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and substitution of (5.31) in (5.30) gives

$$V_o = -R_L I_Q\left(\exp\frac{V_s}{V_T} - 1\right) \tag{5.32}$$

Expansion of the exponential term in (5.32) in a power series gives

$$V_o = -R_L I_Q\left[\frac{V_s}{V_T} + \frac{1}{2}\left(\frac{V_s}{V_T}\right)^2 + \frac{1}{6}\left(\frac{V_s}{V_T}\right)^3 + \cdots\right] \tag{5.33}$$

$$= a_1 V_s + a_2 V_s^2 + a_3 V_s^3 + \cdots \tag{5.34}$$

where

$$a_1 = -\frac{R_L I_Q}{V_T} \tag{5.35}$$

$$a_2 = -\frac{R_L I_Q}{2V_T^2} \tag{5.36}$$

$$a_3 = -\frac{R_L I_Q}{6V_T^3} \tag{5.37}$$

Equation 5.33 allows calculation of distortion in the common-emitter stage. For values of $V_s/V_T \ll 1$ the first term in parentheses dominates and the circuit is essentially linear. However, as V_s becomes comparable to V_T, the terms involving V_s^2 and V_s^3 become significant and distortion products are generated as will now be illustrated. A common method of describing the nonlinearity of an amplifier is the specification of *harmonic distortion*. This is defined for a single sinusoidal input applied to the amplifier. Thus let

$$V_s = \hat{V}_s \sin\omega t \tag{5.38}$$

and substitution of (5.38) in (5.34) gives

$$V_o = a_1\hat{V}_s \sin\omega t + a_2\hat{V}_s^2 \sin^2\omega t + a_3\hat{V}_s^3\sin^3\omega t + \cdots$$

$$= a_1\hat{V}_s\sin\omega t + \frac{a_2\hat{V}_s^2}{2}(1 - \cos 2\omega t)$$

$$+ \frac{a_3\hat{V}_s^3}{4}(3\sin\omega t - \sin 3\omega t) + \cdots \tag{5.39}$$

are not present in the input signal. Second-harmonic distortion HD_2 is defined as the ratio of the amplitude of the output-signal component at frequency 2ω to the amplitude of the first harmonic (or fundamental) at frequency ω. For small distortion, the term $\frac{3}{4}a_3\hat{V}_s^3\sin\omega t$ in (5.39) is small compared to $a_1\hat{V}_s\sin\omega t$ and the amplitude of the fundamental is approximately $a_1\hat{V}_s$. Again for small distortion, higher-order terms in (5.39) may be neglected and

$$HD_2 = \frac{a_2\hat{V}_s^2}{2}\frac{1}{a_1\hat{V}_s} = \frac{1}{2}\frac{a_2}{a_1}\hat{V}_s \tag{5.40}$$

Note that with the assumptions made, HD_2 varies *linearly* with signal level $\hat{V}_s$. The value of HD_2 can be expressed in terms of known parameters by substituting (5.35) and (5.36) in (5.40) to give

$$HD_2 = \frac{1}{4}\frac{\hat{V}_s}{V_T} \tag{5.41}$$

This important result shows that second-harmonic distortion in *any* voltage-driven bipolar transistor at low frequencies depends only on the normalized input voltage. This result shows that 10 percent of second-harmonic distortion ($HD_2 =$ 0.1) occurs for $\hat{V}_s = 0.1 \times 4 \times 26$ mV $\simeq 10$ mV.

Third-harmonic distortion HD_3 is defined as the ratio of the output signal component at frequency 3ω to the first harmonic. From (5.39) and assuming small distortion

$$HD_3 = \frac{a_3\hat{V}_s^3}{4}\frac{1}{a_1\hat{V}_s} = \frac{1}{4}\frac{a_3}{a_1}\hat{V}_s^2 \tag{5.42}$$

With the assumptions made, HD_3 varies as the *square* of the signal amplitude. The value of HD_3 can be expressed in terms of known parameters by substituting (5.35) and (5.37) in (5.40) to give

$$HD_3 = \frac{1}{24}\left(\frac{\hat{V}_s}{V_T}\right)^2 \tag{5.43}$$

Thus HD_3 also depends on the normalized input voltage amplitude. For $\hat{V}_s = 10$ mV, $HD_3 = 0.62$ percent.

Equations 5.41 and 5.43 can be used to calculate harmonic distortion in *any voltage-driven*, common-emitter transistor stage (assuming saturation does not occur). However the presence of finite source resistance will change the situation, and in the extreme case of a current drive to Q_1 the distortion will be that due to variation of β_F with I_C, which is usually much less than distortion created by the

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Consider a nonlinear system[3] with an input signal, $v_{in}(t)$, and an output signal, $v_o(t)$. The output signal can be written as a Taylor series expansion of the input signal:

$$v_o(t) = a_1 v_{in}(t) + a_2 v_{in}^2(t) + a_3 v_{in}^3(t) + a_4 v_{in}^4(t) + \cdots \quad (15.181)$$

Here, the linear term is a_1, whereas a_2, a_3, and a_4 characterize the second-, third-, and fourth-order distortion terms, respectively. As mentioned previously, in fully differential circuits, all even terms (i.e., a_2 and a_4) are small, so typically a_3 dominates and we approximate $v_o(t)$ as

$$v_o(t) \cong a_1 v_{in}(t) + a_3 v_{in}^3(t) \quad (15.182)$$

If $v_{in}(t)$ is a sinusoidal signal given by

$$v_{in}(t) = A\cos(\omega t) \quad (15.183)$$

the output signal can be shown to be approximated by

$$v_o(t) \cong a_1 A\cos(\omega t) + \frac{a_3}{4}A^3[3\cos(\omega t) + \cos(3\omega t)] \quad (15.184)$$

where we see a fundamental term and a third-harmonic term. Defining H_{D1} and H_{D3} to be the amplitudes of the fundamental and third-harmonic terms, respectively, we can write

$$v_o(t) \equiv H_{D1}\cos(\omega t) + H_{D3}\cos(3\omega t) \quad (15.185)$$

Since, typically, $(3/4)a_3A^3 << a_1A$, one usually approximates the linear component of the output signal as

$$H_{D1} = a_1 A \quad (15.186)$$

and the third-harmonic term as

$$H_{D3} = \frac{a_3}{4}A^3 \quad (15.187)$$

Finally, we see that the third-order distortion term results in power at the third harmonic frequency and the ratio of H_{D3}/H_{D1} is defined as the third-order harmonic distortion ratio, given by

$$HD_3 \equiv \frac{H_{D3}}{H_{D1}} = \left(\frac{a_3}{a_1}\right)\left(\frac{A^2}{4}\right) \quad (15.188)$$

Unfortunately, as just noted, this distortion term lies at $3\omega t$ for a single sinusoidal input, and thus we resort to an intermodulation test to move the distortion term back near the frequency of the input signals.

Consider now the case of an intermodulation test, where the input signal consists of two equally sized sinusoidal signals and is written as

$$v_{in}(t) = A\cos(\omega_1 t) + A\cos(\omega_2 t) \quad (15.189)$$

3. We assume here that the nonlinear system is memoryless and time invariant. Unfortunately, filters are not memoryless, and a Volterra series should be used; however, this assumption simplifies the analysis and usually results in good approximations for distortion figures

In this case, the output signal can be shown to be approximated by

$$\begin{aligned} v_o(t) \cong\ & \left(a_1 A + \frac{9a_3}{4}A^3\right)[\cos(\omega_1 t) + \cos(\omega_2 t)] \\ & + \frac{a_3}{4}A^3[\cos(3\omega_1 t) + \cos(3\omega_2 t)] \\ & + \frac{3a_3}{4}A^3[\cos(2\omega_1 t + \omega_2 t) + \cos(2\omega_2 t + \omega_1 t)] \\ & + \frac{3a_3}{4}A^3[\cos(\omega_1 t - \Delta\omega t) + \cos(\omega_2 t + \Delta\omega t)] \end{aligned} \quad (15.190)$$

where $\Delta\omega$ is defined to be the difference between the input frequencies (i.e., $\Delta\omega \equiv \omega_2 - \omega$) which we assume to be small. Here, we see that the first line of (15.90) is the fundamental components, the second line shows the levels at three times the fundamentals, the third line also describes distortion at nearly three times the fundamentals, and the fourth line describes the distortion levels at two new frequencies that are close to the input frequencies (slightly below ω_1 and slightly above ω_2). As a result, for a narrowband or low-pass filter, these two new distortion components (due to third-order distortion) fall in the passband and can be used to predict the third-order distortion term.

Using the same notation as in the harmonic distortion case, we have the intermodulation distortion levels given by

$$I_{D1} = a_1 A \quad (15.191)$$

and

$$I_{D3} = \frac{3a_3}{4}A^3 \quad (15.192)$$

The ratio of these two is the third-order intermodulation value, given by

$$ID_3 = \frac{I_{D3}}{I_{D1}} = \left(\frac{a_3}{a_1}\right)\left(\frac{3A^2}{4}\right) \quad (15.193)$$

From (15.191) and (15.192), note that, as the amplitude of the input signal, A, is increased, the level of the fundamental rises linearly, whereas the I_{D3} rises in a cubic fashion. For example, if A is increased by 1 dB, then I_{D1} also increases by 1 dB while I_{D3} increases by 3 dB. Thus, when the fundamental and intermodulation levels are plotted in a dB scale against the signal amplitude, A, the two curves are linear, but with different slopes, as shown in Fig. 15.54. The third-order intercept point is defined to be the intersection of these two lines. Note, however, that as the signal amplitude rises, the linear relationships of I_{D1} and I_{D3} are no longer obeyed due to the large amount of nonlinearities violating some of our original assumptions. For example, in (15.190), we ignored the cubic A term in estimating the fundamental level, I_{D1}. Also, other distortion terms that were ignored now become important. As a result, it is impossible to directly measure the third-order intercept point, and thus it must be extrapolated from the measured data. The third-order intercept point results in two

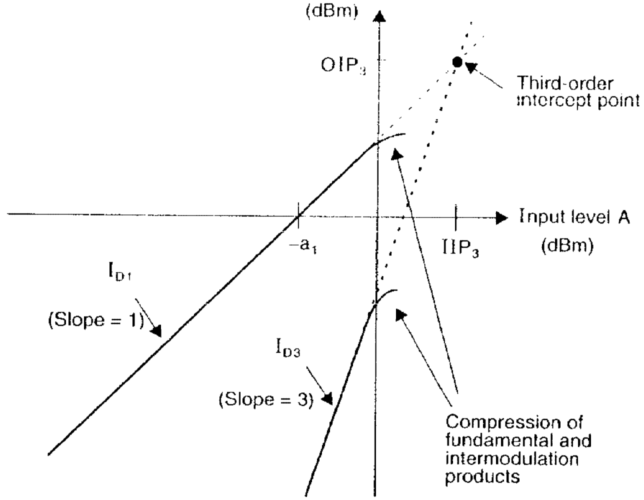

Fig. 15.54 Graphical illustration of the third-order intercept-point. IIP_3 and OIP_3 are the input and output third-order intercept points, respectively. They cannot be measured directly due to compression of the fundamental and intermodulation products at high-power levels.

values—IIP_3 and OIP_3, which are related to the input and output levels, respectively. If the linear gain term, a_1, is unity (or, equivalently, 0 dB), then $IIP_3 = OIP_3$. However, if a_1 is not unity, one must be careful to state which of the two intercept points is being reported.

Knowledge of the third-order intercept point is quite useful in determining what signal level should be chosen to achieve a desired intermodulation ratio, ID_3. Specifically, we see from (15.193) that ID_3 improves by 2 dB for every 1 dB of signal level decrease (since it is related to A^2) and that OIP_3 is defined to be the I_{D1} point where $ID_3 = 0$ dB. Thus, we have the simple relationship (all in decibels)

$$OIP_3 = I_{D1} - \frac{ID_3}{2} \tag{15.194}$$

EXAMPLE 15.10

If $OIP_3 = 20$ dBm, what output signal level should be used such that the third-order intermodulation products are 60 dB below the fundamental?

Solution

Using (15.194) with $ID_3 = -60$ dB, we have

$$I_{D1} = OIP_3 + \frac{ID_3}{2} = -10 \text{ dBm} \tag{15.195}$$

Thus, an output level of −10 dBm should be used.

Spurious-Free Dynamic Range (SFDR)

Spurious-free dynamic range (SFDR) is defined to be the signal-to-noise ratio when the power of the third-order intermodulation products equals the noise power. In Fig. 15.55, the filter's output noise power is shown along the vertical axis as N_o. If a low enough signal level is used, I_{D3} will be well below the noise floor. However, since I_{D3} rises 3 dB for every 1 dB of signal-level increase, there will soon be a point where I_{D3} is equal to the noise power. As the figure shows, SFDR is defined to be the output SNR ratio when I_{D3} is equal to N_o. Alternatively, one can measure SFDR using the input-signal levels as the difference between the level that results in $I_{D3} = N_o$ and the level A_{No} that results in a fundamental output level equal to N_o.

To find a formula relating SFDR, OIP_3, and N_o (all in dB units), we first note that

$$SFDR = I^{*}_{D1} - N_o = I^{*}_{D1} - I^{*}_{D3} \tag{15.196}$$

where I^{*}_{D1} and I^{*}_{D3} refer to the output and distortion levels when $I_{D3} = N_o$, as shown in Fig. 15.55. Since the units are assumed to be in dB, we also have, from (15.193),

$$ID_3 = I_{D3} - I_{D1} \tag{15.197}$$

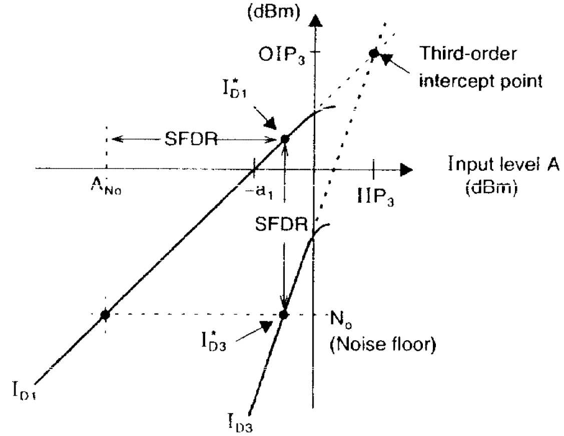

Fig. 15.55 Graphical illustration of spurious-free dynamic range (SFDR).

Using (15.194), we have

$$OIP_3 = I_{D1}^{*} - \frac{(N_o - I_{D1}^{*})}{2} \qquad (15.198)$$

and substituting in $I_{D1}^{*} = SFDR + N_o$ from (15.196) and rearranging, we have

$$SFDR = \frac{2}{3}(OIP_3 - N_o) \qquad (15.199)$$

EXAMPLE 15.11

At an input-signal level of 0 dBm, an intermodulation ratio of –40 dB was measured in an filter with a gain of 2 dB. Calculate the value of the input and output third-order intercept points. What input-signal level should be applied if one wants an intermodulation ratio of –45 dB? If the noise power at the output is measured to be –50 dBm, what is the expected SFDR, and what output-signal level does it correspond to?

Solution

With an input level of 0 dBm and a gain of 2 dB, the output level is $I_{D1} = 2$ dBm with a measured value of $ID_3 = -40$ dB. Using (15.194), we have

$$OIP_3 = I_{D1} - \frac{ID_3}{2} = 22 \text{ dBm} \qquad (15.200)$$

whereas the IIP_3 is 2 dB lower (i.e., $IIP_3 = 20$ dBm).

For signal levels corresponding to an intermodulation of $ID_3 = 45$ dB, we have

$$I_{D1} = OIP_3 + \frac{ID_3}{2} = 22 - \frac{45}{2} = -0.5 \text{ dBm} \qquad (15.201)$$

However, this value is the level of the output signal, so the input signal should be 2 dB lower or, equivalently, the input-signal level should be at –2.5 dBm. Note that this result could have been obtained more directly by noting that a 5-dB improvement in distortion was desired, implying that the signal level should be decreased by (5 dB)/2.

Finally, we use (15.199) to find

$$SFDR = \frac{2}{3}(22 + 50) = 48 \text{ dB} \qquad (15.202)$$

from which the output level is given by

$$I_{D1}^{*} = SFDR + N_o = -2 \text{ dBm} \qquad (15.203)$$

Thus, when the output level is at –2 dBm, the third-order intermodulation power equals the noise power. If the output level is increased, the distortion products will increase and limit the dynamic-range performance. However, if the output level is decreased, the distortion products will be buried below the noise floor, and the noise will limit dynamic-range performance. Therefore, optimum dynamic-range performance is obtained at an output level of –2 dBm. However, note that the dynamic-range performance is actually 3 dB below the SFDR value since the dynamic range is based on the ratio of the signal power to the distortion plus the noise power (which are both equal at this point).

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