Temporal Logic Point Processes

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Outline

Motivation

- modeling continuous-time event data becomes increasingly important to understand the underlying systems, to make accurate predictions, and to regulate these systems towards desired states
- recurrent Marked point processes; neural Hawkes processes; policy-like generative point processes: require lots of data; difficult to interpret
- in some cases, **interpretability is more imporant than prediction**
- Very often, there already exists a rich collection of prior knowledge from a particular domain, and we want to incorporate them to improve the interpretability and generalizability of the event models
- When the amount of data is small and noisy, it will also be challenging to accurately recover these rules via data-driven models

Contributions

In addition to the interpretability, TLPS framework has the following characteristics and advantages:

1. tolerance of uncertainty: use soften constraints
2. temporal relation constraints: temporal constraints as part of the logic functions
3. continuous-time reasoning process
4. Small data and knowledge transfer
5. many existing point process models can be recovered as special cases of this framework

Temporal Logic

Definition (First-order Logic)

A predicate such as $Smokes(c)$ *or* $Friend(c, c0)$*, denoted as* $x(\cdot)$*, is a logic function defined over a set of entities* $\mathcal{C} = \left\{c_1, c_2, ..., c_{|\mathcal{C}|}\right\}$*, i.e.,*

$$x(\cdot) : \mathcal{C} \times \mathcal{C} \times \cdots \times \mathcal{C} \to \{0, 1\} \tag{1}$$

The predicates indicate the property or relation of entities. A first-order logic rule is a logical connectives of predicates, such as

$$\begin{aligned} f_1 :& \forall c, Cancer(c) \leftarrow Smokes(c); \\ f_2 :& \forall c, c', Smokes(c') \leftarrow Friends(c, c') \wedge Smokes(c) \end{aligned} \tag{2}$$

Commonly used logic connectives are $\vee$ for conjunction, $\wedge$ for disjunction, $\leftarrow$ for implication, and $\neg$ for negation. It is often convenient to convert logic rules to a *clausal form*, which is a conjunction or disjunction of predicates. (i.e. $x_B \leftarrow x_A$ is logically equivalent to the clausal form $\neg x_A \vee x_B$)

Temporal Logic Predicate

Definition (Temporal Logic Predicate)

A temporal predicate is a logic function $x(\cdot, \cdot)$ *defined over the set of entities* $\mathcal{C} = \{c_1, c_2, ..., c_{|\mathcal{C}|}\}$ *and time* $t \in [0, \infty)$*, i.e.,*

$$x(c, t) : \mathcal{C} \times \mathcal{C} \times \cdots \times \mathcal{C} \times [0, \infty) \to \{0, 1\} . \tag{3}$$

The trajectory of a grounded temporal predicate $\{x(c, t)\}_{t \geq 0}$ can also be viewed as a continuous-time two-state stochastic process

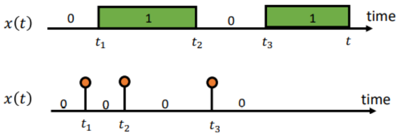


Figure 1. Illustration of grounded two-state (top) and point-based (bottom) temporal predicates.

Temporal Relation

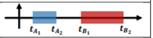
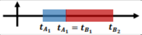
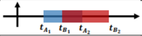
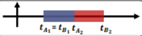
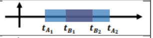
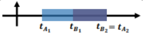
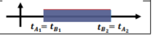
Definition (Temporal Relation)

a temporal relation is a logic function defined as

$$r(\cdot) : (t_{A_1}, t_{A_2}, t_{B_1}, t_{B_2}) \rightarrow \{0, 1\} \tag{4}$$

which can be mathematically evaluated by a step function $g(s)$ and an indicator function $\kappa(s)$

Table 2. Interval-based temporal relation constraints and their illustrative figures.

Temporal Relation	Logic Function $r(\cdot)$	Illustration
r_b: A before B	$g(t_{B_1} - t_{A_2})$	
r_m: A meets B	$\kappa(t_{A_2} - t_{B_1})$	
r_o: A overlaps B	$g(t_{B_1} - t_{A_1}) \cdot g(t_{B_1} - t_{A_2}) \cdot g(t_{B_2} - t_{A_2})$	
r_s: A starts B	$\kappa(t_{A_1} - t_{B_1}) \cdot g(t_{B_2} - t_{A_2})$	
r_c: A contains B	$g(t_{B_1} - t_{A_1}) \cdot g(t_{A_2} - t_{B_2})$	
r_f: A finished-by B	$g(t_{B_1} - t_{A_1}) \cdot \kappa(t_{A_2} - t_{B_2})$	
r_e: A equals B	$\kappa(t_{A_1} - t_{B_1}) \cdot \kappa(t_{A_2} - t_{B_2})$	

Temporal Logic Formula

Definition

A temporal logic formula is a logical composition of temporal logic predicates and temporal relations, $f(\mathcal{X}_f, \mathcal{T}_f) \in \{0, 1\}$, where

1. $\mathcal{X}_f = \{x_u(t)\}$ *is a set of temporal predicates used to define the formula* f
2. $\mathcal{T}_f = \{\tau_u\}$ *is a set of time intervals, with each* $x_u \in \mathcal{X}_f$ *associated with a time interval* $\tau_u = [t_{u_1}, t_{u_2}]$

a temporal logic formula has a generic form

$$f(\mathcal{X}_f, \mathcal{T}_f) := \left(\left(\vee_{x_u \in \mathcal{X}_f^+} x_u(t_u)\right) \vee \left(\vee_{x_v \in \mathcal{X}_f^-} \neg x_v(t_v)\right)\right) \wedge \left(\wedge_{x_u, x_v \in \mathcal{X}_f} r_i(\tau_u, \tau_v)\right) \tag{5}$$

References I