+ EIGEN & DIAGONALES

Friday, 31 May 2024 13.56

Eigenvalues (Square matrices)

Hermetian C^n $\bar{\bar{A}}^{*T} = \bar{\bar{A}}$
(symmetric, R^n)
λ_i real (0)

Skew-Hermetian C^n $\bar{\bar{A}}^{*T} = -\bar{\bar{A}}$
(skew-symmetric, R^n)
λ_i imaginary (0)

Unitary C^n $\bar{\bar{A}}^{*T} = \bar{\bar{A}}^{-1}$
(orthogonal, R^n)
$|\lambda_i| = 1$

* complex conjugate

Aalborg University, WCN – lineær algebra og dynamiske systemer slide

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Find Eigen values and basis

Eks

$$\bar{\bar{A}} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}$$

$$\det(\bar{\bar{A}}) = (a-\lambda)(d-\lambda) - (c \cdot b)$$

Som man isolere og for $\underline{\lambda_1}$ og $\underline{\lambda_2}$

DETTE ER EIGEN VÆRDIER

Eigenbase for λ_1

$$\begin{bmatrix} a-\lambda_1 & b \\ c & d-\lambda_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$$\left.\begin{aligned} a-\lambda_1 \underline{x_1} + b\underline{x_2} &= 0 \\ c\underline{x_1} + d-\lambda_1 \underline{x_2} &= 0 \end{aligned}\right\}$$ Så løser man for $\underline{x_1}$ og $\underline{x_2}$

hvor $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ er vores svar for λ_1

Og så gør man det samme for λ_2

Diagonalisering!

for eig1 for eig2

$$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \qquad P = \begin{bmatrix} x_1\lambda_1 & x_1\lambda_2 \\ x_2\lambda_1 & x_2\lambda_2 \end{bmatrix}$$

Pure eigenvalues Pure eigenbasis

$$\bar{\bar{D}} = \bar{\bar{X}}^{-1}\,\bar{\bar{A}}\,\bar{\bar{X}} = \bar{\bar{P}}^{-1}\bar{\bar{A}}\,\bar{\bar{P}}$$

Diagonalization

$$\bar{\bar{D}} = \bar{\bar{X}}^{-1}\,\bar{\bar{A}}\,\bar{\bar{X}}$$

Husk Rækkefølgen!

$$\begin{bmatrix} \lambda_1 & & \bar{\bar{0}} \\ & \ddots & \\ \bar{\bar{0}} & & \lambda_n \end{bmatrix}$$ (diagonal) matrix of eigenvalues

matrix of eigenvectors $[\bar{x}_1, \bar{x}_2, \ldots \bar{x}_n]$

$\bar{\bar{X}}$ is said to diagonalize $\bar{\bar{A}}$

$\bar{\bar{D}}$ and $\bar{\bar{A}}$ are called similar* in that they have the same eigenvalues

(* generally, any non-singular matrix $\bar{\bar{X}}$ sufficise

FIND D	FROM D $\Longrightarrow$ A
$\bar{\bar{D}} = \bar{\bar{P}}^{-1}\,\bar{\bar{A}}\,\bar{\bar{P}}$	$\bar{\bar{A}} = \bar{\bar{P}}\,\bar{\bar{D}}\,\bar{\bar{P}}^{-1}$
Husk række følgen!	Husk række følgen!

Spectral radius is the LARGEST abs. value of your D eigenvectors in that form

So for $(\sqrt{-1}\,A) = (\sqrt{-1}\,D)$ as they are unitary the same as we can diagonalize it

$(\sqrt{-1}\cdot 0)^5 = 0$; $(\sqrt{-1}\cdot 4)^5 = 1024 I$; $(\sqrt{-1}\cdot -4)^5 = -1024 I$

So spectre is $\underline{\underline{1024}}$

be specific to [illegible]