Visualization
– Illustrative Visualization (Questions)

J.-Prof. Dr. habil. Lawonn

Curvature Measurement

- What is P?

$$P = I - nn^T$$

Curvature Measurement

- What is P?
- P is an operator that projects every point on the tangent plane defined by the normal n

Curvature Measurement

- Calculate the projection operator with $n = (2,1,3)^T$

Curvature Measurement

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$$\|n\| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{14}$$

$$n_0 = \frac{1}{\sqrt{14}} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

Curvature Measurement

- Calculate the projection operator with n=(2,1,3) $P = I - nn^T$

$$n_0 = \frac{1}{\sqrt{14}} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 2/\sqrt{14} \\ 1/\sqrt{14} \\ 3/\sqrt{14} \end{pmatrix} \begin{pmatrix} 2/\sqrt{14} & 1/\sqrt{14} & 3/\sqrt{14} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 4/14 & 2/14 & 6/14 \\ 2/14 & 1/14 & 3/14 \\ 6/14 & 3/14 & 9/14 \end{pmatrix}$$

$$= \begin{pmatrix} 10/14 & -2/14 & -6/14 \\ -2/14 & 13/14 & -3/14 \\ -6/14 & -3/14 & 5/14 \end{pmatrix}$$

Curvature Measurement

- Calculate the projection operator with n=(2,1,3)

$$P = I - nn^T$$

$$n_0 = \frac{1}{\sqrt{14}} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$$P = \begin{pmatrix} 5/7 & -1/7 & -3/7 \\ -1/7 & 13/14 & -3/14 \\ -3/7 & -3/14 & 5/14 \end{pmatrix}$$

Curvature Measurement

- Determine the curvatures:

$$G = \begin{pmatrix} 4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Curvature Measurement

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$$G = \begin{pmatrix} \boxed{4} & 0 & 0 \\ 0 & \boxed{-2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Curvature Measurement

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$$G = \begin{pmatrix} 2 & 2 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Curvature Measurement

- Determine the curvatures:

$$G = \begin{pmatrix} 2 & 2 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$T = trace(G) = \kappa_1 + \kappa_2$$

$$F = |G|_F = \sqrt{\kappa_1^2 + \kappa_2^2}$$

Curvature Measurement

- Determine the curvatures:

$$G = \begin{pmatrix} \boxed{2} & 2 & 1 \\ 1 & \boxed{-2} & 0 \\ 0 & 1 & \boxed{1} \end{pmatrix}$$

$$T = 2 + (-2) + 1 = 1$$

Curvature Measurement

- Determine the curvatures:

$$G = \begin{pmatrix} 2 & 2 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$F = \sqrt{2^2 + 2^2 + 1^2 + 1^2 + (-2)^2 + 0^2 + 0^2 + 1^2 + 1^2} = 4$$

Curvature Measurement

- Determine the curvatures:

$$G = \begin{pmatrix} 2 & 2 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

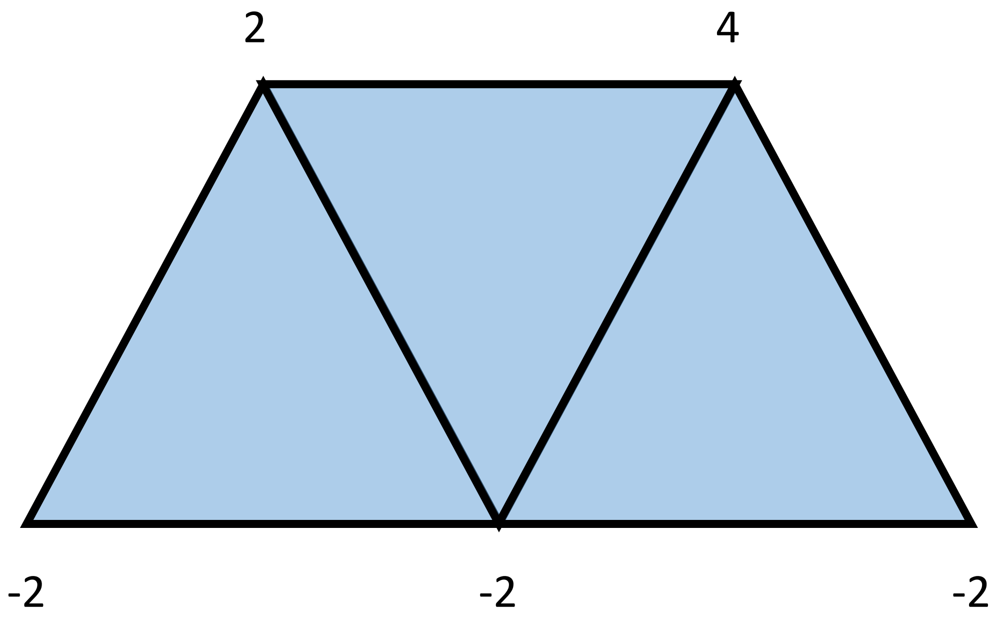
$$T = 1$$

$$F = 4$$

$$\kappa_{1/2} = \frac{T \pm \sqrt{2F^2 - T^2}}{2} = \frac{1 \pm \sqrt{2 \cdot 4^2 - 1^2}}{2} = \frac{1 \pm \sqrt{31}}{2}$$

Isolines on Triangulated Surfaces

- Determine the isolines with value 0.



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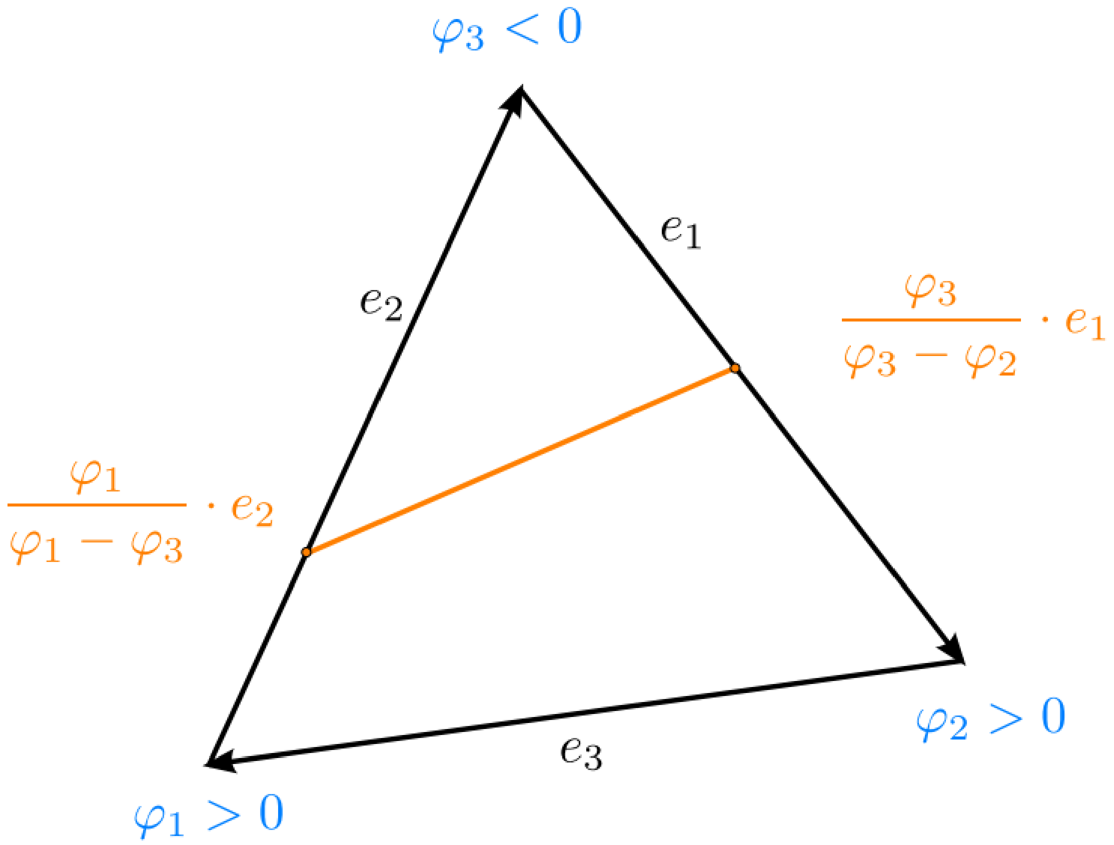

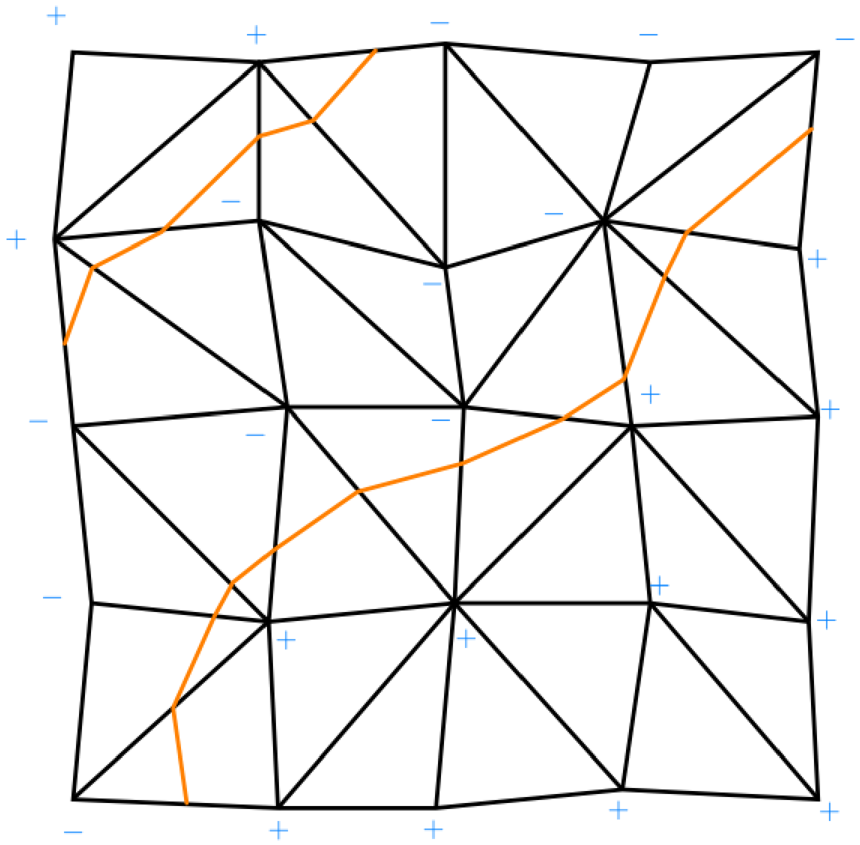

Isolines on Triangulated Surfaces

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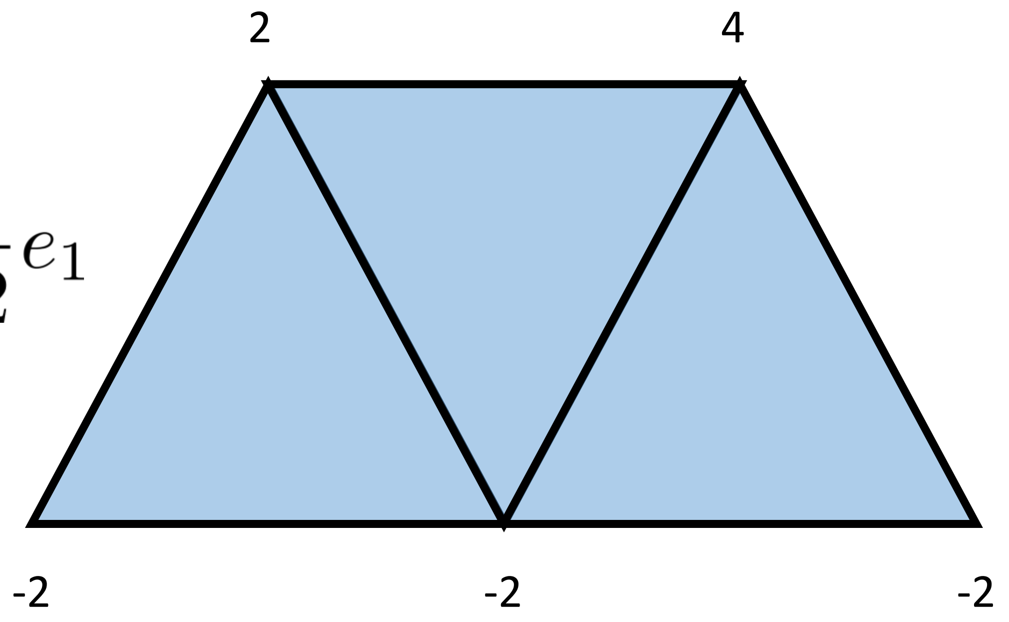
$$\frac{-2}{-2-2}e_1$$



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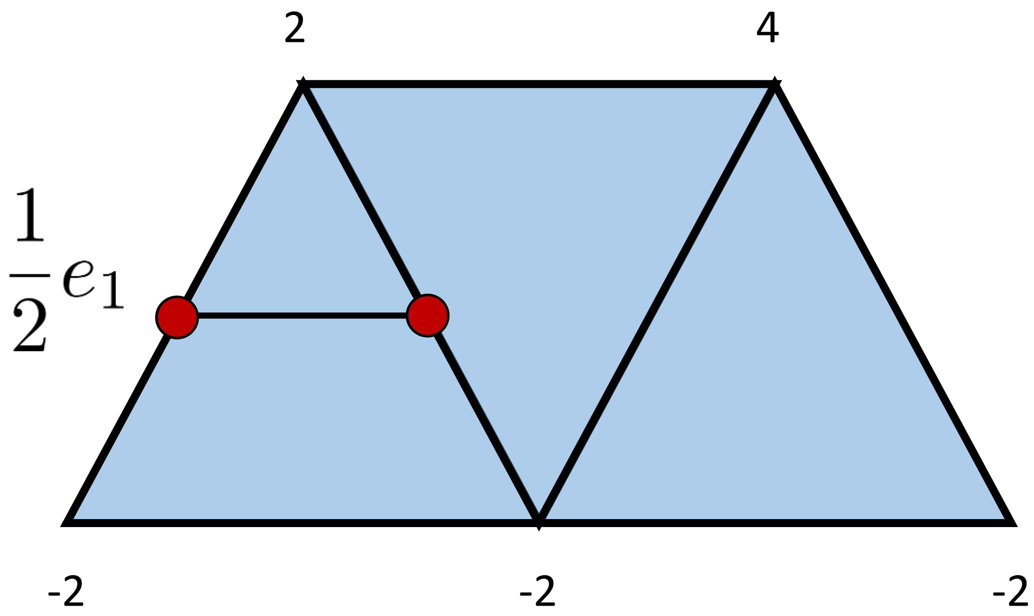

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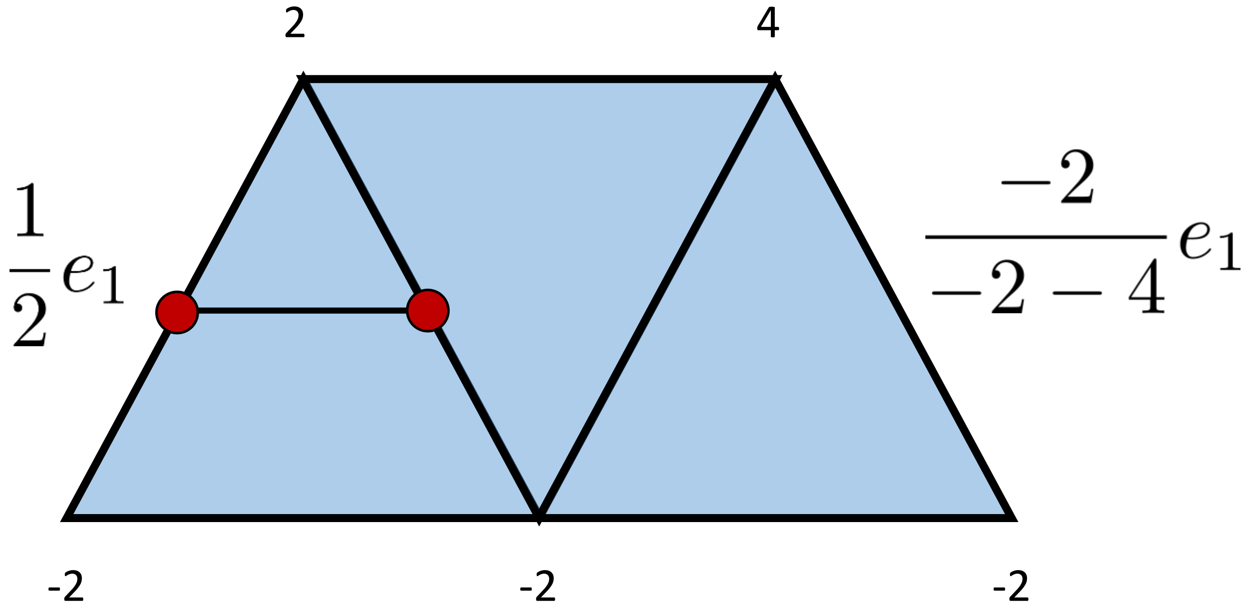

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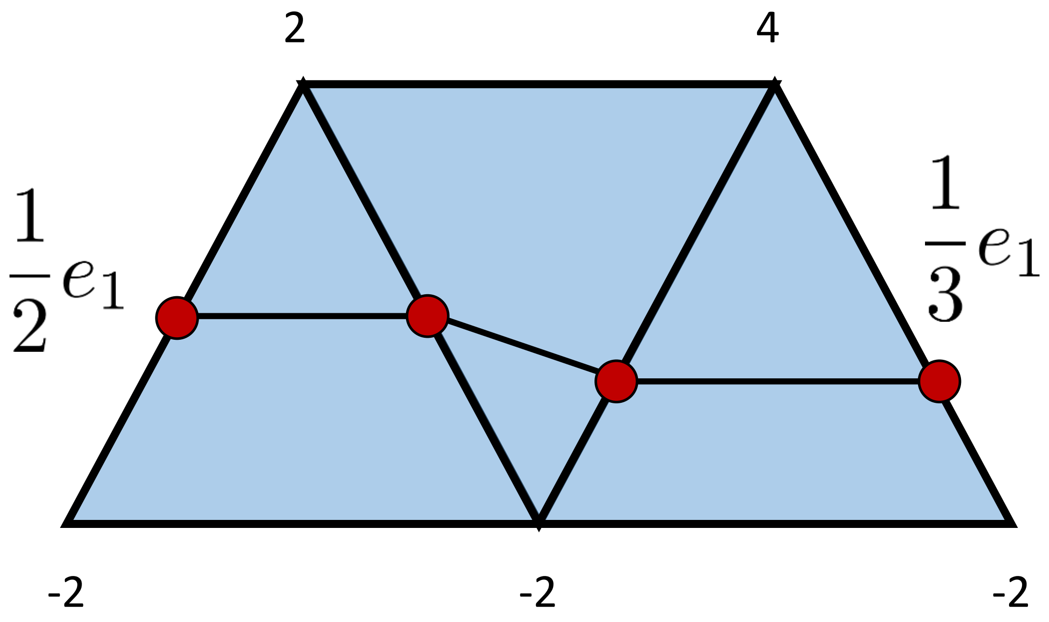

Covariant Derivative

- Determine the covariant derivative.

$$f(x,y) = 2 + x^2 - y$$

$$v(x,y) = \begin{pmatrix} -x \\ 0 \end{pmatrix}$$

$$D_{v(x,y)} f(x,y)$$

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Covariant Derivative

- What is the covariant derivative of a scalar field along isolines?

Covariant Derivative

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$$0$$

Feature Lines

- Compare two feature line techniques (of your choice) according to the order of derivatives, view-dependencies, and features.

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Name	Order	View-dep	Sharp Edges	Round Edges	Bumps (s.w.)	Bumps (top)	Contour	Deformation
Contours	1	yes	☒	☒	☒	☒	☑	☑
Crease Lines	1	no	☑	☒	☒	☒	☒	☑
Ridges & Valleys	3	no	☑	☑	☒	☑	☒	☒
Suggestive Contours	2	yes	☒	☒	☑	☑	☒	☑
Apparent Ridges	3	yes	☑	☑	☑	☑	☑	☒
Photic Extremum Lines	3	yes	☑	☒	☒	☑	☑	☑
Demarcating Curves	3	no	☒	☒	☒	☑	☒	☒
Laplacian Lines	3	yes	☑	☒	☒	☑	☑	☒