QF4102 Assignment 1 Report

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28 September 2024

1 Question 1

1.1 (i)

BS_doCall.m submitted.

1.2 (ii) & (iii)

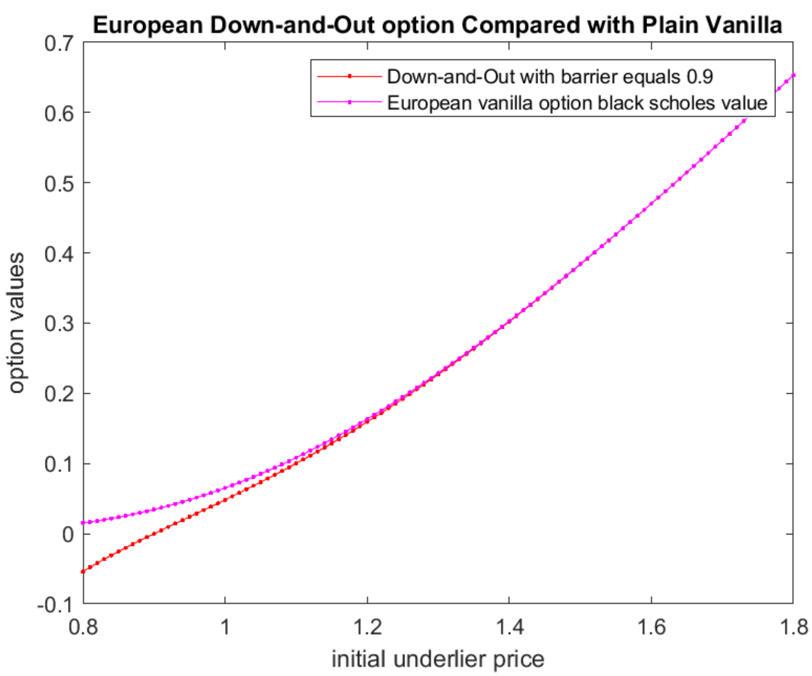

Figure 1: Q1ii and Q1iii

Comment:

- Down-and-Out call options with $S_0 < H$ possess negative values. This is because the option cannot be exercised after it's price falls below H. As S0 increases, holding all other parameters the same, the value of down-and-out call option increases too. This is because value of a call option is given by max(S-X,0). As the underlying price increases, investor confidence will also increase and hence, the value of option increases.
- In general, plain-vanilla price is higher than Down-and-Out price (both being European call options) with same underlier price and maturity time, especially when S_0 is low. This is because with the barrier, some branches of price path will be lower than H and hence render it of no value. This is why value of Down-and-Out call options will be lower than that of plain vanilla's.

1.3 (iv)

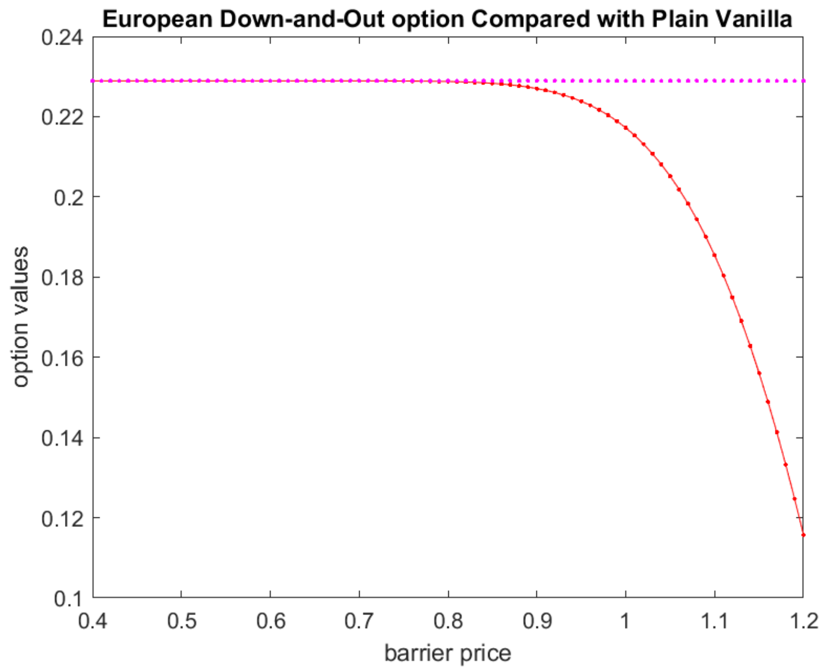

Figure 2: Q1iv

Comment:

- As barrier price of a down-out call option increase and nears the strike price, value of the option falls. This is especially so when the gap slowly narrows. This is because with a higher barrier, the option has a larger probability of not being able to be exercised at expiry, which leads to a lower option value. Pink line is plain vanilla price and red line is down-and-out call price. Plain vanilla price is consistent as it is not affected by barrier. Down-and-out call price will decrease as barrier increase and starts nearing strike price.

1.4 (v)

btm_doCall.m submitted.

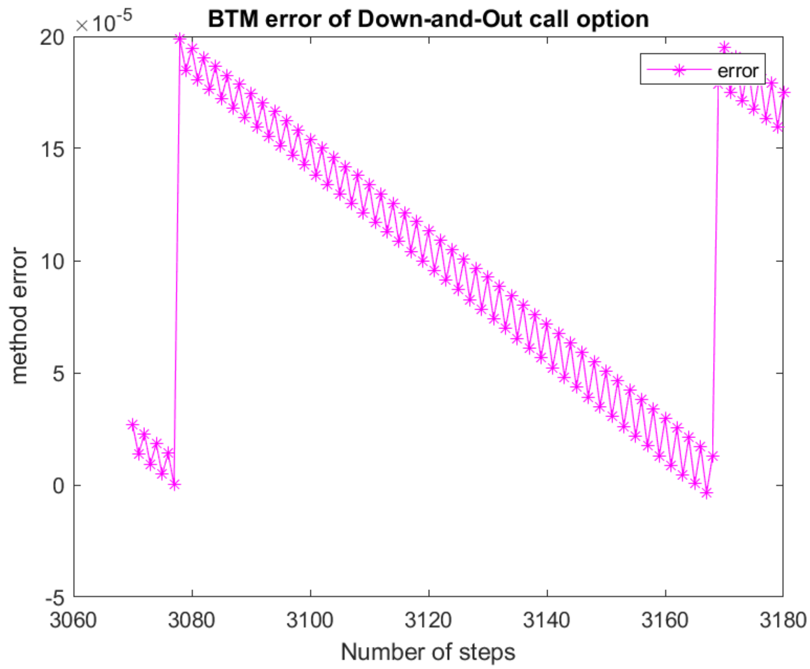

Figure 3: Q1v

Comment:

- Error pattern seem highly cyclical. The error gradually decreases to a local minimum value each cycle and then increase sharply at next N value. The sharp increase marks the start of a new cycle. During each cycle, artificial barrier will slowly move closer to actual barrier as N increases. After it reaches closest position, next N will result in artificial barrier being furthest away from actual barrier.

1.5 (vi)

$i = \sqrt{N_{min\_err}} \cdot \frac{-\ln\left(\frac{H}{S_0}\right)}{\sigma \cdot \sqrt{T}}$

Substituting all known parameters in, we get $i = \sqrt{N_{min\_err}} \cdot \frac{-\ln\left(\frac{0.9}{1.3}\right)}{0.3 \cdot \sqrt{1}}$ where N_{min_err} are 3077 and 3167 from the graph in part (v). Hence, $i = 67.9932$ and 68.9804. Rounding to nearest whole numbers, we get $i = 68$ and 69.

2 Question 2

2.1 (i)

Initialize

$x_i^n = -i\Delta x$, $i = \max(k-N,0),\ldots,k+N+1$, $n = 0,1,\ldots,N$

where k = floor of the height of running minimum.

for $i = \max(k-n,0),\ldots,k+N$ **do**

$W_i^N = 1 - \exp(x_i^N)$

for $n = N-1, N-2, \ldots, 0$ **do**

for $i = \max(k-n,0),\ldots,k+n+1$ **do**

$W_i^n = e^{-r\Delta t}\left[puW_{i+1}^{n+1} + (1-p)dW_{i-1}^{n+1}\right]$

Interpolate between

$$-k\Delta x, -\text{exact height of runmin}\Delta x, -(k+1)\Delta x$$

Output: $S_0 \cdot$ interpolated value

2.2 (ii)

btm_1vFlLookbackCall.m submitted.

2.3 (iii) & (iv)

btm_1vFlLookbackCall.m modified by setting default runmin value to S_0.

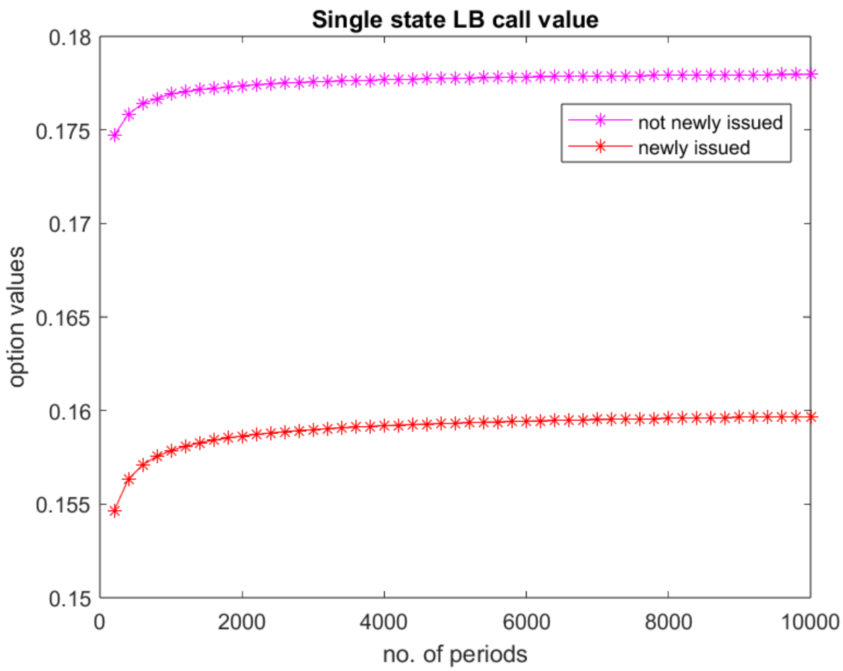

Figure 4: Q2iii and Q2iv

Comment:

- Non-newly issued option value increases with decreasing rate and converges quickly to a stable value around 0.18 as shown in the graph. As number of periods increase, the increase in option value becomes very small and thus insignificant.

- Newly issued converges to 0.16. Trend of option value is similar to that of non-newly issued ones. The magnitude of increase in option value decreases as the number of periods become larger. Option value converges to around 0.16. Comparing, we see that the value of newly issued lookback options are lower than that of non-newly issued ones. This arises from a higher probability of getting a higher payoff, as the value of running minimum could be less than S_0 at $t = 0$. Since the payoff function is $\max(S_T - runmin, 0)$, the smaller the running minimum, the higher the profit, and hence higher option value.

3 Question 3

3.1 (i)

btm_fixGeomAsianCall.m submitted.

3.2 (ii)

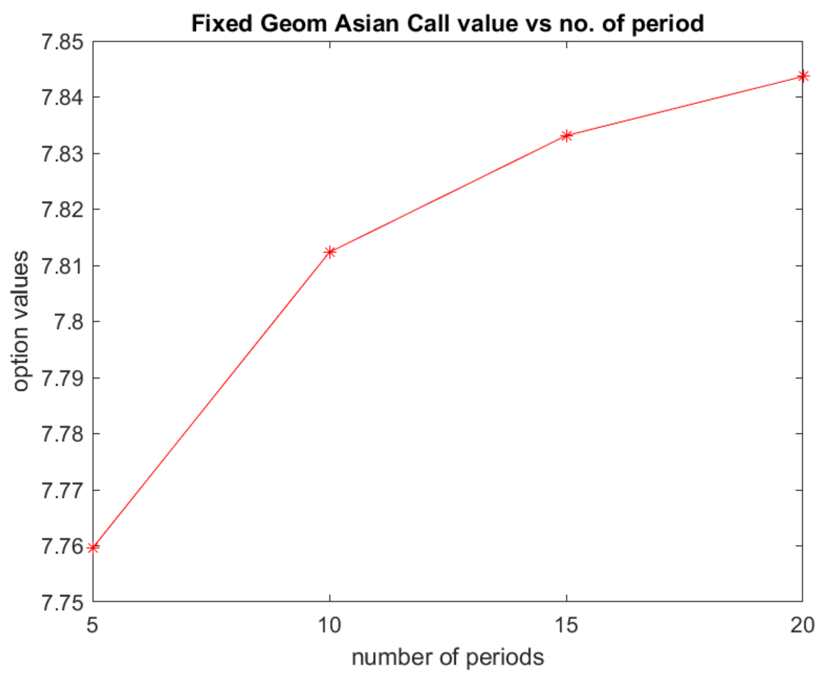

Figure 5: Q3iia

Comment:

- Option value increases 'concavely' as number of time periods increase. It increases faster at the start then slows down as number of periods increases. This is because as the time period increases, the averaging effect becomes more pronounced. The geometric average of a larger number of observations (i.e., the average price over a longer period) will tend to become more stable and converge to the expected geometric mean. This reduces the variability and uncertainty in the profit and hence the option value.

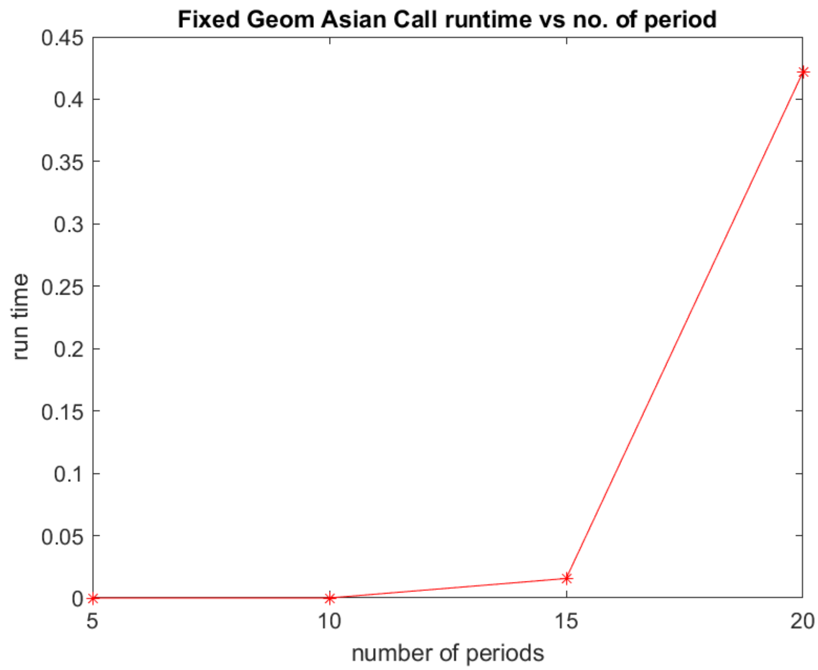

Figure 6: Q3iib

Comment:

- Runtime of 2 state variable btm fixed geom asian call function grows exponentially as the number of periods N increases. This is due to 2^N number of splits for average as time period grows. Hence, the FSG method can be used to save huge amount of computation time due to exponential growth of tree nodes.