Special matrices (square)

Aalborg University, WCN - lineær algebra og dynamiske systemer

slide

Transposée d'une matrice carrée

$$\begin{pmatrix} \boxed{1 & 5} \\ \boxed{6 & 8} \end{pmatrix}^T = \begin{pmatrix} 1 \\ 5 \\ \boxed{8} \end{pmatrix}$$

$$\begin{pmatrix} 9 & 7 & 5 \\ 1 & 0 & 7 \\ 4 & 2 & 6 \end{pmatrix}^T = \begin{pmatrix} 9 & 1 & 4 \\ 7 & 0 & 2 \\ 5 & 7 & 6 \end{pmatrix}$$



Special matrices (square)

Symmetric
$$\bar{A}^T = \bar{\bar{A}}$$

e.g.
$$\begin{bmatrix} 5 & -1 & 0 \\ -1 & 9 & 5 \\ 0 & 5 & 7 \end{bmatrix}$$

e.g.
$$\begin{bmatrix} 0 & -6 & 2 \\ 6 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

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OBS	
9 0.	1 - 10
what's	particular
0	5 72

about it?

Taxunomy for normal matrices

Reel matrix	Kompleks matrix	Generel	Normal ⁽²⁾	Egenværdier
Symmetrisk	Hermitesk			
	(diagonal er reel)	Selvadjungeret ⁽¹⁾	Ja	Reelle (inkl. 0)
$A^{T} = A$	$A^{*T} = A$			
Skævsymmetrisk	Skævhermitesk			
(diagonal = 0)	(diagonal imaginær eller 0)	Skævadjungeret	Ja	Imaginære (inkl. 0)
$A^{T} = -A$	$A^{*T} = -A$			
Ortogonal	Unitær			
$(\Delta A = \pm 1)$	$(\Delta A = 1)$	Isometrisk	Ja	Absolut værdi 1
$A^{T} = A^{-1}$	$\mathbf{A}^{*T} = \mathbf{A}^{-1}$			

- (1) Den (komplekst) konjugerede transponerede, A^{*†}, kaldes for den adjungerede til A (og deraf betegnelsen selvadjungeret i dette tilfælde); mere formelt kaldes den komplekst konjugerede transponerede for den hermitesk adjungerede, hvor hermitesk adjungering er analogt til kompleks konjugering.
- (2) En normal matrix er en (generelt) kompleks kvadratisk matrix der kommuterer med sin adjungerede, dvs. opfylder A*TA = AA*T.

Example [edit]

For example, the following matrix is skew-Hermitian

$$A = \left[egin{matrix} -i & +2+i \ -2+i & 0 \end{array}
ight]$$

because

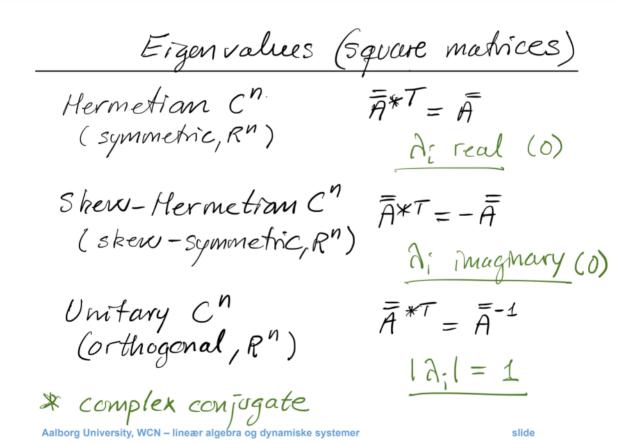
$$-A = egin{bmatrix} i & -2-i \ 2-i & 0 \end{bmatrix} = egin{bmatrix} \overline{-i} & \overline{-2+i} \ \overline{2+i} & \overline{0} \end{bmatrix} = egin{bmatrix} \overline{-i} & \overline{2+i} \ \overline{-2+i} & \overline{0} \end{bmatrix}^\mathsf{T} = A^\mathsf{H}$$

Basic remarks [edit]

A square matrix ${f A}$ with entries a_{ij} is called

- ullet Hermitian or self-adjoint if ${f A}={f A}^{
 m H}$; i.e., $a_{ij}=\overline{a_{ji}}$.
- ullet Skew Hermitian or antihermitian if ${f A}=-{f A}^{
 m H}$; i.e., $a_{ij}=-\overline{a_{ji}}$
- Normal if $\mathbf{A}^H \mathbf{A} = \mathbf{A} \mathbf{A}^H$
- Unitary if $\mathbf{A}^{\mathrm{H}}=\mathbf{A}^{-1}$, equivalently $\mathbf{A}\mathbf{A}^{\mathrm{H}}=\mathbf{\emph{I}}$, equivalently $\mathbf{A}^{\mathrm{H}}\mathbf{A}=\mathbf{\emph{I}}$.

Even if $\bf A$ is not square, the two matrices $\bf A^H \bf A$ and $\bf A \bf A^H$ are both Hermitian and in fact positive semi-definite matrices.



Invers matrix

$$\begin{array}{c}
\overrightarrow{A} = \begin{cases}
\alpha_1 & \alpha_2 & \alpha_3 \\
b_1 & b_2 & b_3
\end{cases} \Rightarrow \begin{cases}
\alpha_1 & \alpha_2 & \alpha_3 \\
b_1 & b_2 & b_3
\end{cases} = 0 \\
C_1 & C_2 & C_3
\end{cases}$$

$$\begin{array}{c}
C_1 & C_2 & C_3
\end{array}$$

$$\begin{array}{c}
C_2 & C_3
\end{array}$$

$$\begin{array}{c}
C_3 & C_2
\end{array}$$

$$\begin{array}{c}
C_4 & C_2
\end{array}$$