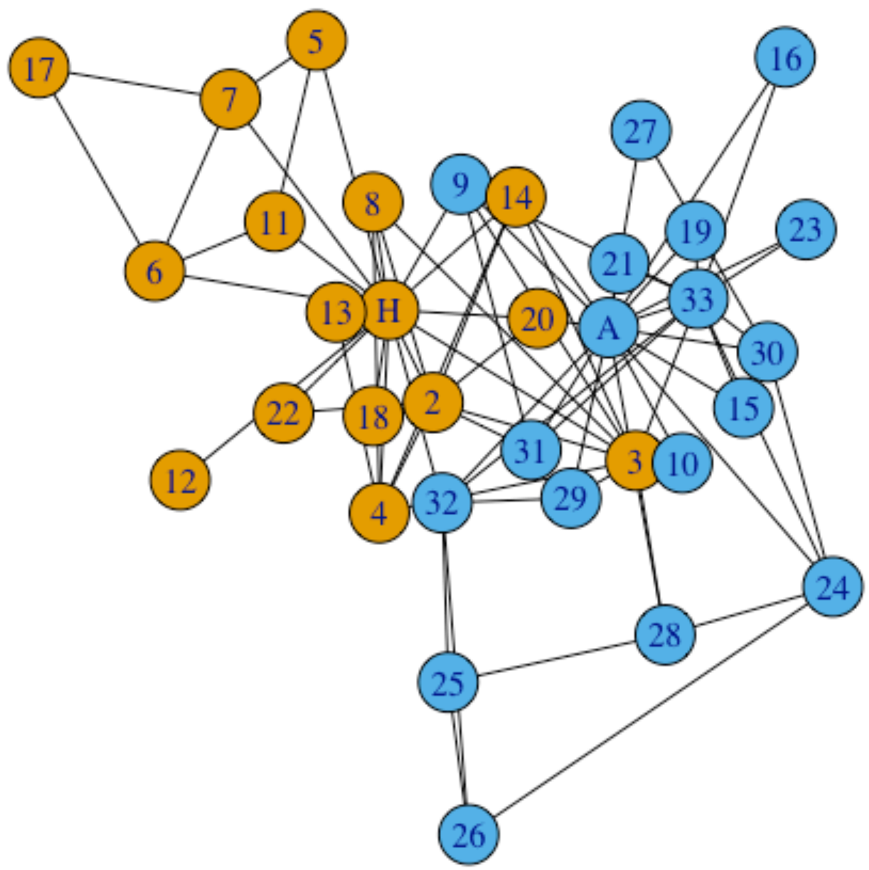


Visualization

Plotting parameters

Globally

```
igraph_options(edge.color = "black")
data(karate) ; par(mar=c(0,0,0,0)); plot(karate)
```



Graph parameter

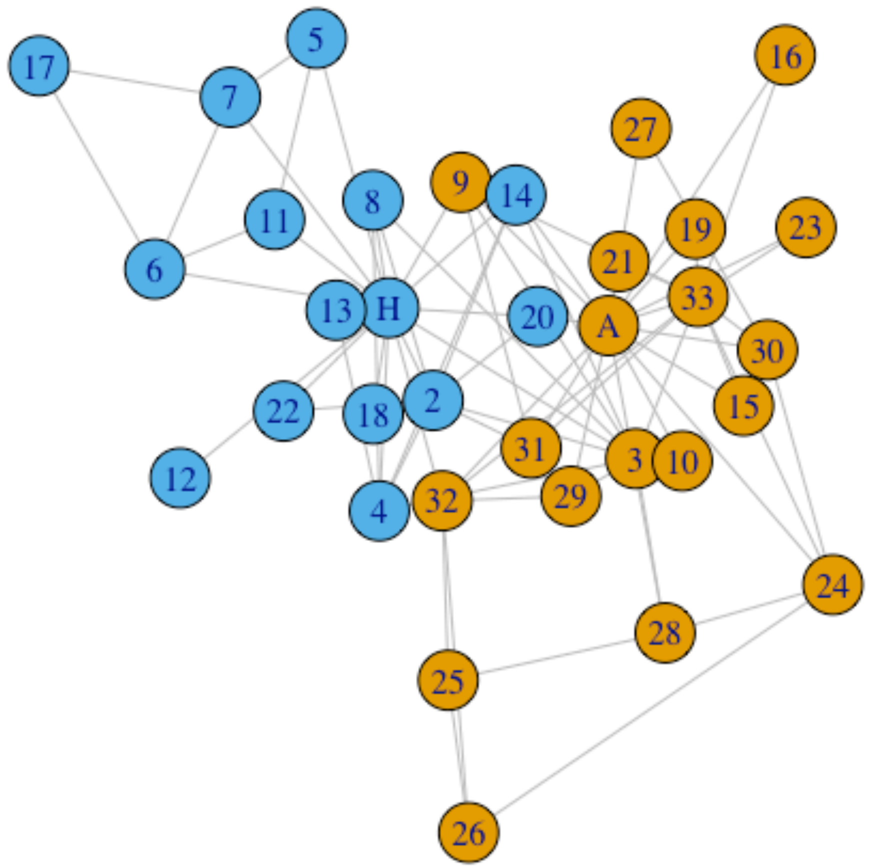
```
V(karate)$color <- "DarkOliveGreen" ; E(karate)$color <- "grey"
par(mar=c(0,0,0,0)) ; plot(karate)
```

As an argument to `plot()`:

```
par(mar = c(0,0,0,0))
plot(karate, edge.color = "black", vertex.color = "#00B7FF",
     vertex.label.color = "black")
```

igraph color palettes

```
karate$palette <- categorical_pal(length(clustering))
par(mar = c(0,0,0,0)); plot(karate, vertex.color = membership(clustering))
```



Others: `r_pal()`, `sequential_pal()`, `diverging_pal()`.

Graphical parameters

Vertices: `size`, `size`, `color`, `frame.color`, `shape` (circle, square, rectangle, pie, raster, none), `label`, `label.family`, `label.font`, `label.cex`, `label.dist`, `label.degree`, `label.color`.

Edges: `color`, `width`, `arrow.size`, `arrow.width`, `lty`, `label`, `label.family`, `label.font`, `label.cex`, `label.color`, `label.x`, `label.y`, `curved`, `arrow.mode`, `loop.angle`, `loop.angle2`.

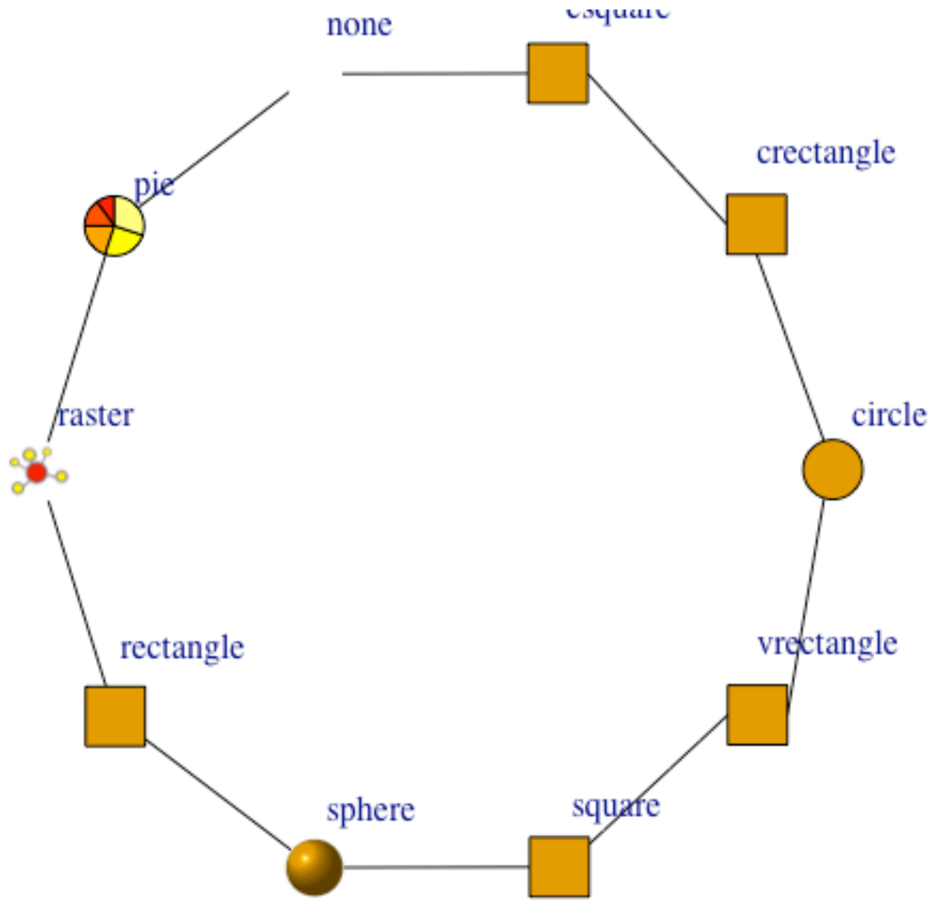
Graph: `layout` (a numeric matrix), `margin`, `palette` (for vertex color), `rescale`, `asp`, `frame`, `main` (title), `sub` (title), `xlab`, `ylab`.

Vertex shapes

```
shapes()
```

```
#>  [1] "circle"      "crectangle" "csquare"     "none"        "pie"
#>  [6] "raster"      "rectangle"  "sphere"      "square"      "vrectangle"
```

```
plot(g, vertex.shape=shapes, vertex.label=shapes, vertex.label.dist=1,
     vertex.size=15, vertex.size2=15,
     vertex.pie=lapply(shapes, function(x) if (x=="pie") 2:6 else 0),
     vertex.pie.color=list(heat.colors(5)))
```


none
csquare
crectangle
pie
raster
circle
rectangle
vrectangle
sphere
square

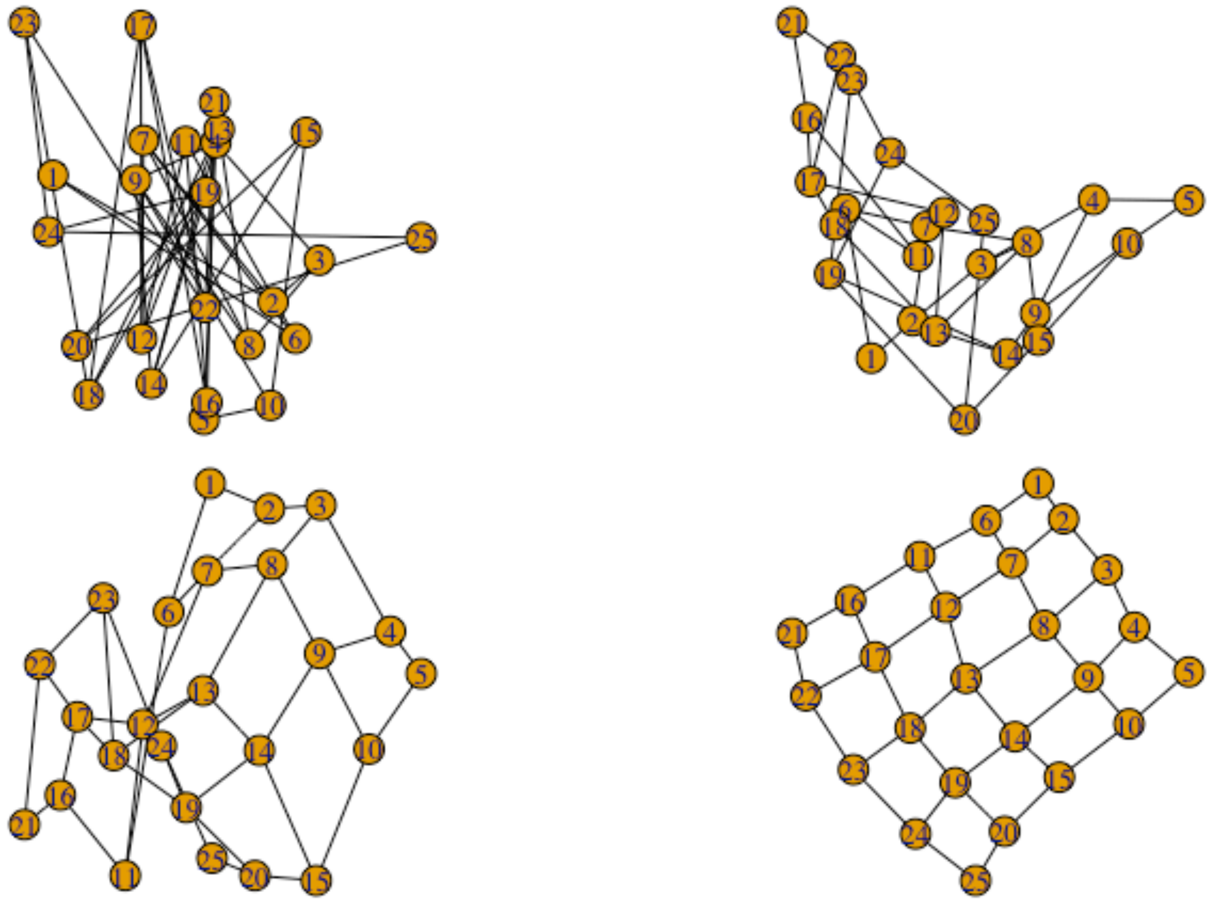
Layout algorithms

Layout algorithm: place the vertices in a way, such that

- nodes are distributed evenly
- edges have about the same length
- connected vertices are closer to each other
- edges are not crossing

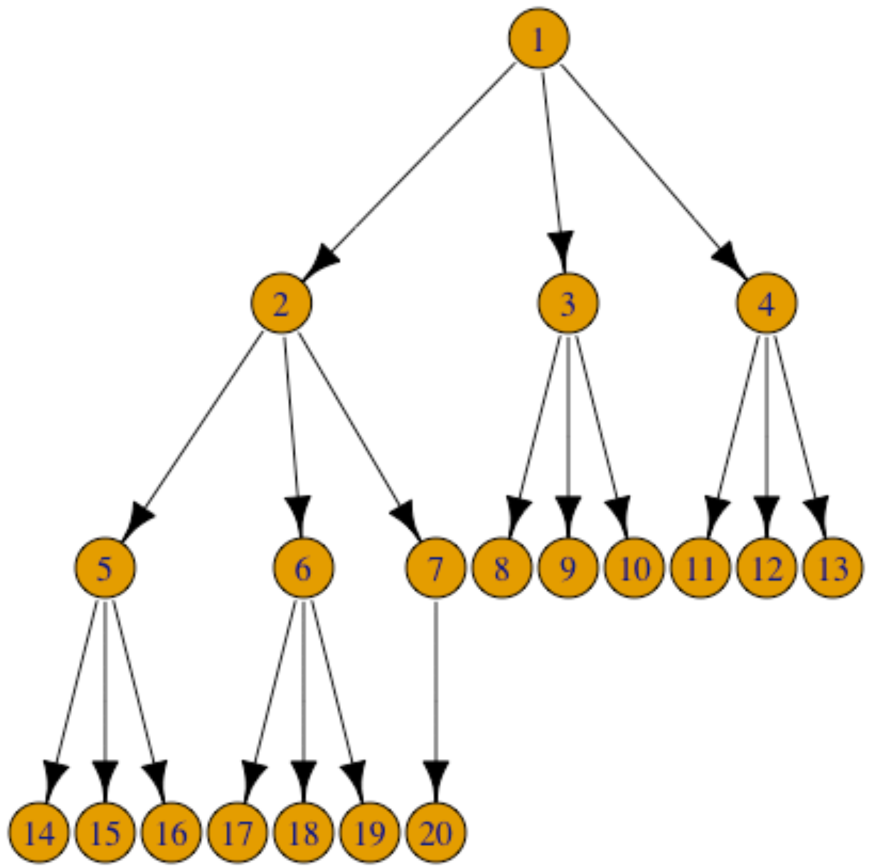
This is really hard, often impossible!

Force-directed algorithms

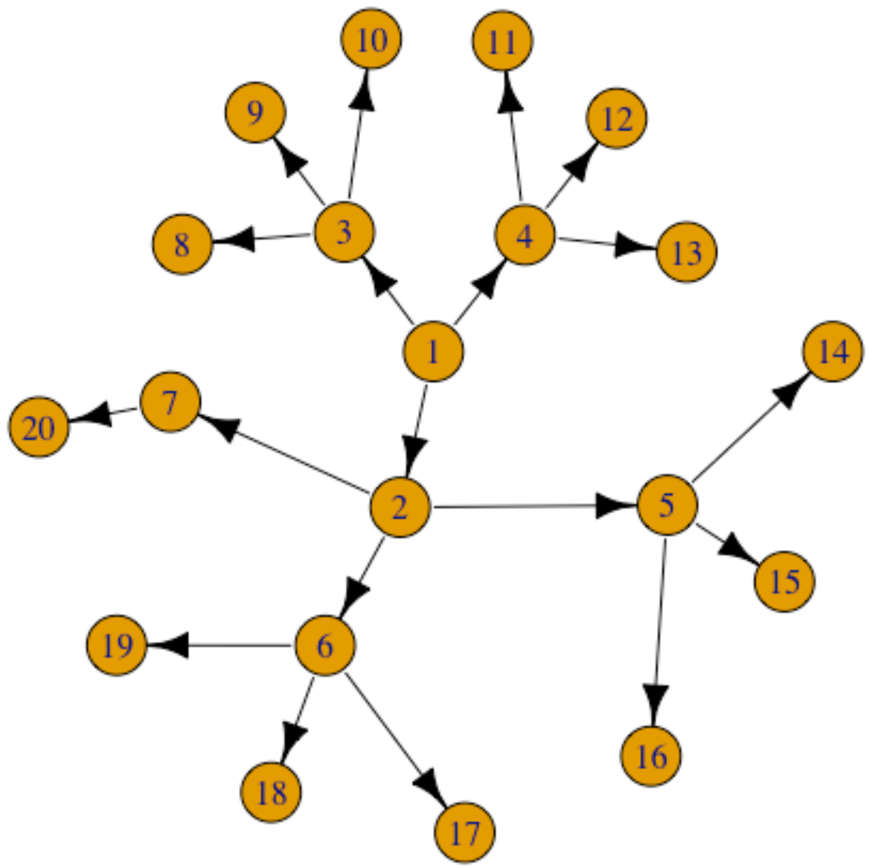


Trees

```
tree <- make_tree(20, 3)
par(mar = c(0,0,0,0)); plot(tree, layout=layout_as_tree)
```



```r
l <- layout_as_tree(tree, circular = TRUE)
par(mar = c(0,0,0,0)); plot(tree, layout = l)
```



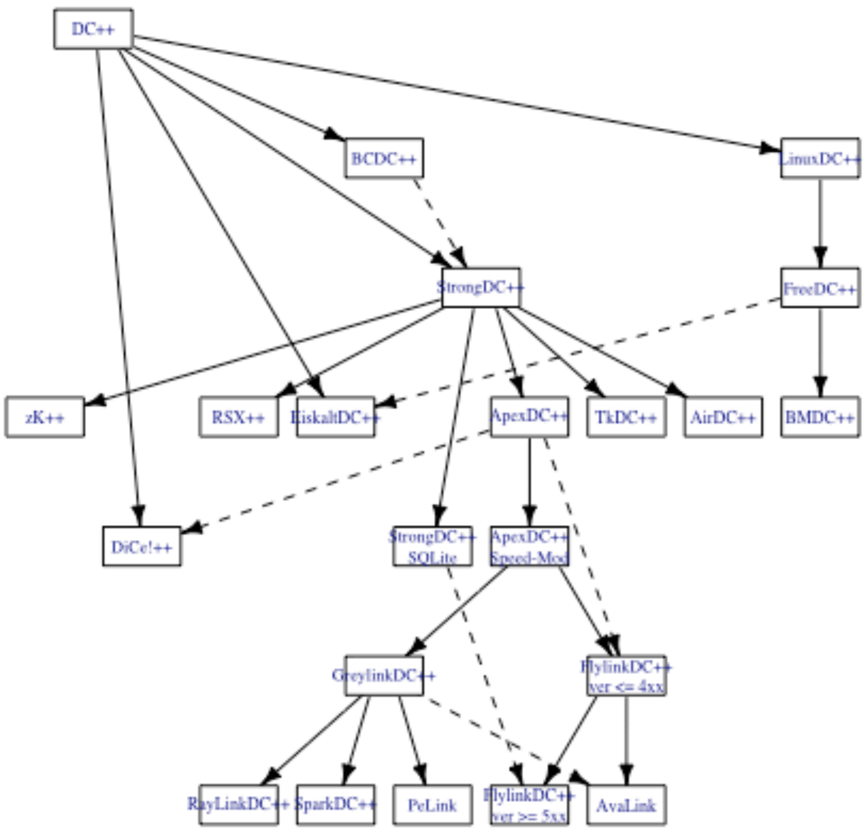
```
#> [1] TRUE
```

```
summary(DC)
```

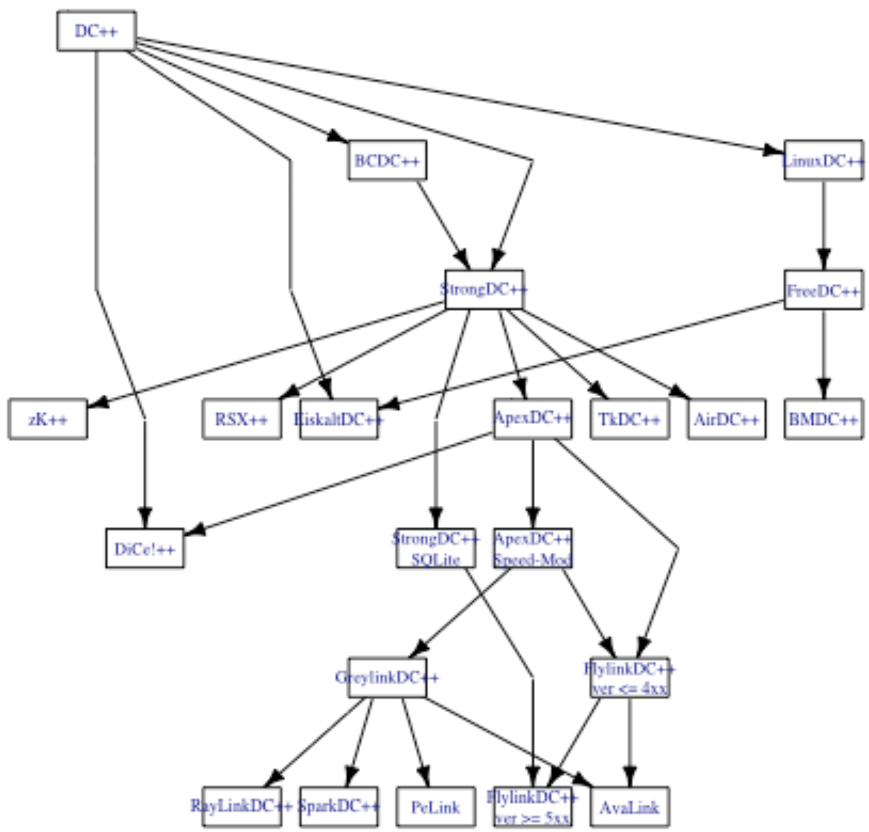
```
#> IGRAPH DN-- 22 27 --
#> + attr: name (v/c), color (v/c), shape (v/c), size (v/n), size2
#> | (v/n), label (v/x), lty (e/n), arrow.size (e/n)
```

```
lay1 <-  layout_with_sugiyama(DC, layers=apply(sapply(layers,
                        function(x) V(DC)$name %in% x), 1, which))
```

```r
par(mar = rep(0, 4))
plot(DC, layout = lay1$layout, vertex.label.cex = 0.5)
```



```r
par(mar = c(0,0,0,0)); plot(lay1$extd_graph, vertex.label.cex=0.5)
```

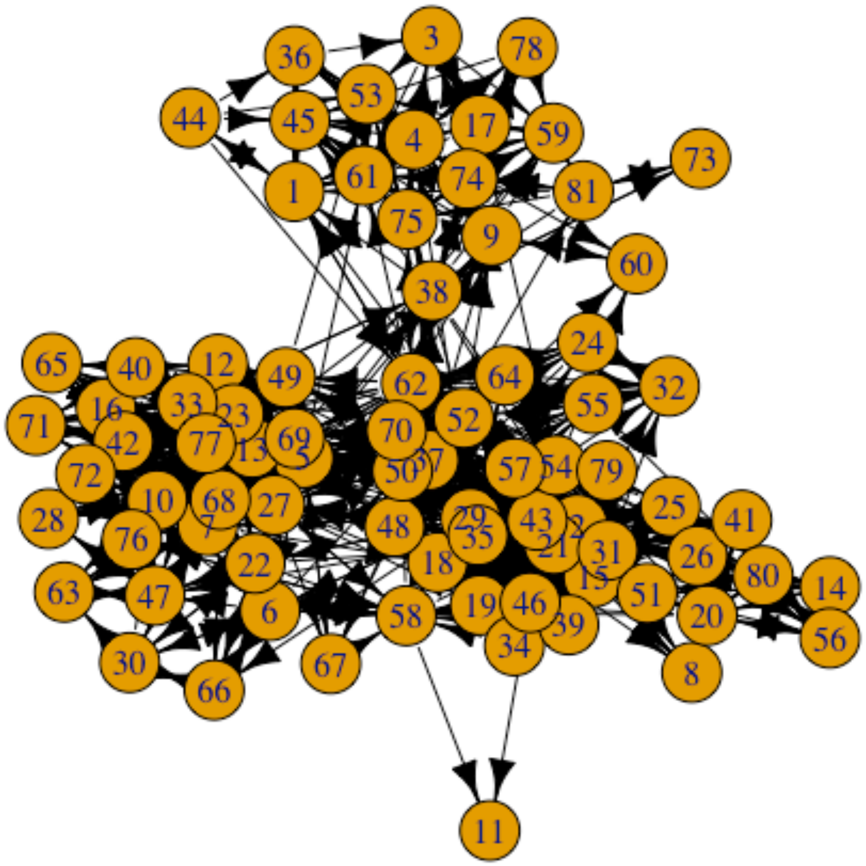


Slightly bigger networks

```
data(UKfaculty)
UKfaculty
```

```
#> IGRAPH D-W- 81 817 --
#> + attr: Type (g/c), Date (g/c), Citation (g/c), Author (g/c),
#> | Group (v/n), weight (e/n)
#> + edges:
#>  [1] 57->52 76->42 12->69 43->34 28->47 58->51  7->29 40->71  5->37
#> [10] 48->55  6->58 21-> 8 28->69 43->21 67->58 65->42  5->67 52->75
#> [19] 37->64  4->36 12->49 19->46 37-> 9 74->36 62-> 1 15-> 2 72->49
#> [28] 46->62  2->29 40->12 22->29 71->69  4-> 3 37->69  5-> 6 77->13
#> [37] 23->49 52->35 20->14 62->70 34->35 76->72  7->42 37->42 51->80
#> [46] 38->45 62->64 36->53 62->77 17->61  7->68 46->29 44->53 18->58
#> [55] 12->16 72->42 52->32 58->21 38->17 15->51 22-> 7 22->69  5->13
#> + ... omitted several edges
```

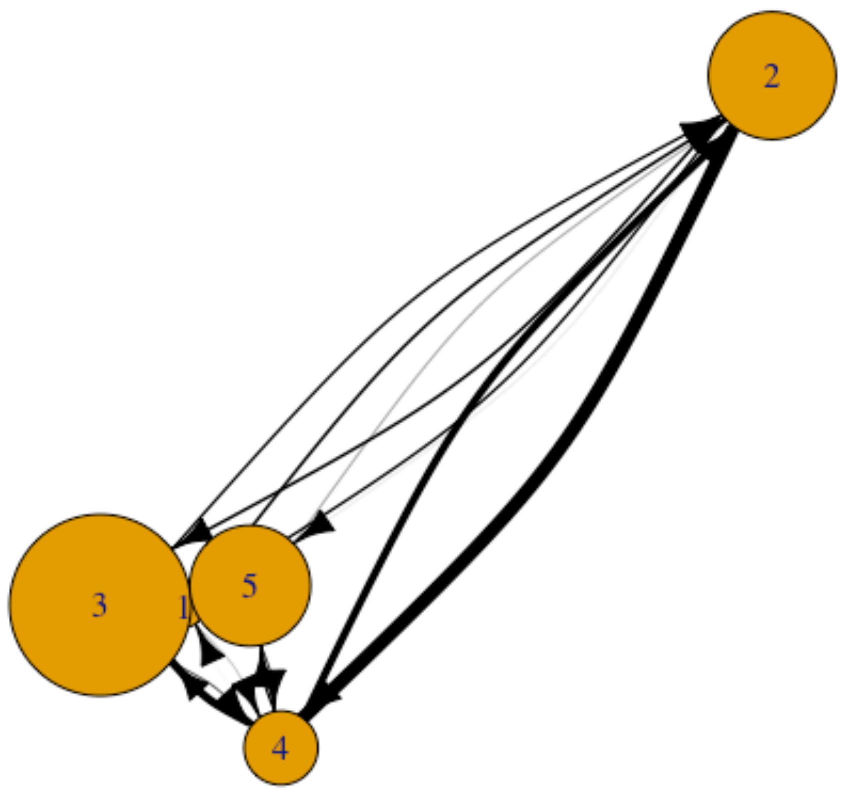
```r
par(mar = c(0,0,0,0)); plot(UKfaculty, layout = layout_with_graphopt)
```



```
cl_uk <- cluster_louvain(as.undirected(UKfaculty))
cl_gr <- contract(UKfaculty, mapping = cl_uk$membership)
E(cl_gr)$weight <- count_multiple(cl_gr)
cl_grs <- simplify(cl_gr)
E(cl_grs)$weight
```

```
#>  [1]  289    1   49  256  289 1296   16  256  144   16    4  729  784
#> [14]  256    1   81  121  169
```

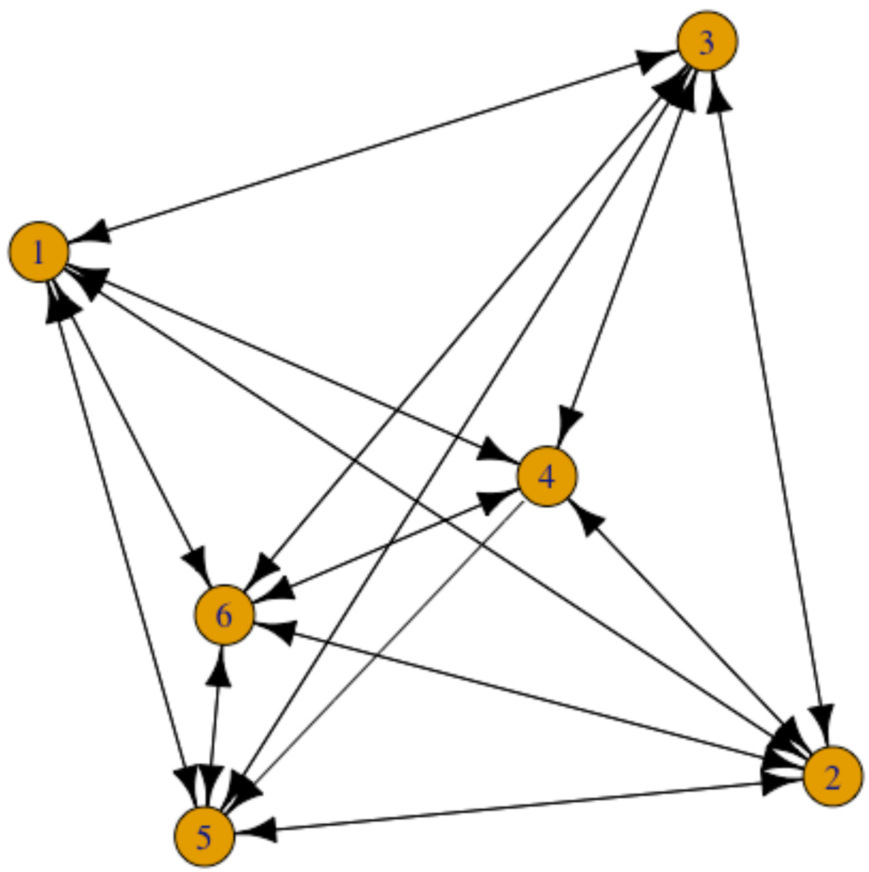
```r
par(mar = c(0,0,0,0)); plot(cl_grs, edge.width=E(cl_grs)$weight / 200,
                   edge.curved = .2, vertex.size = sizes(cl_uk) * 2)
```



```r
subs <- lapply(groups(cl_uk), induced_subgraph, graph = UKfaculty)
summary(subs[[1]])
```

```
#> IGRAPH D-W- 6 29 --
#> + attr: Type (g/c), Date (g/c), Citation (g/c), Author (g/c),
#> | Group (v/n), weight (e/n)
```

```r
par(mar=c(0,0,0,0)); plot(subs[[1]])
```



Exercise

A minimum spanning tree is a graph without cycle, that has the minimal weight sum among all spanning trees of the graph.

Try to visualize the airport network using the minimal spanning tree. `mst()` calculates the (or a) minimum spanning tree. Hint: what will you use as weight? Do you really want a minimum spanning tree, or a maximum spanning tree?

Exporting and importing graphs

`read_graph()` and `write_graph()`.

Imports: edge list, Pajek, GraphML, GML, DL, …

Exports: edge list, Pajek, GraphML, GML, DOT, Leda, …

Helpful packages: `rgexf`, `intergraph`, `DiagrammeR`, `networkD3`.

The `networkD3` package

```
library(networkD3)
d3_net <- simpleNetwork(as_data_frame(karate, what = "edges")[, 1:3])
d3_net
```

The **`DiagrammeR`** package

```
library(DiagrammeR)
```

```
#>
#> Attaching package: 'DiagrammeR'
#>
#> The following object is masked from 'package:igraph':
#>
#>     add_edges
```

```r
df_kar <- as_data_frame(karate, what = "both")
df_kar$vertices <- cbind(node = rownames(df_kar$vertices),
                         df_kar$vertices)
dg <- create_graph(
  nodes_df = df_kar$vertices,
  edges_df = df_kar$edges
)
render_graph(dg, width = 800, height = 600)
```

```
Error: No such file or directory
```

How to export to Gephi

```
library(rgexf)
```

```
#> Loading required package: XML
#> Loading required package: Rook
```

```
df_fac <- as_data_frame(UKfaculty, what = "both")
df_fac$vertices <- cbind(seq_len(gorder(UKfaculty)), df_fac$vertices)
output <- "images/UKfaculty.gexf"
write.gexf(nodes = df_fac$vertices, edges = df_fac$edges[,1:2],
           edgesAtt = df_fac$edges[,-(1:2), drop = FALSE],
           output = output)
```

```
#> GEXF graph successfully written at:
#> /Users/gaborcsardi/works/igraph/netuser15/images/UKfaculty.gexf
```

A network viz tutorial

Highly recommended:

https://github.com/kateto/R-Network-Visualization-Workshop

Questions?

Ask a question:

http://igraph.org/r/#help

Report a bug:

http://igraph.org/r/#contribute