+ SOLVE y(n)^2

Sunday, 2 June 2024 12.18

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Convolusion not multiplien

1) Rewrite for $n \to k$

$$x(n) = 5u[n] \implies x(k) = 5u[k]$$

$$h(n) = 3^n u[n-1] \implies h(k) = 3^{n-k} u[n-1-k]$$

2) Insert in Σ

$$\sum_{k=-\infty}^{\infty} x(k)\, h(k) = \sum_{k=-\infty}^{\infty} 5u[k] \cdot 3^{n-k} u[n-1-k]$$

$$\frac{3^n}{3^k}$$

3) Extract constants out of Σ

$$\sum_{k=-\infty}^{\infty} 5u[k] \cdot \frac{3^n}{3^k} \cdot u[n-1-k]$$

$$5 \cdot 3^n \sum_{k=-\infty}^{\infty} u[k] \cdot \frac{1}{3^k} \cdot u[n-1-k]$$

$u[k]$ enten 1 eller 0
$u[n-1-k]$ enten 1 eller 0

afskriver jeg dem

$$5 \cdot 3^n \sum \frac{1}{3^k}$$

4) Quick removal of $\sum$

$$\sum_{n=0}^{n-1} r^k = \frac{1-r^n}{1-r} \quad \text{for } r \neq 1$$

$$5 \cdot 3^n \sum \frac{1}{3^k} \qquad \frac{1}{3^k} = \frac{1^k}{3^k} = \left(\frac{1}{3}\right)^k$$

$$5 \cdot 3^n \sum \left(\frac{1}{3}\right)^k = \boxed{5 \cdot 3^n \cdot \frac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}}$$

This you can maple plugg!

$$\frac{1-\left(\frac{1}{3}\right)^n}{\frac{2}{3}}$$

as $1-\frac{1}{3} = \frac{3}{3} - \frac{1}{3} = \frac{2}{3}$

$$5 \cdot 3^n \frac{1-\left(\frac{1}{3}\right)^n}{\frac{2}{3}} = \frac{5 \cdot 3^n}{2/3} - \frac{5 \cdot 3^n \left(\frac{1}{3}\right)^n}{2/3}$$

$$\frac{5 \cdot 3^n \cdot 3}{2} - \frac{5 \cdot 3^n \cdot 3 \left(\frac{1}{3}\right)^n}{2}$$

$$\frac{5 \cdot 3^n \cdot 3 \left(\frac{1^n}{3^n}\right)}{2} \rightarrow \frac{5 \cdot 3 \cdot 3^n \left(\frac{1}{3^n}\right)}{2} = \frac{5 \cdot 3 \left(\frac{3^n}{3^n}\right)}{2} = \frac{5 \cdot 3}{2}$$

$$\frac{5 \cdot 3 \cdot 3^n}{2} - \frac{5 \cdot 3}{2} = \frac{15}{2} 3^n - \frac{15}{2}$$

5) Back sub u[n] into it

$$\frac{15}{2} 3^n u[n] - \frac{15}{2} u[n]$$

And this is the answer ☺