QF4102 Financial Modelling and Computation

Assignment 2

AY24/25 Semester 1

1. (i) Implement the explicit difference scheme (II), that is, the backward-time scheme with V_n^i in place of V_{n+1}^i, to estimate the value of a European vanilla call option. Your function should have the header

```
function v = FD_eds_call(S0,X,r,T,sigma,q,N,I)
```

Use *linear interpolation* if the current input values do not lie exactly on the grid.

(ii) Use your Matlab function to estimate the value of the option with $S_0 = X = 1$, and a time to maturity of 0.5 year. The current risk free rate is 2%, the volatility of the asset is 0.5, and the dividend yield is 3%. Obtain the estimate with $\Delta t = 0.01$ and $h = 0.05$. Compare your estimate with the exact Black-Scholes price.

(iii) Determine a lower bound for $N = T/\Delta t$ so that all coefficients in the finite difference equation are nonnegative.

(iv) Use the lower bound in (iii) to obtain a new estimate using the explicit difference scheme. Compare your estimate with the Black-Scholes price.

(v) Gradually reduce the value of N, and through multiple calls to the above function, determine the cut-off value for N where the option estimates becomes meaningless. Comment on the value and estimates obtained.

2. (i) Write down the algorithm to estimate a fixed-strike *arithmetic* Asian call option using a two-state-variable forward shooting grid method. Assume that the option is newly issued.

(ii) Implement the above algorithm in a Matlab function. Use the following header for your function:

```
function v = fsg_fixArithAsianCallNew(S0,X,r,T,sigma,q,N,L)
```

and save the function in its own file.

(iii) Test your implementation with $S_0 = X = 100$, $r = 0.03$, $T = 1$, $N = 4$ and $L = 2$, to verify the correctness of your implementation. Assume that the underlying asset pays no dividends, and that the volatility of the asset is 0.22.

Note: In your implementation, you need not store option values for all time points with a 3-dimensional array. Instead, for the backward time iterations, you only need to store values with two 2-dimensional arrays for the two successive time points.

(iv) Modify your algorithm and implementation in parts (i) and (ii) to estimate the value of a arithmetic Asian option that is *not newly issued*, with a running average `runavg` based on `Nhist` time periods. Use the following header for your function:

```
function v = fsg_fixArithAsianCall(S0,X,r,T,sigma,q,N,L,runavg,Nhist
    )
```

and save the function in its own file.

(v) Using the above function, obtain estimates of the value of a arithmetic average Asian call option, which was issued 2 months ago, with 1/2 year to expiry. The current underlier price is \$95, and the strike price of the option is \$90. The risk free rate is 4%, the volatility of the underlier is 30%, the historical average of the underlier is \$93, and the underlying asset pays no dividends. Use $N = 60, 120, 180, 240$ time periods in your implementation, and for each value of N, obtain option value estimates for $\rho = 1, \frac{1}{2}, \frac{1}{4}$.

Tabulate your numerical results, and comment on the values obtained. Also obtain the runtimes for each value of N and ρ, and tabulate the results, and comment.

(vi) Plot the runtimes versus N. Comment on the plot obtained, and making reference to the two-state-variable binomial tree method in the previous assignment, comment on the computational efficiency of the forward shooting grid method.

Submission

Your submission must be a .zip folder, containing the following:

- A pdf document consisting of all written responses, results, figures, and comments.
- Two Matlab scripts — one for each question — containing all the code that reproduces your results and figures.
  - Name these files `assm2_q1.m`, and `assm2_q2.m`, respectively.

  These scripts will be run during the grading process.
- All required Matlab function files.
- All supporting Matlab files which are required for the execution of the above mentioned Matlab scripts and functions.

Submit your .zip folder to Canvas by the due date.

Note: Plagiarism will not be tolerated. In the event of a violation of the academic integrity policy, all parties involved will be penalized severely, and referred for further action.