REGNEREGLER

Thursday, 30 May 2024 10.11

Rules of Exponents or Laws of Exponents	
Multiplication Rule	$a^x \times a^y = a^{x+y}$
Division Rule	$a^x \div a^y = a^{x-y}$
Power of a Power Rule	$\left(a^x\right)^y = a^{xy}$
Power of a Product Rule	$(ab)^x = a^x b^x$
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-x} = \frac{1}{a^x}$
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

$$\sum_{n=0}^{n-1} r^k = \frac{1-r^n}{1-r} \quad \text{for } r \neq 1$$

Summation Shortcuts

$$\sum_{n=1}^{k} n = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$$

$$\sum_{n=1}^{k} n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6}$$

$$\sum_{n=1}^{k} n^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4+2n^3+n^2}{4}$$

$$\sum_{n=1}^{k} n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{6n^5+15n^4+10n^3-n}{30}$$

$$\sum_{n=1}^{k} n^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12} = \frac{2n^6+6n^5+5n^4-n^2}{12}$$

Brøker

Brøker er tal på formen

$$\frac{a}{b},$$

hvor a, b er tal samt $b \neq 0$. a er *tælleren* og b er *nævneren*.

Regneregler

Der gælder

$$\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}, \quad \frac{a}{b}\frac{c}{d} = \frac{ac}{bd}, \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc},$$

$$a\frac{b}{c} = \frac{ab}{c}, \quad \frac{\frac{a}{b}}{c} = \frac{a}{bc}, \quad \frac{a}{\frac{b}{c}} = \frac{ac}{b}.$$

Forkorte/Forlænge Brøker

Fælles faktorer kan forkortes:

$$\frac{a}{b} = \frac{ac}{bc}$$

Potenser

Potenser er tal på formen x^a, x er *grundtallet* og a er *eksponenten*.

Regneregler

Der gælder

$$x^a x^b = x^{a+b}, \quad \frac{x^a}{x^b} = x^{a-b}, \quad (xy)^a = x^a y^a,$$

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}, \quad (x^a)^b = x^{ab}, \quad x^{-a} = \frac{1}{x^a}.$$

Rødder

Hvis $x \geq 0$ og $n \in \mathbf{Z}_+$ så findes et tal $\sqrt[n]{x} > 0$ så

$$(\sqrt[n]{x})^n = x.$$

Bemærk at $\sqrt[n]{x} = x^{\frac{1}{n}}$.

Regneregler

Der gælder

$$\sqrt[n]{x} = x^{\frac{1}{n}}, \quad \sqrt[n]{x^m} = x^{\frac{m}{n}} = (\sqrt[n]{x})^m,$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}, \quad \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}.$$

Kvadratsætninger

Der gælder

$$(a+b)^2 = a^2 + b^2 + 2ab$$
$$(a-b)^2 = a^2 + b^2 - 2ab$$
$$(a+b)(a-b) = a^2 - b^2.$$

Ligninger

Ligninger kan reduceres med følgende regler:

1. Man må lægge til/trække fra med det samme tal på begge sider af et lighedstegn.
2. Man må gange/dividere med det samme tal (undtagen 0) på begge sider af et lighedstegn.

Andengradsligninger

Andengradsligninger er på formen

$$ax^2 + bx + c = 0, \tag{1}$$

Løsningerne til (1) er

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Faktorisering

Hvis $ax^2 + bx + c = 0$ har rødder r_1 og r_2 så gælder.

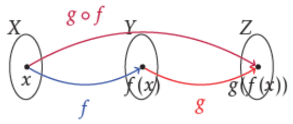
$$ax^2 + bx + c = a(x - r_1)(x - r_2).$$

Funktioner

En funktion $f : X \to Y$ tildeler alle $x \in X$ *præcis ét* element $f(x) \in Y$.

Sammensatte funktioner

Hvis $f : X \to Y$ og $g : Y \to Z$ defineres sammensætningen $g \circ f : X \to Z$ ved $(g \circ f)(x) = g(f(x))$. f er den *indre funktion*, g er den *ydre funktion*



Inverse funktioner

To funktioner $f : X \to Y$ og $g : Y \to X$ er hinandens *inverse* hvis

$$f(g(y)) = y, \quad \text{og} \quad g(f(x)) = x$$

for alle x i X og y i Y.

Polynomier

Et førstegradspolynomium har forskrift:

$$f(x) = ax + b.$$

Et andengradspolynomium har forskrift:

$$f(x) = ax^2 + bx + c.$$

Logaritmer og eksponentialfunktioner

Logaritmen med grundtal a, $\log_a :]0, \infty[\to \mathbf{R}$ er invers til eksponentialfunktionen $f_a(x) = a^x$ ($a > 0$, $a \neq 1$). Der gælder at

$$\log_a(a^x) = x \quad \text{og} \quad a^{\log_a(y)} = y$$

og vi har

$$\ln x = \log_e x, \qquad \log x = \log_{10} x$$

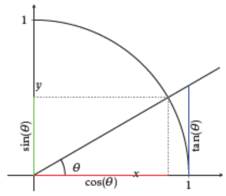
Regneregler

Der gælder

$$\log_a(xy) = \log_a(x) + \log_a(y),$$
$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y),$$
$$\log_a(x^r) = r\log_a(x).$$

Trigonometriske funktioner

De trigonometriske funktioner er defineret ud fra enhedscirklen:



Der gælder at $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ samt

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-

Differentialregning

Den afledede af f skrives som $f' = \frac{d}{dx}f = \frac{df}{dx}$.

Regneregler

Der gælder at

$f(x)$	$f'(x)$
c	0
x	1
x^n	nx^{n-1}
e^x	e^x
e^{cx}	ce^{cx}
a^x	$a^x \ln a$
$\ln x$	$\frac{1}{x}$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$1 + \tan^2(x)$

Generelle regneregler

Der gælder at

$$(cf)'(x) = cf'(x)$$
$$(f \pm g)'(x) = f'(x) \pm g'(x)$$
$$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$$
$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$
$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x).$$

Den sidste regneregel kaldes *kædereglen*.

Ubestemte integraler

En funktion f har *stamfunktion* F hvis

$$F'(x) = f(x).$$

Det ubestemte integral af f er

$$\int f(x)\,dx = F(x) + k,$$

hvor $F'(x) = f(x)$ og $k \in \mathbf{R}$.

Generelle regneregler

$$\int cf(x)\,dx = c\int f(x)\,dx$$

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx.$$

$$\int f(x)g(x)\,dx = f(x)G(x) - \int f'(x)G(x)\,dx$$

$$\int f(g(x))g'(x)\,dx = F(g(x)) + k.$$

Den 3. regel kaldes *delvis integration* og den sidste kaldes *integration ved substitution*.

Regneregler

Der gælder at

$f(x)$	$\int f(x)\,dx$
c	$cx + k$
x	$\frac{1}{2}x^2 + k$
x^n	$\frac{1}{n+1}x^{n+1} + k$
e^x	$e^x + k$
e^{cx}	$\frac{1}{c}e^{cx} + k$
$\frac{1}{x}$	$\ln(\lvert x\rvert) + k$
$\ln x$	$x\ln(x) - x + k$
$\cos x$	$\sin x + k$
$\sin x$	$-\cos x + k$
$\tan x$	$-\ln(\lvert\cos(x)\rvert) + k$

Integration ved substitution

Givet et integral på formen $\int f(g(x))g'(x)\,dx$ anvendes metoden:

1. Lad $u = g(x)$.
2. Udregn $\frac{du}{dx}$ og isoler dx.
3. Substituer $g(x)$ og dx.
4. Udregn integralet mht. u.
5. Substituer tilbage.

Besemte integraler

Det bestemte integral af f i intervallet $[a,b]$ til

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a),$$

hvor F er en stamfunktion til f.

Generelle regneregler

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b f(x)g(x)\,dx = [f(x)G(x)]_a^b - \int_a^b f'(x)G(x)\,dx$$

$$\int_a^b f(g(x))g'(x)\,dx = [F(x)]_{g(a)}^{g(b)}.$$

Integration ved substitution

Givet et integral på formen $\int_a^b f(g(x))g'(x)\,dx$ anvendes metoden

1. Lad $u = g(x)$.
2. Udregn $\frac{du}{dx}$ og isoler dx.
3. Substituer $g(x)$, dx samt grænser.
4. Udregn integralet mht. u.

Differentialligninger

Løsningsformler

Differentiallign.	Fuldstændig løsn.
$f'(x) = k$	$f(x) = kx + c$
$f'(x) = h(x)$	$f(x) = \int h(x)\,dx$
$f'(x) = kf(x)$	$f(x) = ce^{kx}$
$f'(x) + af(x) = b$	$f(x) = \frac{b}{a} + ce^{-ax}$

Panserformlen

Differentialligningen

$$f'(x) + a(x)f(x) = b(x)$$

har fuldstændig løsning

$$f(x) = e^{-A(x)}\int b(x)e^{A(x)}\,dx + ce^{-A(x)},$$

hvor $A'(x) = a(x)$.

Vektorer i planen

En vektor $\vec{u}$ i planen skrives som $\vec{u} = [x,y]$ hvor $x,y \in \mathbf{R}$.

Regneregler

For $\vec{u} = [x_1,y_1]$, $\vec{v} = [x_2,y_2]$, $c \in \mathbf{R}$ er

$$\vec{u} \pm \vec{v} = \begin{bmatrix} x_1 \pm x_2 \\ y_1 \pm y_2 \end{bmatrix}, \qquad \vec{u}\cdot\vec{v} = x_1x_2 + y_1y_2,$$

$$c\vec{u} = \begin{bmatrix} cx_1 \\ cy_1 \end{bmatrix}, \quad \det(\vec{u},\vec{v}) = x_1y_2 - x_2y_1$$

Længden af $\vec{u}$ er $\|\vec{u}\| = \sqrt{x_1^2 + y_1^2}$.

Vinklen mellem to vektorer

For vinklen θ mellem $\vec{u}$, $\vec{v}$ er

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}, \quad \sin\theta = \frac{\det(\vec{u},\vec{v})}{\|\vec{u}\|\,\|\vec{v}\|}$$

Yderligere gælder

1. $\vec{u}$ og $\vec{v}$ er ortogonale $\Leftrightarrow \vec{u}\cdot\vec{v} = 0$.
2. $\vec{u}$ og $\vec{v}$ er parallelle $\Leftrightarrow \det(\vec{u},\vec{v}) = 0$.

Vektorer i rummet

En vektor $\vec{u}$ i rummet skrives som $\vec{u} = [x,y,z]$ hvor $x,y,z \in \mathbf{R}$.

Regneregler

For $\vec{u} = [x_1,y_1,z_1]$, $\vec{v} = [x_2,y_2,z_2]$ og $c \in \mathbf{R}$ gælder

$$\vec{u} \pm \vec{v} = \begin{bmatrix} x_1 \pm x_2 \\ y_1 \pm y_2 \\ z_1 \pm z_2 \end{bmatrix}, \qquad c\vec{u} = \begin{bmatrix} cx_1 \\ cy_1 \\ cz_1 \end{bmatrix},$$

$$\vec{u}\cdot\vec{v} = x_1x_2 + y_1y_2 + z_1z_2.$$

Længden af $\vec{u}$ er $\|\vec{u}\| = \sqrt{x_1^2 + y_1^2 + z_1^2}$.
Krydsproduktet er givet ved

$$\vec{u}\times\vec{v} = \begin{bmatrix} y_1z_2 - z_1y_2 \\ z_1x_2 - x_1z_2 \\ x_1y_2 - y_1x_2 \end{bmatrix}$$

Vinklen mellem to vektorer

For vinklen θ mellem $\vec{u}$ og $\vec{v}$ gælder

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}, \quad \sin\theta = \frac{\|\vec{u}\times\vec{v}\|}{\|\vec{u}\|\,\|\vec{v}\|}$$

Yderligere gælder

1. $\vec{u}$ og $\vec{v}$ er ortogonale $\Leftrightarrow \vec{u}\cdot\vec{v} = 0$.
2. $\vec{u}$ og $\vec{v}$ er parallelle $\Leftrightarrow \vec{u}\times\vec{v} = 0$.

Linjer og Planer

Planen/linjen gennem punktet med stedvektor $\vec{x}_0$ med normalvektor $\vec{n}$ beskrives ved alle vektorer $\vec{x}$ der løser ligningen

$$\vec{n}\cdot(\vec{x} - \vec{x}_0) = 0$$

En linje i rummet/planen gennem punktet med stedvektor $\vec{x}_0$ og retning $\vec{r}$ har parameterfremstilling

$$\vec{x}_0 + t\vec{r}, \quad t \in \mathbf{R}.$$

+ TYPES OF MATRIXES

Friday, 31 May 2024 12.29

Special matrices (square)

Diagonal

$$\begin{bmatrix} a & & & \bar{\bar{0}} \\ & b & & \\ & & c & \\ \bar{\bar{0}} & & & \ddots \; z \end{bmatrix}$$

Triangular

lower/upper

$$\begin{bmatrix} 0 & & \bar{\bar{d}} \\ \vdots & \ddots & \\ 0 & \cdots & 0 \end{bmatrix}$$

Scalar

$$\begin{bmatrix} k & & & \bar{\bar{0}} \\ & k & & \\ & & \ddots & \\ \bar{\bar{0}} & & & k \end{bmatrix} = k \cdot \bar{\bar{I}}$$

Identity

$$\begin{bmatrix} 1 & & & \bar{\bar{0}} \\ & 1 & & \\ & & \ddots & \\ \bar{\bar{0}} & & & 1 \end{bmatrix} = \bar{\bar{I}}$$

Null (in various forms) $\bar{\bar{0}}$?

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Transposée d'une matrice carrée

$$\begin{pmatrix} 1 & 5 \\ 6 & 8 \end{pmatrix}^T = \begin{pmatrix} 1 & 6 \\ 5 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 7 & 5 \\ 1 & 0 & 7 \\ 4 & 2 & 6 \end{pmatrix}^T = \begin{pmatrix} 9 & 1 & 4 \\ 7 & 0 & 2 \\ 5 & 7 & 6 \end{pmatrix}$$

Étapes :

- La 1ère ligne devient la 1ère colonne
- La 2 -ème ligne devient la 2 -ème colonne
- La 3 -ème ligne devient la 3 -ème colonne
- Ainsi de suite…

$$\begin{matrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{matrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}^T$$

Special matrices (square)

Symmetric $\bar{\bar{A}}^T = \bar{\bar{A}}$

e.g. $\begin{bmatrix} 5 & -1 & 0 \\ -1 & 9 & 5 \\ 0 & 5 & 7 \end{bmatrix}$

Skew-symmetric $\bar{\bar{A}}^T = -\bar{\bar{A}}$

e.g. $\begin{bmatrix} 0 & -6 & 2 \\ 6 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$

OBS! what's particular about it?

Taxonomy for normal matrices

Reel matrix	Kompleks matrix	Generel	Normal[2]	Egenværdier
Symmetrisk $A^T = A$	Hermitesk (diagonal er reel) $A^{*T} = A$	Selvadjungeret[1]	Ja	Reelle (inkl. 0)
Skævsymmetrisk (diagonal = 0) $A^T = -A$	Skævhermitesk (diagonal imaginær eller 0) $A^{*T} = -A$	Skævadjungeret	Ja	Imaginære (inkl. 0)
Ortogonal ($\Delta A = \pm 1$) $A^T = A^{-1}$	Unitær ($\lvert \Delta A \rvert = 1$) $A^{*T} = A^{-1}$	Isometrisk	Ja	Absolut værdi 1

(1) Den (komplekst) konjugerede transponerede, A^{*T}, kaldes for den adjungerede til A (og deraf betegnelsen selvadjungeret i dette tilfælde); mere formelt kaldes den komplekst konjugerede transponerede for den hermitesk adjungerede, hvor hermitesk adjungering er analogt til kompleks konjugering.

(2) En normal matrix er en (generelt) kompleks kvadratisk matrix der kommuterer med sin adjungerede, dvs. opfylder $A^{*T}A = AA^{*T}$.

$\Delta \bar{\bar{A}} = \det(\bar{\bar{A}})$

Example [edit]

For example, the following matrix is skew-Hermitian

$$A = \begin{bmatrix} -i & +2+i \\ -2+i & 0 \end{bmatrix}$$

because

$$-A = \begin{bmatrix} i & -2-i \\ 2-i & 0 \end{bmatrix} = \begin{bmatrix} \overline{-i} & \overline{-2+i} \\ \overline{2+i} & \overline{0} \end{bmatrix} = \begin{bmatrix} \overline{-i} & \overline{2+i} \\ \overline{-2+i} & \overline{0} \end{bmatrix}^{\mathsf{T}} = A^{\mathsf{H}}$$

Basic remarks [edit]

A square matrix $\mathbf{A}$ with entries a_{ij} is called

- Hermitian or self-adjoint if $\mathbf{A} = \mathbf{A}^{\mathrm{H}}$; i.e., $a_{ij} = \overline{a_{ji}}$.
- Skew Hermitian or antihermitian if $\mathbf{A} = -\mathbf{A}^{\mathrm{H}}$; i.e., $a_{ij} = -\overline{a_{ji}}$.
- Normal if $\mathbf{A}^{\mathrm{H}}\mathbf{A} = \mathbf{A}\mathbf{A}^{\mathrm{H}}$.
- Unitary if $\mathbf{A}^{\mathrm{H}} = \mathbf{A}^{-1}$, equivalently $\mathbf{A}\mathbf{A}^{\mathrm{H}} = \boldsymbol{I}$, equivalently $\mathbf{A}^{\mathrm{H}}\mathbf{A} = \boldsymbol{I}$.

Even if $\mathbf{A}$ is not square, the two matrices $\mathbf{A}^{\mathrm{H}}\mathbf{A}$ and $\mathbf{A}\mathbf{A}^{\mathrm{H}}$ are both Hermitian and in fact positive semi-definite matrices.

Eigenvalues (square matrices)

Hermetian C^n (symmetric, R^n) $\quad \bar{\bar{A}}^{*T} = \bar{\bar{A}}$

λ_i real (0)

Skew-Hermetian C^n (skew-symmetric, R^n) $\quad \bar{\bar{A}}^{*T} = -\bar{\bar{A}}$

λ_i imaginary (0)

Unitary C^n (orthogonal, R^n) $\quad \bar{\bar{A}}^{*T} = \bar{\bar{A}}^{-1}$

$|\lambda_i| = 1$

* complex conjugate

Invers matrix

$$\bar{\bar{A}} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|ccc} a_1 & a_2 & a_3 & 1 & 0 & 0 \\ b_1 & b_2 & b_3 & 0 & 1 & 0 \\ c_1 & c_2 & c_3 & 0 & 0 & 1 \end{array}\right]$$

Så laver man reduced row echelon på den til man for

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & A_1 & A_2 & A_3 \\ 0 & 1 & 0 & B_1 & B_2 & B_3 \\ 0 & 0 & 1 & C_1 & C_2 & C_3 \end{array}\right] \rightarrow = \bar{\bar{A}}^{-1}$$

Pa

$$\bar{\bar{A}}\,\bar{\bar{A}}^{-1} = \bar{\bar{I}}$$

Identitets matrica

Eigenvalues (square matrices)

Hermetian C^n (symmetric, R^n) $\quad \bar{\bar{A}}^{*T} = \bar{\bar{A}}$

λ_i real (0)

Skew-Hermetian C^n (skew-symmetric, R^n) $\quad \bar{\bar{A}}^{*T} = -\bar{\bar{A}}$

λ_i imaginary (0)

Unitary C^n (orthogonal, R^n) $\quad \bar{\bar{A}}^{*T} = \bar{\bar{A}}^{-1}$

$|\lambda_i| = 1$

* complex conjugate

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Find Eigen values and basis

Eks

$$\bar{\bar{A}} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}$$

$$\det(\bar{\bar{A}}) = (a-\lambda)(d-\lambda) - (c \cdot b)$$

Som man isolere og for λ_1 og λ_2

DETTE ER EIGEN VÆRDIER

Eigenbase for λ_1

$$\begin{bmatrix} a-\lambda_1 & b \\ c & d-\lambda_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$$\left.\begin{aligned} a-\lambda_1 \underline{x_1} + b\underline{x_2} &= 0 \\ c\underline{x_1} + d-\lambda_1 \underline{x_2} &= 0 \end{aligned}\right\}$$

Så løser man for $\underline{x_1}$ og $\underline{x_2}$

hvor $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ er vores svar for λ_1

Og så gør man det samme for λ_2

Diagonalisering!

for eig1

for eig2

$$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \qquad P = \begin{bmatrix} x_1\lambda_1 & x_1\lambda_2 \\ x_2\lambda_1 & x_2\lambda_2 \end{bmatrix}$$

Pure eigenvalues

Pure eigenbasis

$$\bar{\bar{D}} = \bar{\bar{X}}^{-1}\,\bar{\bar{A}}\,\bar{\bar{X}} = \bar{\bar{P}}^{-1}\,\bar{\bar{A}}\,\bar{\bar{P}}$$

Diagonalization

$$\bar{\bar{D}} = \bar{\bar{X}}^{-1} \bar{\bar{A}} \bar{\bar{X}}$$

Husk Rækkefølgen!

$$\begin{bmatrix} \lambda_1 & & \bar{\bar{0}} \\ & \ddots & \\ \bar{\bar{0}} & & \lambda_n \end{bmatrix}$$ (diagonal) matrix of eigenvalues

matrix of eigenvectors $[\bar{X}_1, \bar{X}_2, \ldots \bar{X}_n]$

$\bar{\bar{X}}$ is said to diagonalize $\bar{\bar{A}}$

$\bar{\bar{D}}$ and $\bar{\bar{A}}$ are called similar* in that they have the same eigenvalues

(* generally, any non-singular matrix $\bar{\bar{X}}$ soffice

FIND D	FROM D $\Rightarrow$ A
$\bar{\bar{D}} = \bar{\bar{P}}^{-1} \bar{\bar{A}} \bar{\bar{P}}$	$\bar{\bar{A}} = \bar{\bar{P}} \bar{\bar{D}} \bar{\bar{P}}^{-1}$
Husk rækkefølgen!	Husk rækkefølgen!

Spectral radius is the LARGEST abs. value of your D eigenvectors in that form

So for $(\sqrt{-1}\, A) = (\sqrt{-1}\, D)$ as they are unitary the same as we can diagonalize it

$(\sqrt{-1} \cdot 0)^5 = 0$; $(\sqrt{-1} \cdot 4)^5 = 1024 I$; $(\sqrt{-1} \cdot -4)^5 = -1024 I$

So spectre is $\underline{\underline{1024}}$

[illegible] spec[illegible] [illegible] [illegible]

+ SOLVE of ODE'S

Wednesday, May 29, 2024 9:18 AM

$$x^{(4)} - 7x^{(3)} + 4x^{(2)} + 5x^{(1)} - 2x = 0$$

$$x^{(4)} = +2x - 5x^{(1)} - 4x^{(2)} + 7x^{(3)}$$

$x = v_1$ $\quad x^{(1)} = v_2$ $\quad x^{(2)} = v_3$ $\quad x^{(3)} = v_4$ $\quad x^{(4)} = v_5$

$$x^{(4)} = +2v_1 - 5v_2 - 4v_3 + 7v_4 = v_4'$$

$x = v_1$ $\quad x' = v_2 = v_1'$ $\quad x^2 = v_3 = v_2'$ $\quad x^3 = v_4 = v_3'$ $\quad x^4 = v_5 = v_4'$

$$v_1' = -v_2$$

$$v_2' = -v_3$$

$$v_3' = v_4$$

$$v_4' = 2v_1 - 5v_2 - 4v_3 + 7x_4$$

$$y'' + 4y = 0$$

$$y'' = -4y$$

$Y = v_1$

$y' = v_2 = v_1'$

$y'' = v_3 = v_2'$

$y''' = v_4 = v_3'$

$$\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$v_2' = -4v_1 + 0v_2$$

... vi sagde : vores omskrivningen

$V_2' = -4v_1 + 0v_2$

$V_1' = 0v_1 + 1_2$ som vi sagde : vores omskrivingen

$$Ay''' + By'' + Cy' + Dy = 0$$

$$\begin{bmatrix} V_1' \\ V_2' \\ V_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{D}{A} & -\frac{C}{A} & \frac{B}{A} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$y = V_1$

$y' = V_2 = V_1'$

$y'' = V_3 = V_2'$

$y''' = V_4 = V_3'$

So this is the basis way to do it

$$\boxed{\bar{y} = C_1 \bar{V}_1 e^{\lambda_1 t} + C_2 \bar{V}_2 e^{\lambda_2 t} + C_3 \bar{V}_3 e^{\lambda_3 t} \ldots\ldots}$$

Basis formen husk den nu forfanen!

+ SOLVE y(n)^2

Sunday, 2 June 2024 12.18

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Convolusion not multiplien

1) Rewrite for $n \rightarrow k$

$$x(n) = 5u[n] \Longrightarrow x(k) = 5u[k]$$

$$h(n) = 3^n u[n-1] \Longrightarrow h(k) = 3^{n-k} u[n-1-k]$$

2) Insert in Σ

$$\sum_{k=-\infty}^{\infty} x(k)\, h(k) = \sum_{k=-\infty}^{\infty} 5u[k] \cdot 3^{n-k} u[n-1-k]$$

$$\frac{3^n}{3^k}$$

3) Extract constants out of Σ

$$\sum_{k=-\infty}^{\infty} 5u[k] \cdot \frac{3^n}{3^k} \cdot u[n-1-k]$$

$$5 \cdot 3^n \sum_{k=-\infty}^{\infty} u[k] \cdot \frac{1}{3^k} \cdot u[n-1-k]$$

$u[k]$ enten 1 eller 0

$u[n-1-k]$ enten 1 eller 0

afskriver jeg dem

$$5 \cdot 3^n \sum \frac{1}{3^k}$$

4) Quick removal of $\sum$

$$\sum_{n=0}^{n-1} r^k = \frac{1-r^n}{1-r} \quad \text{for } r \neq 1$$

$$5 \cdot 3^n \sum \frac{1}{3^k} \qquad \frac{1}{3^k} = \frac{1^k}{3^k} = \left(\frac{1}{3}\right)^k$$

$$5 \cdot 3^n \sum \left(\frac{1}{3}\right)^k = \boxed{5 \cdot 3^n \cdot \frac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}}$$

This you can maple plugg!

$$\frac{1-\left(\frac{1}{3}\right)^n}{\frac{2}{3}}$$

as $1-\frac{1}{3} = \frac{3}{3} - \frac{1}{3} = \frac{2}{3}$

$$5 \cdot 3^n \frac{1-\left(\frac{1}{3}\right)^n}{\frac{2}{3}} = \frac{5 \cdot 3^n}{2/3} - \frac{5 \cdot 3^n \left(\frac{1}{3}\right)^n}{2/3}$$

$$\frac{5 \cdot 3^n \cdot 3}{2} - \frac{5 \cdot 3^n \cdot 3 \left(\frac{1}{3}\right)^n}{2}$$

$$\frac{5 \cdot 3^n \cdot 3 \left(\frac{1^n}{3^n}\right)}{2} \rightarrow \frac{5 \cdot 3 \cdot 3^n \left(\frac{1}{3^n}\right)}{2} = \frac{5 \cdot 3 \left(\frac{3^n}{3^n}\right)}{2} = \frac{5 \cdot 3}{2}$$

$$\frac{5 \cdot 3 \cdot 3^n}{2} - \frac{5 \cdot 3}{2} = \frac{15}{2} 3^n - \frac{15}{2}$$

5) Back sub $u[n]$ into it

$$\frac{15}{2} 3^n u[n] - \frac{15}{2} u[n]$$

And this is the answer :)

+ FOLDNINGER

Friday, 31 May 2024 15.13

Når der i opgaven står x[n-2] betyder det at vi skubber pulstoget 2 timestamps frem, da vi så modtager pulstoget 2 timestamps senere.

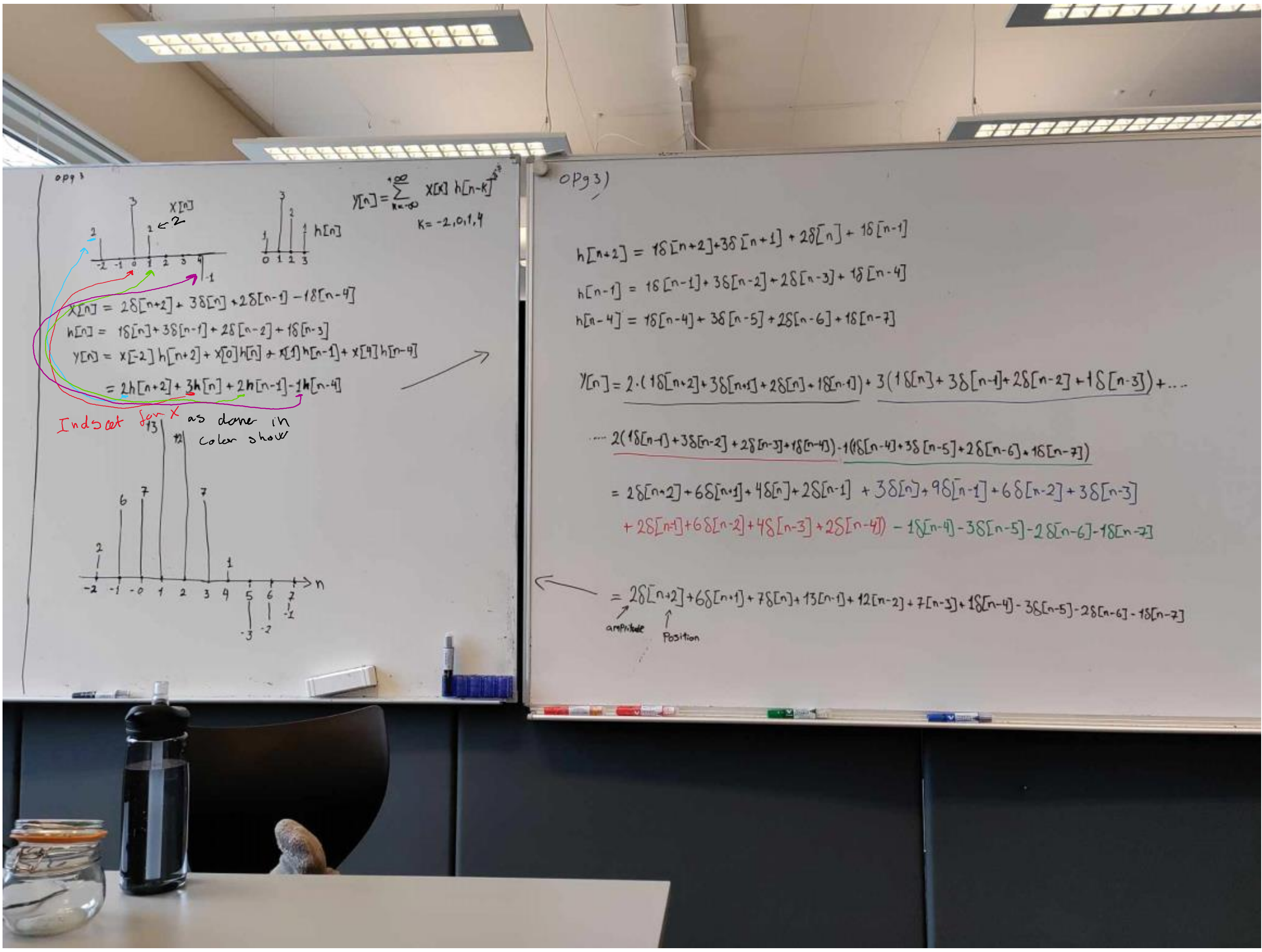
Altså rykker vi pulstoget mod højre på y-aksen.

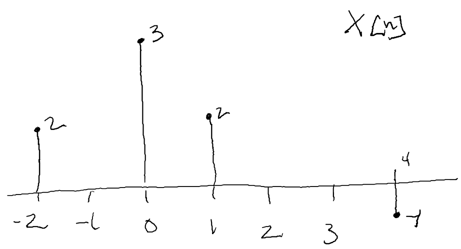
Hvis der til gengæld står + et eller andet skal vi rykke pulstoget mod venstre.

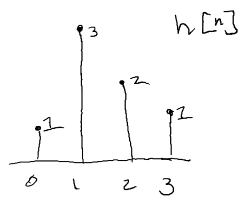
Når der står -n, skal hele pulstoget flippes, da det så er et negativt/modsatrettet pulstog vi har med at gøre. I eksemplet fra opgaven er det nemmest at omskrive til formen: x[-n+1], da det beskriver for os at vi har et flippet pulstog, og skal rykke 1 mod venstre.

OG HUSK! SHIFT and FLIP!
Ikke flip and shift!

Når vi har med convultions at gøre



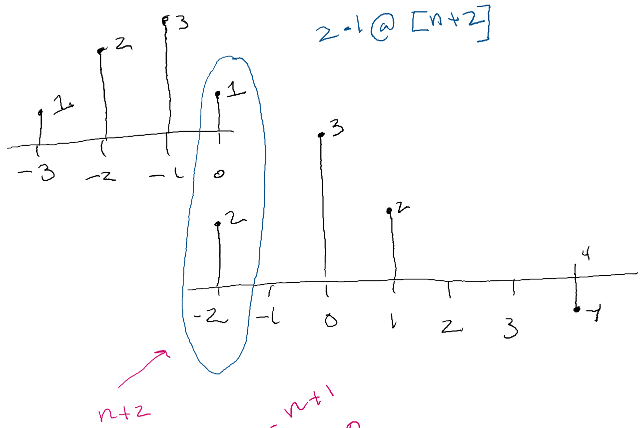




— Now for the Convoluting —

— Now for the convoluting —

$2\cdot 1 @ [n+2]$



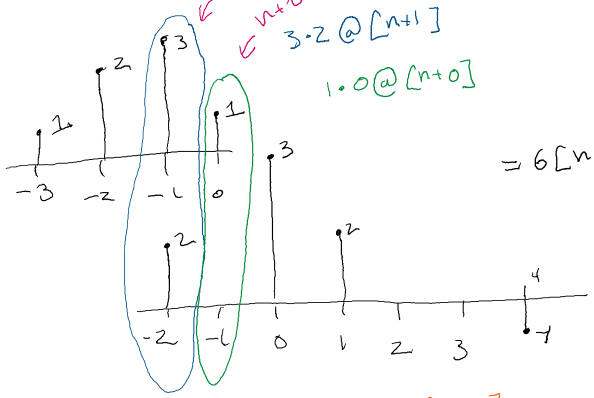
$= 2\cdot 1 [n+2]$

$n+1$

$n+0$

$3\cdot 2 @ [n+1]$

$1\cdot 0 @ [n+0]$

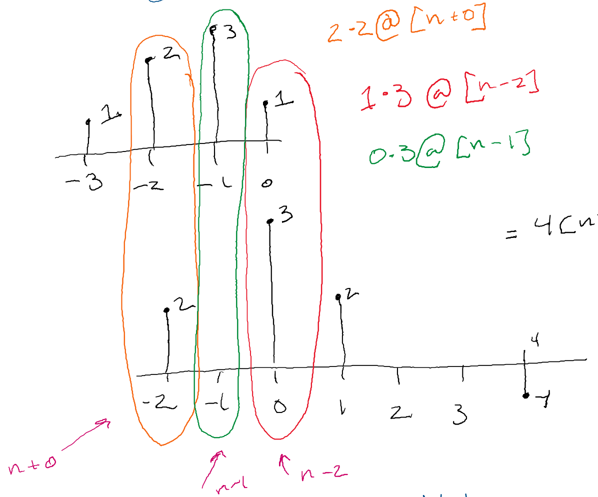


$= 6[n+1]$

$2\cdot 2 @ [n+0]$

$1\cdot 3 @ [n-2]$

$0\cdot 3 @ [n-1]$

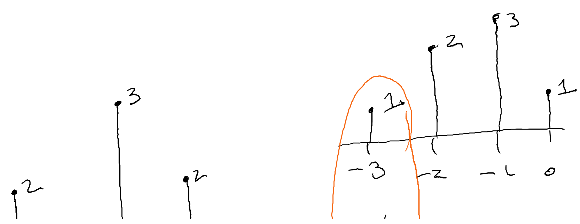


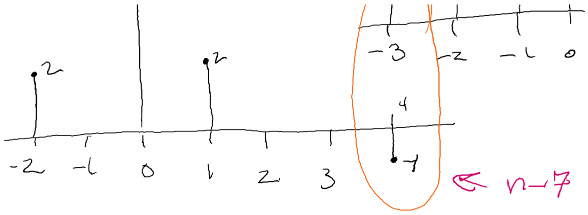
$= 4[n] + 3[n-2]$

osv osv osv indtil man er henl

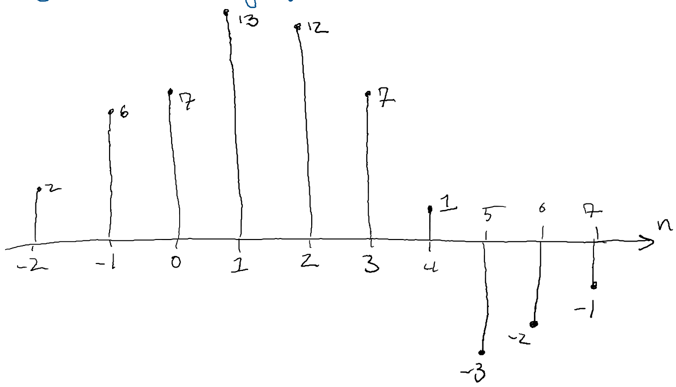
Man følger bagerste indtil den man folder ind oven er fuldt overlappet og ligger 1 til indtil man er igennem hele ens spectre

$1\cdot 1[n-7]$





Så ligger man alle ens [n-?] sammen i størrelse og for!



$$\text{Len}(y[n]) = (\text{len}(h[n]) + \text{len}(x[n])) - 1$$

+ Z-Transform and INVERSE Z-Transform

Friday, 31 May 2024 09.41

The z-transform

AALBORG UNIVERSITY DENMARK

TABLE 3.1 SOME COMMON z-TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3. $-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m>0$) or ∞ (if $m<0$)
5. $a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
6. $-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
7. $na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
8. $-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
9. $\cos(\omega_0 n)u[n]$	$\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
10. $\sin(\omega_0 n)u[n]$	$\frac{\sin(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
11. $r^n\cos(\omega_0 n)u[n]$	$\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
12. $r^n\sin(\omega_0 n)u[n]$	$\frac{r\sin(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
13. $\begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$

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Pole finding:

$$\frac{1}{2-6z^{-1}}$$

$2\cdot 6z^{-1}=0 \Leftrightarrow 2=6z^{-1}$

$z^{-1}=\frac{2}{6} \Leftrightarrow z=3$

$|z|<|a|$ and leftside

$\frac{1}{2-\frac{1}{2}z^{-1}} \rightarrow 2=\frac{1}{2}z^{-1} \rightarrow$

$4=z^{-1} \Rightarrow z=\frac{1}{4}$

$|z|>|a|$ and right sided

Negativ / standart

$$\frac{Q z^{-M}}{1-Az^{-1}} \xRightarrow{\text{using 5}} Q A^{n-M} u[n-M]$$

Husk fortejn for M. Som standart er neg

$$\frac{Q z^{-M}}{1-Az^{-1}} \xRightarrow{\text{using 6}} Q\cdot-(A)^{n-M} u[-n-1--M]$$

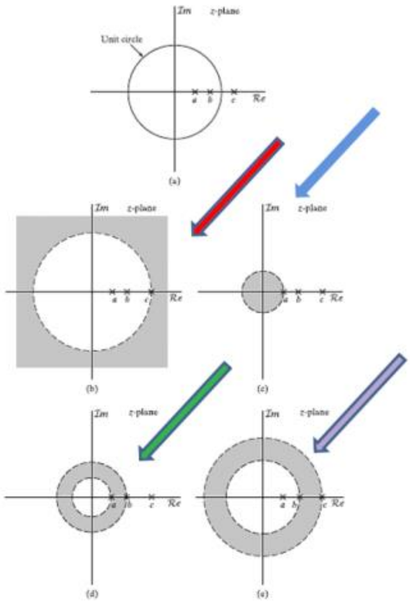
Positive / -- Standart

$$\frac{Q z^{+M}}{1-Az^{-1}} \Rightarrow A^{n+M} u[n+M]$$

Her er den positiv

$$\frac{Q z^{+M}}{1-Az^{-1}} \Rightarrow Q\cdot-(A)^{n+M} u[-n-1-M]$$

The z-transform



Examples of four z-transforms with the same pole–zero locations, illustrating the different possibilities for the ROC, each of which corresponds to a different sequence:
(b) to a right-sided sequence,
(c) to a left-sided sequence,
(d) to a two-sided sequence, and
(e) to a two-sided sequence.

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The z-transform

TABLE 3.2 SOME z-TRANSFORM PROPERTIES

Property Number	Section Reference	Sequence	Transform	ROC
		$x[n]$	$X(z)$	R_x
		$x_1[n]$	$X_1(z)$	R_{x_1}
		$x_2[n]$	$X_2(z)$	R_{x_2}
1	3.4.1	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$	Contains $R_{x_1} \cap R_{x_2}$
2	3.4.2	$x[n-n_0]$	$z^{-n_0}X(z)$	R_x, except for the possible addition or deletion of the origin or ∞
3	3.4.3	$z_0^n x[n]$	$X(z/z_0)$	$\|z_0\|R_x$
4	3.4.4	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	R_x
5	3.4.5	$x^*[n]$	$X^*(z^*)$	R_x
6		$\mathcal{R}e\{x[n]\}$	$\frac{1}{2}[X(z)+X^*(z^*)]$	Contains R_x
7		$\mathcal{I}m\{x[n]\}$	$\frac{1}{2j}[X(z)-X^*(z^*)]$	Contains R_x
8	3.4.6	$x^*[-n]$	$X^*(1/z^*)$	$1/R_x$
9	3.4.7	$x_1[n]*x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{x_1} \cap R_{x_2}$

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$$H(z) = \frac{Y(z)}{X(z)}$$

; $Y(z)(a+b+c\ldots) = X(z)(d+e+f)\ldots$

$$\frac{Y(z)}{X(z)} = \frac{d+ef}{a+b+c} = H(z)$$

How to inverse z-transform!

$$X(z) = \frac{2z^{-1}}{1-z^{-1}} + \frac{z^3}{1-\frac{4}{7}z^{-1}}$$ given ROC $|z|>1$

Det betyder vi bruger denne basis

5) $a^n u[n] \Leftrightarrow \frac{1}{1-az^{-1}}$

$$\frac{2z^{-1}}{1-z^{-1}} = 2\frac{z^{-1}}{1-1\cdot z^{-1}} \quad ; \quad \frac{z^3}{1-\frac{4}{7}z^{-1}}$$

$$2\frac{z^{-1}}{1-1z^{-1}} \qquad \frac{z^3}{1-\frac{4}{7}z^{-1}}$$

$$2 \cdot 1^{n-1}[n-1] \qquad \frac{4}{7}^{n+3} u[n+3]$$

$$2[n-1]$$

$$\underline{2[n-1] + \left(\frac{4}{7}\right)^{n+3} u[n+3]}$$

Let's inverse some more

$$X(z) = \frac{3}{1-2z^{-1}} + \frac{z^{-1}}{1-z^{-1}}$$ ROC $|z|<1$

Which means we use this

6) $-a^n u[-n-1] \Leftrightarrow \frac{1}{1-az^{-1}}$

$$\frac{3}{1-2z^{-1}} + \frac{z^{-1}}{1-z^{-1}}$$

de bliver

-- = +

da har det

der minus

$$3 \; -(2)^n u[-n-1] + -1^{n-1} \underbrace{u[-n-1--1]}_{-n-1+1=-n}$$

from formula

$-1^{n-1} = 1$

$$\underline{\underline{-3(2)^n u[-n-1] - u[-n]}}$$

A stable LTI system is characterized by this difference equation:

$$y[n] - 3y[n-1] = 5(x)[n-2]$$

$$y[n-0] - 3y[n-1] = 5(x)[n-2]$$

$$\mathcal{Z}\{\Updownarrow\}$$

$$Y(z)\cdot z^0 - 3Y(z)\cdot z^{-1} = 5X(z)\cdot z^{-2}$$

$$Y(z)\cdot(z^0 - 3\cdot z^{-1}) = X(z)(5\cdot z^{-2})$$

And then we know that $H(z) = \frac{Y(z)}{X(z)}$

$X(z)$

$$\frac{Y(z)}{X(z)}(1-3z^{-1}) = 5z^{-2} \Rightarrow \frac{Y(z)}{X(z)} = \frac{5z^{-2}}{1-3z^{-1}} = H(z)$$

THIS IS IMPORTANT

$$H(z) = \frac{Y(z)}{X(z)} = \frac{5 \cdot z^{-2}}{1-3z^{-1}}$$

og det kan vi bruge inverse Z transform på

ROC = 3 which means leftsided and therefon

$|z| < |a|$

Som betyder vi bruger

$$-a^n u[-n-1] \Rightarrow \frac{1}{1-az^{-1}}$$

$$5 \cdot \frac{z^{-2}}{1-3z^{-1}} \rightarrow Z^{-1}\{5 - 3^{n-2} u[-n-1--2]\}$$

$$-5(3)^{n-2} u[-n+1]$$

and is non causal as $[-n+1]$ is positive

Apply $Z^{-1}\{\ \}$ to

$$H(z) = \frac{1-3z^{-1}}{2-4z^{-1}} = \frac{1}{2-4z^{-1}} - \frac{3z^{-1}}{2-4z^{-1}}$$

for ROC $2-4z^{-1} \Rightarrow 2-4\frac{1}{2}z^{-1} = 0$

Then ROC $|z| < |a|$

6) $-a^n u[-n-1] \Leftrightarrow \frac{1}{1-az^{-1}}$

$$\frac{1}{2-4z^{-1}} - 3\frac{z^{-1}}{2-4z^{-1}} = \frac{1}{2}\frac{1}{1-2z^{-1}} - \frac{3}{2}\frac{z^{-1}}{1-2z^{-1}}$$

$$\frac{1}{2}\cdot -(2)^n u[-n-1] - \frac{3}{2}\cdot -(2)^{n+1} u[-n-1-1]$$

$$-\frac{1}{2}2^n u[-n-1] + \frac{3}{2}(2)^{n+1} u[-n]$$

5 $a^n u[n] \Leftrightarrow \frac{1}{1-az^{-1}}$ ROC $|z| > |a|$

$$\frac{1}{2}\frac{1}{1-2z^{-1}} - \frac{3}{2}\frac{z^{-1}}{1-2z^{-1}}$$

$$\frac{1}{2}2^n[n] - \frac{3}{2}2^{n-1}[n-1]$$

Opgaveeks. 7

Given a causal linear time invariant system with the following transfer function:

$$H(z) = \frac{1-3z^{-1}}{2-4z^{-1}},$$

calculate the impulse response $h_i[n]$ of the inverse system.

Solution: $h_i[n] = 2(3)^n u[n] - 4(3)^{n-1}u[n-1]$, or $h_i[n] = -2(3)^n u[-n-1] + 4(3)^{n-1}u[-n]$.

Så opdag nu lige at det skal
være den inverse

$$H(z) = \mathcal{Z}^{-1}\left\{\frac{1}{H(z)}\right\}$$

$$H(z) = \mathcal{Z}^{-1}\left\{\frac{1}{H(z)}\right\}$$

$$H(z)^{-1} = \frac{2-4z^{-1}}{1-3z^{-1}} = 2\frac{1}{1-3z^{-1}} - \frac{4z^{-1}}{1-3z}$$

$$2(3)^{n}[n] - 4(3)^{n-1}[n-1]$$ formel 5)

OR

$$-2(3)^{n}[-n-1] + 4(3)^{n-1}[-n]$$ formel 6)

Stability and eigenvalues

$$\det(\bar{\bar{A}} - \lambda \bar{\bar{I}}) = \begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix}$$

$$= \lambda^2 - (a_{11} + a_{22})\lambda + \det(\bar{\bar{A}}) = 0$$

From LA we know:

$$\mathrm{trace}(\bar{\bar{A}}) = \sum_i a_{ii} = a_{11} + a_{22} = \sum_i \lambda_i \equiv p$$

$$\det(\bar{\bar{A}}) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \prod_i \lambda_i \equiv q$$

$$\det(\bar{\bar{A}} - \lambda \bar{\bar{I}}) = \lambda^2 - p\lambda + q \quad , D = p^2 - 4q$$

Nodes and eigenvalues

$D \geq 0$ (real eigenvalues)

$$q = \prod_i \lambda_i > 0 \overset{(p \neq 0)}{\Longrightarrow} \text{node}$$

$$q = \prod_i \lambda_i < 0 \Longrightarrow \text{saddle}$$

except $q \leq 0$

except $p = 0$

$$p = \sum_i \lambda_i = 0 \overset{(q>0)}{\Longrightarrow} \text{center}$$

$$p = \sum_i \lambda_i \neq 0 \Longrightarrow \text{spiral point}$$

$D < 0$ (complex eigenvalues)

Sommering

Phase plane - stability

Phase plane - stability

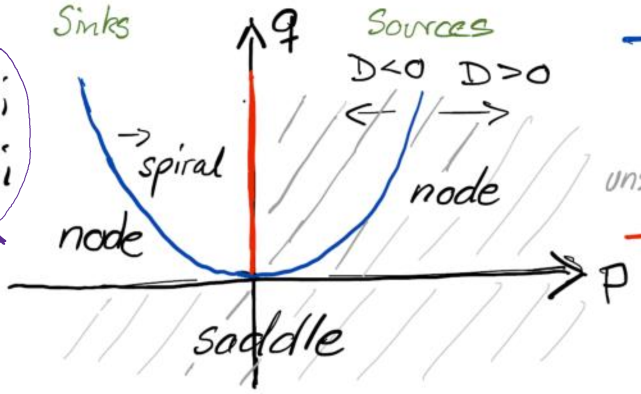
Sommering

$p = \sum \lambda_i$

$q = \prod_i \lambda_i$

Produkt af det hele

Eigen vals



— degenerate (+ proper)

$D = 0$

unstable

— center ($p = 0$)

- unstable if $q < 0$ or $p > 0$
- stable if $q > 0$ and $p \leq 0$
- stable if $q > 0$ and $p < 0$ (attractive or asymptotically stable)

holds also for $n > 2$

+ NUL/SUB SPACE

Saturday, 1 June 2024 10.32

Argument that:

$3x_1 + x_3$ is a subspace in $\mathbb{R}^3$

and find the basis for it

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Gauss elimination

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1/3\, x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

Isolate for $x_1 = -1/3\, x_3$

These exist but only for them selves

hence $x_2 = x_2$ and $x_3 = x_3$

$$\begin{bmatrix} -\frac{1}{3}x_3 \\ x_2 \\ x_3 \end{bmatrix} \qquad x_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad x_3 = \begin{bmatrix} -\frac{1}{3} \\ 0 \\ 1 \end{bmatrix}$$

da x_1 ikke er tilstede

da x_2 er til ved sig selv

da x_3 er både ved "x_1" og "x_3"

+ BASIC STUFF

Sunday, 2 June 2024 10.36

Determinante of a 4x4

Berechnung der Determinante von (4x4)-Matrizen (Laplace)

$$\det\begin{pmatrix} 1 & 2 & 0 & 2 \\ 4 & 0 & 0 & 5 \\ 1 & 2 & 1 & 1 \\ 1 & 4 & 3 & 2 \end{pmatrix} = +0 \cdot \det\begin{pmatrix} 4 & 0 & 5 \\ 1 & 2 & 1 \\ 1 & 4 & 2 \end{pmatrix} - 0 \cdot \det\begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & 1 \\ 1 & 4 & 2 \end{pmatrix} + 1 \cdot \det\begin{pmatrix} 1 & 2 & 2 \\ 4 & 0 & 5 \\ 1 & 4 & 2 \end{pmatrix} - 3 \cdot \det\begin{pmatrix} 1 & 2 & 2 \\ 4 & 0 & 5 \\ 1 & 2 & 1 \end{pmatrix}$$

Entw. n. 3.Sp.

$$4 \cdot ((2 \cdot 2) - 4 \cdot 1) - 1 \cdot ((0 \cdot 2) - (4 \cdot 1)) + 1 \cdot ((0 \cdot 1) - 2 \cdot 5))$$

$0 \cdot$ 0>V 0≤V