# Visualization - Flow Visualization (Questions)

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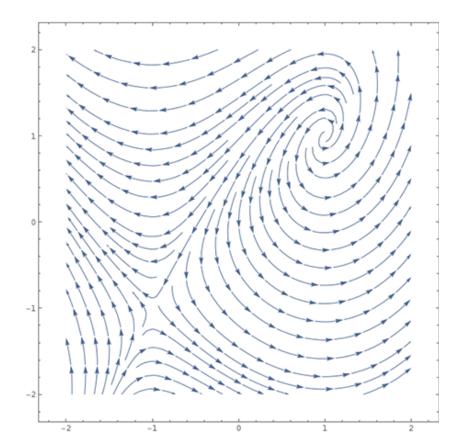
• Given is the following vector field  $v(x, y, z) = (x - y, x^2 - 1, x + z)^T$ . Determine the Jacobian, the divergence, and the curl.

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$$\mathbf{J} = \begin{pmatrix} 1 & -1 & 0 \\ 2x & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$div \ v(x, y, z) = 2$$

$$curl \ \boldsymbol{v}(x, y, z) = \begin{pmatrix} 0 \\ -1 \\ 2x + 1 \end{pmatrix}$$



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#### • Stream lines:

- Trajectories of massless particles in a "frozen" (steady) vector field
- Trajectories of massless particles at one time step

#### • Path lines:

- Trajectories of massless particles in (unsteady/time-varying) flow
- Follow one particle through time and space

• Given is the 2D vector field  $v(x,y,t)=(-y, x/2)^T$ . Perform the Euler integration at  $x_0=\begin{pmatrix} 0\\-1 \end{pmatrix}$  with  $\Delta t=0.5$ .

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• See lecture

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- $v(x,y) = 0 \rightarrow (x,y) = (0,0)$
- $\mathbf{J} = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$

Compute eigenvalues  $\lambda_1$ ,  $\lambda_2$ :

$$\det\begin{bmatrix} \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix} = \det\begin{pmatrix} 1 - \lambda & 1 \\ 4 & -2 - \lambda \end{pmatrix} = \lambda^2 + \lambda - 6$$
 
$$\Rightarrow \lambda^2 + \lambda - 6 = 0$$
 
$$\Rightarrow \lambda_{1,2} = \frac{-1 \pm \sqrt{1 + 24}}{2}$$
 Saddle point 
$$\text{We get real eigenvalues: } \lambda_1 = -3, \ \lambda_2 = 2 \Rightarrow \text{Im}(\lambda_{1,2}) = 0$$
 
$$\lambda_1 \lambda_2 < 0$$

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