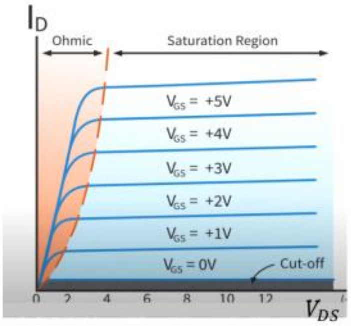
Operation regions of MOSFET



MOSFET transconductance parameter:

$$k_n = \mu_n C_{ox} W/L$$

- μ_n: mobility of the electrons at the surface of the channel
- C_{ox}: oxide capacitance
- W & L: width & length of the channel

Overdrive voltage: $V_{ov} = V_{GS} - V_{TH}$

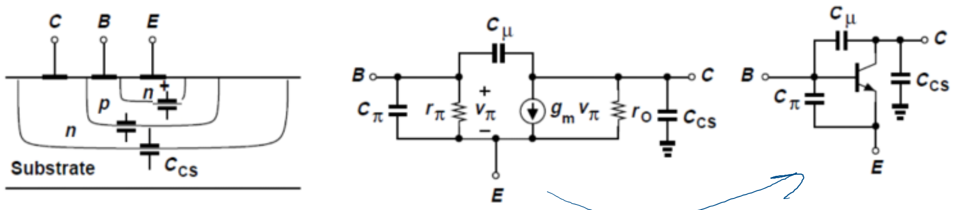
Threshold voltage: V_{TH}----0.3 V ~ 1V

$$I_D = \begin{cases} 0, & \text{cut-off: } V_{GS} < V_{TH} \\ k_n[(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2], & \text{triode: } V_{GD} > V_{TH} \\ \frac{1}{2}k_n(V_{GS} - V_{TH})^2(1 + \lambda V_{DS}), & \text{saturation: } V_{GD} < V_{TH} \end{cases}$$

Triode and cut-off region: switching devices

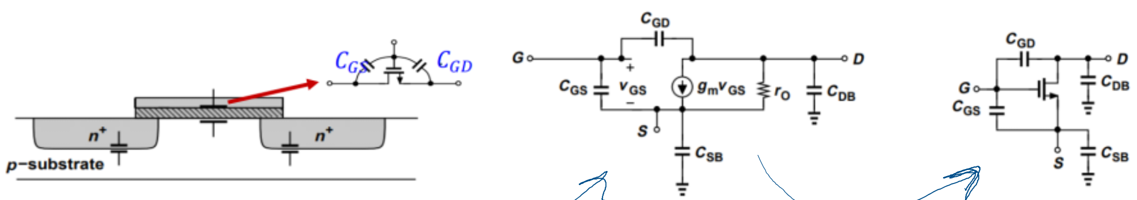
Saturation region: amplifier

High-frequency models of BJT and MOSFET



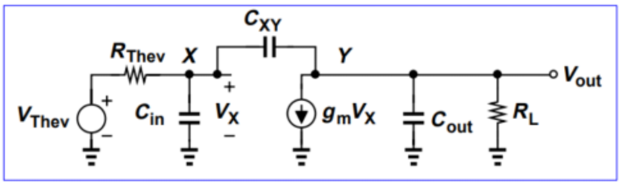
(a) (b) (c)

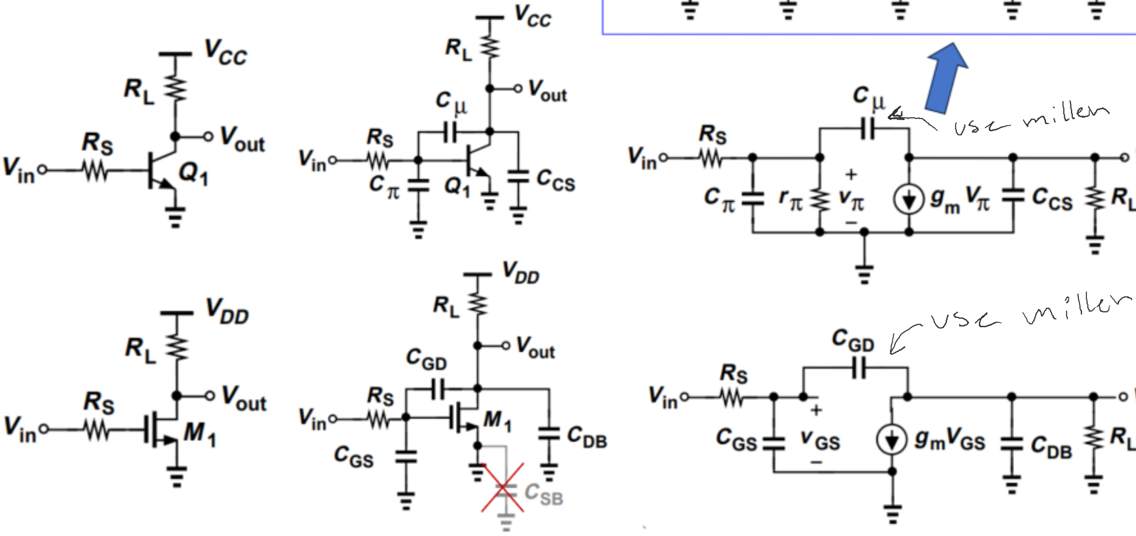
R becomes wires



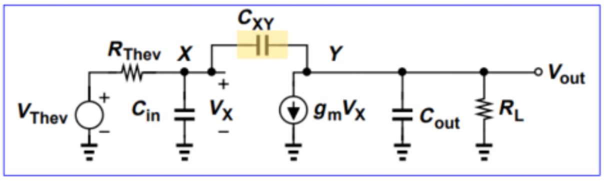
R becomes wires

CE/CS frequency response



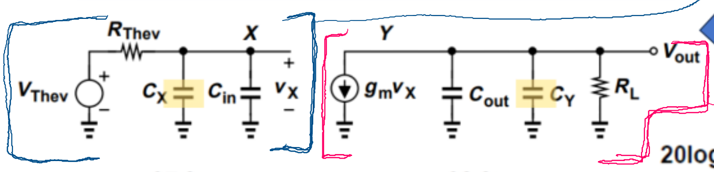


Approach I: finding poles by inspection

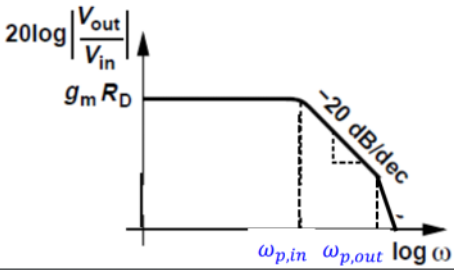


Apply Miller's theorem to C_{XY}

$$|\omega_{p,in}| = \frac{1}{R_{Thev}[C_{in} + C_{XY}(1 + g_m R_L)]}$$



$$|\omega_{p,out}| = \frac{1}{R_L[C_{out} + C_{XY}(1 + \frac{1}{g_m R_L})]}$$



CE Stage

$V_{Thev} = V_{in}\dfrac{r_\pi}{r_\pi + R_S}$

$R_{Thev} = R_S \| r_\pi$

$C_X = C_\mu(1 + g_m R_L)$

$C_Y = C_\mu(1 + \dfrac{1}{g_m R_L})$

CS Stage

$V_{Thev} = V_{in}$

$R_{Thev} = R_S$

$C_X = C_{GD}(1 + g_m R_L)$

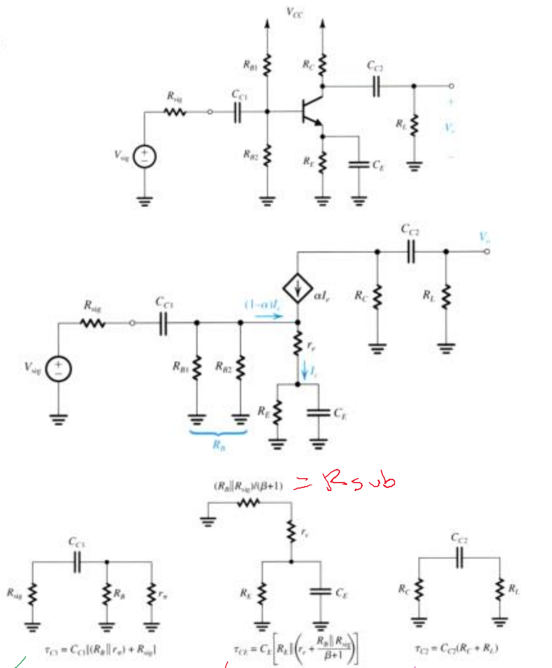
$C_Y = C_{GD}(1 + \dfrac{1}{g_m R_L})$

OR $C_{in\,Miller} = C_f(1 - A_v)$

$C_{out\,Miller} = C_f(1 - \frac{1}{A_v})$

Check maple doc

Low-frequency response-BJT



$$\frac{V_o}{V_{sig}} = A_M \frac{s}{s+\omega_{p1}} \frac{s+\omega_z}{s+\omega_{pE}} \frac{s}{s+\omega_{p2}}$$

$$\omega_{p1} = \frac{1}{\tau_{c1}} = \frac{1}{C_{C1}(R_B||r_\pi + R_{sig})}$$

$$\omega_{pE} = \frac{1}{\tau_{cE}} = \frac{1}{C_E[R_E||\left(\frac{1}{g_m} + \frac{R_B||R_{sig}}{\beta+1}\right)]}$$ Dominant pole

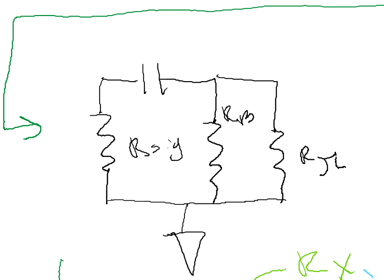
$$\omega_{p2} = \frac{1}{\tau_{c2}} = \frac{1}{C_{C2}(R_D + R_L)}$$

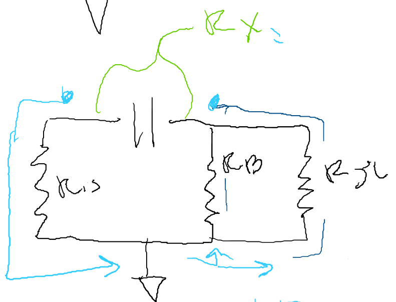
$$\omega_z = \frac{1}{C_E R_E}$$

$R_B = R_{B1}||R_{B2}$ $\qquad r_\pi = \frac{\beta}{g_m} = \beta r_e$

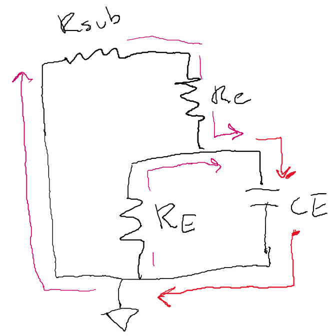
Short-circuit time constant method:

- Short other capacitors
- turn off sources:
  - Voltage source → short circuit
  - Current source → open circuit

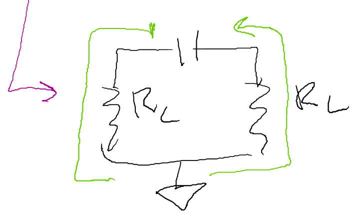




Rsig + RB || Rπ



RE || (Rsub + Re)



Rc + RL

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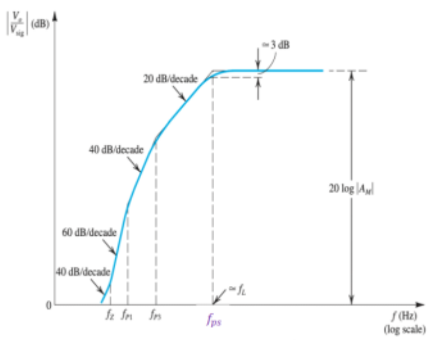
Low-frequency response--MOSFET



Dominant pole

$$\frac{V_o}{V_{sig}} = A_M \frac{s}{s+\omega_{p1}} \frac{s+\omega_z}{s+\omega_{ps}} \frac{s}{s+\omega_{p2}}$$

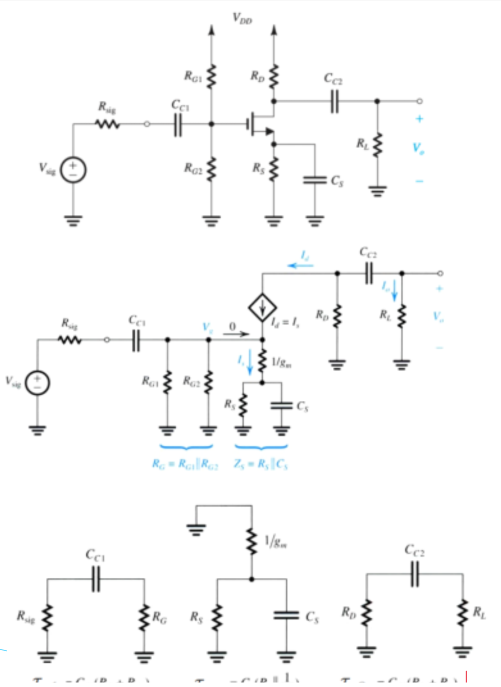
$$\omega_{p1} = \frac{1}{\tau_{c1}} = \frac{1}{C_{C1}(R_G + R_{sig})}$$

$$\omega_{ps} = \frac{1}{\tau_{cs}} = \frac{1}{C_S(R_S||\frac{1}{g_m})}$$

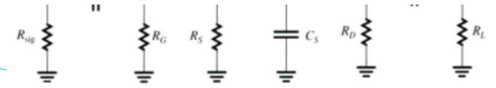
$$\omega_{p2} = \frac{1}{\tau_{c2}} = \frac{1}{C_{C2}(R_D + R_L)}$$

$$\omega_z = \frac{1}{C_S R_S}$$

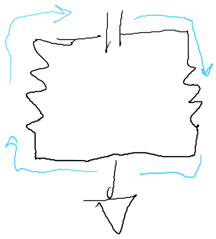
$R_G = R_{G1}||R_{G2}$



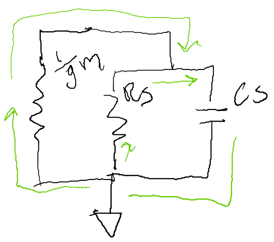
$R_G = R_{G1}||R_{G2}$



$\tau_{C1} = C_{C1}(R_G + R_{sig})$ $\tau_{CS} = C_S(R_S \| \frac{1}{g_m})$ $\tau_{C2} = C_{C2}(R_D + R_L)$

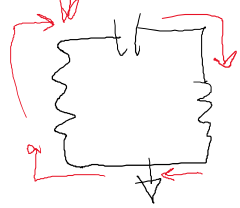


$\tau_1 = C \cdot (R_G + R_{sig})$



$C_S \cdot (R_S || \frac{1}{g_m})$

$C(R_L + R_D)$



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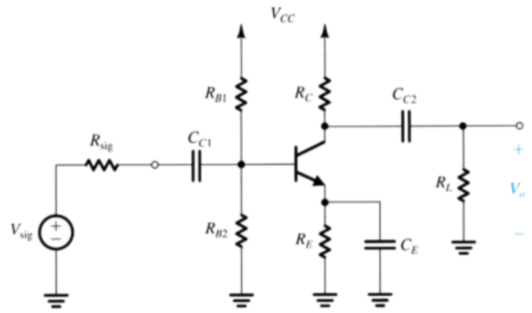
Low frequency design

- Design the C_{C1}, C_{CE} and C_{C1} to achieve a given f_L

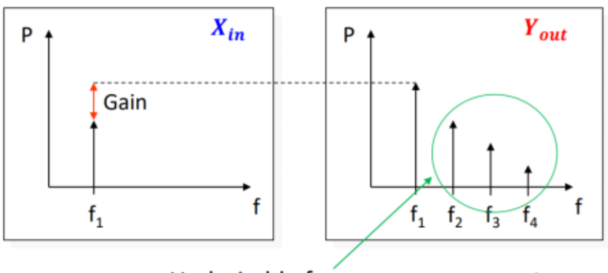
$f_{C1} = f_{C2} = 0.1 f_L$
$f_{CE} = 0.8 f_L$



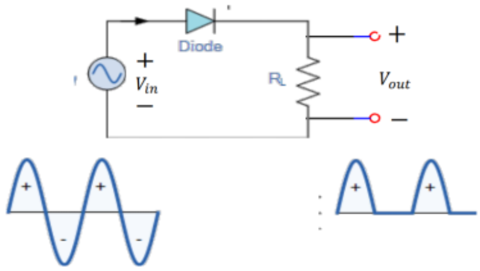
- $\omega_{p1} = 2\pi f_{C1} = \frac{1}{C_{C1}(R_B||r_\pi + R_{sig})}$
- $\omega_{pE} = 2\pi f_{CE} = \frac{1}{C_E[R_E||\left(\frac{1}{g_m} + \frac{R_B||R_{sig}}{\beta+1}\right)]}$
- $\omega_{p2} = 2\pi f_{C2} = \frac{1}{C_{C2}(R_D + R_L)}$

Nonlinear system

- $Y_{out} = f(X_{in})$: nonlinear function
  - X and Y could be **V** or **I**
  - $Y_{out} = \alpha_1 X_{in} + \alpha_2 X_{in}^2$, $Y_{out} = e^{\frac{X_{in}}{a}}$ ...
  - Capacitors, diodes, transistor



Undesirable frequency components



General nonlinear system

$$Y_{out} = f(X_{in}) = f(a) + f'(a)(X_{in} - a) + \frac{f''(a)}{2!}(X_{in} - a)^2 + \ldots$$

$$X_{in} = X_{in,Q} + x_{in}$$

In small-signal model, assume x_{in} is a sin wave centered at $a = 0$

$$Y_{out} = f(0) + f'(0)x_{in} + \frac{f''(0)}{2!}x_{in}^2 + \frac{f'''(0)}{3!}x_{in}^3 + \ldots$$

$$= \alpha_0 + \alpha_1 x_{in} + \alpha_2 x_{in}^2 + \alpha_3 x_{in}^3 + \ldots$$

General nonlinear system

$$Y_{out} = \alpha_0 + \alpha_1 x_{in} + \alpha_2 x_{in}^2 + \alpha_3 x_{in}^3 + \ldots$$

- $x_{in} = A\cos(\omega t)$
- $x_{in}^2 = A^2\cos^2(\omega t) = A^2\,\frac{1+\cos(2\omega t)}{2}$
- $x_{in}^3 = A^3\cos^3(\omega t) = A^3\,\frac{3\cos(\omega t)+\cos(3\omega t)}{4}$

DC:

$$\beta_0 = \alpha_0 + \alpha_2\frac{A^2}{2}$$

2nd Harmonic:

$$\beta_2 = \alpha_2\frac{A^2}{2}$$

Fundamental:

$$\beta_1 = \alpha_1 A + \alpha_3\frac{3A^3}{4} \approx \alpha_1 A$$

3rd Harmonic:

$$\beta_3 = \alpha_3\frac{A^3}{4}$$

Total harmonic distortion (THD):

$$THD = \sqrt{HD_2^2 + HD_3^2 + HD_4^2 + \cdots}$$

Harmonic distortion (HD):

2nd Harmonic:

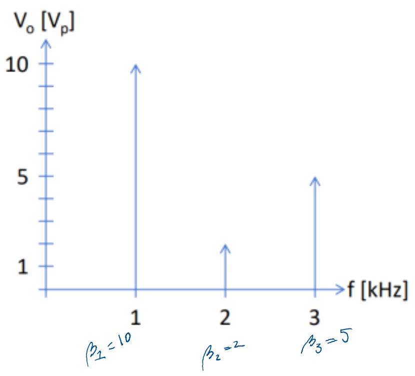
$$HD_2 = \frac{\beta_2}{\beta_1} \approx \frac{\alpha_2}{2\alpha_1}A$$

3rd Harmonic:

$$HD_3 = \frac{\beta_3}{\beta_1} \approx \frac{\alpha_3}{4\alpha_1}A^2$$

Example for harmonic distortion calculation

Here β = "height"

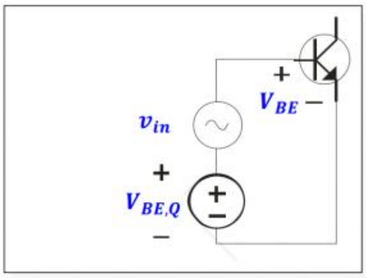


$$HD_2 = \frac{\beta_2}{\beta_1} = \frac{2}{10} = 0.2 \longrightarrow 20\%$$

$$HD_3 = \frac{\beta_3}{\beta_1} = \frac{5}{10} = 0.5 \longrightarrow 50\%$$

$$THD = \sqrt{HD_2^2 + HD_3^2} = \sqrt{0.2^2 + 0.5^2} = 0.54 \to 54\%$$

Nonlinear system example--BJT



Taylor series expansion:

$$e^{x/a} = 1 + \frac{1}{a}x + \frac{1}{2!*a^2}x^2 + \cdots + \frac{1}{n!*a^n}x^n + \cdots$$

$$I_C = I_s\, e^{V_{BE}/V_T} = I_s e^{(V_{BE,Q}+v_{in})/V_T} = I_S\, e^{V_{BE,Q}/V_T}\ e^{v_{in}/V_T} = I_{CQ}\, e^{v_{in}/V_T}$$

$$= I_{CQ}\left(1 + \frac{1}{V_T}v_{in} + \frac{1}{2! * V_T^2}v_{in}^2 + \frac{1}{3! * V_T^3}v_{in}^3 + \ldots\right)$$

$$= I_{CQ} + \frac{I_{CQ}}{V_T}v_{in} + \frac{I_{CQ}}{2! * V_T^2}v_{in}^2 + \frac{I_{CQ}}{3! * V_T^3}v_{in}^3 + \ldots$$

$$= \alpha_0 \ + \alpha_1 \boldsymbol{v_{in}} \ + \boldsymbol{\alpha_2 v_{in}^2} \ + \boldsymbol{\alpha_3 v_{in}^3} + \ldots$$

DC:

$$\beta_0 = \alpha_0 + \alpha_2 \frac{A^2}{2}$$

2nd Harmonic:

$$\beta_2 = \alpha_2 \frac{A^2}{2}$$

Fundamental:

$$\beta_1 = \alpha_1 A + \alpha_3 \frac{3A^3}{4} \approx \alpha_1 A$$

3rd Harmonic:

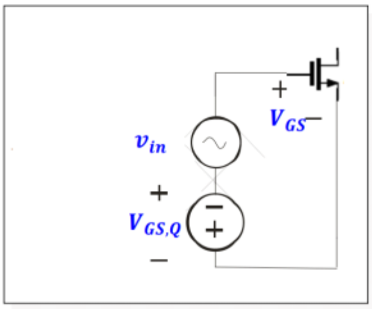
$$\beta_3 = \alpha_3 \frac{A^3}{4}$$

Total harmonic distortion (THD):

$$THD = \sqrt{HD_2^2 + HD_3^2 + HD_4^2 + \cdots}$$

$$= \sqrt{\left(\frac{\beta_2}{\beta_1}\right)^2 + \left(\frac{\beta_3}{\beta_1}\right)^2 + \left(\frac{\beta_4}{\beta_1}\right)^2 + \cdots}$$

Nonlinear system example--MOSFET



$$I_D = \frac{1}{2}k_n(V_{GS} - V_{TH})^2 = \frac{1}{2}k_n(V_{GS,Q} - V_{TH} + v_{in})^2$$

$$= \frac{1}{2}k_n(V_{GS,Q} - V_{TH})^2\left(1 + \frac{v_{in}}{V_{GS,Q} - V_{TH}}\right)^2$$

$$= I_{DQ}\left(1 + \frac{2}{V_{GS,Q} - V_{TH}}v_{in} + \frac{1}{(V_{GS,Q} - V_{TH})^2}v_{in}^2\right)$$

$$= I_{DQ} + \frac{2I_{DQ}}{V_{GS,Q} - V_{TH}}v_{in} + \frac{I_{DQ}}{(V_{GS,Q} - V_{TH})^2}v_{in}^2$$

$$= \alpha_0 \ + \alpha_1 \boldsymbol{v_{in}} \ + \boldsymbol{\alpha_2 v_{in}^2}$$

DC:

$$\beta_0 = \alpha_0 + \alpha_2 \frac{A^2}{2}$$

2nd Harmonic:

$$\beta_2 = \alpha_2 \frac{A^2}{2}$$

Fundamental:

$$\beta_1 = \alpha_1 A$$

3rd Harmonic:

$$\beta_3 = 0$$

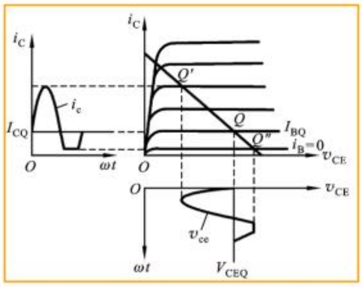
Total harmonic distortion (THD):

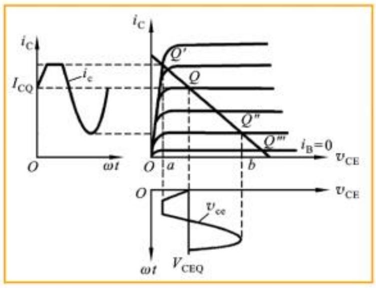
$$THD = \sqrt{HD_2^2 + HD_3^2 + HD_4^2 + \cdots}$$

$$= \frac{\beta_2}{\beta_1} = HD_2$$

Design consideration

- Properly bias
  - BJT
    - Active forward region ➔ $\boldsymbol{V_{BE} \geq 0.7\,V}$ & $\boldsymbol{V_{CE} \geq V_{BE}}$
  - MOSFET
    - Saturation ➔ $\boldsymbol{V_{DS} > V_{GS} - V_{TH}}$
- Select suitable Q point (clip distortion)
- Small-signal assumption
  - BJT
    - Small-signal assumption ➔ $v_{in} < 0.2\,V_T$
  - MOSFET
    - Small-signal assumption ➔ $v_{in} < 0.2\,(V_{GS,Q} - V_{TH})$





Assignment

Assignments:

9.1:

Use the short-circuit time constant method to determine the capacitor values of C_1, C_2 and C_3, so that $f_L = 100$ Hz can be achieved for the circuit in Fig. 1. Where $g_m = 3.82 \cdot 10^{-2}$, $\beta = 301$.

Hint: $r_\pi = \beta / g_m$, $r_e = 1/g_m$.

(1) C_1, C_2 and C_3 = ?

(2) Run the LTspice simulation to check your calculation by comparing with the simulated f_L.

(3) Run the LTspice simulation to find the f_H.

(4) Run the LTspice simulation to observe harmonic distortion.

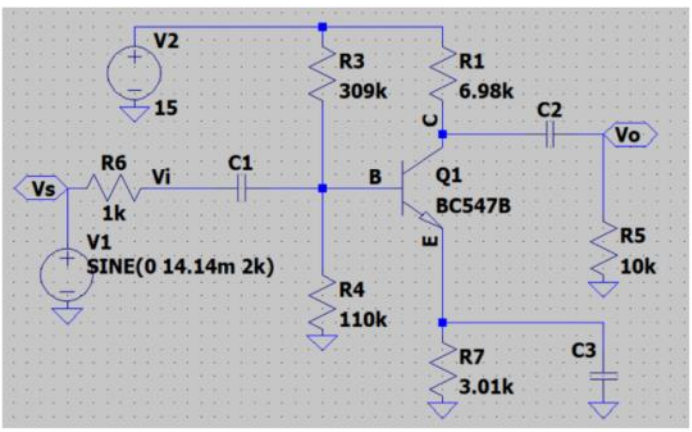


Fig. 1 A CE stage

Fig. 1 A CE stage

Solution:

(1)

$f_{C_1} = f_{C_2} = 0.1f_L$= 10 Hz

$f_{C_E} = 0.8f_L = 80$ Hz

$r_\pi = \dfrac{\beta}{g_m} = 7.88\ K\Omega$

$R_B = R_3||R_4 = 81.1\ K\Omega$

$R_E = R_7 = 3.01\ K\Omega$

$R_{C_1} = (R_B||r_\pi) + R_{sig} = 8.18\ K\Omega$

$R_{C_E} = R_E||\left(\dfrac{1}{g_m} + \dfrac{R_B||R_{sig}}{\beta + 1}\right) = 29.16\Omega$

$R_{C_2} = R_C + R_L = R_1 + R_5 = 16.98\ K\Omega$

$2\pi f_{C_1} = \frac{1}{C_{C_1}R_{C_1}} \Rightarrow C_{C_1} = \frac{1}{2\pi f_{C_1}R_{C_1}} = 1.94\ \mu F$

$2\pi f_{C_E} = \frac{1}{C_{C_E}R_{C_E}} \Rightarrow C_{C_E} = \frac{1}{2\pi f_{C_E}R_{C_E}} = 68.2\ \mu F$

$2\pi f_{C_2} = \frac{1}{C_{C_2}R_{C_2}} \Rightarrow C_{C_2} = \frac{1}{2\pi f_{C_2}R_{C_2}} = 0.94\ \mu F$

Check:
classic BJT Design.mw
(maple doc)
for how to do this.