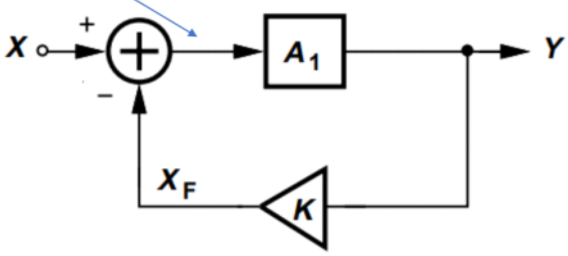
Transfer function of closed-loop system

Error signal = $\boldsymbol{X - X_F}$



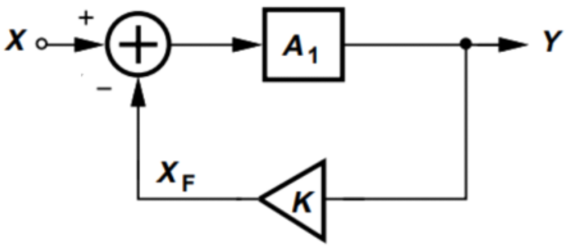
$A_{cL} = \frac{Y}{X} = \frac{A_1}{1+KA_1}$ --- closed-loop gain

A_1 is the open-loop gain
K is the feedback factor

$KA_1 > 0$ ➔ $|A_{cL}| < |A_1|$

Example

$\boldsymbol{A_1 = 100}$



$\boldsymbol{K = 0.1}$

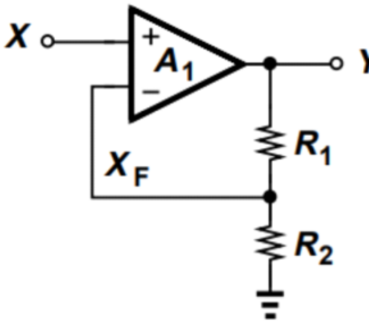
Nominal gain of an amp: $\boldsymbol{A_1 = 100}$
Actual gain in application: $\boldsymbol{A'_1 = 50}$

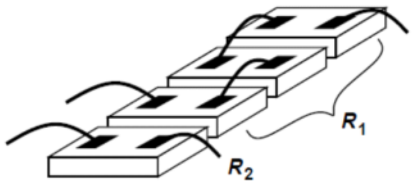
How much does $\boldsymbol{A_1}$ change?
How much does the closed-loop gain $\boldsymbol{A_{cL}}$ change?

$\boldsymbol{A_{cL}}$ = $\boldsymbol{A_1}$ /(1+K $\boldsymbol{A_1}$) = 100/11 = 9.09
$\boldsymbol{A_{cL}}$' = $\boldsymbol{A_1}$ '/(1+K $\boldsymbol{A_1}$ ') = 50/6 = 8.33

$\boldsymbol{A_{cL}}$ change: 8.3%

Feedback system example





The op amp A1 performs two functions:

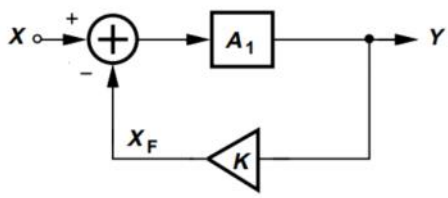
- substraction
- Amplification

What is the closed-loop gain?

$$A_{cL} = \frac{Y}{X} = \frac{A_1}{1 + KA_1} = \frac{A_1}{1 + \frac{R_2}{R_1 + R_2} A_1}$$

$$A_{cL} \approx \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}, \text{ if } \frac{R_2}{R_1 + R_2} A_1 \gg 1$$

Loop-gain

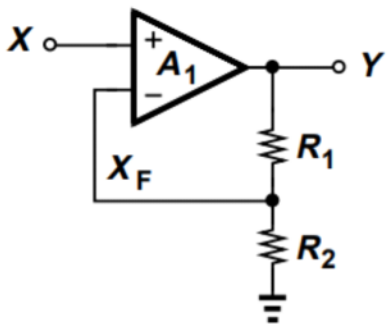


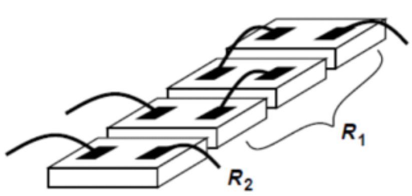
- Open-loop gain: A_1
- Closed-loop gain: $A_{cL} = \frac{Y}{X} = \frac{A_1}{1+KA_1}$
- Loop-gain: KA_1
  - Procedure to measure loop-gain
    - Set the input X to zero (X is voltage → AC ground; if X is current→ open)
    - Break the loop at an arbitrary point
    - Apply a test signal $\boldsymbol{V_{test}}$ at one terminal and measure the signal $\boldsymbol{V_F}$ at the other terminal
    - Calculate the loop-gain $-\frac{V_F}{V_{test}} = KA_1$
      - $\frac{V_F}{V_{test}} < 0$ → negative feedback
      - $\frac{V_F}{V_{test}} > 0$ → positivie feedback

Summary of negative feedback concept

- Sacrifice the open-loop gain A_1 to benefit from negative feedback
- The feedback signal X_F is a good copy of input signal X
- The feedback factor K is normally independent to frequency
  - Make Y a good (scaled by $1/\mathrm{K}$) copy of X
    - Wider frequency band
    - Better linearity
- If loop-gain $KA_1 \gg 1$ ➔ $A_{cL} \approx \frac{1}{K}$, relatively independent of A_1
  - Factors that cause A_1 to vary have less impact on the closed-loop gain
  - Factors: temperature, supply voltage, frequency, load impedance

Gain desensitization example



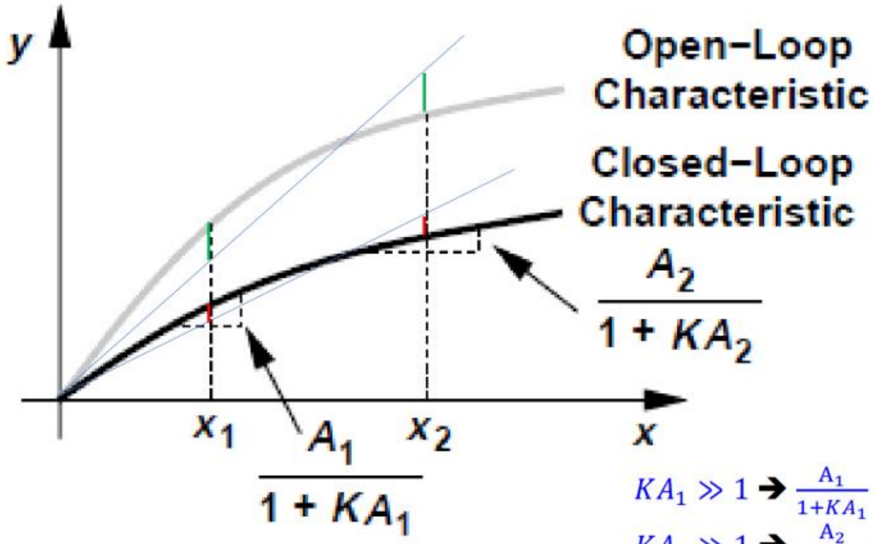


Assume the nominal gain of A_1=100 and $\frac{\mathrm{R}_1}{R_2} = 3.$ Due to e.g., temperature, supply voltage, frequency and loading imdedance, A_1 drops to 50.

- How does the closed-loop gain A_{cL} change?

$$A_{cL} = \frac{Y}{X} = \frac{A_1}{1 + KA_1} = \frac{A_1}{1 + \frac{R_2}{R_1 + R_2} A_1}$$

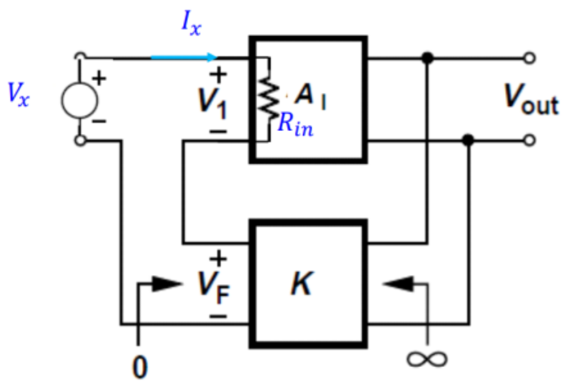
Linearity improvement



$KA_1 \gg 1 \rightarrow \frac{A_1}{1+KA_1} \approx K, when\ KA_1 \gg 1$

$KA_2 \gg 1 \rightarrow \frac{A_2}{1+KA_2} \approx K, when\ KA_2 \gg 1$

Modification of input and output impedance Voltage-voltage FB



$V_1 = I_x R_{in}$

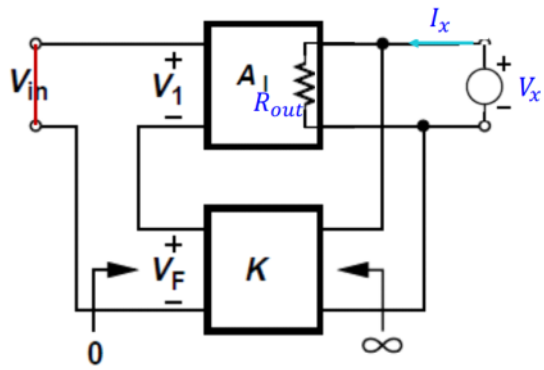
$V_{out} = A_1 I_x R_{in}$

$V_F = KA_1 I_x R_{in}$

$V_x = V_1 + V_F$

$\rightarrow R_{in,CL} = V_x / I_x = R_{in}(1 + KA_1)$

A better voltage sensor



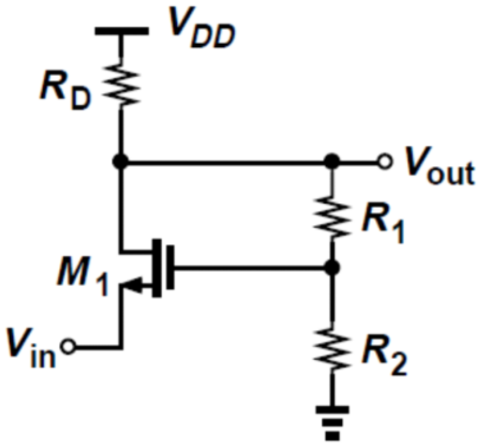
$V_x = I_x R_{out}$

$V_1 = -V_F = -KV_x$

$\rightarrow R_{out,CL} = V_x / I_x = R_{out}/(1 + KA_1)$

A better voltage source

Modification of input and output impedance

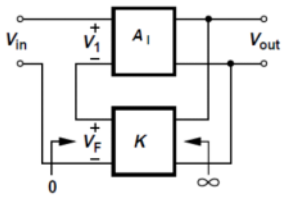


$A_1 = g_m R_D$
$K = \dfrac{R_2}{R_1 + R_2}$
$A_{CL} = \dfrac{A_1}{1 + KA_1}$
$R_{in} = 1/g_m$
$R_{out} = R_D$
$R_{in,CL} = R_{in}(1 + KA_1)$
$R_{out,CL} = R_{out}/(1 + KA_1)$

Assume $R_1 + R_2 >> R_D$

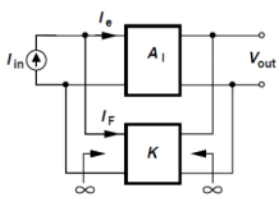
Feedback topologies

$V_o - V_i$



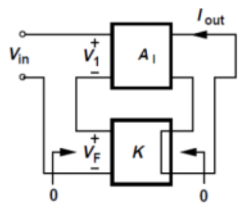
$R_{in,CL} = R_{in}(1 + KA_1)$
$R_{out,CL} = R_{out}/(1 + KA_1)$

$V_o - I_i$



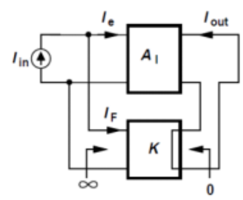
$R_{in,CL} = R_{in}/(1 + KA_1)$
$R_{out,CL} = R_{out}/(1 + KA_1)$

$I_o - V_i$



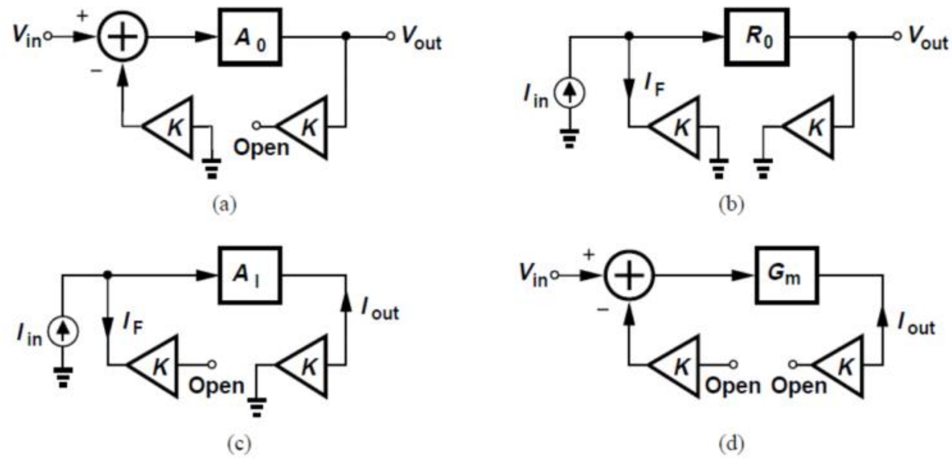
$R_{in,CL} = R_{in}(1 + KA_1)$
$R_{out,CL} = R_{out}(1 + KA_1)$

$I_o - I_i$



$R_{in,CL} = R_{in}/(1 + KA_1)$
$R_{out,CL} = R_{out}(1 + KA_1)$

Rules for breaking the feedback network(self-study)



Assignments:

10.1:

A CE stage circuit without (by setting $R_f = 0$) or with feedback (by setting $R_f = 20\ \Omega$) is shown in Fig. 1.

(1) Set $R_f = 0$, i.e., without negative feedback, run the simulation and find out $A_v = ?$ $f_L = ?$ $f_H = ?$ THD = ? @ 20KHz.

(2) Set $R_f = 20$, i.e., with negative feedback, run the simulation and find out $A_v = ?$ $f_L = ?$ $f_H = ?$ THD = ? @ 20KHz.

(3) Comparing the results obtained in (1) and (2), discuss what is the advantages and disadvantages by introducing negative feedback into the circuit.

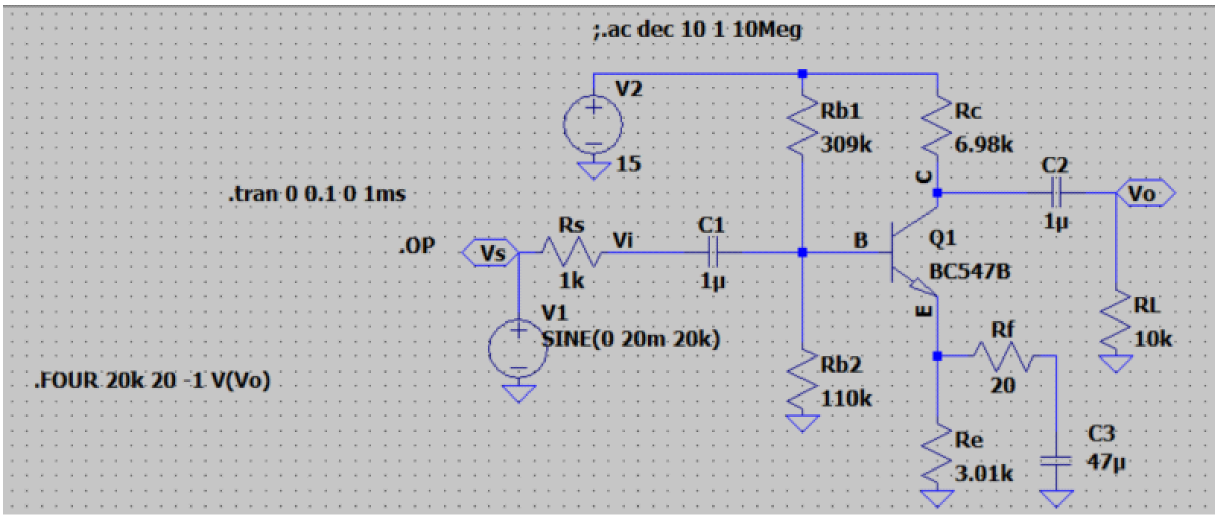


Fig. 1 A CE stage

Solution:

(1) For $R_f = 0\ \Omega$, A_V_simulated = 127.6 --(42 dB); f_L = 127 Hz; f_H = 543 KHz; THD = 10.6%

(2) For $R_f = 20\ \Omega$, A_V_simulated = 78.5 --(37.9 dB); f_L = 78.1 Hz; f_H = 865 KHz; THD = 4.45%

(3) Disadvantage: gain A_v is dropped. Advantages: frequency bandwidth becomes wider; harmonic distortion becomes smaller.