

Online Appendix

Dry Firms, Deep Recessions: Corporate Payouts and Aggregate Dynamics

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A Data Construction and Cleaning

A.1 Definition of liquid assets from the Flow of Funds

The nonfinancial corporate business holdings of liquid assets are defined both as a narrow measure and a broad measure. Liquid assets is defined as the sum of, private foreign deposits (FL103091003.Q), checkable deposits and currency (FL103020005.Q), total time and savings deposits (FL103030003.Q) and money market fund shares (FL103034000.Q).

A.2 Compustat Data Construction and Cleaning

We construct a panel of U.S. non-financial, non-utility firms using the Compustat North America annual fundamentals file. The cleaning procedure follows standard practices. The main steps are as follows:

1. **Sample restrictions:** We retain only firm-year observations that satisfy the following:
 - Report in U.S. dollars (curcd = USD), use standard data format (datafmt = STD), industrial format (indfmt = INDL), and consolidated accounting (consol = C).
 - Incorporated in the United States (fic = USA).
 - Have non-missing accounting dates (datadate).
2. **Firm-year identification:** We ensure each firm-year is uniquely identified. For firms with multiple observations in a fiscal year, we keep the latest observation based on reporting month.
3. **Industry exclusions:** We exclude firms in the financial and utility sectors, defined as those with NAICS codes beginning in 22 (utilities), 52 (finance), or 53 (real estate). Multinational firms (NAICS = 999) are also excluded. Observations with missing or overly coarse (e.g., 2-digit) NAICS codes are dropped.

4. **Data validity and trimming:** We apply the following filters to address data quality and outliers:

- Observations must have strictly positive values for sales and total assets.
- The cash-to-assets ratio must lie within $[0, 1]$.
- Observations with negative debt (short or long term) are excluded.
- Leverage (defined as total debt over assets) is trimmed at the 99th percentile.
- Dividends-to-assets ratios exceeding 10 are excluded.
- Sales values are winsorized by dropping the top and bottom 1% of the distribution.

A.3 PSID Data Construction and Cleaning

Our empirical analysis draws on the Panel Study of Income Dynamics (PSID) Family and Individual Files from 2005 to 2021. We construct a panel dataset of U.S. households with a rich set of variables on income, wealth, consumption, and demographics. This appendix summarizes the key steps involved in preparing the dataset used for the regression analysis in Section 6.

Data Sources and Structure

We use the PSID Family Files to construct household-level variables on consumption, wealth, and various forms of income. These files are extracted for each survey year, standardized, and appended. We merge the Family Files with the PSID Individual File to recover detailed demographic characteristics and household composition, particularly the number of dependents and the identity of household heads and spouses.

Cleaning and Harmonization Procedures

- **Expenditure Variables:** We construct total household consumption by aggregating over categories such as food at home and away, rent, utilities, health care, transportation, education, and durable goods. Where expenditures are reported on dif-

ferent time scales (weekly, monthly, annually), we convert all values to annual figures.

- **Income and Dividend Variables:** We compute household-level dividend income as the sum of head and spouse self-reported income from dividends. Careful attention is given to avoid double-counting in cases where joint asset ownership is reported. We similarly construct interest, rent, and trust income, along with earned labor income and transfer payments.
- **Wealth Variables:** Wealth is calculated as the sum of financial (cash, stocks, bonds), real (real estate, vehicles), and business assets, net of outstanding debts such as mortgages and other liabilities.
- **Deflation:** All nominal variables are deflated using the December values of the Consumer Price Index (CPI-U) to reflect constant 2005 dollars. Forward-looking deflation is applied to match the retrospective nature of many income variables in the PSID.
- **Sample Restrictions:** We restrict the sample to households with a valid head, non-missing expenditure and dividend data, and at least one year of observation between 2005 and 2021. Implausible or top-coded values (e.g., dividends > \$10 million) are set to missing, and observations with zero or negative consumption or assets are excluded.

Key Variables for Analysis

The core regression examines the relationship between *real dividend income* and *real household expenditure*. The key variables used include:

- `divtotal_real`: Sum of head and spouse dividend income, deflated.
- `totexpenditure_real`: Total household expenditure, deflated using future CPI.
- `wealth_real, hlabinc_real, wlabinc_real`: Real wealth and labor income of household members.

- id, year : Household and year identifiers used for fixed effects.

A.4 Model validation - Cash holdings across the firm distribution

To test the validity of the micro mechanism highlighted in Section 4.1—that the standard deviation of cash holdings should be decreasing in firm size/productivity—we look at the standard deviation of the cash-to-total-assets and cash-to-sales ratios at each percentile of the assets and sales distributions, respectively. The raw distributions are plotted in Figure A1, with each dot representing a percentile of the assets or sales distributions and the corresponding standard deviation of the cash ratio for that respective percentile. In line with the model prediction, the standard deviation of cash holdings is negatively correlated with size/productivity.

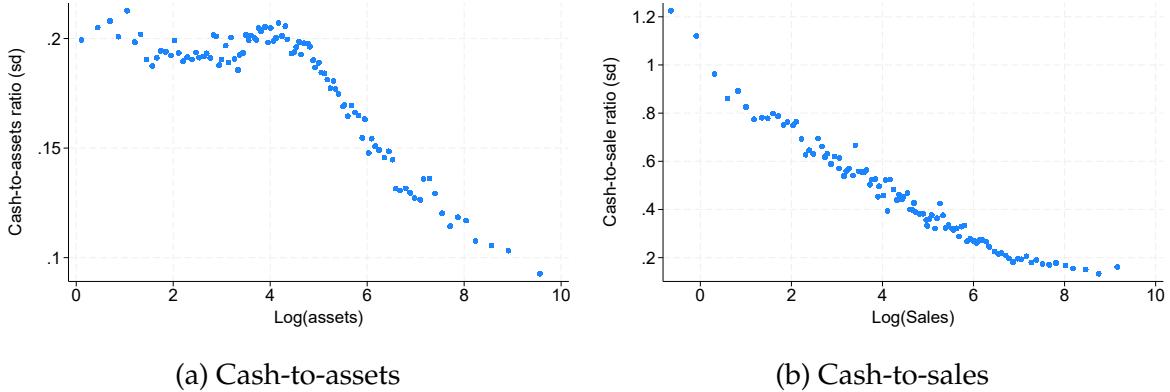


Figure A1: Standard deviation of cash-to-asset and cash-to-sales ratios over the asset and sales distribution

Notes: On the left panel, the figure plots the standard deviation of the cash-to-assets ratio, on the y-axis, at each percentile of the log of assets distribution, on the x-axis. On the right panel, the figure plots the standard deviation of the cash-to-sales ratio, on the y-axis, at each percentile of the log of sales distribution, on the x-axis. Each dot represents a percentile of the variable on the x-axis, and the respective standard deviation of the variable on the y-axis.

To guarantee the closest comparison to the model, we additionally clean the data of factors not explicitly considered in the model. These include firm and sector fixed effects, as firms in the model are ex-ante identical and the model does not feature multiple sectors. In addition, to compare to the steady state distribution, we also control for time fixed

effects. As different sectors can have different exposures to the business cycle, we include sector-time fixed effects. We run the following specification

$$n_{it} = \Gamma_h \mathbf{X}_{it-1} + \alpha_i + \lambda_{st} + \epsilon_{it} \quad (\text{A.1})$$

where n_{it} is either the cash-to-asset or cash-to-sales ratio, by firm i in year t . \mathbf{X}_{it-1} is a vector of controls that includes the lag of log real total assets, leverage, and real sales growth. α_i and λ_{st} are the firm and sector-year fixed effects. We then take the residuals and compute the standard deviation of the residuals for each percentile of the assets and sales distribution. Figure A2 illustrates that results are robust even after controlling for firm and sector-time fixed effects.

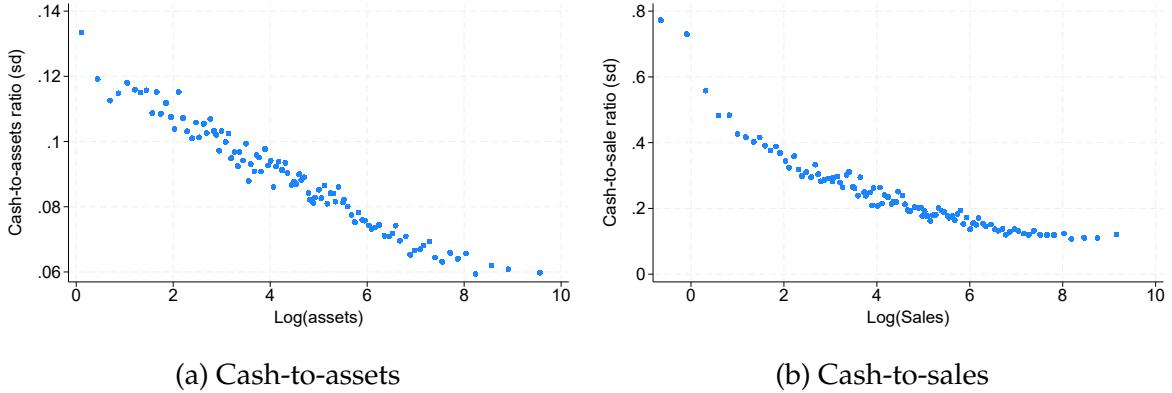


Figure A2: Standard deviation of cash-to-asset and cash-to-sales ratios residuals over the asset and sales distribution

Notes: On the left panel, the figure plots the standard deviation of the cash-to-assets ratio residuals, on the y-axis, at each percentile of the log of assets distribution, on the x-axis. On the right panel, the figure plots the standard deviation of the cash-to-sales ratio residuals, on the y-axis, at each percentile of the log of sales distribution, on the x-axis. Each dot represents a percentile of the variable on the x-axis, and the respective standard deviation of the variable on the y-axis.

Next, we plot the model correlation between the cash-to-sales standard deviation and log sales. Figure A3 illustrates that the model exhibits a correlation similar to that observed in the data as predicted in Section 4.1. The dispersion in terms of cash-to-sales ratio is considerably higher at the bottom of the sales distribution than at the top.

Finally, to ensure that it is not only the dispersion of the cash-to-sales ratio that is in

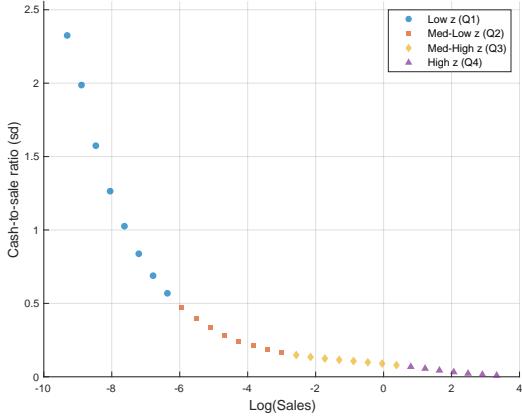


Figure A3: Standard deviation of cash-to-sales ratio over the sales distribution in the model

Notes: The figure plots the the standard deviation of the cash-to-sales ratio residuals, on the y-axis, and the log of sales distribution, on the x-axis. Each dot represents a productivity grid point, and the respective standard deviation of the variable on the y-axis.

line with the data, we also look at the average cash-to-sales ratio over the sales distribution, both in the model and in the data. Figure A4 illustrates that both in the model and in the data cash-to-sales ratio has a negative correlation with sales, considerably higher at the bottom than at the top of the distribution.

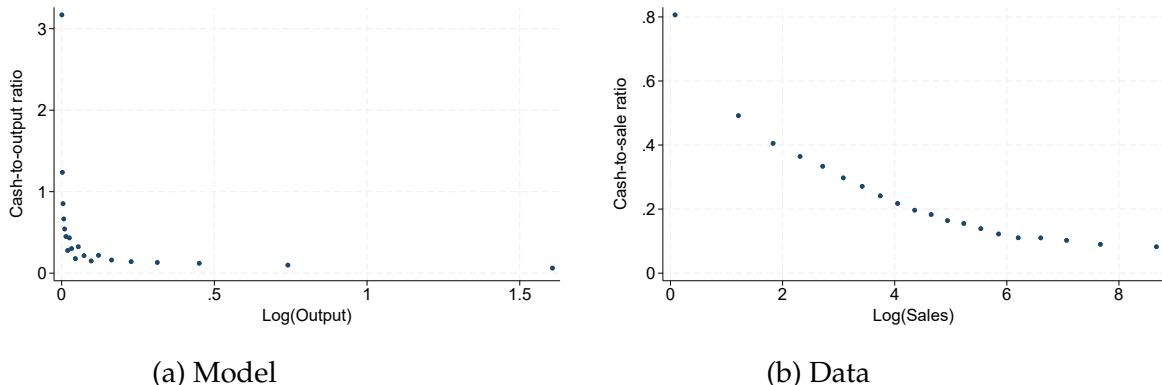


Figure A4: Correlation between cash-to-output ratio and output in the model (panel a) and in the data (panel b).

Notes: On the left panel, the figure plots the the correlation between cash-to-output ratio, on the y-axis, and log of output, on the x-axis, in the baseline model. On the right panel, the figure plots the empirical correlation between cash-to-sales ratio, on the y-axis, and log of sales, on the x-axis.

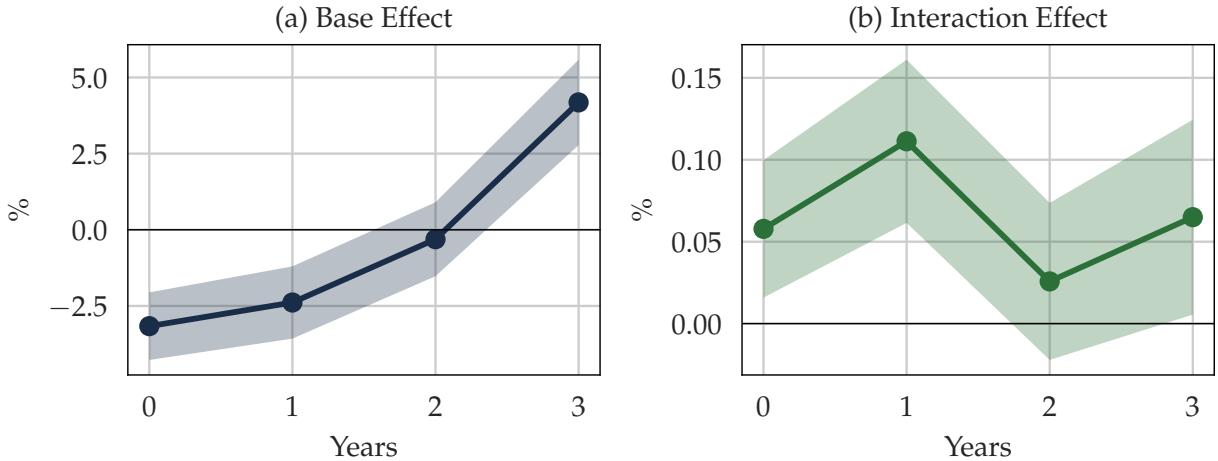


Figure A5: Dynamic dividend reaction to negative GDP and interaction with cash to assets

Note. The figure plots the impact of recessions on panel (a) up to three years after the recession. Panel (b) plots the coefficient associated with the interaction term between cash to total assets ratio and the recession indicator. Shaded bands represent 90% confidence intervals. Log of one plus real dividends considered.

A.5 Robustness exercise for Section 6

We run the following local projection

$$d_{it+h} = \beta \text{rec}_t + \delta n_{it-1} + \gamma \text{rec}_t \times n_{it-1} + \Gamma_h \mathbf{X}_{it-1} + \alpha_i + \epsilon_{it} \quad (\text{A.2})$$

where d_{it+h} is the log of the real dividends plus one, so that zero dividend observations are included, by firm i in year $t+h$, rec_t is an indicator variable that takes the value of 1 whenever real GDP growth is negative, n_{it-1} represents the ratio of cash and short-term investments to total assets, and X_{it-1} is a vector of controls that includes the lag of log real total assets, leverage, and real sales growth.¹ α_i represents the firm fixed effects and standard errors are clustered at the firm level.

Panel (a) on Figure A5 plots the base effect (β) of recessions on dividends on the left panel, and on the right panel the effect of cash holdings on dividends during recessions (γ). For firms with no cash holdings, dividends drop by approximately 3.1 percentage points during a recession, but recover after two years. Importantly, panel (b) illustrates that firms which have a higher cash-to-asset ratio do not reduce their dividend issuance

¹All variables except cash are deflated using the CPI.

as much after a year of negative GDP growth, as the coefficient is positive and statistically significant in both Year 0 and Year 1. One year after the recession, a 1 percentage point increase in the cash-to-assets ratio mitigates the dividend drop by 0.06 percentage points.

B Two period model extensions

B.1 Linear equity issuance cost

The firm's problem is to maximize its value, considering current and future dividends net of equity issuance costs. The cost of equity issuance $C(d)$ is given by:

$$C(d) = \begin{cases} \mu_1 d + \frac{1}{2}\mu_2 d^2 & \text{if } d < 0 \\ 0 & \text{if } d \geq 0 \end{cases}$$

where d represents dividends. This cost is incurred only when the firm issues equity (i.e., when dividends are negative).

The firm chooses its cash holding n_1 in period 1. The dividends in period 1 (d_1) and in period 2 (\tilde{d}_2) are functions of n_1 and productivity shocks.

The firm's objective is to maximize its value, which depends on its current cash holdings n_1 . The firm's value function $V(n_1)$ is given by:

$$V(n_1) = (A_1 - w_1 - n_1\beta^n + n_0) - C(d_1) + \mathbb{E}[m(\tilde{A}_2)(\tilde{d}_2 - C(\tilde{d}_2))]$$

Case 1: n_1^* is such that $d_2^L \geq 0$ (i.e., $n_1^* \geq n_1^{max}$)

Let $n_1^{max} = w_2 - A_1 + \Delta$. In this scenario, $C(d_2^L) = 0$ (since $d_2^L \geq 0$), and thus $C'(d_2^L) = 0$.

The first-order condition becomes:

$$-\beta^n + \frac{1}{2}m(A_1 + \Delta) + \frac{1}{2}m(A_1 - \Delta) = -\beta^n + \mathbb{E}[m(\tilde{A}_2)] = 0$$

However, we assume $\beta^n > \mathbb{E}[m(\tilde{A}_2)]$, meaning $-\beta^n + \mathbb{E}[m(\tilde{A}_2)] < 0$. This implies that the firm has a satiation point at $n_1 = n_1^{max}$.

Case 2: $d_i < 0$

The first-order condition is:

$$-\beta^n + \mu_1\beta^n + \mu_2\beta^n d_1 + \mathbb{E}[m(\tilde{A}_2)](1 - \mu_1 - \mu_2 d_2) = 0$$

Solving for n_1^* :

$$n_1 = \frac{\mu_2 \left[\beta^n (A_1 - w_1 + n_0) + \mathbb{E} \left(m(\tilde{A}_2)(w_2 - \tilde{A}_2) \right) \right] + \left(\beta^n - \mathbb{E}(m(\tilde{A}_2)) \right) (\mu_1 - 1)}{\left(\mathbb{E}(m(\tilde{A}_2)) + (\beta^n)^2 \right) \mu_2}$$

Proposition A.1 (Precautionary savings and linear equity issuance cost.).

As $\beta^n > \mathbb{E}(m(\tilde{A}_2))$ an increase in μ_1 leads to an increase in cash holdings.

B.2 Optimal savings

Social planner The social planner maximizes the present discounted value of the household's utility:

$$\max_{c_1, c_2} \log(c_1) + \beta \mathbb{E} \log(\tilde{c}_2), \quad (\text{A.3})$$

$$\text{s.t. } c_1 = A_1 - n_1\beta^n + n_0, \quad (\text{A.4})$$

$$\tilde{c}_2 = \tilde{A}_2 + n_1, \quad (\text{A.5})$$

$$n_1 \geq 0 \quad (\text{A.6})$$

Consumption in period 1 equals production net of changes in cash holdings, and consumption in period 2 includes stochastic output and cash. Since the equity issuance cost is rebated, it does not appear in the planner's constraints.

Proposition A.2 (Inefficiency of the decentralized cash holdings).

Socially optimal cash holdings are equal to zero.

Proof. The social planner first order condition implies

$$\beta^n = \beta \mathbb{E} \frac{A_1 - \beta^n n_1 + n_0}{\tilde{A}_2 + n_1} + \lambda (A_1 - \beta^n n_1 + n_0). \quad (\text{A.7})$$

Notice the term $\beta \mathbb{E} \frac{A_1 - \beta^n n_1 + n_0}{\tilde{A}_2 + n_1}$ is equal to the stochastic discount factor and as β^n is strictly lower than the SDF, this implies $\lambda > 0$ and $n_1 = 0$ for any level of n_0 . ■

The social planner wants zero cash holdings due to the externality of cash holdings. When firms increase their cash buffers, they depress household wealth by reallocating resources toward lower-return assets, thereby leading to a decline in aggregate consumption. This externality leads to the social planner strictly preferring zero cash to avoid the externality. Despite the cost of holding cash, the firms still prefer strictly positive amounts of cash for precautionary motives as they do not internalize that the equity issuance cost is not a social cost. If firms fully internalized the externality, the decentralized and planner solutions would coincide.

This proposition equally holds for the quantitative model, as the planner's problem is similar due to the presence of the wedge between cash return and the stochastic discount factor and a representative household.

Pecuniary externality The above identified cash externality could also be seen as a pecuniary externality of holding cash, if equity prices were considered. As dividends become on average lower with higher cash holdings, this depresses the equity prices, which leads to a lower level of average consumption. The social planner maximizes the presented discounted value of the household's utility:

$$\begin{aligned} \max_{c_1, c_2} \quad & \log(c_1) + \beta \mathbb{E} \log(\tilde{c}_2), \\ \text{s.t.} \quad & c_1 + q_2 = q_1 + w_1, \\ & \tilde{c}_2 = q_2 + w_2, \\ & q_1 = d_1 + \mathbb{E}(m(\tilde{A}_2)d_2), \\ & q_2 = \mathbb{E}(m(\tilde{A}_2)d_2). \end{aligned} \quad (\text{A.8})$$

Consumption in period 1 equals labor income w_1 net of changes in equity holdings $q_1 - q_2$, and consumption in period 2 includes labor income w_2 and the equity prices q_2 in period 2. Since the equity issuance cost is rebated, it does not appear in the planner's constraints and does not affect equity prices.

Proposition A.3 (Inefficiency of the decentralized cash holdings).

Socially optimal cash holdings are equal to zero.

Proof. The social planner first order condition implies

$$\beta^n = \beta \mathbb{E} \frac{A_1 - \beta^n n_1 + n_0}{\tilde{A}_2 + n_1} + \lambda(A_1 - \beta^n n_1 + n_0). \quad (\text{A.9})$$

Notice the term $\beta \mathbb{E} \frac{A_1 - \beta^n n_1 + n_0}{\tilde{A}_2 + n_1}$ is equal to the stochastic discount factor and as β^n is strictly lower than the SDF, this implies $\lambda > 0$ and $n_1 = 0$ for any level of n_0 . ■

The result is still the same as before, but the externality now is a pecuniary externality as it operates via equity prices.

C Note on computation

Then, the equilibrium price p_t is determined from the following variant of the non-trivial market clearing condition:²

$$p = \arg_{\tilde{p}} \left\{ c(\tilde{p}) - \int [d(x; X, \tilde{p}) + w(X, \tilde{p})l(x; X, \tilde{p})] d\Phi = 0 \right\}.$$

This is a fixed-point problem and computationally costly to solve. Instead, the repeated transition method uses the implied price p^* , which is obtained as follows:

$$\begin{aligned} p^* &= \arg_{\tilde{p}} \left\{ c(\tilde{p}) - \int [d(x; X, p^{(n)}) + w(X, p^{(n)})l(x; X, p^{(n)})] d\Phi = 0 \right\} \\ &= 1 / \int [d(x; X, p^{(n)}) + w(X, p^{(n)})l(x; X, p^{(n)})] d\Phi, \end{aligned}$$

where $p^{(n)}$ is the guessed price in the n^{th} iteration. We take the sufficient statistic approach, and the aggregate cash holdings N_t (the first moment of the distribution of cash holding) is the sufficient statistic.

²This condition is derived from combining the household's budget constraint and the equity market clearing condition. For a sharp illustration, the time subscript is lifted.

D Recovering the true nonlinear law of motion

In this section, we recover the true law of motion from the converged equilibrium outcomes over the simulated path. Then, we test the validity of the true law of motion by fitting the law of motion into the out-of-sample simulated path.

Specifically, the following laws of motion are studied:

$$N_{t+1} = G_N(N_t, N_{t-1}, N_{t-2}, \dots, N_{t-n}; A_t)$$

$$p_t = G_p(N_t, N_{t-1}, N_{t-2}, \dots, N_{t-n}; A_t).$$

Table D1 reports the goodness of fitness (R^2) of the different specifications. The first five rows report the fitness when only the contemporaneous aggregate cash stock N_t is considered up to different polynomial orders. When a single argument is considered without the higher-order polynomials, R^2 gets as low as 0.8956 for G_N if the contemporaneous productivity state is G .³ As the more higher-order polynomials are included, the better the fitness becomes. However, the fitness of the specification of G_p stops improving after a certain threshold. This shows that the true law of motion can be recovered only by including further historical allocations.

The bottom seven rows of Table D1 report the fitness of the law of motion when the additional lagged terms of the aggregate cash stock are considered. Up to the third order polynomials are included for each of lagged terms on top of the polynomial terms of the contemporaneous cash stocks up to the fifth order.⁴ As more lagged terms are considered in the law of motion, the fitness improves, especially in G_p . However, only after the polynomials of the seven-period lagged aggregate cash stock are included in the law of motion, the accurate law of motion is recovered.

This exercise shows the substantial nonlinearity of the aggregate fluctuations in this model. In the repeated transition method, the contemporaneous aggregate cash stock is used as a sufficient statistic of each period's cross-section. However, this does *not* imply that the true aggregate law of motion is a function of only the contemporaneous cash

³This pure linear specification is different from the log-linear specification in Section 5.3.

⁴The results only negligibly change over different order choices.

Table D1: The fitness of the law of motion across different specifications

	# of lagged	order	Goodness of fitness: R^2		
			$CA_{t+1} : Good$	$CA_{t+1} : Bad$	$p_t : Good$
Contemp.	0	1	0.8956	0.9452	0.9922
	0	2	0.9839	0.9952	0.9927
	0	3	0.9973	0.9995	0.9930
	0	4	0.9993	0.9999	0.9932
	0	5	0.9996	1.0000	0.9933
Add. history	1	3	0.9999	1.0000	0.9987
	2	3	0.9999	1.0000	0.9997
	3	3	0.9999	1.0000	0.9998
	4	3	0.9999	1.0000	0.9998
	5	3	0.9999	1.0000	0.9998
	6	3	0.9999	1.0000	0.9998
	7	3	0.9999	1.0000	0.9997

Notes: The first column (# of lagged) reports the number of lagged terms included in the specification. The second column (order) reports the highest order of polynomials considered. When lagged terms are included, contemporaneous terms are considered up to the fifth polynomial for all specifications. The third and fourth column reports the R^2 of the law of motion of N_{t+1} for Good and Bad shock realizations. The fifth and sixth column reports the R^2 of the law of motion of p_t for Good and Bad shock realizations.

stock. The contemporaneous cash stock is rather a labeling of each period that correctly sorts the rankings of the value functions across the periods in the repeated transition method.

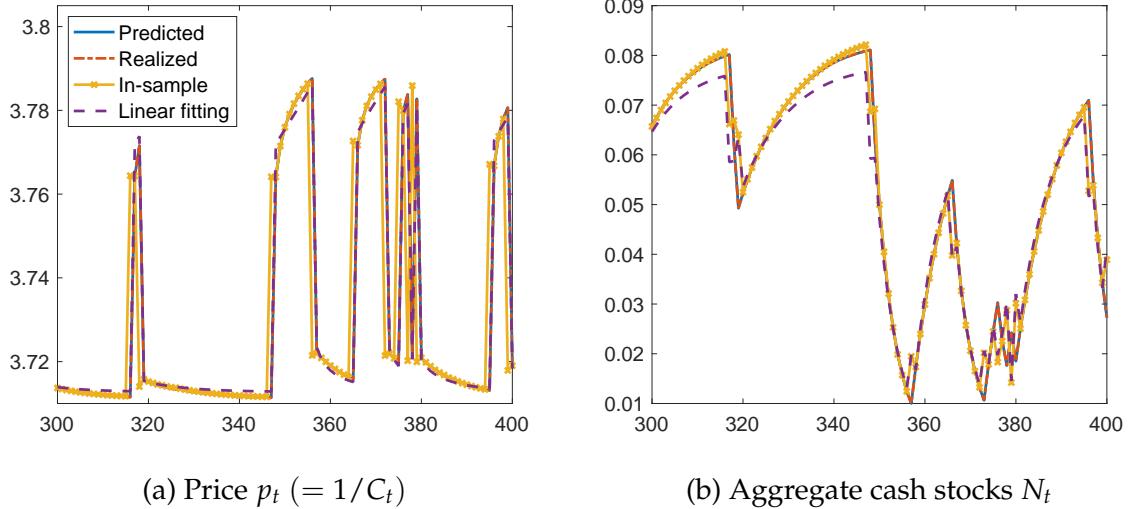


Figure D1: Fitting into the out-of-sample path

Notes: The figure plots the time series of the price p_t the aggregate cash stock N_t in the baseline model for an out-of-sample simulated path of the aggregate shocks. In both panels, the solid line is the predicted time series (n^{th} guess) $\{p_t^{(n)}, N_t^{(n)}\}_{t=300}^{400}$; the dash-dotted line is the realized time series $\{p_t^*, N_t^*\}_{t=300}^{400}$; the line with the star tick mark is the predicted time series implied by the exact law of motion recovered from the in-sample path; the dashed line is the predicted time series implied by the linear law of motion.

We validate the recovered law of motion by fitting it on the out-of-sample simulated path.⁵ Specifically, we solve the model on another simulated path to obtain the converged equilibrium dynamics using the repeated transition method and compare the dynamics with the implied dynamics in the recovered true law of motion on the in-sample path. Figure D1 plots p_t (panel (a)) and N_t (panel (b)) for 1) predicted time series (solid line), 2) realized time series (dot-dashed line), 3) time series implied by the recovered in-sample law of motion (solid line with ticks), and 4) time series implied by the linear law of motion (dashed line). The predicted time series and the realized time series are indistinguishably close to each other due to the convergence requirement of the repeated transition method. The time series implied by the recovered law of motion also closely tracks the converged equilibrium dynamics, validating the specification. The goodness of fitness (R^2) in the time series implied by the recovered law of motion is greater than 0.999 for both G_N and G_p in all shock realizations.

⁵The recovered true of motion refers to the specification that considers up to the seven-period lagged aggregate cash stocks, where R^2 is the highest in Table D1.

E Social planner's problem

The social planner's problem can be formulated in the following recursive form:

$$V^H(n; A) = \max_{c, n', L^H} \left[\log(c) - \frac{\eta}{1 + \frac{1}{\chi}} (L^H)^{1 + \frac{1}{\chi}} + \beta \mathbb{E} [V^H(n'; A')] \right] \quad (\text{A.10})$$

subject to:

$$c + q^n(X)n' = f(L^H) + n$$

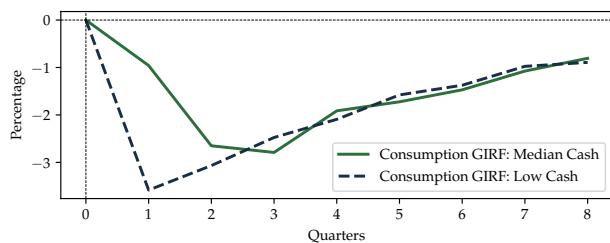
$$A' \sim \Gamma(A)$$

The social planner's state variables are the aggregate cash holding n and the exogenous productivity A . The cash return is determined by $q^n(X)$ as in the baseline model.

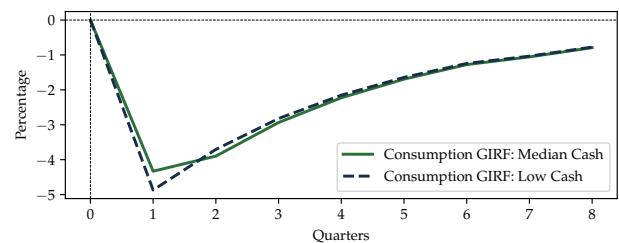
F Additional model results

F.1 Representative firm model

Figure F1 plots the GIRF to a negative TFP shock when cash is at median and low levels over the business cycle. Panel (a) plots the GIRF for the baseline model with heterogeneous firms. Panel (b) plots the GIRF for the representative firm model, a model where there is only aggregate TFP shocks and no idiosyncratic TFP shocks. The Figure illustrates that the nonlinearity, while it is still present in the representative firm model, is severely amplified by firm heterogeneity. This comes from the fact that the representative firm is rarely at the two extremes of the cash policy function, where the nonlinearity is stronger, resulting in an almost linear business cycle.



(a) Heterogeneous firms



(b) Representative firm

Figure F1: GIRF: Consumption responses to negative shocks for median and low cash in the baseline model (panel a) and the representative firm model (panel b).

Notes: The figure plots the consumption % change to a negative aggregate TFP shocks when cash is at median level (green solid line) vs lowest level (blue dashed line) over the entire simulation.