Cairo University
Faculty of Engineering
Electrical Power Engineering

3rd year
Digital Control Systems
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Introduction

A model of a flexible robot arm is shown in Figure 1. A motor with current constant k_I drives a load consisting of two masses coupled with a spring with a spring constant k. It is assumed that friction and damping can be neglected. The input signal is the motor current I. The angular velocities and the angles of the masses are $\omega_1,\ \omega_2, \varphi_1,$ and φ_2. The moments of inertia are J_1 and J_2. It is assumed that there is a relative damping, d, in the spring and the mass may be disturbed by a torque v. Finally, the output of the process is the angular velocity ω_2.

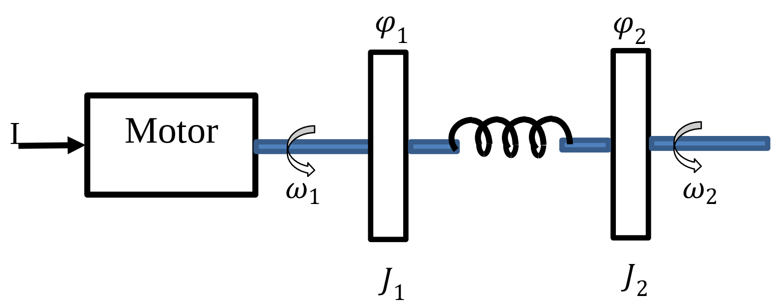

Figure 1 Model of a flexible robot arm

Define the system states as

$$x_1 = \varphi_1 - \varphi_2$$
$$x_2 = \frac{\omega_1}{\omega_0}$$
$$x_3 = \frac{\omega_2}{\omega_0}$$

where $\omega_0 = \sqrt{k(J_1 + J_2)/(J_1 J_2)}$.

System Dynamics

The system is described by the following equations

$$\frac{d\boldsymbol{x}}{dt} = \omega_0 \begin{bmatrix} 0 & 1 & -1 \\ \alpha - 1 & -\beta_1 & \beta_1 \\ \alpha & \beta_2 & -\beta_2 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ \gamma \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ \delta \\ 0 \end{bmatrix} v$$
$$y = [0 \quad 0 \quad \omega_0]\ \boldsymbol{x}$$

where $\boldsymbol{x} = [x_1 \quad x_2 \quad x_3]^T$.
Define

$$\begin{aligned} \alpha &= J_1/(J_1 + J_2) \\ \beta_1 &= d/J_1\omega_0 \\ \beta_2 &= d/J_2\omega_0 \\ \gamma &= k_I/J_1\omega_0 \\ \delta &= 1/J_1\omega_0 \end{aligned}$$

Assume the following numerical values.
$J_1 = 10/9, J_2 = 10$, $k = 1$, $d = 0.1$, and k_I=1.

Continuous System Analysis and Control Design Using the Transfer Function Approach

1- Find the transfer function $\frac{Y}{U}$. Plot the root locus and find the stability range of a gain controller.
2- Use the root locus to design a lead compensator that achieves a damping ratio 0.7 and a natural frequency 0.5 rad/sec.
3- Use the Bode plots to design a lead compensator that achieves a damping ratio 0.7 and a natural frequency 0.5 rad/sec.
4- Simulate the system with both controllers (designed in parts 2 and 3 above) assuming the reference is a unit step and the disturbance is a pulse of magnitude 1 that appears from $20 < t < 22$ secs.
5- Plot the output and the control signals for both controllers. Comment on the results.

Digital Control Design Using the Transfer Function Approach

Note: In all the simulations required in this part, the system model must be represented by a continuous-time model. Only, the controller will be in discrete-time (or z-domain). The output of the controller should go through a zero-order-hold before it is applied to the system.

In all simulations, assume the reference is a unit step and the disturbance is a pulse of magnitude 1 that appears from $20 < t < 22$ secs.

1- Show the effect of selecting short and long sampling interval on the resulting poles and zeros in the discrete model. Comment on your results.
2- Select an appropriate sampling interval and design a digital controller (using the direct digital control design) to stabilize the system and ensure a damping ratio 0.7 and a natural frequency 0.5 rad/sec.
3- Repeat in question #2 using a design by emulation.

4- Compare the results due to the design by emulation to those obtained by the direct design. Comment on the results.

Continuous State Space Representation

In all simulations, assume the reference is a unit step and the disturbance is a pulse of magnitude 1 that appears from $20 < t < 22$ secs.

1- Using the continuous system dynamics given, develop and Simulink model for open loop system. Draw the system states (Call this representation "rep A").
2- Find appropriate state feedback gain to place the poles of the system in suitable places (You can use the same requirements as mentioned in the transfer function approach).
3- Implement the feedback signals using the Simulink. Draw the states and output versus time. Comment on the results.

Discrete State Space Representation

1- Choose a suitable sampling period and find the discrete form for "rep A" (Call this representation "rep B"). Write the state space representation in the controllable canonical form (Call this representation "rep C"). Draw the system states for both representations for open loop case. Comment on the results.
2- Design state feedback vector for "rep B" to achieve same transient response specifications as before, and use reference manipulation gain to achieve zero steady state error. Implement your controller on the continuous-time model (rep A). Draw the states and output versus time. Comment on the results.
3- Implement same controller designed on (2) using the states measurements from (rep C). Draw the system states, and comment on the results.
4- Using rep A and the controller designed in 2, assuming that the only measurement available is the output ω_2 , design an appropriate observer for the system and implement the state feedback. Draw the system states (starting from initial condition $x = [3 \quad 5 \quad 0]^T$) and the estimated states. Comment on the results.
5- For the controller in 4, if the sampling interval is changed to double its value, what will be the effect of this change on the feed-back system response with the same state feedback gain? Study this effect and draw the system states.