+ TYPES OF MATRIXES

Friday, 31 May 2024 12.29

Special matrices (square)

Diagonal

$$\begin{bmatrix} a & & & & \bar{\bar{0}} \\ & b & & & \\ & & c & & \\ & & & \ddots & \\ \bar{\bar{0}} & & & & z \end{bmatrix}$$

Triangular
lower/upper

$$\begin{bmatrix} 0 & & & \bar{\bar{d}} \\ \vdots & \ddots & & \\ 0 & \cdots & 0 & \end{bmatrix}$$

Scalar

$$\begin{bmatrix} k & & & \bar{\bar{0}} \\ & k & & \\ & & \ddots & \\ \bar{\bar{0}} & & & k \end{bmatrix} = k \cdot \bar{\bar{I}}$$

Identity

$$\begin{bmatrix} 1 & & & \bar{\bar{0}} \\ & 1 & & \\ & & \ddots & \\ \bar{\bar{0}} & & & 1 \end{bmatrix} = \bar{\bar{I}}$$

Null (in various forms) $\bar{\bar{0}}$?

Aalborg University, WCN – lineær algebra og dynamiske systemer slide

Transposée d'une matrice carrée

$$\begin{pmatrix} 1 & 5 \\ 6 & 8 \end{pmatrix}^T = \begin{pmatrix} 1 & 6 \\ 5 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 7 & 5 \\ 1 & 0 & 7 \\ 4 & 2 & 6 \end{pmatrix}^T = \begin{pmatrix} 9 & 1 & 4 \\ 7 & 0 & 2 \\ 5 & 7 & 6 \end{pmatrix}$$

Étapes :

- La 1ère ligne devient la 1ère colonne
- La 2 -ème ligne devient la 2 -ème colonne
- La 3 -ème ligne devient la 3 -ème colonne
- Ainsi de suite…

$$\begin{matrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{matrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}^T$$

Special matrices (square)

Symmetric $\bar{\bar{A}}^T = \bar{\bar{A}}$

$$\text{e.g.} \quad \begin{bmatrix} 5 & -1 & 0 \\ -1 & 9 & 5 \\ 0 & 5 & 7 \end{bmatrix}$$

Skew-symmetric $\bar{\bar{A}}^T = -\bar{\bar{A}}$

$$\text{e.g.} \quad \begin{bmatrix} 0 & -6 & 2 \\ 6 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

OBS!
what's particular about it?

Taxonomy for normal matrices

Reel matrix	Kompleks matrix	Generel	Normal[2]	Egenværdier
Symmetrisk $A^T = A$	Hermitesk (diagonal er reel) $A^{*T} = A$	Selvadjungeret[1]	Ja	Reelle (inkl. 0)
Skævsymmetrisk (diagonal = 0) $A^T = -A$	Skævhermitesk (diagonal imaginær eller 0) $A^{*T} = -A$	Skævadjungeret	Ja	Imaginære (inkl. 0)
Ortogonal ($\Delta A = \pm 1$) $A^T = A^{-1}$	Unitær ($\lvert \Delta A \rvert = 1$) $A^{*T} = A^{-1}$	Isometrisk	Ja	Absolut værdi 1

(1) Den (komplekst) konjugerede transponerede, A^{*T}, kaldes for den adjungerede til A (og deraf betegnelsen selvadjungeret i dette tilfælde); mere formelt kaldes den komplekst konjugerede transponerede for den hermitesk adjungerede, hvor hermitesk adjungering er analogt til kompleks konjugering.

$$\Delta \bar{\bar{A}} = \det(\bar{\bar{A}})$$

(2) En normal matrix er en (generelt) kompleks kvadratisk matrix der kommuterer med sin adjungerede, dvs. opfylder $A^{*T}A = AA^{*T}$.

Example [edit]

For example, the following matrix is skew-Hermitian

$$A = \begin{bmatrix} -i & +2+i \\ -2+i & 0 \end{bmatrix}$$

because

$$-A = \begin{bmatrix} i & -2-i \\ 2-i & 0 \end{bmatrix} = \begin{bmatrix} \overline{-i} & \overline{-2+i} \\ \overline{2+i} & \overline{0} \end{bmatrix} = \begin{bmatrix} \overline{-i} & \overline{2+i} \\ \overline{-2+i} & \overline{0} \end{bmatrix}^{\mathsf{T}} = A^{\mathsf{H}}$$

Basic remarks [edit]

A square matrix $\mathbf{A}$ with entries a_{ij} is called

- Hermitian or self-adjoint if $\mathbf{A} = \mathbf{A}^{\mathrm{H}}$; i.e., $a_{ij} = \overline{a_{ji}}$.
- Skew Hermitian or antihermitian if $\mathbf{A} = -\mathbf{A}^{\mathrm{H}}$; i.e., $a_{ij} = -\overline{a_{ji}}$.
- Normal if $\mathbf{A}^{\mathrm{H}}\mathbf{A} = \mathbf{A}\mathbf{A}^{\mathrm{H}}$.
- Unitary if $\mathbf{A}^{\mathrm{H}} = \mathbf{A}^{-1}$, equivalently $\mathbf{A}\mathbf{A}^{\mathrm{H}} = \boldsymbol{I}$, equivalently $\mathbf{A}^{\mathrm{H}}\mathbf{A} = \boldsymbol{I}$.

Even if $\mathbf{A}$ is not square, the two matrices $\mathbf{A}^{\mathrm{H}}\mathbf{A}$ and $\mathbf{A}\mathbf{A}^{\mathrm{H}}$ are both Hermitian and in fact positive semi-definite matrices.

Eigenvalues (square matrices)

Hermetian C^n (symmetric, R^n) — $\bar{\bar{A}}^{*T} = \bar{\bar{A}}$ — λ_i real (0)

Skew-Hermetian C^n (skew-symmetric, R^n) — $\bar{\bar{A}}^{*T} = -\bar{\bar{A}}$ — λ_i imaginary (0)

Unitary C^n (orthogonal, R^n) — $\bar{\bar{A}}^{*T} = \bar{\bar{A}}^{-1}$ — $|\lambda_i| = 1$

* complex conjugate

Invers matrix

$$\bar{\bar{A}} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|ccc} a_1 & a_2 & a_3 & 1 & 0 & 0 \\ b_1 & b_2 & b_3 & 0 & 1 & 0 \\ c_1 & c_2 & c_3 & 0 & 0 & 1 \end{array}\right]$$

så laver man reduced row echelon på den
til man får

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & A_1 & A_2 & A_3 \\ 0 & 1 & 0 & B_1 & B_2 & B_3 \\ 0 & 0 & 1 & C_1 & C_2 & C_3 \end{array}\right] \rightarrow = \bar{\bar{A}}^{-1}$$

på

$$\bar{\bar{A}}\,\bar{\bar{A}}^{-1} = \bar{\bar{I}}$$

Identitets matrice