QF4102 Financial Modelling and Computation

Assignment 1

AY24/25 Semester 1

1. There are a series of closed-form solutions for Barrier options. In (Hull, 2012, Section 25.8), the Black-Scholes price for a down-and-in European call option when $H \leq X$ is given by

$$c_{di} = S_0 e^{-qT}(H/S_0)^{2\lambda} N(y) - X e^{-rT}(H/S_0)^{2\lambda-2} N(y - \sigma\sqrt{T}),$$

where

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2}$$
$$y = \frac{\ln[H^2/(S_0 X)]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T},$$

and the value of a down-and-out option c_{do} and c_{di} satisfy the parity relation

$$c_{do} = c - c_{di},$$

where c is the value of a vanilla European call option.

(i) Implement the down-out call price formula as a Matlab function `BS_doCall`, using the header

```
function c = BS_doCall(S0,X,r,T,sigma,q,H)
```

when $H \leq X$.

Your function must allow for the input arguments `S0` and `H` to be vectors of compatible sizes.

(ii) Use your Matlab function to obtain the price of a down-out barrier option with a time to maturity of 1 year, where the strike price is \$1.2, volatility of the underlier is 30%, a dividend yield of 1%, and the risk free rate is 5%.

Compute the option values with $H = \$0.9$, and with S_0 on a grid with the range \$0.8 to \$1.8, in increments of \$0.01. Plot the option values versus S_0; and comment on the graph obtained.

(iii) For the same range of S_0, add on the Black-Scholes prices for a vanilla European call onto the graph in (ii), and comment.

(iv) For the same set of parameters in (ii), compute the value of the option with $S_0 = \$1.3$, and with the barrier H over the range from \$0.4 to \$1.2, in increments of \$0.01. Plot the option values versus H; and comment on the graph obtained.

Add in the Black-Scholes vanilla European call price to the graph in (iv), and comment.

(v) Implement the binomial tree method to approximate the price of a down-out call in a Matlab function `btm_doCall`, using the header

```
function c = btm_doCall(S0,X,r,T,sigma,q,H,N)
```

Using the same parameters above, make use of your function to compute the option values when the barrier is at \$0.9, for $N = 3070, \ldots, 3180$.

Obtain the absolute errors of your estimates, and plot these versus N. Comment on the graph obtained.

(vi) Which values of N correspond to the local minimum absolute errors? For these values of N, determine the corresponding values of i for the artificial barrier. Include all relevant workings to support your answer.

2. (i) For the floating-strike lookback **call** option (that is, a lookback on minimum), a reduction in variables can be applied to reduce the two state variables A, S to a single state variable $x = \ln(A/S)$, where $V(S, A, t) = S\,W(x, t)$.

Provide an algorithm which uses a single-state-variable binomial tree method to price a floating strike European lookback *call* option which is *not newly issued*.

(ii) Implement the algorithm as a Matlab function with the header

```
function v = btm_1vFlLookbackCall(S0,r,T,sigma,q,N,runmin)
```

where the argument `runmin` corresponds to the running minimum of the underlying asset.

(iii) Use your Matlab function in (ii) to obtain estimates of the value of a previously issued European floating strike lookback call. The option has 6 months to maturity, with a current underlier price of \$0.85, a running minimum of \$0.75, and a volatility of 0.35. The underlying asset pays no dividends, and the risk free rate is 3%.

Obtain estimates where the number of time steps N ranges from 200 to 10000, in increments of 200. Plot the option values versus N; and comment on the graph obtained.

(iv) Modify your Matlab function to allow for a default value for `runmin`, such that it can also be used to estimate the value of a newly issued lookback call option.

Obtain estimates for an option with the same parameters in (iii), using the same range of values for N, but for a newly issued option. Plot the option values versus N, and comment. Compare your results with that in (iii), and comment.

3. (i) Implement the two-state-variable binomial tree method to estimate the value of a fixed-strike geometric average Asian call option in a Matlab function. Assume that the option is newly issued. Use the following header for your function:

```
function v = btm_fixGeomAsianCall(S0,X,r,T,sigma,q,N)
```

(ii) Using the above function, obtain estimates of the value of a geometric average Asian call option, with a current underlier price of \$95, strike price of \$90, with the risk free rate of 4%, a time to maturity of 0.5 years, where the volatility of the underlier is 30%, and where the underlying asset pays no dividends. Use $N = 5, 10, 15, 20$ periods in your implementation, and plot the option values obtained versus N. Also obtain the runtimes for each value of N, and plot the runtimes versus N. Comment on the plots obtained, and on the computational efficiency of the binomial tree method.

Submission

Your submission must be a .zip folder, containing the following:

- A pdf document consisting of all written responses, results, figures, and comments.
- Three Matlab scripts — one for each question — containing all the code that reproduces your results and figures.
  - Name these files `assm1_q1.m`, `assm1_q2.m`, and `assm_q3.m` respectively.

  These scripts will be run during the grading process.
- All required Matlab function files.
- All supporting Matlab files which are required for the execution of the above mentioned Matlab scripts and functions.

Submit your .zip folder to Canvas by the due date.

Note: Plagiarism will not be tolerated. In the event of a violation of the academic integrity policy, all parties involved will be penalized severely, and referred for further action.

References

John Hull. *Options, futures, and other derivatives.* Pearson College Division, 8th edition, 2012.