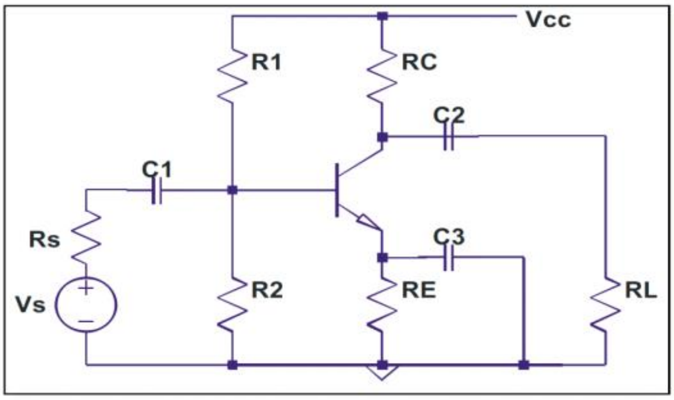
Gain optimization for BJT



Definition & assumption:

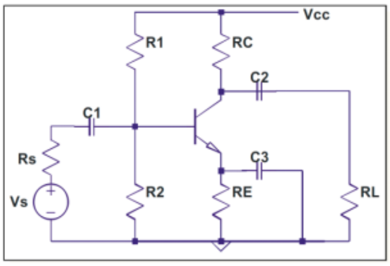
$$v_S' = \frac{R_B}{R_B + R_S} v_S$$

$$R_S' = R_S \parallel R_B$$

$$r_o >> R_C \parallel R_L$$

$$R_B = R_1 || R_2$$

Optimize R_C to maximize gain for the fixed V_{R_C}



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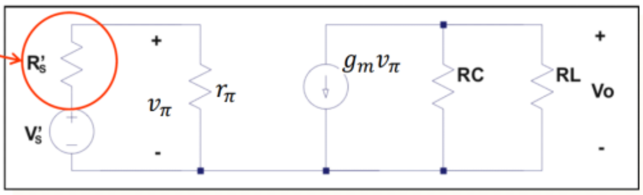


$$A_v = \frac{v_o}{v_s} = \frac{v_o}{v_S'} \cdot \frac{v_S'}{v_s} = A_v' \cdot \frac{R_B}{R_B + R_S}$$

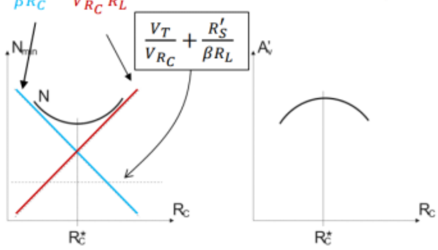
$$A_v' = -\frac{g_m (R_C || R_L)}{1 + \frac{R_S'}{r_\pi}} = -\frac{1}{\left(\frac{1}{g_m} + \frac{R_S'}{g_m r_\pi}\right)\left(\frac{1}{R_C} + \frac{1}{R_L}\right)}$$

$g_m = \frac{I_C}{V_T}$ and $\beta = g_m r_\pi$

$$\rightarrow A_v' = -\frac{1}{\left(\frac{V_T}{I_C} + \frac{R_S'}{\beta}\right)\left(\frac{1}{R_C} + \frac{1}{R_L}\right)} = -\frac{1}{\frac{V_T}{I_C R_C} + \frac{R_S'}{\beta R_C} + \frac{V_T}{I_C R_L} + \frac{R_S'}{\beta R_L}}$$

$$= -\frac{1}{\frac{V_T}{V_{R_C}} + \frac{R_S'}{\beta R_C} + \frac{R_C V_T}{V_{R_C} R_L} + \frac{R_S'}{\beta R_L}} = -\frac{1}{N}$$

Minimize $\frac{R_S'}{\beta R_C} + \frac{R_C V_T}{V_{R_C} R_L}$ → *minimize* N → *maximize* A_v'



Optimized R_C

$$\frac{\partial N}{\partial R_C} = 0 \Rightarrow R_C^* = \sqrt{\frac{R_S' \cdot R_L \cdot V_{R_C}}{\beta \cdot V_T}} \rightarrow \left|A_{v,\max}'\right| = \frac{1}{\left(\sqrt{\frac{V_T}{V_{R_C}}} + \sqrt{\frac{R_S'}{R_L \cdot \beta}}\right)^2}$$

Gain optimization

- Max gain based *exclude* $r_o = \frac{V_A}{I_C}$

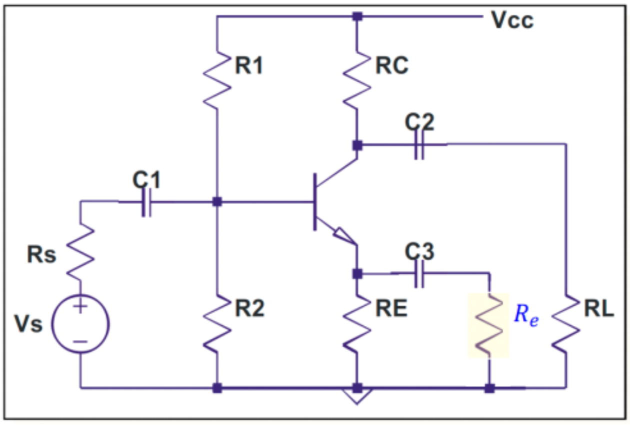
$$\left|A'_{v,\max}\right| = \frac{1}{\left(\sqrt{\frac{V_T}{V_{R_C}}} + \sqrt{\frac{R'_S}{R_L \cdot \beta}}\right)^2} \Rightarrow R^*_C = \sqrt{\frac{R'_S \cdot R_L \cdot V_{R_C}}{\beta \cdot V_T}}$$

- Max gain based *include* $r_o = \frac{V_A}{I_C}$

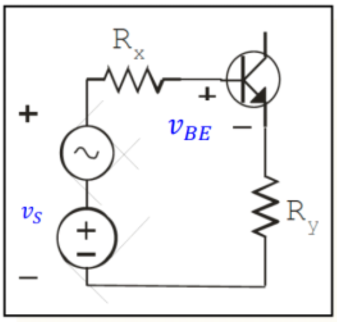
$$\left|A'_v\right| = \frac{r_o \parallel R_C \parallel R_L}{\frac{1}{g_m} + \frac{R'_S}{\beta}} \Rightarrow R^*_C = \sqrt{\frac{R'_S \cdot R_L \cdot V_{R_C}}{\beta \cdot V_T}\left(1 + \frac{V_{R_C}}{V_A}\right)}$$

- Adding emitter resistor R_e reduce the gain

$$\left|A'_v\right| = \frac{r_o \parallel R_C \parallel R_L}{R'_e + \frac{1}{g_m} + \frac{R'_S}{\beta}} \Rightarrow R'_e = \frac{r_o \parallel R_C \parallel R_L}{A'_v} - \frac{1}{g_m} - \frac{R'_S}{\beta} = R_e \parallel R_E$$



Improving the THD for a BJT



Large-signal model
$v_S = V_S + \Delta V_S$
$i_C = I_C + \Delta I_C$

- $i_C = f(v_S)$ must be found and then $f^{(1)}$and $f^{(2)}$are found at the operating point given by I_C
- Determine $f^1$and f^2 by implicitly differentiation

$$v_S = \frac{i_C}{\beta} R_x + v_{BE} + i_C R_y$$

$$i_C = I_S \cdot e^{\left(\frac{v_{BE}}{V_T}\right)}$$

$$= \left(\frac{R_x}{\beta} + R_y\right) i_C + V_T \ln\left(\frac{i_C}{I_S}\right)$$

Improving the THD for a BJT

Determination of $f^{(1)}$:

$$v_S = \left(\frac{R_x}{\beta}+R_y\right)i_C + V_T \ln\left(\frac{i_C}{I_S}\right)$$

$$\Downarrow$$

$$1 = \left(\frac{R_x}{\beta}+R_y\right)\frac{\partial i_C}{\partial v_S} + 0 + V_T \frac{1}{\frac{i_C}{I_S}}\frac{1}{I_S}\frac{\partial i_C}{\partial v_S}$$

$$\Downarrow$$

$$1 = \left(\frac{R_x}{\beta}+R_y+\frac{V_T}{i_C}\right)\frac{\partial i_C}{\partial v_S} = \left(\frac{R_x}{\beta}+R_y+\frac{V_T}{i_C}\right)f^1$$

$$\Downarrow$$

$$\boxed{f^{(1)}(i_C) = \frac{1}{\left(\frac{R_x}{\beta}+R_y+\frac{V_T}{i_C}\right)}}$$

Diff. With v_S on both sides

(fg)'=(f)'g+f(g)'

(f(g))'=(f(g))'g'

Determination of $f^{(2)}$:

$$1 = \left(\frac{R_x}{\beta}+R_y+\frac{V_T}{i_C}\right)\frac{\partial i_C}{\partial v_s}$$

$$\Downarrow$$

$$0 = \left(\frac{R_x}{\beta}+R_y\right)\frac{\partial^2 i_C}{\partial v_s^2} + \frac{V_T}{i_C}\frac{\partial^2 i_C}{\partial v_s^2} + \frac{\partial i_C}{\partial v_s}\left(-\frac{V_T}{i_C^2}\right)\frac{\partial i_C}{\partial v_s}$$

$$\Downarrow$$

$$0 = \left(\frac{R_x}{\beta}+R_y+\frac{V_T}{i_C}\right)\frac{\partial^2 i_C}{\partial v_s^2} - \left(\frac{\partial i_C}{\partial v_s}\right)^2\left(\frac{V_T}{i_C^2}\right)$$

$$\Downarrow$$

$$\boxed{f^{(2)}(i_C) = \frac{(f^1)^2\frac{V_T}{i_C^2}}{\frac{R_x}{\beta}+R_y+\frac{V_T}{i_C}}}$$

Diff. With v_s on both sides one more time

(fg)'=(f)'g+f(g)'

(f(g))'=(f(g))'g'

Improving the THD for a BJT

We have now determined $f^{(1)}$ and of $f^{(2)}$ and are thus able to assess which effect *Rx* and *Ry* might have on, for example, HD2

$$\Downarrow$$

$$\boxed{f^{(1)}(i_C) = \frac{1}{\left(\frac{R_x}{\beta}+R_y+\frac{V_T}{i_C}\right)}}$$

$$\Downarrow$$

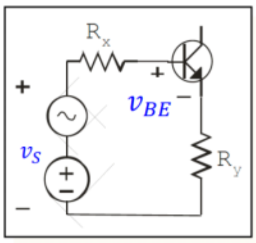
$$\boxed{f^{(2)}(i_C) = \frac{(f^1)^2\frac{V_T}{i_C^2}}{\frac{R_x}{\beta}+R_y+\frac{V_T}{i_C}}}$$

$$HD_2 = \frac{1}{4}\left|\frac{f^{(2)}}{f^{(1)}}\right|A = \frac{1}{4}\frac{\left(f^{(1)}\right)\frac{V_T}{i_C^2}}{\frac{R_x}{\beta}+R_y+\frac{V_T}{i_C}}A = \frac{1}{4}\frac{\frac{V_T^2}{i_C^2}}{\left(\frac{R_x}{\beta}+R_y+\frac{V_T}{i_C}\right)^2}\frac{A}{V_T}$$

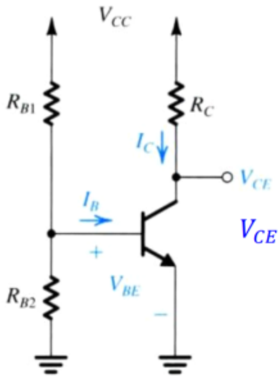
$$= \frac{1}{4}\frac{1}{\left(\frac{i_C}{V_T}\left(\frac{R_x}{\beta}+R_y\right)+1\right)^2}\frac{A}{V_T} = \frac{\frac{1}{4}\frac{A}{V_T}}{F^2}$$

$i_C = I_C + \Delta I_C \approx I_C$

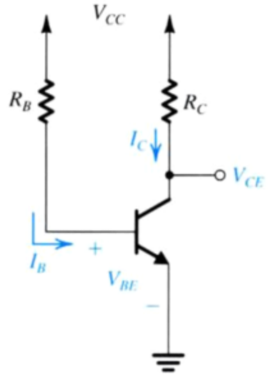
$$F = \frac{i_C}{V_T}\left(\frac{R_x}{\beta}+R_y\right)+1 = g_m\left(\frac{R_x}{\beta}+R_y\right)+1$$



Bias circuit – BJT



$V_{CE} = V_{CC} - I_C R_C$



Fix V_{BE} is not a good design:

$$I_C = I_s\, e^{\frac{V_{BE}}{V_T}}$$

- Small variation in V_{BE} ➔ large variation in I_C
- I_s and V_T is temperature dependent

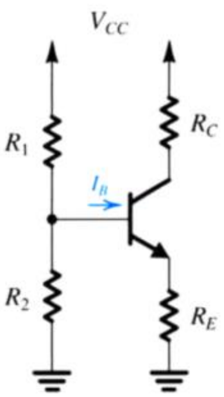
Fix I_B is not a good design:

$$I_C = \beta I_B$$

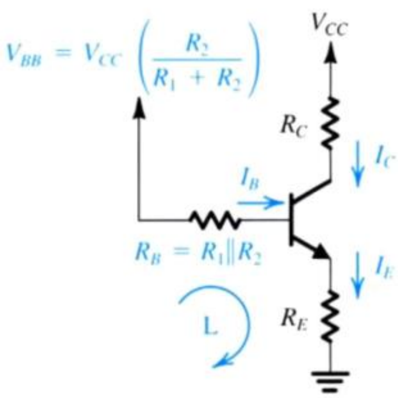
- Large variation in β among units of the same device type ➔ large variation in I_C

The classical discrete circuit bias design--BJT

Single-power supply



(a)



(b)

$$V_{BB} = V_{CC}\frac{R_2}{R_1 + R_2}$$

$$R_B = R_1||R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

To make I_E insensitive to

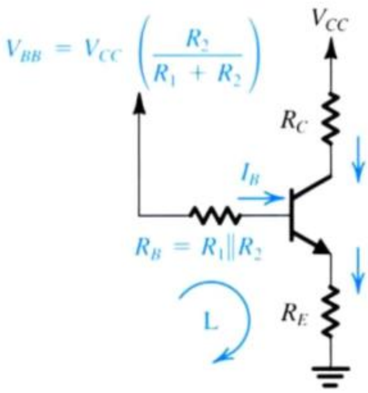
- Temperature (V_{BE}) ➔ $V_{BB} \gg$ VBE
- β ➔ $R_E \gg \frac{R_B}{\beta+1}$

Fix I_E is a good design: $I_E \approx \frac{V_{BB}}{R_E}$

The classical discrete circuit bias design -BJT

Stable operating point despite of uncertainty in V_{BE} & β

$$I_E = \frac{V_{BB}-V_{BE}}{R_E+\frac{R_B}{\beta+1}} \approx \frac{V_{BB}}{R_E}$$



Cond 1: $V_{BB} \gg V_{BE}$

$V_{CC} = V_{R_C} + V_{CB} + V_{BB}$

- $V_{BB} \uparrow$ → $(V_{R_C} + V_{CB})\downarrow$
- V_{R_C} needs be large to have a large $A_v \approx -g_m R_C = -\frac{I_C}{V_T} R_C = -\frac{V_{RC}}{V_T}$ (CE, R_E bypassed in AC)
- V_{CB} needs to be large → large signal swing

→ *trade-off* in designing V_{R_C} & V_{CB} & V_{BB}

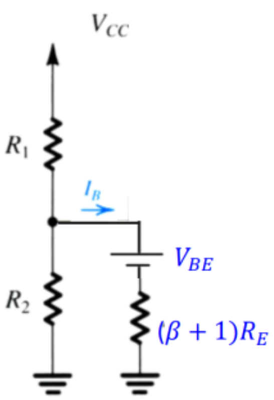
Cond 2: $R_E \gg \frac{R_B}{\beta+1}$

- small R_B → small R_1 & R_2 :
  - large bias current drained from V_{CC}
  - Smaller input impedance

→ *trade-off* in design R_1 & R_2

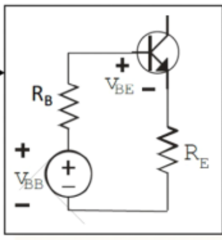
Rules of thumb for practical bias design: BJT amplifiers

Using these rules, a design based on data for a **BC547b** transistor can ensure ~ 3% spread at operating point due to V_{BE} and β tolerances



$$V_{BB} >> V_{BE} \Downarrow V_{BB} = 5 \cdot V_{BE}$$

→



→

$$V_{R_B} \approx 0$$
$$\Downarrow$$
$$V_{R_E} = V_{BB} - V_{BE} = 5 \cdot V_{BE} - V_{BE} = 4 \cdot V_{BE}$$
$$\Downarrow$$
$$\boxed{V_{R_E} \approx [2.4V, 3V]}$$

$$R_E >> \frac{R_B}{\beta} \Downarrow R_E \geq 10 \cdot \frac{R_B}{\beta}$$

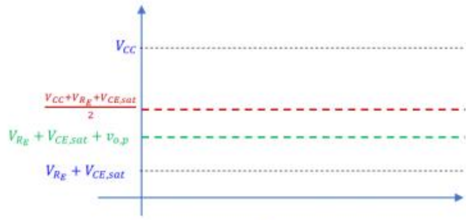
→

$$\boxed{R_B \leq \frac{\beta \cdot R_E}{10}}$$

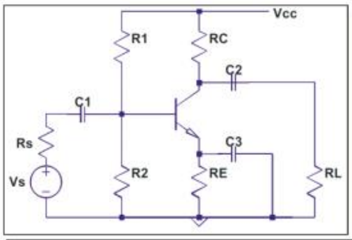
→

$$\boxed{R_B = R_1 \| R_2 \quad \Rightarrow \quad \begin{cases} I_{R_1} = 10 \cdot I_B \\ I_{R_2} = 9 \cdot I_B \end{cases}}$$

Design procedure - BJT



- Determine V_C or V_{R_C}: $V_{R_E}+V_{CE,sat} \le V_C \le V_{CC}$
  - *Maximum output swing case*: $V_C = \frac{V_{CC}+V_{R_E}+V_{CE,sat}}{2} \rightarrow V_{R_C} = V_{CC}-V_C = \frac{V_{CC}-(V_{R_E}+V_{CE,sat})}{2}$
  - *Known $v_{o,p}$ for highest A_v case*: $V_C = V_{CE,sat}+V_{R_E}+v_{o,p} \rightarrow V_{R_C} = V_{CC}-V_C = V_{CC}-V_{CE,sat}-V_{R_E}-v_{o,p}$
- Determine R_C to achieve maximum gain: $R_C = \sqrt{\frac{R_S' \cdot R_L \cdot V_{R_C}}{\beta V_T}} \approx \sqrt{\frac{R_S \cdot R_L \cdot V_{R_C}}{\beta V_T}}$, as $R_S \ll R_B$
- Determine $I_C = V_{R_C}/R_C$
- Determine $R_E = \frac{V_{R_E}}{I_C} = \frac{3V}{I_C}$
- Determine $R_B = \beta R_E/10$ -- to approx $R_E \gg R_B/\beta$
- Calculate $V_{BB} = I_C R_E + V_{BE} + \frac{I_C}{\beta} R_B$
- Determine R_1 and R_2: $R_B = \frac{R_1 R_2}{R_1+R_2}$ & $V_{BB} = V_{CC}\frac{R_2}{R_1+R_2}$
- Calculate the gain and compare with requirement
- Check harmonic distortion



Definitioner/antagelser:

$v_S' = \frac{R_B}{R_B+R_S} v_S$

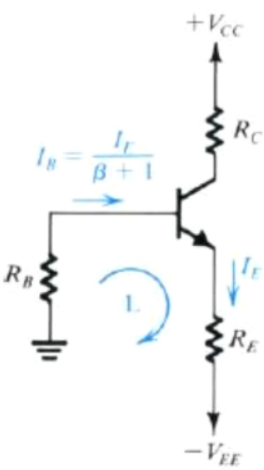
$R_S' = R_S \parallel R_B$

$r_o \gg R_C \parallel R_L$

$A_v = -\frac{R_{in}}{R_{in}+R_S} g_m (R_C||R_L)$ -- $R_{in} = R_1||R_2||r_\pi$

Or $A_v = A_v' \frac{R_B}{R_B+R_S}$ with $A_v' = \frac{R_C||R_L}{\frac{1}{g_m}+\frac{R_S'}{\beta}}$

Classical configuration with two power supplies



$$I_E = \frac{V_{EE}-V_{BE}}{R_E + \frac{R_B}{\beta+1}}$$

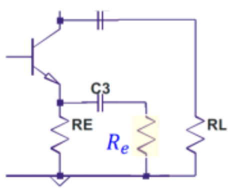
Cond 1: $V_{EE} \gg V_{BE}$ — Cond 2: $R_E \gg \frac{R_B}{\beta+1}$

$V_{EE} \ge 5V_{BE}$ — $R_E \ge 10\frac{R_B}{\beta}$

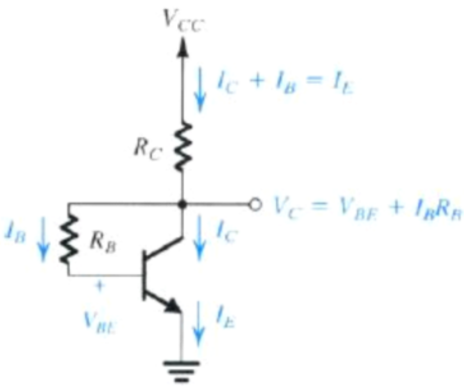
A good configuration:
a stable operating point that is robust to changes in V_{BE}, β and temperature.

$A_v = -g_m R_C$ -- without AC R_e

$A_v = -\frac{R_C}{\frac{1}{g_m}+R_e'}$ -- with AC R_e & $R_e' = R_E||R_e$



Biasing with collector-to-base feedback R_B



$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta + 1}}$$

Cond 1: $V_{CC} \gg V_{BE}$

Cond 2: $R_C \gg \frac{R_B}{\beta+1}$

$$R_C \geq 10\frac{R_B}{\beta}$$

- Small R_B → small signal swing, as $V_{CB} = I_B R_B$
- Large R_C → large V_{CC}

$A_v = -g_m(R_C||R_B)$ -- without AC R_e
Gain drops due to the feedback

Bais design - MOSFET

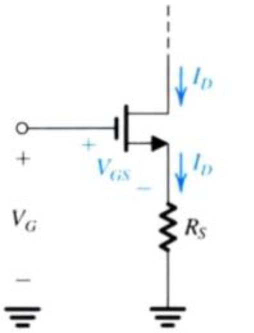
$$I_D = \frac{1}{2}k_n(V_{GS}-V_{TH})^2 = \frac{1}{2}\mu_n C_{ox} W/L\ (V_{GS}-V_{TH})^2$$

- μ_n: mobility of the electrons at the surface of the channel
- C_{ox}: oxide capacitance
- W & L: width & length of the channel

I_D variation:
- For different devices: V_{TH} , C_{ox} and W/L vary among devices even for devices with the same nominal values due to fabrication.
- For the same device due to temperature

Fix V_{GS} is not good.

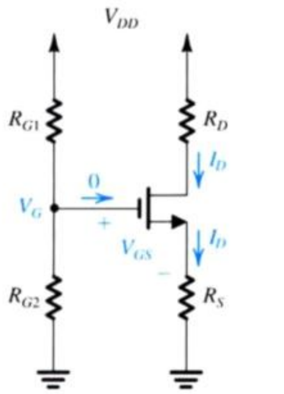
Bias design - MOSFET



Fix V_G and connecting R_S in source lead

$V_G = V_{GS} + I_D R_S \approx I_D R_S$

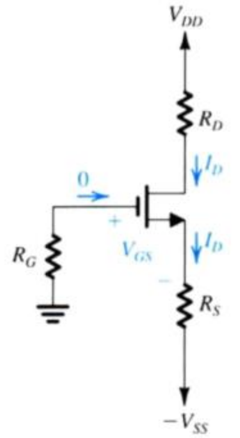
Cond : $V_G \gg V_{GS}$



$$V_G = V_{DD} \frac{R_{G_2}}{R_{G_1} + R_{G_2}}$$

$V_G = V_{GS} + I_D R_S$

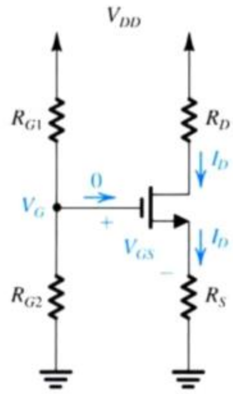
Cond : $V_G \gg V_{GS}$



$V_{SS} = V_{GS} + I_D R_S \approx I_D R_S$

Cond : $V_{SS} \gg V_{GS}$

Bias design procedure - MOSFET



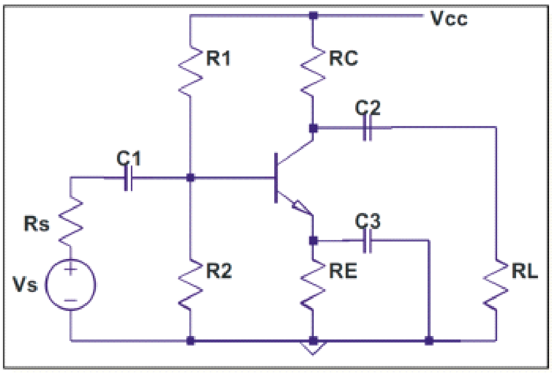
- Determine V_{GS} , for a required $THD = HD_2 = \frac{A}{4(V_{GS}-V_{TH})}$, A input amplitude
- Determine $\boldsymbol{I_D} = \frac{1}{2} k_n (V_{GS} - V_{TH})^2$, k_n and V_{TH} from datasheet
- Determine $V_G = 5\, V_{GS}$ to approx $V_G \gg V_{GS}$
- Determine $\mathrm{R_S} = \frac{\mathrm{V_G - V_{GS}}}{\mathrm{I_D}}$
- Determine V_D or V_{R_D}: $V_G \le V_D \le V_{DD}$
  - *Maximum output swing case:* $V_D = \frac{V_{DD}+V_G}{2} \rightarrow V_{R_D} = V_{DD} - V_D = \frac{V_{DD}-V_G}{2}$
  - *Known $v_{o,p}$ for highest A_v case:* $V_D = V_G + v_{o,p} \rightarrow V_{R_D} = V_{DD} - V_D = V_{DD} - V_G - v_{o,p}$
- Determine $R_D = \frac{V_{R_D}}{I_D}$
- Determine R_{G_1} and R_{G_1} in $\mathrm{M\Omega}$ *range (provide high input impedance) using* $V_G = V_{DD} \frac{R_{G_2}}{R_{G_1}+R_{G_2}}$
- Calculate the gain and compare with requirement
- Check harmonic distortion

11.1:

Design an amplifier by using a BC547B transistor that can provide a voltage gain of $A_v = -50$. $R_S = 1\ K\Omega$, $R_L = 10\ K\Omega$, $V_{CC} = 15$ V and the output swing peak value $v_{o,p} = 3$ V. In addition, we assume that $\beta = 300$, $V_{CE,sat} \leq 0.3$ V and $r_o \gg R_L$. Try to design the amplifier with lowest distortion for the given gain requirement.

Verify using LTspice gain and distortion for your design when fully equipped and optimize. If necessary, modify your design and verify again gain and distortion. 1kHz is used as test frequency.

Solution:



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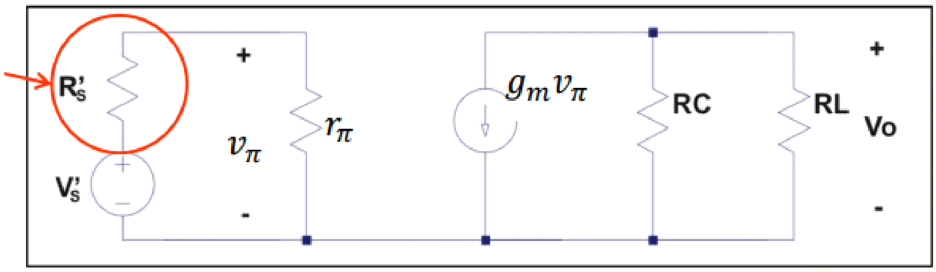
$$R_S' = R_S \parallel R_B$$

$$r_o >> R_C \parallel R_L$$

$$R_B = R_1 || R_2$$



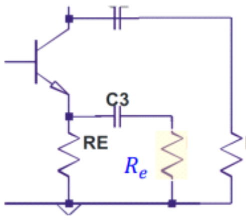
1. Determine $V_{R_C} = V_{CC} - V_{R_E} - V_{CE,sat} - V_{o,p} = 15 - 3 - 0.3 - 3 = 8.7$ V
2. Determine optimal R_C to maximize gain for the fixed V_{R_C}:

$$R_C = \sqrt{\frac{R_S' R_L V_{R_C}}{\beta V_T}} \approx \sqrt{\frac{R_S R_L V_{R_C}}{\beta V_T}} = 3.34\ K\Omega$$

3. Determine $I_C = \frac{V_{R_C}}{R_C} = \frac{8.7\ V}{3.34\ K\Omega} = 2.6\ mA$
4. Determine $R_E = \frac{V_{R_E}}{I_C} = \frac{3\ V}{2.6\ mA} \approx 1.2\ K\Omega$
5. Determine $R_B \approx \frac{\beta}{10} R_E = 36\ K\Omega$
6. Calculate $R_S' = R_S || R_B \approx 0.973\ K\Omega$, recalculate $R_C = \sqrt{\frac{R_S' R_L V_{R_C}}{\beta V_T}} = 3.29\ K\Omega$
7. The gain $A_v = \frac{v_o}{v_s} = \frac{v_o}{v_s'} \cdot \frac{v_s'}{v_s} = A_v' \cdot \frac{R_B}{R_B + R_S} = -\frac{1}{\left(\frac{1}{g_m} + \frac{R_S'}{g_m r_\pi}\right)\left(\frac{1}{R_C} + \frac{1}{R_L}\right)} * \frac{R_B}{R_B + R_S} = -181.9$

   $|A_v|$ is much larger than 50. So we can add a resistor in emitter to reduce the total harmonic distortion.

8. Determine the resistance R_e': $A_v = \frac{v_o}{v_s} = \frac{v_o}{v_s'} \cdot \frac{v_s'}{v_s} = A_v' \cdot \frac{R_B}{R_B + R_S} = -50$ ➔$A_v' =$
   $-50 \cdot \frac{36+1}{36} = -51.39 = -(R_C||R_L)/(\frac{1}{g_m} + R_e' + \frac{R_S'}{\beta})$ ➔$R_e' = -\frac{(R_C||R_L)}{A_v'} - \frac{1}{g_m} -$
   $\frac{R_S'}{\beta} = 34.9\ \Omega$. $R_e' = R_E || R_e$ ➔$R_e = \frac{1}{\frac{1}{R_e'} - \frac{1}{R_E}} = 35.9\Omega \approx 36\ \Omega$



9. Calculate the amplitude (i.e. peak value) of the v_S', $v_{s,p}' = \frac{v_{o,p}}{A_v'} = \frac{3\ V}{51.39} = 58.4\ mV$.

   Calculate the harmonic distortion term $F = 1 + g_m \left(\frac{R_S'}{\beta} + R_e'\right) = 1 +$
   $\frac{I_C}{V_T}\left(\frac{R_S'}{\beta} + R_e'\right) = 4.81$ ➔$HD_2 = \frac{\frac{1}{4}\frac{v_{s,p}'}{V_T}}{F^2} = 2.42\%$

10. Determine R_1 and R_2.

$$V_{BB} = I_C R_E + V_{BE} + \frac{I_C}{\beta} R_B = 4.13\ V$$
$$R_B = \frac{R_1 R_2}{R_1 + R_2} \text{ and } V_{BB} = V_{CC}\frac{R_2}{R_1 + R_2}$$
➔$R_1 = \frac{V_{CC}}{V_{BB}} R_B = 130.1\ K\Omega$ and $R_2 = R_1 \frac{V_{BB}}{V_{CC} - V_{BB}} = 49.4\ K\Omega$

11. Check by the LTspice
    Is the gain larger or smaller than 50?
    If it is smaller, try to modify R_e to have the gain >=50.