Stability and eigenvalues

$$\det(\bar{\bar{A}} - \lambda \bar{\bar{I}}) = \begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix}$$

$$= \lambda^2 - (a_{11} + a_{22})\lambda + \det(\bar{\bar{A}}) = 0$$

From LA we know:

$$\mathrm{trace}(\bar{A}) = \sum_i a_{ii} = a_{11} + a_{22} = \sum_i \lambda_i \equiv p$$

$$\det(\bar{\bar{A}}) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \prod_i \lambda_i \equiv q$$

$$\det(\bar{\bar{A}} - \lambda \bar{\bar{I}}) = \lambda^2 - p\lambda + q \quad , \; D = p^2 - 4q$$

Nodes and eigenvalues

$D \geq 0$ (real eigenvalues)

$q = \prod_i \lambda_i > 0 \overset{(p \neq 0)}{\Longrightarrow}$ node

$q = \prod_i \lambda_i < 0 \Longrightarrow$ saddle

except $q \leq 0$

except $p = 0$

$p = \sum_i \lambda_i = 0 \overset{(q>0)}{\Longrightarrow}$ center

$p = \sum_i \lambda_i \neq 0 \Longrightarrow$ spiral point

$D < 0$ (complex eigenvalues)

Sommering

Phase plane - stability

Sommering

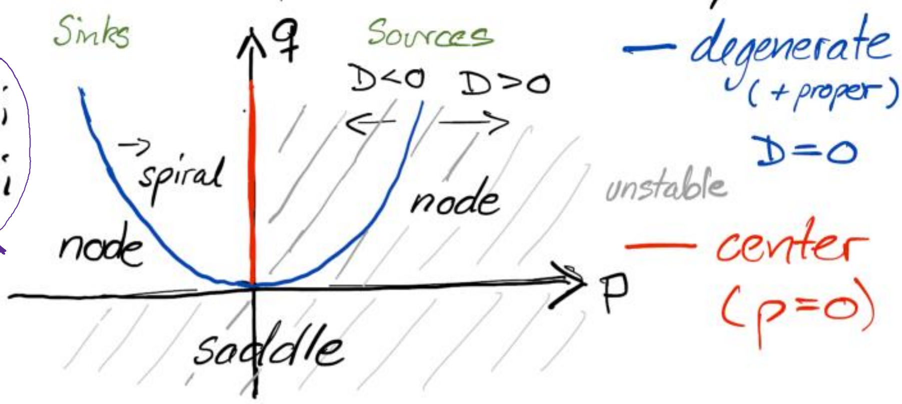
Phase plane - stability

$p = \sum \lambda_i$

$q = \prod_i \lambda_i$

Produkt af det hele

Eigen vals



- unstable if $q<0$ or $p>0$
- stable if $q>0$ and $p \leq 0$
- stable if $q>0$ and $p<0$ (attractive or asymptotically stable)

holds also for $n>2$