

REGNEREGLER

Thursday, 30 May 2024 10.11

Rules of Exponents or Laws of Exponents	
Multiplication Rule	$a^x \times a^y = a^{x+y}$
Division Rule	$a^x \div a^y = a^{x-y}$
Power of a Power Rule	$(a^x)^y = a^{xy}$
Power of a Product Rule	$(ab)^x = a^x b^x$
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-x} = \frac{1}{a^x}$
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

$$\sum_{n=0}^{n-1} r^n = \frac{1 - r^n}{1 - r} \quad \text{for } r \neq 1$$

Summation Shortcuts

$$\sum_{n=1}^k n = \frac{n(n+1)}{2} = \frac{n^2 + n}{2}$$

$$\sum_{n=1}^k n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3 + 3n^2 + n}{6}$$

$$\sum_{n=1}^k n^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4 + 2n^3 + n^2}{4}$$

$$\sum_{n=1}^k n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{6n^5 + 15n^4 + 10n^3 - n}{30}$$

$$\sum_{n=1}^k n^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12} = \frac{2n^6 + 6n^5 + 5n^4 - n^2}{12}$$

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Brøker

Brøker er tal på formen

$$\frac{a}{b},$$

hvor a, b er tal samtidig $b \neq 0$. a er tælleren og b er nævneren.**Regneregler**

Der gælder

$$\begin{aligned} \frac{a+b}{c} &= \frac{a}{c} + \frac{b}{c}, & \frac{a}{b} \cdot \frac{c}{d} &= \frac{ac}{bd}, & \frac{\frac{a}{b}}{\frac{c}{d}} &= \frac{ad}{bc}, \\ \frac{ab}{c} &= \frac{a}{c} \cdot b, & \frac{b}{c} &= \frac{a}{bc}, & \frac{a}{\frac{b}{c}} &= \frac{ac}{b}. \end{aligned}$$

Forkorte/Forlænge Brøker

Fælles faktorer kan forkortes:

$$\frac{a}{b} = \frac{ac}{bc}$$

PotenserPotenser er tal på formen x^a , x er grundtallet og a er eksponenten.**Regneregler**

Der gælder

$$\begin{aligned} x^a x^b &= x^{a+b}, & \frac{x^a}{x^b} &= x^{a-b}, & (xy)^a &= x^a y^a, \\ \left(\frac{x}{y}\right)^a &= \frac{x^a}{y^a}, & (x^a)^b &= x^{ab}, & x^{-a} &= \frac{1}{x^a}. \end{aligned}$$

RødderHvis $x \geq 0$ og $n \in \mathbb{Z}_+$ så findes et tal $\sqrt[n]{x} > 0$ så

$$(\sqrt[n]{x})^n = x.$$

Bemærk at $\sqrt[n]{x} = x^{\frac{1}{n}}$.**Regneregler**

Der gælder

$$\begin{aligned} \sqrt[n]{x} &= x^{\frac{1}{n}}, & \sqrt[n]{x^m} &= x^{\frac{m}{n}} = (\sqrt[n]{x})^m, \\ \sqrt[n]{xy} &= \sqrt[n]{x} \sqrt[n]{y}, & \sqrt[n]{\frac{x}{y}} &= \frac{\sqrt[n]{x}}{\sqrt[n]{y}}. \end{aligned}$$

Kvadratsætninger

Der gælder

$$\begin{aligned} (a+b)^2 &= a^2 + b^2 + 2ab \\ (a-b)^2 &= a^2 + b^2 - 2ab \\ (a+b)(a-b) &= a^2 - b^2. \end{aligned}$$

Ligninger

Ligninger kan reduceres med følgende regler:

1. Man må lægge til/trække fra med det samme tal på begge sider af et lighedstegn.
2. Man må gange/dividere med det samme tal (undtagen 0) på begge sider af et lighedstegn.

Andengrads ligninger

Andengrads ligninger er på formen

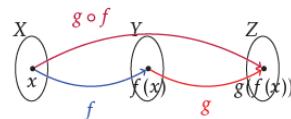
$$ax^2 + bx + c = 0, \quad (1)$$

Løsningerne til (1) er

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

FaktoriseringHvis $ax^2 + bx + c = 0$ har rødder r_1 og r_2 så gælder.

$$ax^2 + bx + c = a(x - r_1)(x - r_2).$$

FunktionerEn funktion $f: X \rightarrow Y$ tildeler alle $x \in X$ præcis ét element $f(x) \in Y$.**Sammensatte funktioner**Hvis $f: X \rightarrow Y$ og $g: Y \rightarrow Z$ defineres sammensætningen $g \circ f: X \rightarrow Z$ ved $(g \circ f)(x) = g(f(x))$. f er den indre funktion, g er den ydre funktion**Inverse funktioner**To funktioner $f: X \rightarrow Y$ og $g: Y \rightarrow X$ er hinandens inverse hvis

$$f(g(y)) = y, \quad \text{og} \quad g(f(x)) = x$$

for alle $x \in X$ og $y \in Y$.**Polynomier**

Et førstegradspolynomium har forskrift:

$$f(x) = ax + b.$$

Et andengradspolynomium har forskrift:

$$f(x) = ax^2 + bx + c.$$

Logaritmer og eksponentalfunktionerLogaritmen med grundtal a , $\log_a:]0, \infty[\rightarrow \mathbb{R}$ er invers til eksponentalfunktionen $f_a(x) = a^x$ ($a > 0$, $a \neq 1$). Der gælder at

$$\log_a(a^x) = x \quad \text{og} \quad a^{\log_a(y)} = y$$

og vi har

$$\ln x = \log_e x, \quad \log x = \log_{10} x$$

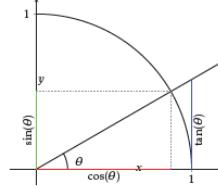
Regneregler

Der gælder

$$\begin{aligned} \log_a(xy) &= \log_a(x) + \log_a(y), \\ \log_a\left(\frac{x}{y}\right) &= \log_a(x) - \log_a(y), \\ \log_a(x^r) &= r \log_a(x). \end{aligned}$$

Trigonometriske funktioner

De trigonometriske funktioner er defineret ud fra enhedscirklen:

Der gælder at $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ samtidig

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-

DifferentialregningDen afledede af f skrives som $f' = \frac{df}{dx}$.**Regneregler**

Der gælder at

$f(x)$	$f'(x)$
c	0
x	1
x^n	nx^{n-1}
e^x	e^x
e^{cx}	ce^{cx}
a^x	$a^x \ln a$
$\ln x$	$\frac{1}{x}$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$1 + \tan^2(x)$

Generelle regneregler

Der gælder at

$$\begin{aligned} (cf)'(x) &= cf'(x) \\ (f \pm g)'(x) &= f'(x) \pm g'(x) \\ (fg)'(x) &= f'(x)g(x) + f(x)g'(x) \\ \left(\frac{f}{g}\right)'(x) &= \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \\ \frac{d}{dx} f(g(x)) &= f'(g(x))g'(x). \end{aligned}$$

Den sidste regneregel kaldes *kædereglen*.

Ubestemte integralerEn funktion f har *stamfunktion* F hvis

$$F'(x) = f(x).$$

Det ubestemte integral af f er

$$\int f(x) dx = F(x) + k,$$

hvor $F'(x) = f(x)$ og $k \in \mathbb{R}$.**Generelle regneregler**

$$\int c f(x) dx = c \int f(x) dx$$

$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx.$$

$$\int f(x)g(x) dx = f(x)G(x) - \int f'(x)G(x) dx$$

$$\int f(g(x))g'(x) dx = F(g(x)) + k.$$

Den 3. regel kaldes *delvis integration* og den sidste kaldes *integration ved substitution*.

Regneregler

Der gælder at

$f(x)$	$\int f(x) dx$
c	$cx + k$
x	$\frac{1}{2}x^2 + k$
x^n	$\frac{1}{n+1}x^{n+1} + k$
e^x	$e^x + k$
e^{cx}	$\frac{1}{c}e^{cx} + k$
$\frac{1}{x}$	$\ln(x) + k$
$\ln x$	$x \ln(x) - x + k$
$\cos x$	$\sin x + k$
$\sin x$	$-\cos x + k$
$\tan x$	$-\ln(\cos(x)) + k$

Integration ved substitution

Givet et integral på formen $\int f(g(x))g'(x) dx$ anvendes metoden:

1. Lad $u = g(x)$.
2. Udregn $\frac{du}{dx}$ og isoler dx .
3. Substituer $g(x)$ og dx .
4. Udregn integralet mht. u .
5. Substituer tilbage.

Besemte integralerDet bestemte integral af f i intervallet $[a, b]$ til

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a),$$

hvor F er en stamfunktion til f .**Generelle regneregler**

$$\int_a^b cf(x) dx = c \int_a^b f(x) dx$$

$$\int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

$$\int_a^b f(x)g(x) dx = [f(x)G(x)]_a^b - \int_a^b f'(x)G(x) dx$$

$$\int_a^b f(g(x))g'(x) dx = [F(x)]_{g(a)}^{g(b)}$$

Givet et integral på formen $\int_a^b f(g(x))g'(x) dx$ anvendes metoden

1. Lad $u = g(x)$.
2. Udregn $\frac{du}{dx}$ og isoler dx .
3. Substituer $g(x)$, dx samt grænser.
4. Udregn integralet mht. u .

Differentialligninger**Løsningsformler**

Differentiallign. Fuldstændig løsn.

$$f'(x) = k \quad f(x) = kx + c$$

$$f'(x) = h(x) \quad f(x) = \int h(x) dx$$

$$f'(x) = kf(x) \quad f(x) = ce^{kx}$$

$$f'(x) + af(x) = b \quad f(x) = \frac{b}{a} + ce^{-ax}$$

Panserformlen

Differentialligningen

$$f'(x) + a(x)f(x) = b(x)$$

har fuldstændig løsning

$$f(x) = e^{-A(x)} \int b(x)e^{A(x)} dx + ce^{-A(x)},$$

hvor $A'(x) = a(x)$.**Vektorer i planen**En vektor \vec{u} i planen skrives som $\vec{u} = [x, y]$ hvor $x, y \in \mathbb{R}$.**Regneregler**For $\vec{u} = [x_1, y_1]$, $\vec{v} = [x_2, y_2]$, $c \in \mathbb{R}$ er

$$\vec{u} \pm \vec{v} = \begin{bmatrix} x_1 \pm x_2 \\ y_1 \pm y_2 \end{bmatrix}, \quad \vec{u} \cdot \vec{v} = x_1x_2 + y_1y_2,$$

$$c\vec{u} = \begin{bmatrix} cx_1 \\ cy_1 \end{bmatrix}, \quad \det(\vec{u}, \vec{v}) = x_1y_2 - x_2y_1$$

$$\text{Længden af } \vec{u} \text{ er } \|\vec{u}\| = \sqrt{x_1^2 + y_1^2}.$$

Vinklen mellem to vektorerFor vinklen θ mellem \vec{u} , \vec{v} er

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}, \quad \sin \theta = \frac{\det(\vec{u}, \vec{v})}{\|\vec{u}\| \|\vec{v}\|}$$

Yderligere gælder

$$1. \vec{u} \text{ og } \vec{v} \text{ er ortogonale } \Leftrightarrow \vec{u} \cdot \vec{v} = 0.$$

$$2. \vec{u} \text{ og } \vec{v} \text{ er parallelle } \Leftrightarrow \det(\vec{u}, \vec{v}) = 0.$$

Vektorer i rummetEn vektor \vec{u} i rummet skrives som $\vec{u} = [x, y, z]$ hvor $x, y, z \in \mathbb{R}$.**Regneregler**For $\vec{u} = [x_1, y_1, z_1]$, $\vec{v} = [x_2, y_2, z_2]$ og $c \in \mathbb{R}$ gælder

$$\vec{u} \pm \vec{v} = \begin{bmatrix} x_1 \pm x_2 \\ y_1 \pm y_2 \\ z_1 \pm z_2 \end{bmatrix}, \quad c\vec{u} = \begin{bmatrix} cx_1 \\ cy_1 \\ cz_1 \end{bmatrix},$$

$$\vec{u} \cdot \vec{v} = x_1x_2 + y_1y_2 + z_1z_2.$$

$$\text{Længden af } \vec{u} \text{ er } \|\vec{u}\| = \sqrt{x_1^2 + y_1^2 + z_1^2}.$$

Krydsproduktet er givet ved

$$\vec{u} \times \vec{v} = \begin{bmatrix} y_1z_2 - z_1y_2 \\ z_1x_2 - x_1z_2 \\ x_1y_2 - y_1x_2 \end{bmatrix}$$

Vinklen mellem to vektorerFor vinklen θ mellem \vec{u} og \vec{v} gælder

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}, \quad \sin \theta = \frac{\|\vec{u} \times \vec{v}\|}{\|\vec{u}\| \|\vec{v}\|}$$

Yderligere gælder

$$1. \vec{u} \text{ og } \vec{v} \text{ er ortogonale } \Leftrightarrow \vec{u} \cdot \vec{v} = 0.$$

$$2. \vec{u} \text{ og } \vec{v} \text{ er parallelle } \Leftrightarrow \vec{u} \times \vec{v} = 0.$$

Linjer og PlanerPlanen/linjen gennem punktet med stedvektor \vec{x}_0 med normalvektor \vec{n} beskrives ved alle vektorer \vec{x} der løser ligningen

$$\vec{n} \cdot (\vec{x} - \vec{x}_0) = 0$$

En linje i rummet/planen gennem punktet med stedvektor \vec{x}_0 og retning \vec{r} har parameterfremstilling

$$\vec{x}_0 + t\vec{r}, \quad t \in \mathbb{R}.$$

Special matrices (square)

Diagonal

$$\begin{bmatrix} a & b & c & \bar{0} \\ \bar{0} & \ddots & \ddots & z \end{bmatrix}$$

Triangular

lower/upper

$$\begin{bmatrix} \bar{0} & \bar{0} & \bar{0} \\ \vdots & \ddots & \bar{0} \\ \bar{0} & \cdots & \bar{0} \end{bmatrix}$$

Scalar

$$\begin{bmatrix} k & k & \bar{0} \\ \bar{0} & \ddots & k \end{bmatrix}$$

Identity

$$\begin{bmatrix} 1 & & \bar{0} \\ \bar{0} & \ddots & 1 \end{bmatrix} = \bar{I}$$

Null (in various forms) $\bar{0}$?

Transposée d'une matrice carrée

$$\begin{pmatrix} 1 & 5 \\ 6 & 8 \end{pmatrix}^T = \begin{pmatrix} 1 & 6 \\ 5 & 8 \end{pmatrix}$$

Étapes :

- La 1^{re} ligne devient la 1^{re} colonne
- La 2 -ème ligne devient la 2 -ème colonne
- La 3 -ème ligne devient la 3 -ème colonne
- Ainsi de suite...

$$\begin{pmatrix} 9 & 7 & 5 \\ 1 & 0 & 7 \\ 4 & 2 & 6 \end{pmatrix}^T = \begin{pmatrix} 9 & 1 & 4 \\ 7 & 0 & 2 \\ 5 & 7 & 6 \end{pmatrix}$$

$$\begin{matrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{matrix}$$

$$= \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}^T$$

Special matrices (square)

Symmetric $\bar{\bar{A}}^T = \bar{\bar{A}}$

e.g. $\begin{bmatrix} 5 & -1 & 0 \\ -1 & 9 & 5 \\ 0 & 5 & 7 \end{bmatrix}$

Skew-Symmetric $\bar{\bar{A}}^T = -\bar{\bar{A}}$

e.g. $\begin{bmatrix} 0 & -6 & 2 \\ 6 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$

OBS what's particular about it?

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Taxonomy for normal matrices

Reel matrix	Kompleks matrix	Generel	Normal ⁽²⁾	Egenværdier
Symmetrisk $A^T = A$	Hermitesk (diagonal er reel) $A^{*T} = A$	Selvadjungeret ⁽¹⁾	Ja	Reelle (inkl. 0)
Skævsymmetrisk (diagonal = 0) $A^T = -A$	Skævhermitesk (diagonal imaginær eller 0) $A^{*T} = -A$	Skævadjungeret	Ja	Imaginære (inkl. 0)
Ortogonal ($ A = \pm 1$) $A^T = A^{-1}$	Unitær ($ A = 1$) $A^{*T} = A^{-1}$	Isometrisk	Ja	Absolut værdi 1

(1) Den (komplekst) konjugerede transponerede, A^{*T} , kaldes for den adjungerede til A (og deraf betegnelsen selvadjungeret i dette tilfælde); mere formelt kaldes den komplekst konjugerede transponerede for den hermitesk adjungerede, hvor hermitesk adjungering er analogt til kompleks konjugering.

(2) En normal matrix er en (generelt) kompleks kvadratisk matrix der kommuterer med sin adjungerede, dvs. opfylder $A^{*T}A = AA^{*T}$.

$$\Delta \bar{\bar{A}} = \det(\bar{\bar{A}})$$

Example [edit]

For example, the following matrix is skew-Hermitian

$$A = \begin{bmatrix} -i & +2+i \\ -2+i & 0 \end{bmatrix}$$

because

$$-A = \begin{bmatrix} i & -2-i \\ 2-i & 0 \end{bmatrix} = \begin{bmatrix} \overline{-i} & \overline{-2+i} \\ \overline{2+i} & \overline{0} \end{bmatrix} = \begin{bmatrix} \overline{-i} & \overline{2+i} \\ \overline{-2+i} & \overline{0} \end{bmatrix}^T = A^H$$

Basic remarks [edit]

A square matrix \mathbf{A} with entries a_{ij} is called

- Hermitian or self-adjoint if $\mathbf{A} = \mathbf{A}^H$; i.e., $a_{ij} = \overline{a_{ji}}$.
- Skew Hermitian or antihermitian if $\mathbf{A} = -\mathbf{A}^H$; i.e., $a_{ij} = -\overline{a_{ji}}$.
- Normal if $\mathbf{A}^H \mathbf{A} = \mathbf{A} \mathbf{A}^H$.
- Unitary if $\mathbf{A}^H = \mathbf{A}^{-1}$, equivalently $\mathbf{A} \mathbf{A}^H = \mathbf{I}$, equivalently $\mathbf{A}^H \mathbf{A} = \mathbf{I}$.

Even if \mathbf{A} is not square, the two matrices $\mathbf{A}^H \mathbf{A}$ and $\mathbf{A} \mathbf{A}^H$ are both Hermitian and in fact positive semi-definite matrices.

Eigenvalues (square matrices)

Hermetian C^n
(symmetric, R^n)

$$\bar{\mathbf{A}}^* T = \bar{\mathbf{A}}$$

λ_i real (o)

Skew-Hermetian C^n
(skew-symmetric, R^n)

$$\bar{\mathbf{A}}^* T = -\bar{\mathbf{A}}$$

λ_i imaginary (o)

Unitary C^n
(orthogonal, R^n)

$$\bar{\mathbf{A}}^* T = \bar{\mathbf{A}}^{-1}$$

$|\lambda_i| = 1$

* complex conjugate

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Invers matrix

Eigenvalues (square matrices)

Hermetian C^n
(symmetric, R^n)

$$\bar{A}^*T = \bar{A}$$

λ_i real (0)

Skew-Hermetian C^n
(skew-symmetric, R^n)

$$\bar{A}^*T = -\bar{A}$$

λ_i imaginary (0)

Unitary C^n
(orthogonal, R^n)

$$\bar{A}^*T = \bar{A}^{-1}$$

$|\lambda_i| = 1$

* complex conjugate

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Find Eigen values and basis

Eks

$$\bar{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}$$

$$\det(\bar{A}) = (a-\lambda)(d-\lambda) - (c \cdot b)$$

Som man isolere og forn λ_1 og λ_2

DETTE ER EIGEN VÆRDIER

Eigen base for λ_1

$$\begin{bmatrix} a - \lambda_1 & b \\ c & d - \lambda_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} a - \lambda_1 \underline{x_1} + b \underline{x_2} &= 0 \\ c \underline{x_1} + d - \lambda_1 \underline{x_2} &= 0 \end{aligned} \quad \text{Si løser man for } \underline{x_1} \text{ og } \underline{x_2}$$

hvor $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ er vøres svan for λ_1

Og så gør man det samme for λ_2



Diagonalisering!

$$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad P = \begin{bmatrix} x_1 \lambda_1 & x_1 \lambda_2 \\ x_2 \lambda_1 & x_2 \lambda_2 \end{bmatrix}$$

Pore eigenvalues

for eig¹

for eig²

Pore eigenbasis

$$\overline{\overline{D}} = \overline{\overline{X}}^{-1} \overline{\overline{A}} \overline{\overline{X}} = \overline{\overline{P}}^{-1} \overline{\overline{A}} \overline{\overline{P}}$$

Diagonalization

$$\bar{D} = \bar{x}^{-1} \bar{A} \bar{x}$$

Husk rækkefølgen!

$\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$ (diagonal) matrix of eigenvalues

$[\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n]$ matrix of eigenvectors

\bar{x} is said to diagonalize \bar{A}

\bar{D} and \bar{A} are called similar* in that they have the same eigenvalues

(*generally, any non-singular matrix \bar{x} suffice)

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<p>FIND D</p> $\bar{D} = \bar{P}^{-1} \bar{A} \bar{P}$ <p>Husk rækkefølgen!</p>	<p>FROM $D \Rightarrow A$</p> $\bar{A} = \bar{P} \bar{D} \bar{P}^{-1}$ <p>Husk rækkefølgen!</p>
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Spectral radius is the LARGEST abs. value of your D eigenvectors in that form

so for $(\sqrt{-1} A) = (\sqrt{-1} D)$ as they are unitary the same as we can diagonalize it

$$(\sqrt{-1} \cdot 0)^5 = 0 \quad ; \quad (\sqrt{-1} \cdot 4)^5 = 1024 \mathbb{I} \quad ; \quad (\sqrt{-1} \cdot 4)^5 = -1024 \mathbb{I}$$

so spectre is 1024

22 Specie in

+ SOLVE of ODE'S

Wednesday, May 29, 2024 9:18 AM

$$x^{(4)} - 7x^{(2)} + 4x^{(0)} + 5x^{(1)} - 2x = 0$$

$$x^{(4)} = +2x - 5x^{(0)} - 4x^{(2)} + 7x^{(3)}$$

$$x = v_1 \quad x^{(1)} = v_2 \quad x^{(2)} = v_3 \quad x^{(3)} = v_4 \quad x^{(4)} = v_5$$

$$x^{(4)} = +2v_1 - 5v_2 - 4v_3 + 7v_4 = v_4'$$

$$x = v_1 \quad x' = v_2 = v_1' \quad x^2 = v_3 = v_2' \quad x^3 = v_4 = v_3' \quad x^4 = v_5 = v_4'$$

$$v_1' = -v_2$$

$$v_2' = -v_3$$

$$v_3' = v_4$$

$$v_4' = 2v_1 - 5v_2 - 4v_3 + 7v_4$$



$$y'' + 4y = 0$$

$$y = v_1$$

$$y' = v_2 = v_1'$$

$$y'' = -4y$$

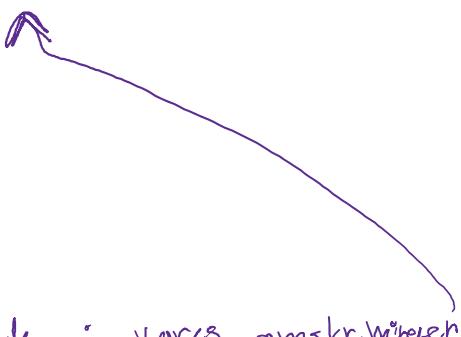
$$y'' = v_3 = v_2'$$

$$y''' = v_4 = v_3'$$

$$\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$v_2' = -4v_1 + 0v_2$$

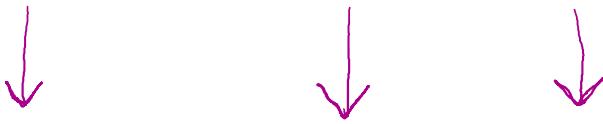
... en vi. snade i varcs omeskri. Målet er



$$V_2' = -4V_1 + 0V_2$$

$$V_1' = 0V_1 + 1V_2$$

Som vi sager: varer omstørrelsen



$$AY''' + BY'' + CY' + DY = 0$$

$$\begin{bmatrix} V_1' \\ V_2' \\ V_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{D}{A} & -\frac{C}{A} & \frac{B}{A} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$y = V_1$ $y' = V_2 = V_1'$ $y'' = V_3 = V_2'$ $y''' = V_4 = V_3'$

So this is the basis way to do it

$$\bar{y} = C_1 \bar{V}_1 e^{\lambda_1 t} + C_2 \bar{V}_2 e^{\lambda_2 t} + C_3 \bar{V}_3 e^{\lambda_3 t} \dots$$

Basis former husk den nu forføren!

$$Y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

Convolution ↑ not multiplien

1) Rewrite for $n \rightarrow k$

$$x(n) = 5u[n] \implies x(k) = 5u[k]$$

$$h(n) = 3^n u[n-1] \implies h(k) = 3^{n-k} u[n-1-k]$$

2) Insert in \sum

$$\sum_{k=-\infty}^{\infty} x(k) h(k) = \sum_{k=-\infty}^{\infty} 5u[k] \cdot 3^{n-k} u[n-1-k]$$

$\frac{3^n}{3^k}$

3) Extract constants out of \sum

$$\sum_{k=-\infty}^{\infty} 5u[k] \cdot \frac{3^n}{3^k} \cdot u[n-1-k]$$

$$5 \cdot 3^n \sum_{k=-\infty}^{\infty} u[k] \cdot \frac{1}{3^k} \cdot u[n-1-k]$$

$u[k]$ enten letter O $u[n-1-k]$ enten letter O

afskriven jeg dem

$$5 \cdot 3^n \sum \frac{1}{3^k}$$

4) Quick removal of \sum

$$\sum_{n=0}^{n-1} r^k = \frac{1-r^n}{1-r} \quad \text{for } r \neq 1$$

$$5 \cdot 3^n \sum \frac{1}{3^k} \quad \frac{1}{3^k} = \frac{1^k}{3^k} = \left(\frac{1}{3}\right)^k$$

$$5 \cdot 3^n \sum \left(\frac{1}{3}\right)^k = \boxed{5 \cdot 3^n \cdot \frac{1 - \left(\frac{1}{3}\right)^n}{1 - \frac{1}{3}}} \quad \text{This you can maple plug!}$$

$$\overbrace{\frac{1 - \left(\frac{1}{3}\right)^n}{\frac{2}{3}}}^{\text{as } 1 - \frac{1}{3} = \frac{3}{3} - \frac{1}{3} = \frac{2}{3}}$$

$$5 \cdot 3^n \frac{\frac{1 - \left(\frac{1}{3}\right)^n}{\frac{2}{3}}}{\frac{2}{3}} = \frac{5 \cdot 3^n}{\frac{2}{3}} - \frac{5 \cdot 3^n \left(\frac{1}{3}\right)^n}{\frac{2}{3}}$$

$$\frac{5 \cdot 3^n \cdot 3}{2} - \frac{5 \cdot 3^n \cdot 3 \left(\frac{1}{3}\right)^n}{2}$$

$$\Rightarrow \frac{5 \cdot 3^n \cdot 3 \left(\frac{1}{3}\right)^n}{2} \rightarrow \frac{5 \cdot 3 \cdot 3^n \left(\frac{1}{3^n}\right)}{2} = \frac{5 \cdot 3 \left(\frac{3^n}{3^n}\right)}{2} = \frac{5 \cdot 3}{2}$$

$$\frac{5 \cdot 3 \cdot 3^n}{2} - \frac{5 \cdot 3}{2} = \frac{15}{2} 3^n - \frac{15}{2}$$

5) Back sub $u[n]$ into it

$$\frac{15}{2} 3^n v[n] - \frac{15}{2} u[n]$$

And this is the
answer 😊

+ FOLDNINGER

Friday, 31 May 2024 15.13

Når der i opgaven står $x[n-2]$ betyder det at vi skubber pulstoget 2 timestamps frem, da vi så modtager pulstoget 2 timestamps senere.

Altså rykker vi pulstoget mod højre på y-aksen.

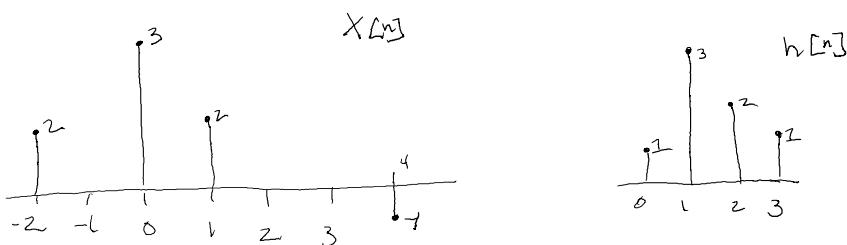
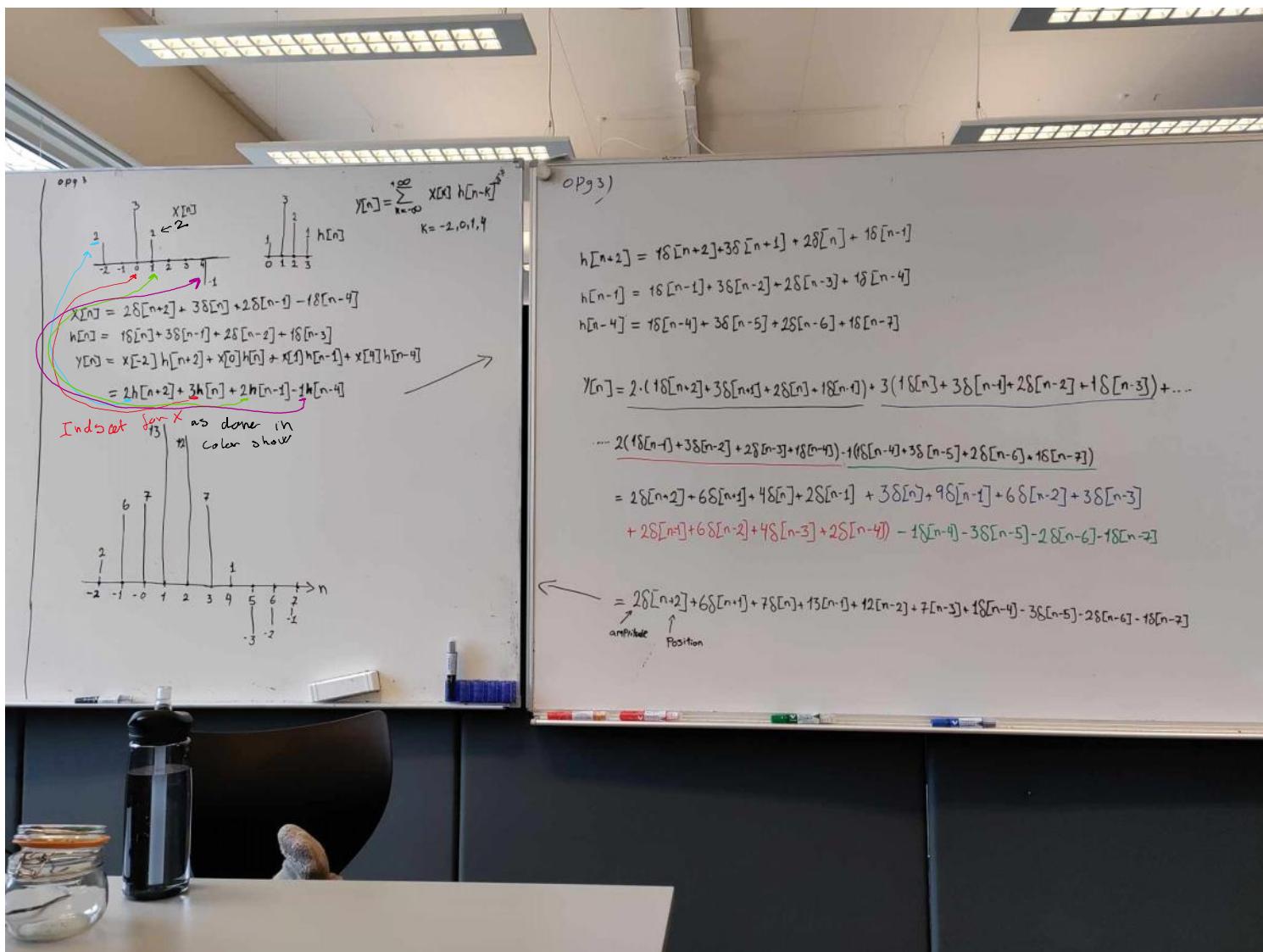
Hvis der til gengæld står + et eller andet skal vi rykke pulstoget mod venstre.

Når der står $-n$, skal hele pulstoget flippes, da det så er et negativt/modsatrettet pulstog vi har med at gøre. I eksemplet fra opgaven er det nemmest at omskrive til formen: $x[-n+1]$, da det beskriver for os at vi har et flippet pulstog, og skal rykke 1 mod venstre.

OG HUSK! SHIFT and FLIP!

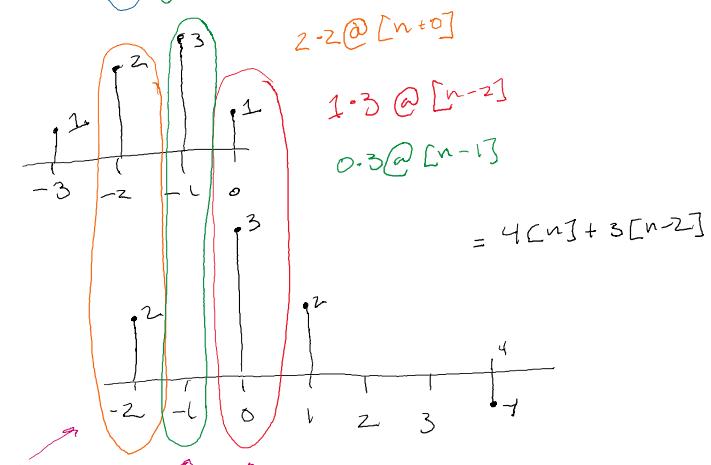
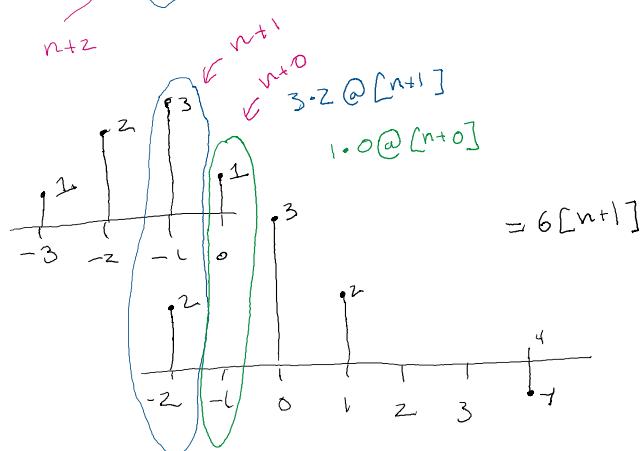
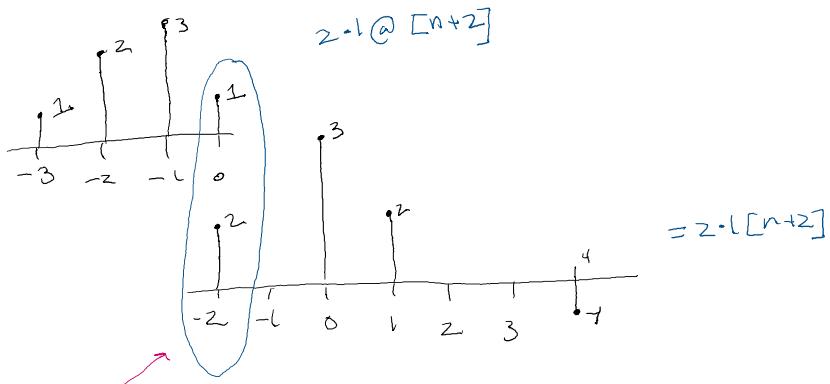
Ikke flip and shift!

Når vi har med convolutions at gøre



Now for the convoluting —

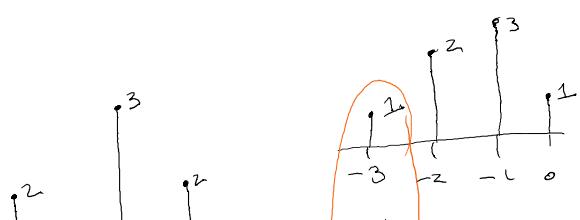
— Now for the convoluting —

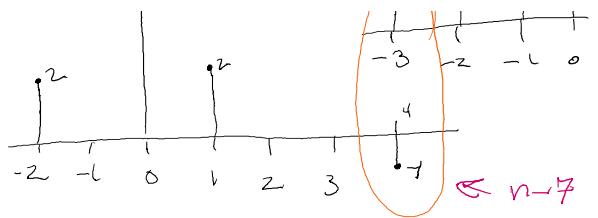


OSV OSV OSV indtil man er her!

Man følger bagrude indtil den man folder ind oven
er fuldt overlappet og ligger 1 til indtil man er
igen nem hele ens spektre

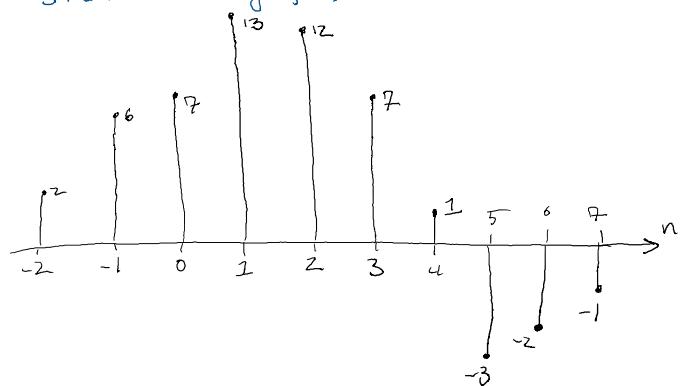
$$1 \cdot 1[n-7]$$





Så ligger man alle ens $[n-3]$ sammen

i størrelse og for!



$$\text{Len}(y[n]) = (\text{len}(h[n]) + \text{len}(x[n])) - 1$$

The z-transform

TABLE 3.1 SOME COMMON z-TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1-z^{-1}}$	$ z > 1$
3. $-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$ z < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\frac{1}{1-az^{-1}}$	$ z > a $
6. $-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$ z < a $
7. $na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$ z > a $
8. $-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$ z < a $
9. $\cos(\omega_0 n)u[n]$	$\frac{1 - \cos(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$	$ z > 1$
10. $\sin(\omega_0 n)u[n]$	$\frac{\sin(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$	$ z > 1$
11. $r^n \cos(\omega_0 n)u[n]$	$\frac{1 - r\cos(\omega_0)z^{-1}}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}}$	$ z > r$
12. $r^n \sin(\omega_0 n)u[n]$	$\frac{r\sin(\omega_0)z^{-1}}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}}$	$ z > r$
13. $\begin{cases} a^n, & 0 \leq n \leq N-1, \\ 0, & \text{otherwise} \end{cases}$	$\frac{1 - a^N z^{-N}}{1 - az^{-1}}$	$ z > 0$

Pole finding:

$$\frac{1}{z - bz^{-1}}$$

$$z - bz^{-1} = 0 \Leftrightarrow z = bz^1$$

$$z^{-1} = \frac{z}{b} \Leftrightarrow z = \underline{\underline{z}}$$

$|z| < |a|$ and left side

$$\frac{1}{z - \frac{1}{2}z^{-1}} \Rightarrow z = \frac{1}{2}z^{-1} \Rightarrow$$

$$4 = z^{-1} \Rightarrow z = \underline{\underline{\frac{1}{4}}}$$

$|z| > |a|$ and right side

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Negativ / standard

$$\frac{Qz^{-M}}{1 - Az^{-1}} \xrightarrow{\text{using 5}} Q A^{n-M} U[n-M]$$

Positive / -- Standard

$$\frac{Qz^{+M}}{1 - Az^{-1}} \Rightarrow A^{n+M} U[n+M]$$

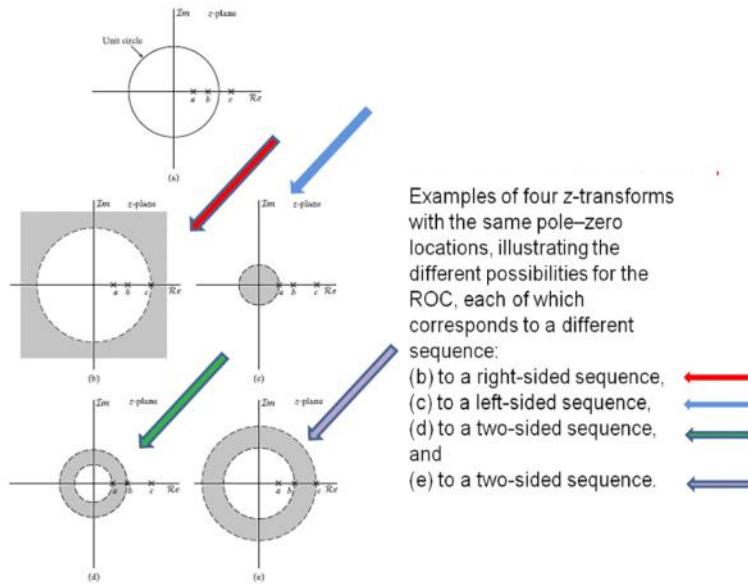
Husk fortegn for M. Som standard er neg

$$\frac{Qz^{-M}}{1 - Az^{-1}} \xrightarrow{\text{using 6}} Q - (A)^{n-M} U[-n-1-M]$$

Her er den positiv

$$\frac{Qz^{+M}}{1 - Az^{-1}} \Rightarrow Q - (A)^{n+M} U[n+1+M]$$

The z-transform



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The z-transform

TABLE 3.2 SOME z-TRANSFORM PROPERTIES

Property Number	Section Reference	Sequence	Transform	ROC
1	3.4.1	$x[n]$	$X(z)$	R_x
		$x_1[n]$	$X_1(z)$	R_{x_1}
		$x_2[n]$	$X_2(z)$	R_{x_2}
2	3.4.2	$x[n - n_0]$	$z^{-n_0}X(z)$	Contains $R_{x_1} \cap R_{x_2}$ except for the possible addition or deletion of the origin or ∞
3	3.4.3	$z_0^n x[n]$	$X(z/z_0)$	$ z_0 R_x$
4	3.4.4	$n x[n]$	$-z \frac{dX(z)}{dz}$	R_x
5	3.4.5	$x^*[n]$	$X^*(z^*)$	R_x
6		$\mathcal{R}e\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	Contains R_x
7		$\mathcal{I}m\{x[n]\}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	Contains R_x
8	3.4.6	$x^*[-n]$	$X^*(1/z^*)$	$1/R_x$
9	3.4.7	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{x_1} \cap R_{x_2}$

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How to inverse z-transform!

$$X(z) = \frac{2z^{-1}}{1-z^{-1}} + \frac{z^3}{1-\frac{4}{7}z^{-1}} \quad \text{given ROC } |z| > 1$$

Det betyder vi bruger denne basis

$$\rightarrow \dots n-1 \dots 1$$

$$5) \quad a^n u[n] \Leftrightarrow \frac{1}{1 - az^{-1}}$$

$$\frac{2z^{-1}}{1 - z^{-1}} = 2 \frac{z^{-1}}{1 - z^{-1}} ; \quad \frac{z^3}{1 - \frac{4}{7}z^{-1}}$$

$$2 \cdot [n-1] \quad 2[n-1]$$

$$\frac{4}{7} u[n+3] \quad \frac{4}{7} u[n+3]$$

$$2[n-1] + \left(\frac{4}{7}\right)^{n+3} u[n+3]$$

Let's inverse some more

$$X(z) = \frac{3}{1 - 2z^{-1}} + \frac{z^{-1}}{1 - z^{-1}} \quad \text{ROC } |z| < 1$$

Which means we use this

$$6) \quad -a^n u[-n-1] \Leftrightarrow \frac{1}{1 - az^{-1}}$$

$$\frac{3}{1-2z^{-1}} + \frac{z^{-1}}{1-z^{-1}}$$

da bliver
-- = t

$3 - (z)^n u[-n-1] + (-1)^{n-1} u[-n-1-1]$ da har det
der minus

from formula

$$-3(z)^n u[-n-1] - u[-n]$$

$-1^{n-1} = 1$ $-n-1+1 = -n$



A stable LTI system is characterized by
this difference equation:

$$y[n] - 3y[n-1] = 5x[n-2]$$

$$y[n-0] - 3y[n-1] = 5x[n-2]$$

$$\mathcal{Z}\{y\}$$

$$Y(z) \cdot z^0 - 3Y(z) \cdot z^{-1} = 5X(z) \cdot z^{-2}$$

$$Y(z) \cdot (z^0 - 3 \cdot z^{-1}) = X_{res} / 5 \cdot z^{-2}$$

$$Y(z) \cdot (z^0 - 3 \cdot z^{-1}) = X(z)(5 \cdot z^2)$$

And then we know that $H(z) = \frac{Y(z)}{X(z)}$

$$\frac{Y(z)}{X(z)} (1 - 3z^{-1}) = 5z^2 \Rightarrow \frac{Y(z)}{X(z)} = \frac{5z^2}{1 - 3z^{-1}} = H(z)$$

THIS IS IMPORTANT

$$H(z) = \frac{Y(z)}{X(z)} = \frac{5 \cdot z^2}{1 - 3z^{-1}}$$

og det kan vi bruge inverse
Z transform på

$ROC = 3$ which means left-sided and therefore
 $|z| < 1$

Som betyder vi bruger

$$-a^n u[-n-1] \Rightarrow \frac{1}{1 - az^{-1}}$$

$$5 \cdot \frac{z^2}{1 - 3z^{-1}} \Rightarrow Z^{-1} \left\{ 5 - 3^{n-2} u[-n-1-2] \right\}$$

$$-5(3)^{n-2} u[-n+1]$$

and is non causal as $[-n+1]$
is positive

oooooooooooo

Apply $\mathcal{Z}^{-1}\{y\}$ to

$$H(z) = \frac{1 - 3z^{-1}}{2 - 4z^{-1}} = \frac{1}{2 - 4z^{-1}} - \frac{\frac{3}{2}z^{-1}}{2 - 4z^{-1}}$$

for ROC $2 - 4z^{-1} \Rightarrow z - 4\frac{1}{2}z^{-1} = 0$

Then ROC $|z| < |a|$

6) $-a^n u[-n-1] \Leftrightarrow \frac{1}{1 - az^{-1}}$

$$\frac{1}{2 - 4z^{-1}} - \frac{\frac{3}{2}z^{-1}}{2 - 4z^{-1}} = \frac{1}{2} \frac{1}{1 - 2z^{-1}} - \frac{\frac{3}{2}}{2} \frac{z^{-1}}{1 - 2z^{-1}}$$

$$\begin{aligned} \frac{1}{2} \cdot -(z)^n u[-n-1] - \frac{\frac{3}{2}}{2} \cdot -(z)^{n+1} u[-n-1-1] \\ - \frac{1}{2} z^n u[-n-1] + \frac{\frac{3}{2}}{2} (z)^{n+1} u[-n] \end{aligned}$$

5 $a^n u[n] \Leftrightarrow \frac{1}{1 - az^{-1}}$ ROC $|z| > |a|$

$$\frac{1}{2} \frac{1}{1 - 2z^{-1}} - \frac{\frac{3}{2}}{2} \frac{z^{-1}}{1 - 2z^{-1}}$$

$$\underline{\underline{\frac{1}{2} 2^n [n] - \frac{3}{2} 2^{n-1} [n-1]}}$$

oooooooooooo

Opgaveeks. 7

Given a causal linear time invariant system with the following transfer function:

$$H(z) = \frac{1 - 3z^{-1}}{2 - 4z^{-1}},$$

calculate the impulse response $h_i[n]$ of the inverse system.

Solution: $h_i[n] = 2(3)^n u[n] - 4(3)^{n-1} u[n-1]$, or $h_i[n] = -2(3)^n u[-n-1] + 4(3)^{n-1} u[-n]$.

Så opdag nu lige at det skal
være den inverse

$$H(z) = \mathcal{Z}^{-1} \left\{ \frac{1}{H(z)} \right\}$$

$$H(z)^{-1} = \frac{z - 4z^{-1}}{1 - 3z^{-1}} = 2 \frac{1}{1 - 3z^{-1}} - \frac{4z^{-1}}{1 - 3z^{-1}}$$

$$2(3)^n [n] - 4(3)^{n-1} [n-1] \quad \text{formel 5)}$$

OR

$$-2(3)^n [-n-1] + 4(3)^{n-1} [-n] \quad \text{formel 6)}$$

Stability and eigenvalues

$$\det(\bar{A} - \lambda \bar{I}) = \begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = \lambda^2 - (a_{11} + a_{22})\lambda + \det(\bar{A}) = 0$$

From LA we know:

$$\text{trace}(\bar{A}) = \sum_i a_{ii} = a_{11} + a_{22} = \sum_i \lambda_i \equiv p$$

$$\det(\bar{A}) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \prod_i \lambda_i \equiv q$$

$$\det(\bar{A} - \lambda \bar{I}) = \lambda^2 - p\lambda + q , D = p^2 - 4q$$

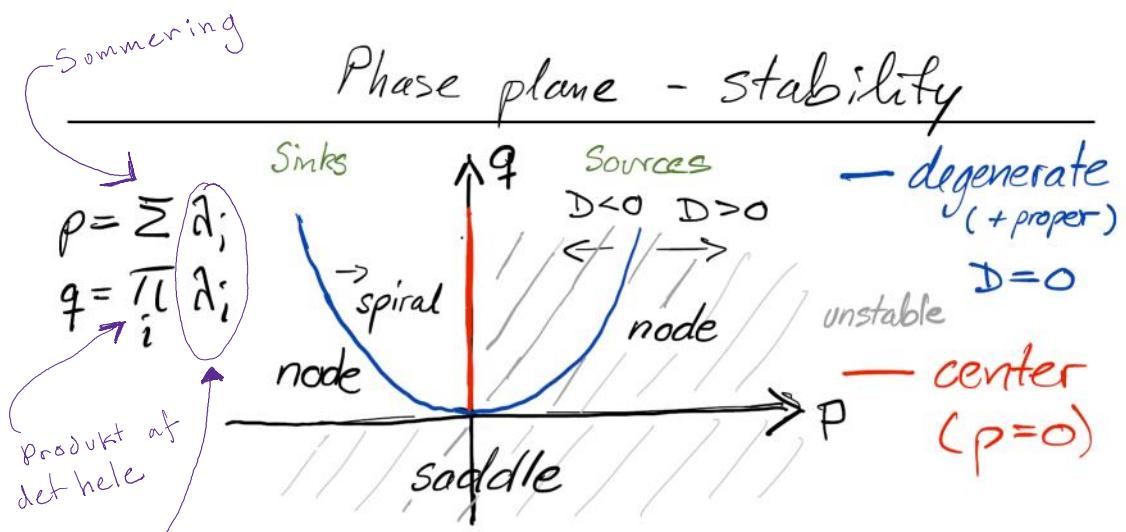
Nodes and eigenvalues

$$\left\{ \begin{array}{l} D \geq 0 \quad (\text{real eigenvalues}) \\ q = \prod_i \lambda_i > 0 \stackrel{(p \neq 0)}{\implies} \text{node} \\ q = \prod_i \lambda_i < 0 \implies \text{saddle} \\ \qquad \qquad \qquad \text{except } q \leq 0 \\ p = \sum_i \lambda_i = 0 \stackrel{(q > 0)}{\implies} \text{center} \\ p = \sum_i \lambda_i \neq 0 \implies \text{spiral point} \\ D < 0 \quad (\text{complex eigenvalues}) \end{array} \right.$$

except $p=0$

F Sommering T
 Phase plane - stability

Phase plane - stability



- unstable if $q < 0$ or $p > 0$ holds also for $n > 2$
- stable if $q > 0$ and $p \leq 0$
- stable if $q > 0$ and $p < 0$

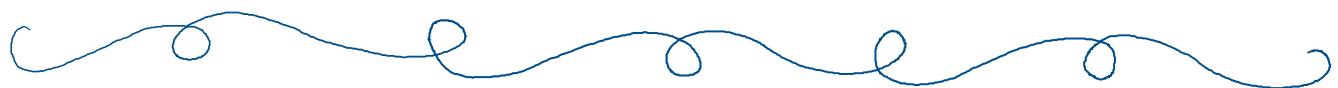
(attractive or asymptotically stable)

+ NUL/SUB SPACE

Saturday, 1 June 2024 10.32

Argument that:

$3x_1 + x_3$ is a subspace in \mathbb{R}^3
and find the basis for it



$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Gauss elimination

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3}x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

Isolate for $x_1 = -\frac{1}{3}x_3$

These exist but only for them selves
hence $x_2 = x_2$ and $x_3 = x_3$

$$\begin{bmatrix} -\frac{1}{3}x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} -\frac{1}{3} \\ 0 \\ 1 \end{bmatrix}$$

da x_1 ikke er tilstede

da x_2 er til ved sig selv

da x_3 er både ved " x_1 " og " x_2 "

+ BASIC STUFF

Sunday, 2 June 2024 10.36

Determinante of a 4x4

Berechnung der Determinante von (4x4)-Matrizen (Laplace)

$$\det \begin{pmatrix} 1 & 2 & 0 & 2 \\ 4 & 0 & 0 & 5 \\ 1 & 2 & 1 & 1 \\ 1 & 4 & 3 & 2 \end{pmatrix} = + 0 \cdot \det \begin{pmatrix} 4 & 0 & 5 \\ 1 & 2 & 1 \\ 1 & 4 & 2 \end{pmatrix} - 0 \cdot \det \begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & 1 \\ 1 & 4 & 2 \end{pmatrix}$$

$$+ 1 \cdot \det \begin{pmatrix} 1 & 2 & 2 \\ 4 & 0 & 5 \\ 1 & 4 & 2 \end{pmatrix} - 3 \cdot \det \begin{pmatrix} 1 & 2 & 2 \\ 4 & 0 & 5 \\ 1 & 2 & 1 \end{pmatrix}$$

Entw. u. 3. Sp.

$$4 \cdot ((2 \cdot 2) - (4 \cdot 1)) - 1 \cdot ((0 \cdot 2) - (4 \cdot 1)) + 1 \cdot ((0 \cdot 1) - 2 \cdot 5)$$

$$0 \quad 0 \geq 0 \quad 0 \leq 0$$