Visualization - Scalar & Vectorfields (Questions)

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• Determine the derivative:

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$$\frac{\partial f(x,y)}{\partial x} = -0.2 \cdot 2 \cdot x \cdot \exp(-0.2 \cdot (x^2 + y^2))$$

$$= -0.4 \cdot x \cdot f(x,y)$$

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$$f'_{\overrightarrow{x}}(x,y) = \frac{f(x,y) - f(x-1,y)}{\Delta x}$$

$$f'_{\overleftarrow{x}}(x,y) = \frac{f(x,y) - f(x+1,y)}{\Delta x}$$

What is the discrete gradient and how can it be calculated?

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 The gradient of a 2D function f is a vector pointing in the direction of the highest slope

$$\nabla f(x,y) = \begin{pmatrix} f'_{\overrightarrow{x}} \\ f'_{\overrightarrow{y}} \end{pmatrix}$$
$$= \begin{pmatrix} f(x,y) - f(x-1,y) \\ f(x,y) - f(x,y-1) \end{pmatrix}$$

• What is an ω -basin?

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 Collection of all points where the water flows to the same local minimum

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• Each region (without boundary) of the Morse-Smale complex is a quadrangle with vertices in this order around the region.

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• Each region (without boundary) of the Morse-Smale complex is a quadrangle with vertices **minimum**, **saddle**, **maximum**, **saddle**, in this order around the region.

Assign the correct words:

Let λ_1 , λ_2 be the eigenvalues of $J_{\mathbf{v}}(\mathbf{x}_0)$ with $Re(\lambda_1) \leq Re(\lambda_2)$:

• $Re(\lambda_i) < 0$

→ | behavior

• $Re(\lambda_i) > 0$

- → behavior
- $Im(\lambda_1) = -Im(\lambda_2) \neq 0$
- → behavior

swirling outflow inflow

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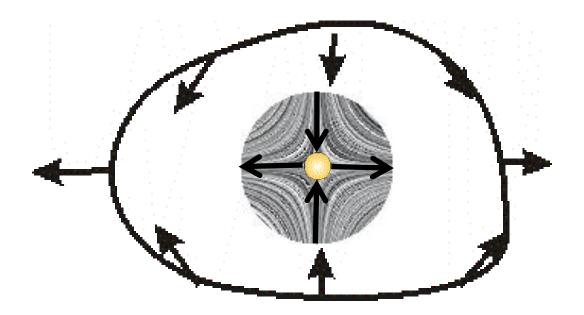
• $Re(\lambda_i) < 0$

→ inflow behavior

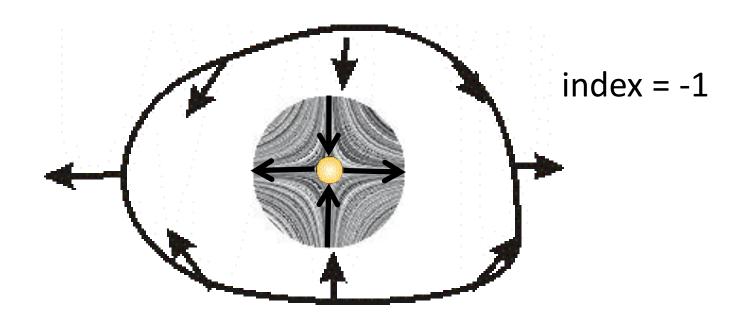
• $Re(\lambda_i) > 0$

- → outflow behavior
- $Im(\lambda_1) = -Im(\lambda_2) \neq 0$
 - → swirling behavior

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