



Digital control systems EPE 3090

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DIGITAL CONTROL PROJECT

FLEXIBLE ROBOTIC ARM



MADE BY:

بنش 33

بنش 34

بنش 35

سكشن 3

سكشن 3

سكشن 3

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CONTINUOUS SYSTEM ANALYSIS AND CONTROL DESIGN USING THE TRANSFER FUNCTION APPROACH

1. Find the transfer function $\frac{Y}{U}$. Plot the root locus and find the stability range of a gain controller.

MATLAB CODE TO OBTAIN THE TRANSFER FUNCTION OF A SYSTEM USING ITS STATE SPACE REPRESENTATION:

CODE

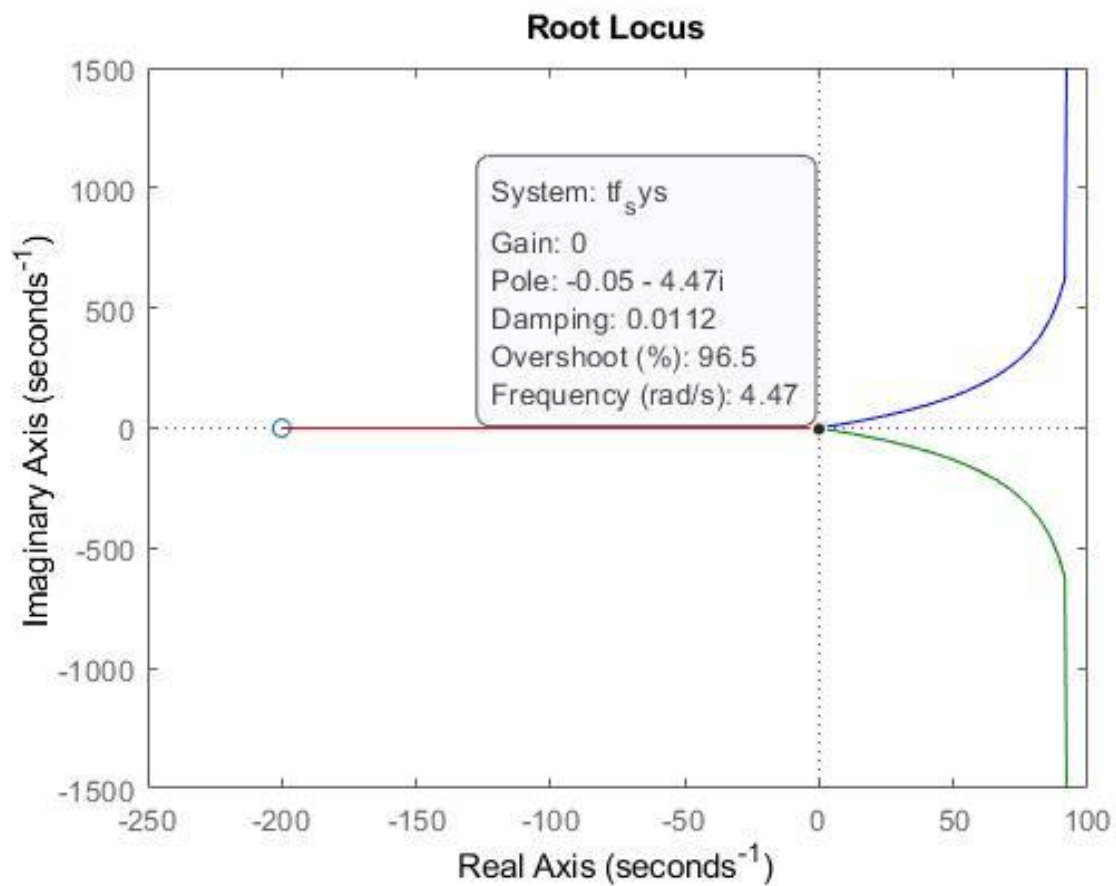
```
J1 = 10/9;  
J2 = 10;  
k = 1;  
d= 0.1;  
ki=1;  
w0 = sqrt(k*(J1 + J2)/(J1*J2));  
  
alpha=J1/(J1+J2);  
beta1=d/(J1*w0);  
beta2=d/(J2*w0);  
gamma=ki/(J1*w0);  
delta=1/(J1*w0);  
  
% Define the system's state space representation  
A = [0 1 -1; alpha-1 -beta1 beta1; alpha beta2 -beta2]; % state matrix  
Anew=w0*A;  
B = [0; gamma; 0;]; % input matrix  
C = [0 0 w0]; % output matrix  
D = 0; % feedthrough matrix  
  
% Convert the state space representation to transfer function  
sys = ss(Anew, B, C, D);  
tf_sys = tf(sys);  
sisotool(tf_sys)
```

OUTPUT

```
tf_sys =  
  
      0.009 s + 0.09  
-----  
s^3 + 0.1 s^2 + s - 4.256e-19  
  
Continuous-time transfer function.
```

ROOT LOCUS FOR THE TRANSFER FUNCTION AND STABILITY RANGE:

ROOT LOCUS



POLES & ZEROS

Zeros:

Zero at $s = -0.09/0.009 = -10$

Poles:

Pole at $s_3 = -0.00233$ (real pole)

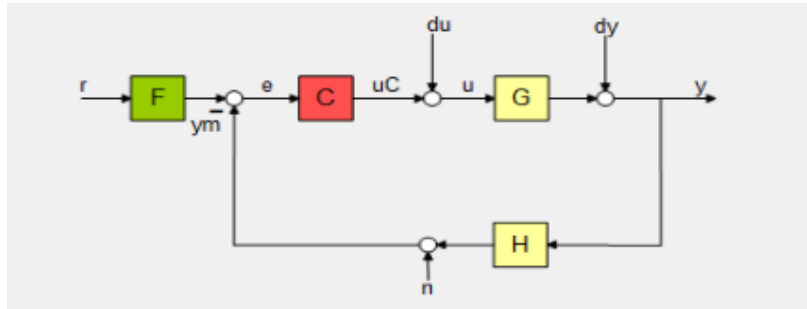
Pole at $s_2 = -0.0499 - 0.999i$ (complex conjugate pole)

Pole at $s_1 = -0.0499 + 0.999i$ (complex conjugate pole)

STABILITY RANGE

$0 < K_p < 1.1529$ (CRITICALLY STABLE)

SYSTEM ARCHITECTURE



2. Use the root locus to design a lead compensator that achieves a damping ratio 0.7 and a natural frequency 0.5 rad/sec.

POLES & ZEROS

We will use a compensator that cancels the TF poles of the system near the imaginary axis (i.e. faster poles) which are at $s = -0.00169 + 1.322j$ & $s = -0.00169 - 1.322j$. So the compensator poles and zeros are:

Controller zeros:

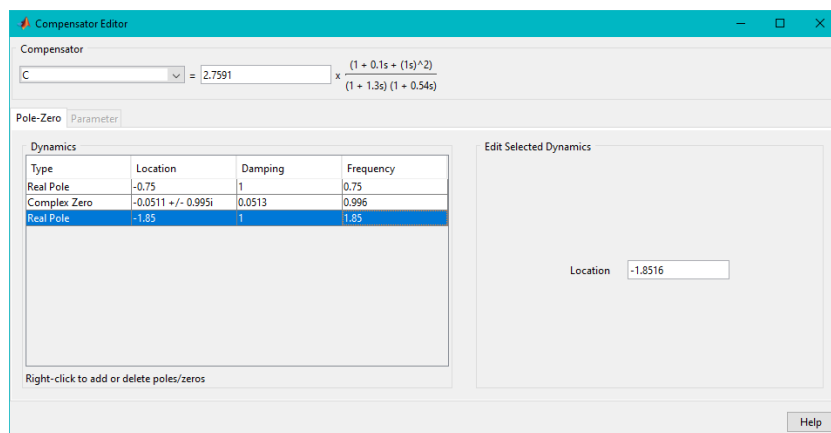
Zero at $s = -0.0021301 + j0.9987$ (complex conjugate pole)

Zero at $s = -0.0021301 - j0.9987$ (complex conjugate pole)

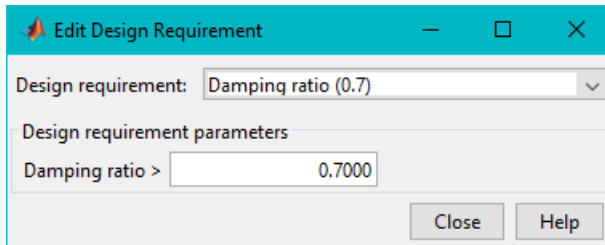
Controller Poles:

Pole at $s = -0.75$ (real pole)

Pole at $s = -1.8516$ (real pole)



DESIGN REQUIREMENTS



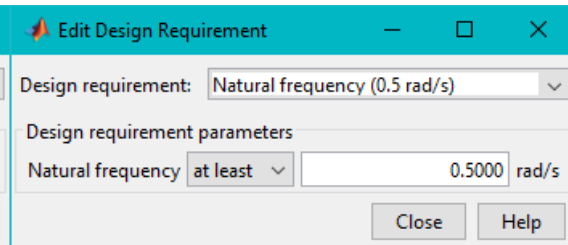
Edit Design Requirement

Design requirement: **Damping ratio (0.7)**

Design requirement parameters

Damping ratio >

Close **Help**



Edit Design Requirement

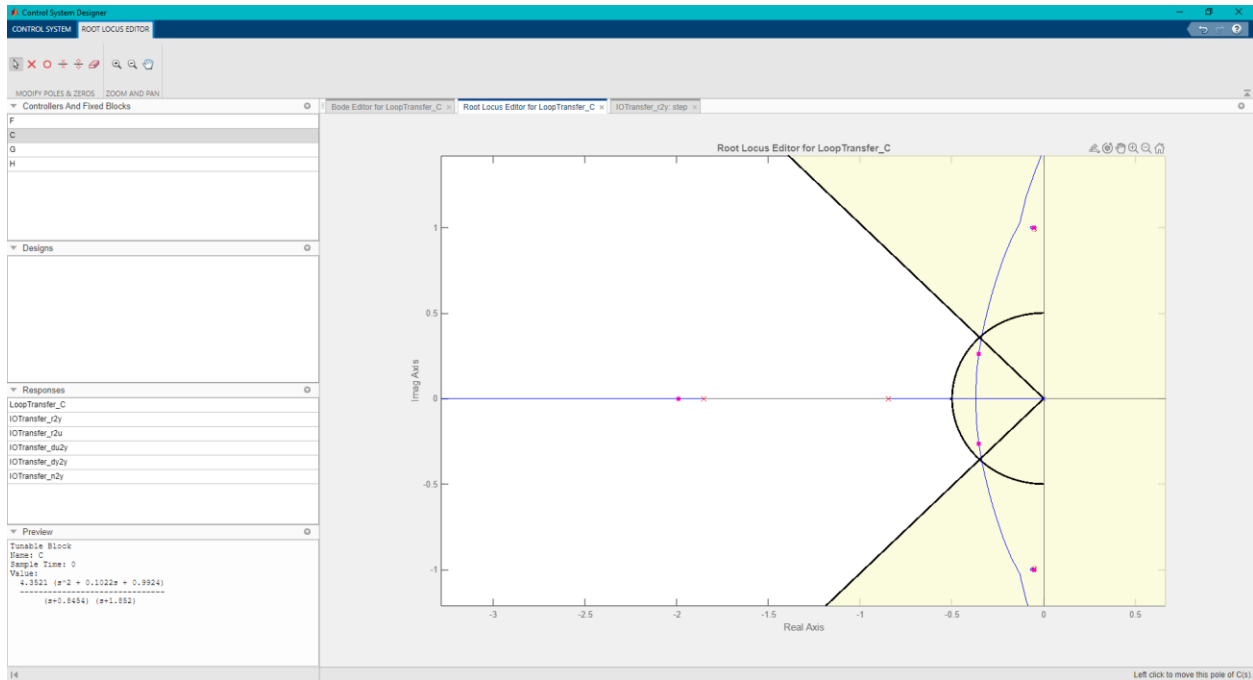
Design requirement: **Natural frequency (0.5 rad/s)**

Design requirement parameters

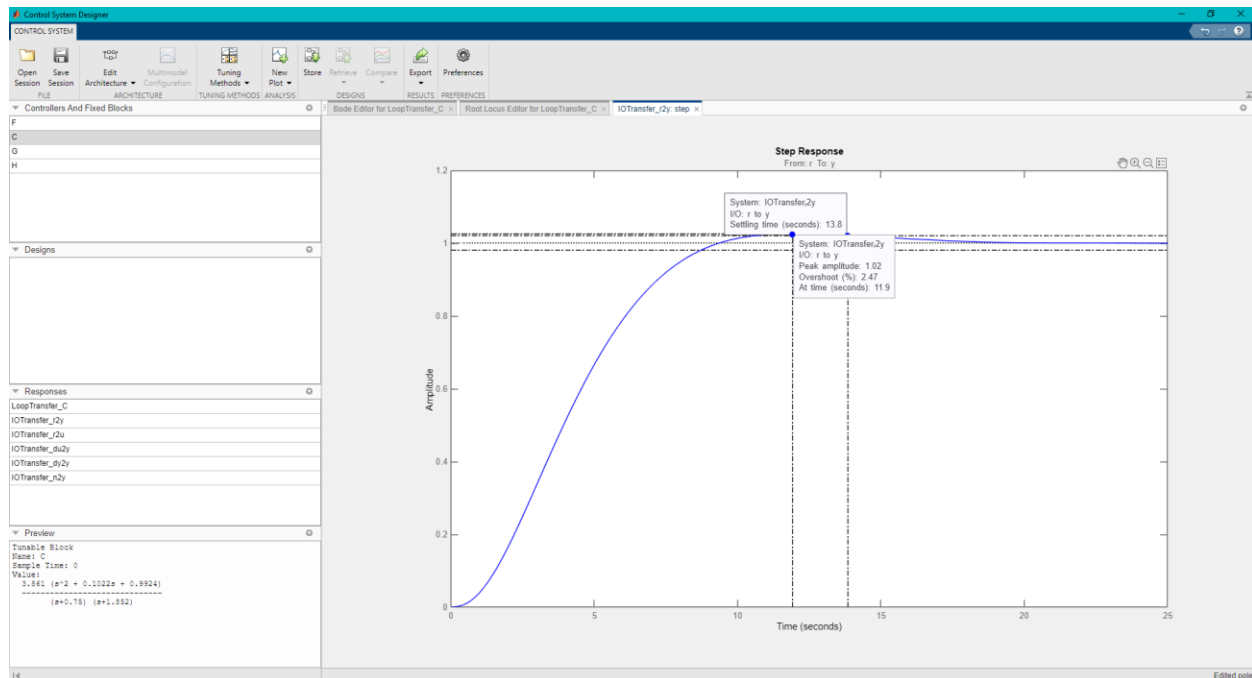
Natural frequency **at least** rad/s

Close **Help**

DESIGN REGION



STEP RESPONSE



3. Use the Bode plots to design a lead compensator that achieves a damping ratio 0.7 and a natural frequency 0.5 rad/sec.

POLES & ZEROS

We will use a compensator that cancels the TF poles of the system near the imaginary axis (i.e. faster poles) which are at $s = -0.00169 + 1.322j$ & $s = -0.00169 - 1.322j$. So the compensator poles and zeros are:

Controller zeros:

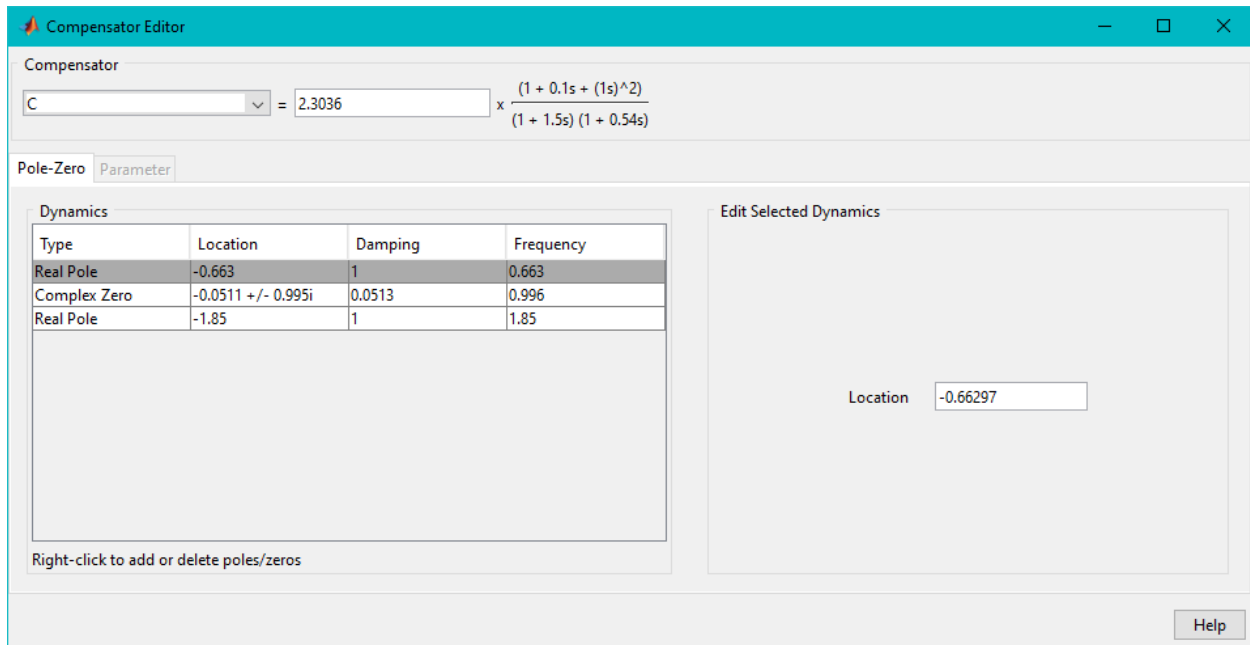
Zero at $s = -0.051121 + j0.99487$ (complex conjugate pole)

Zero at $s = -0.051121 - j0.99487$ (complex conjugate pole)

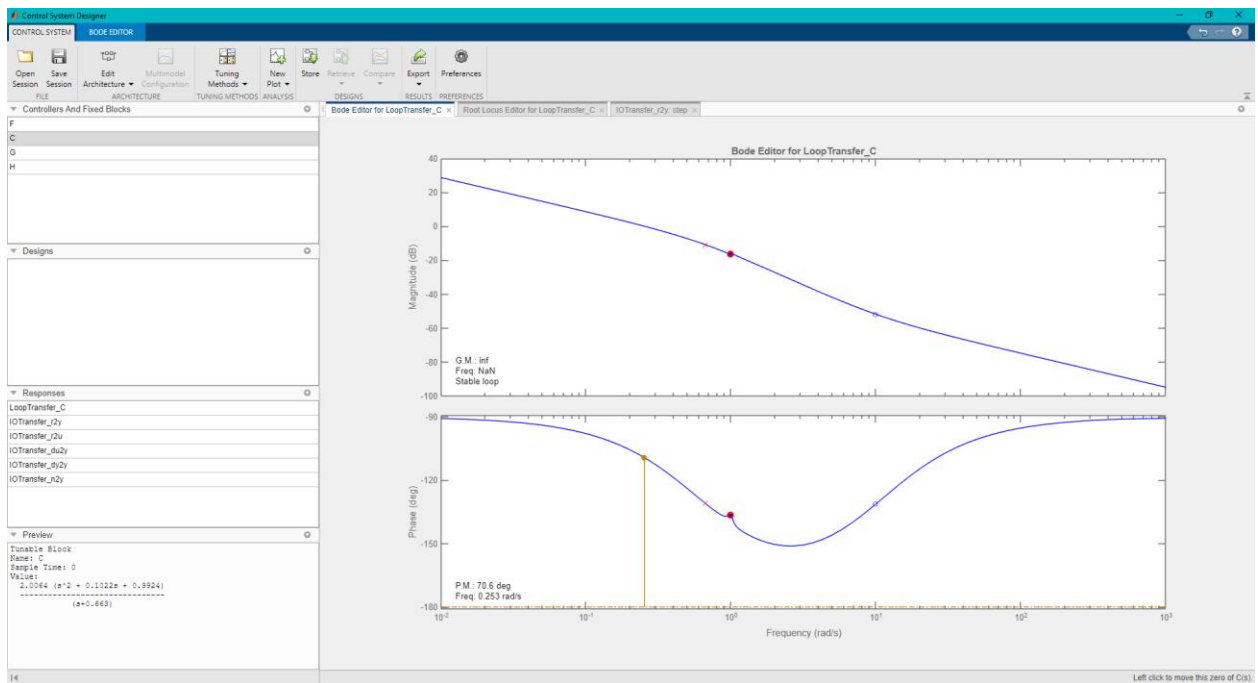
Controller Poles:

Pole at $s = -0.66297$ (real pole)

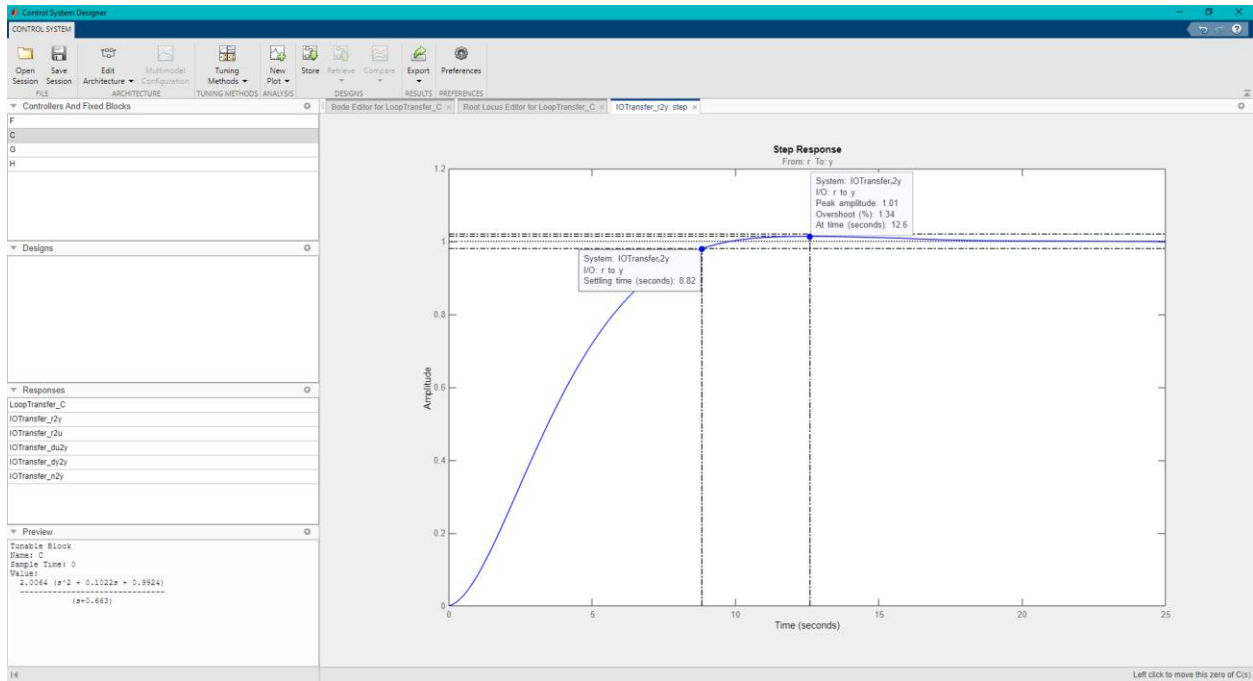
Pole at $s = -1.8516$ (real pole)



BODE PLOT

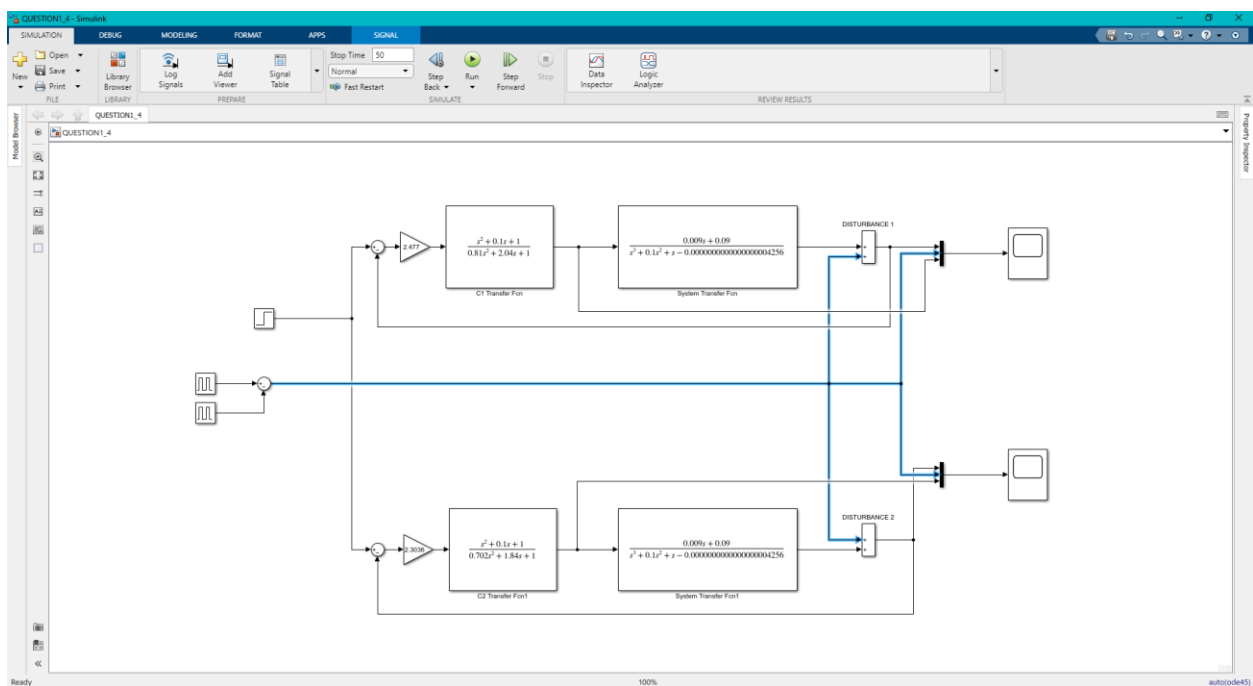


STEP RESPONSE



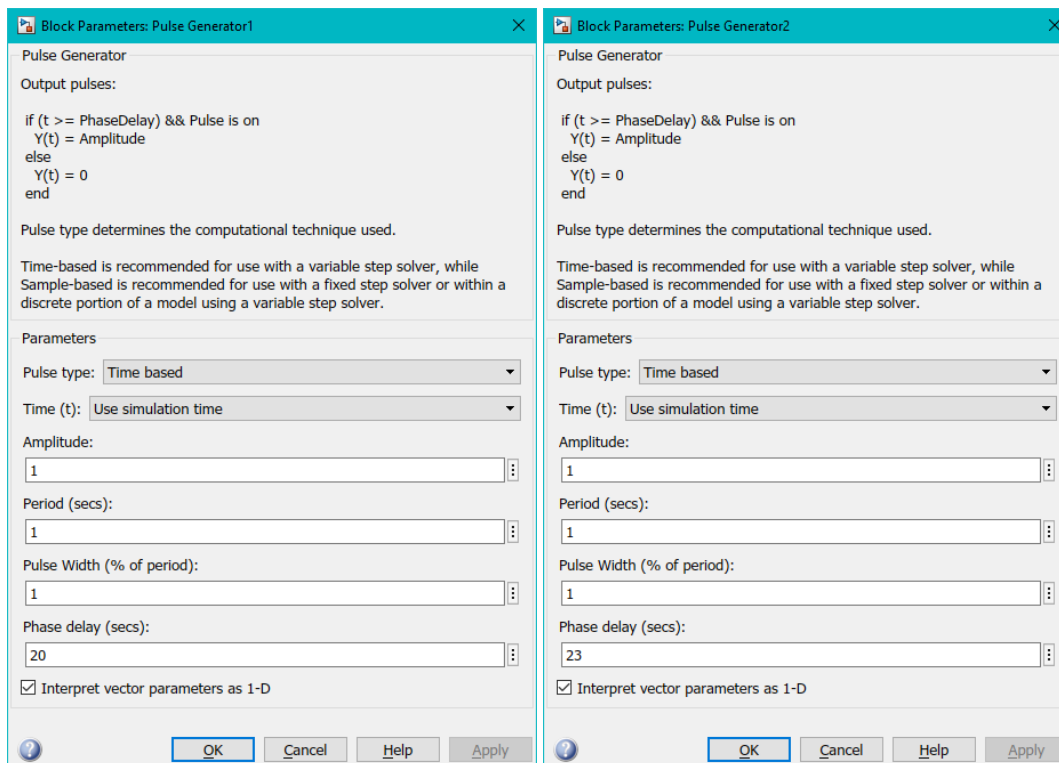
4. Simulate the system with both controllers (designed in parts 2 and 3 above) assuming the reference is a unit step and the disturbance is a pulse of magnitude 1 that appears from $20 < t < 22$ secs.

MODEL




DISTURBANCE PLOT

We create a pulse signal using two pulse generators in Simulink, where the signal is active for a time range from $t=20$ to $t=22$ and the two pulse generators are activated at different times. The output of the difference block will be a pulse signal that starts at time = 20 sec and ends at time = 23 sec, with a pulse width of 0.1 seconds. Note that by subtracting the two pulse signals from each other, the resulting output signal will have a negative pulse of the same width and amplitude as the positive pulse. The amplitude of the negative pulse will depend on the amplitude of the positive pulse and the timing of the second pulse generator. If the second pulse generator is delayed too much, the negative pulse may not completely cancel out the positive pulse, resulting in a smaller amplitude negative pulse. If the second pulse generator is delayed too little, the negative pulse may overlap with the positive pulse, resulting in a larger amplitude negative pulse.



STEP INPUT

 **Block Parameters: Step** ✕

Step

Output a step.

Main Signal Attributes

Step time:

⋮

Initial value:

⋮

Final value:


⋮

Sample time:

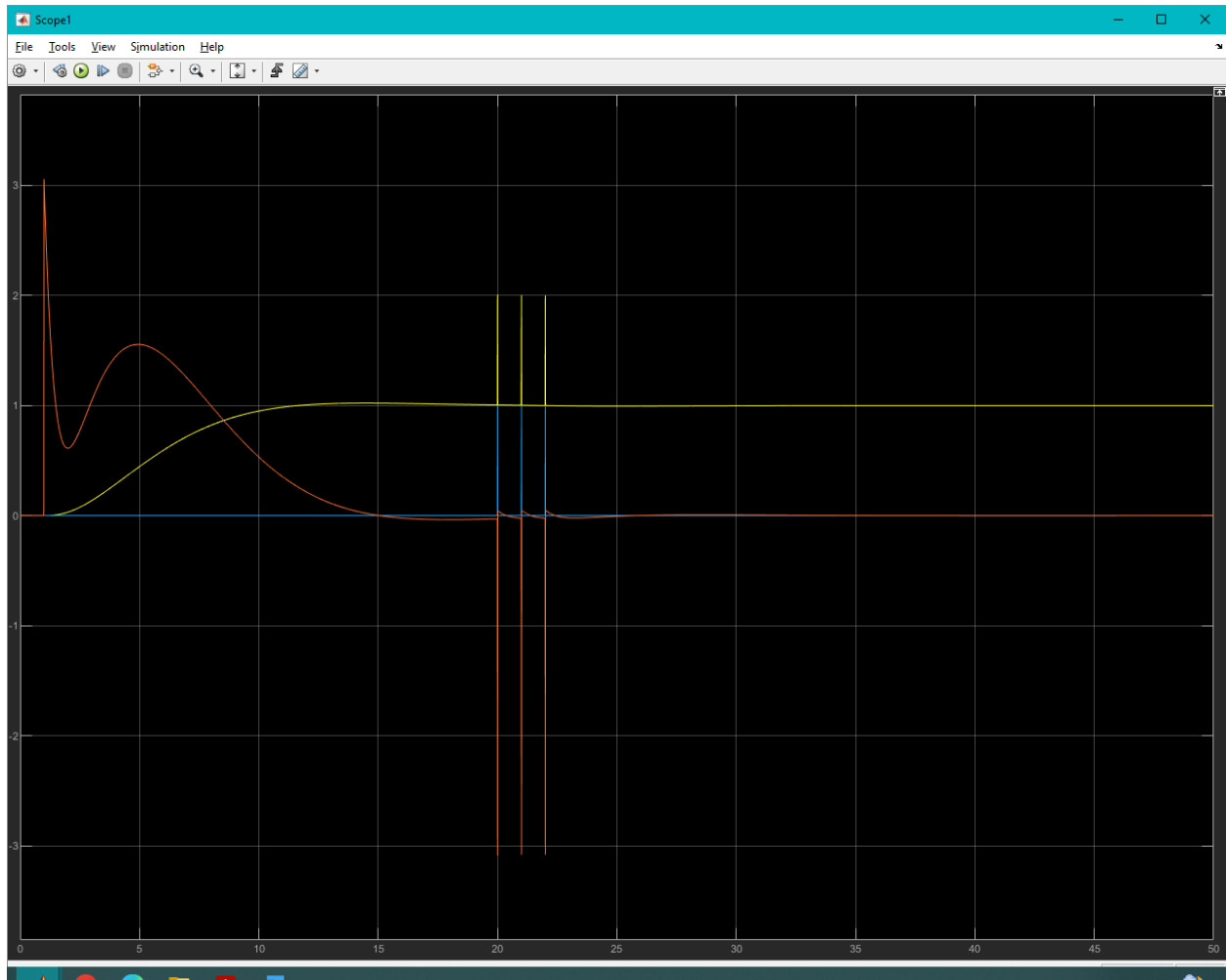
⋮

☒ Interpret vector parameters as 1-D

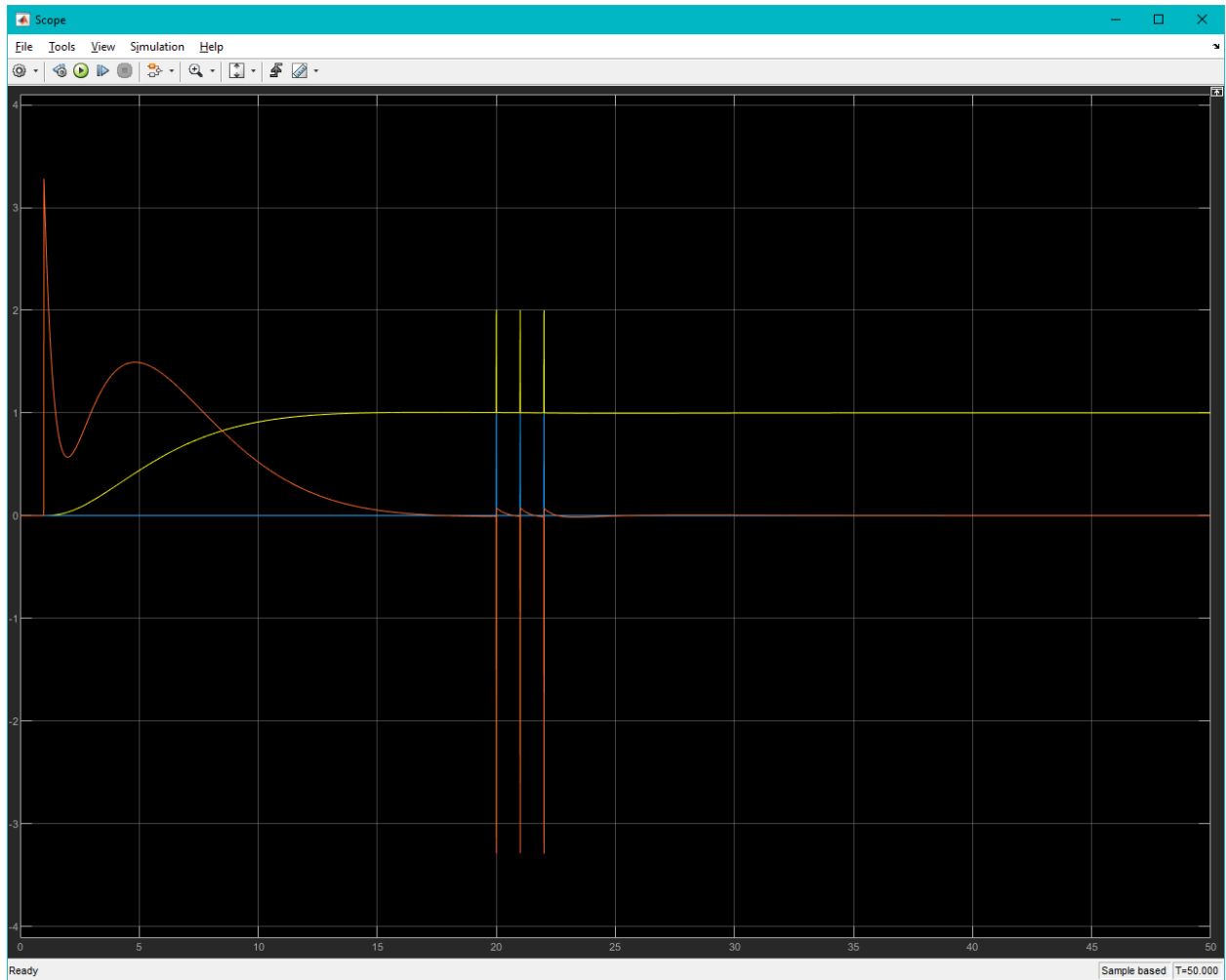
☒ Enable zero-crossing detection

 OK Cancel Help Apply

OUTPUT SIGNAL & CONTROLLER ACTION (ROOT LOCUS) & DISTURBANCE PULSE



OUTPUT SIGNAL & CONTROLLER ACTION (BODE PLOT) & DISTURBANCE PULSE



Digital Control Design Using the Transfer Function Approach

To solve this problem, first we need to get the discrete form of the transfer function we need to follow the following steps:

1. Determine the sampling interval: Once we have the continuous-time model, we need to determine the appropriate sampling interval. A general rule of thumb is to choose a sampling interval that is 10 to 30 times the system's bandwidth.
2. Design a digital controller: We need to design a digital controller to stabilize the system and meet the desired damping ratio and natural frequency. The direct digital control design method could be used for this purpose.
3. Apply zero-order-hold: Before applying the output of the controller to the system, we need to pass it through a zero-order-hold to convert the discrete-time signal back to continuous-time.

Here are the detailed steps for designing the digital controller:

1. Determine the sampling interval: Let's assume that we choose a sampling interval of T_s seconds. Here we use 2 sampling intervals namely 0.1 sec and 0.01 sec.
2. Design a digital controller: To design the digital controller, we can use the direct digital control design method as follows:
 - a. Convert the continuous-time transfer function $G(s)$ to a discrete-time transfer function $G(z)$ using the `c2d` function in MATLAB. The resulting transfer function will have the form:

```
>> c2d(G, 0.1)

ans =

    4.631e-05 z^2 + 5.815e-06 z - 4.318e-05
-----
    z^3 - 2.98 z^2 + 2.97 z - 0.99

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
>> c2d(G, 0.01)

ans =

    4.513e-07 z^2 + 5.847e-09 z - 4.482e-07
-----
    z^3 - 2.999 z^2 + 2.998 z - 0.999

Sample time: 0.01 seconds
Discrete-time transfer function.
```

3. Simulate the system in discrete time using the root locus method and find the step response.
4. Get the desired operating region and design a lead compensator $C(Z) = \kappa \frac{z-\beta}{z-p}$ to satisfy the operating conditions.
5. Apply zero-order-hold: Pass the output of the controller through a zero-order-hold to convert the discrete-time signal back to continuous-time.
6. Simulate the system: Simulate the closed-loop system using the continuous-time model of the system and the designed digital controller to ensure that it meets the desired specifications.

1. Show the effect of selecting short and long sampling interval on the resulting poles and zeros in the discrete model. Comment on your results.

As we have seen, when transferring the system to the z domain and choosing different sampling intervals, the transfer function changes.

Thus, when we chose the $T_s = 0.1$ sec (i.e. long sampling interval), $G(z) =$

$$\frac{4.631 \cdot 10^{-5} z^2 + 5.815 \cdot 10^{-6} z - 4.318 \cdot 10^{-5}}{z^3 - 2.98 z^2 + 2.97 z - 0.99}$$

POLES & ZEROS

Zeros:

Zero at $z_1 = -1.0304$

Zero at $z_2 = 0.9048$

Poles:

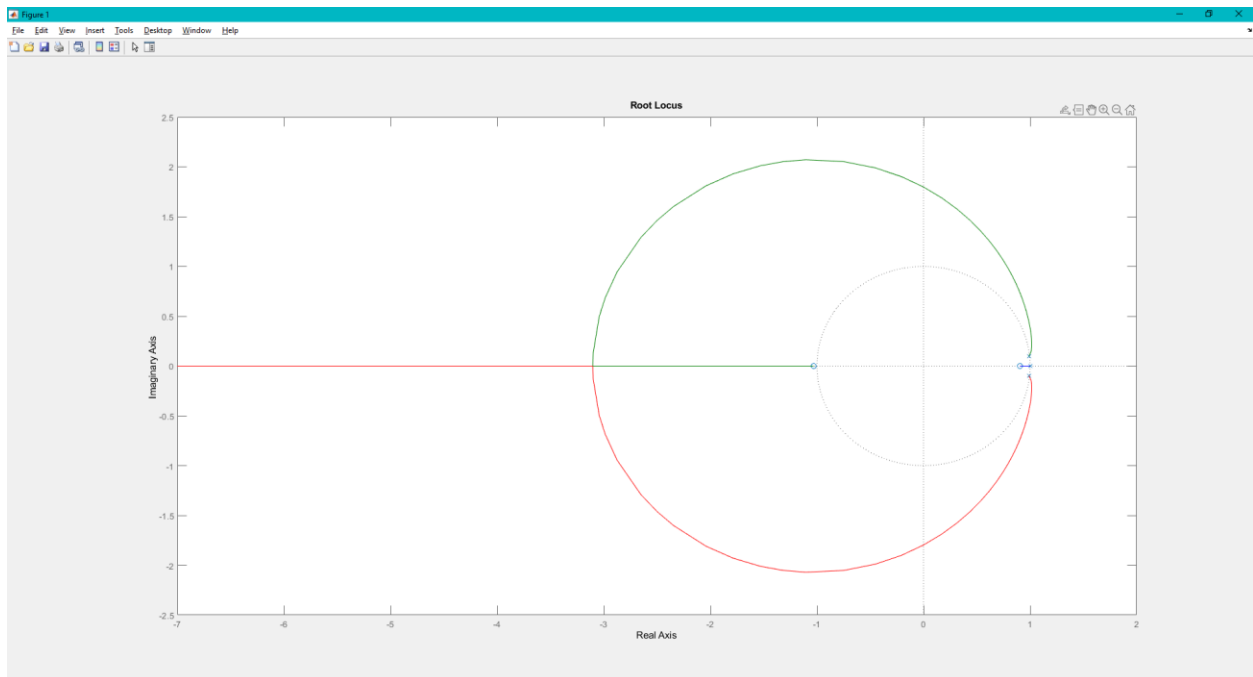
Pole at $p_3 = 1$ (real pole)

Pole at $p_2 = 0.9901 + 0.0992i$ (complex conjugate pole)

Pole at $p_1 = 0.9901 - 0.0992i$ (complex conjugate pole)

STABILITY RANGE

$0 < K_p < 11.432$ (CRITICALLY STABLE)



and when $T_s = 0.01$ sec (i.e. short sampling interval), $G(z) = \frac{4.513 \cdot 10^{-7} z^2 + 5.847 \cdot 10^{-9} z - 4.482 \cdot 10^{-7}}{z^3 - 2.999 z^2 + 2.998 z - 0.999}$

POLES & ZEROS

Zeros:

Zero at $z_1 = -1.0030$

Zero at $z_2 = 0.9900$

Poles:

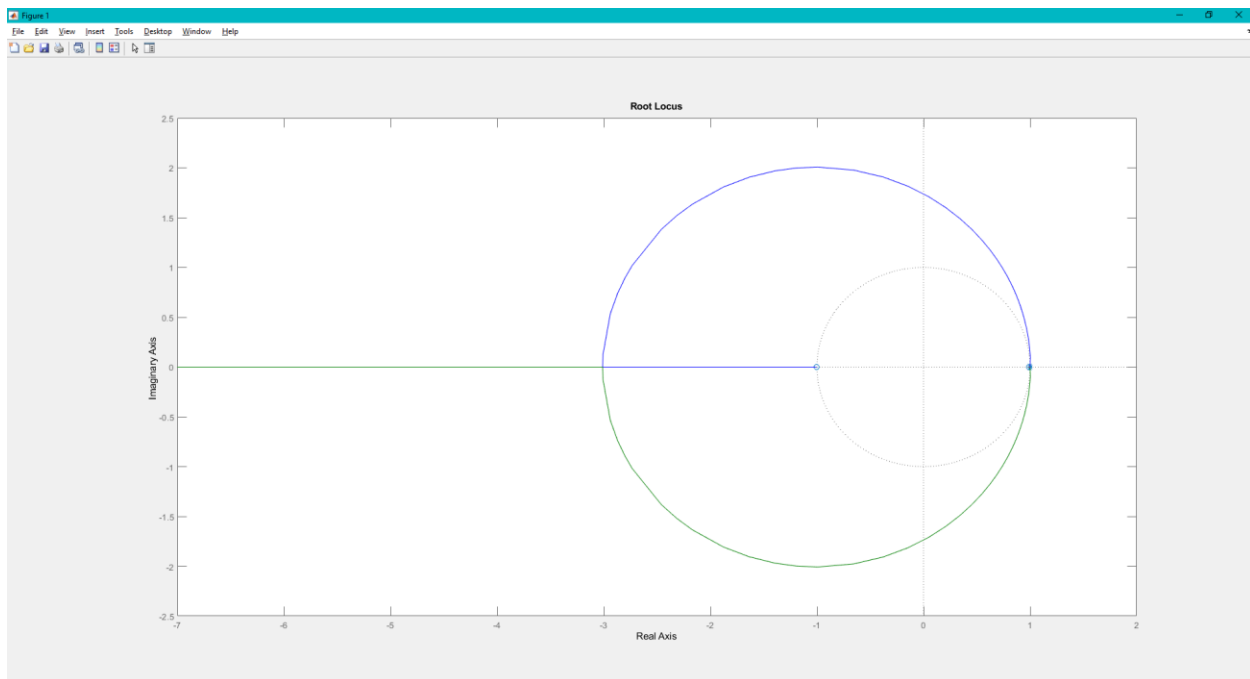
Pole at $p_3 = 1$ (real pole)

Pole at $p_2 = 0.9995 + 0.0100i$ (complex conjugate pole)

Pole at $p_1 = 0.9995 - 0.0100i$ (complex conjugate pole)

STABILITY RANGE

$0 < K_p < 13.554$ (CRITICALLY STABLE)



2. Select an appropriate sampling interval and design a digital controller (using the direct digital control design) to stabilize the system and ensure a damping ratio 0.7 and a natural frequency 0.5 rad/sec.

WE WILL BE USING $T_s=0.01$ sec to design a controller using DDC method:

Controller zeros:

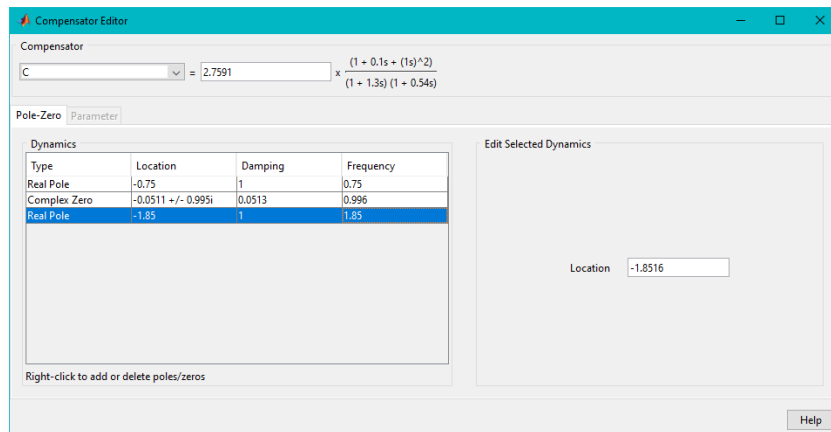
Zero at $z = 0.99945 + j 0.0099757$ (complex conjugate pole)

Zero at $z = 0.99945 - j 0.0099757$ (complex conjugate pole)

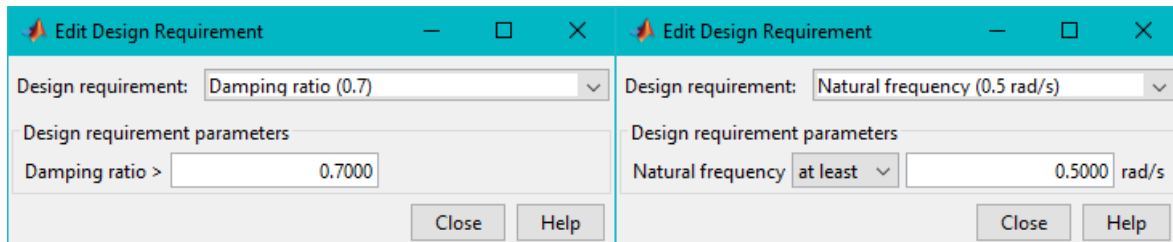
Controller Poles:

Pole at $z = 0.99005$ (real pole)

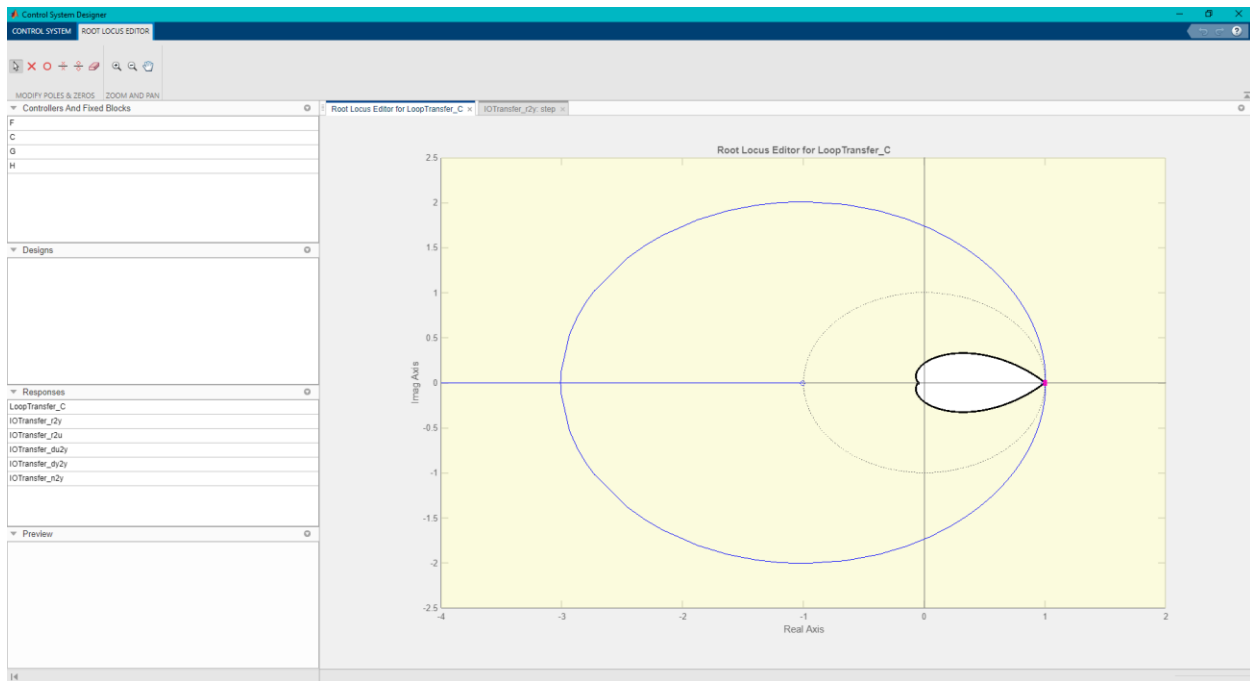
Pole at $z = 0.99304$ (real pole)



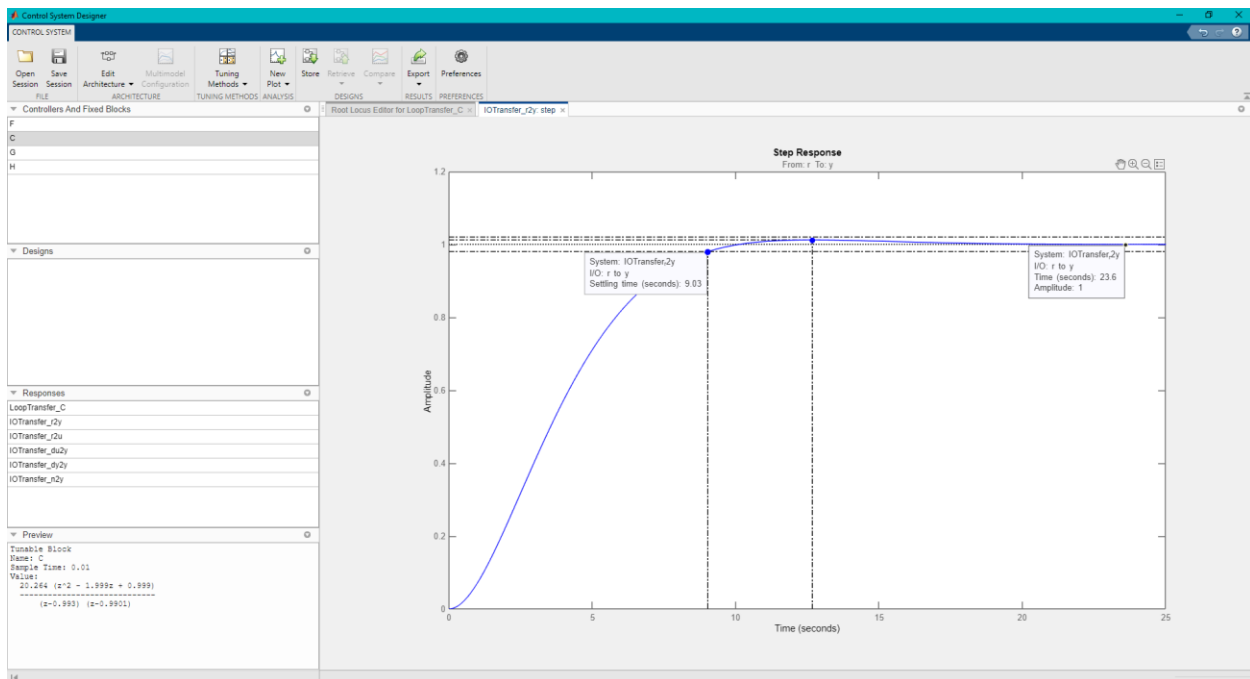
DESIGN REQUIREMENTS



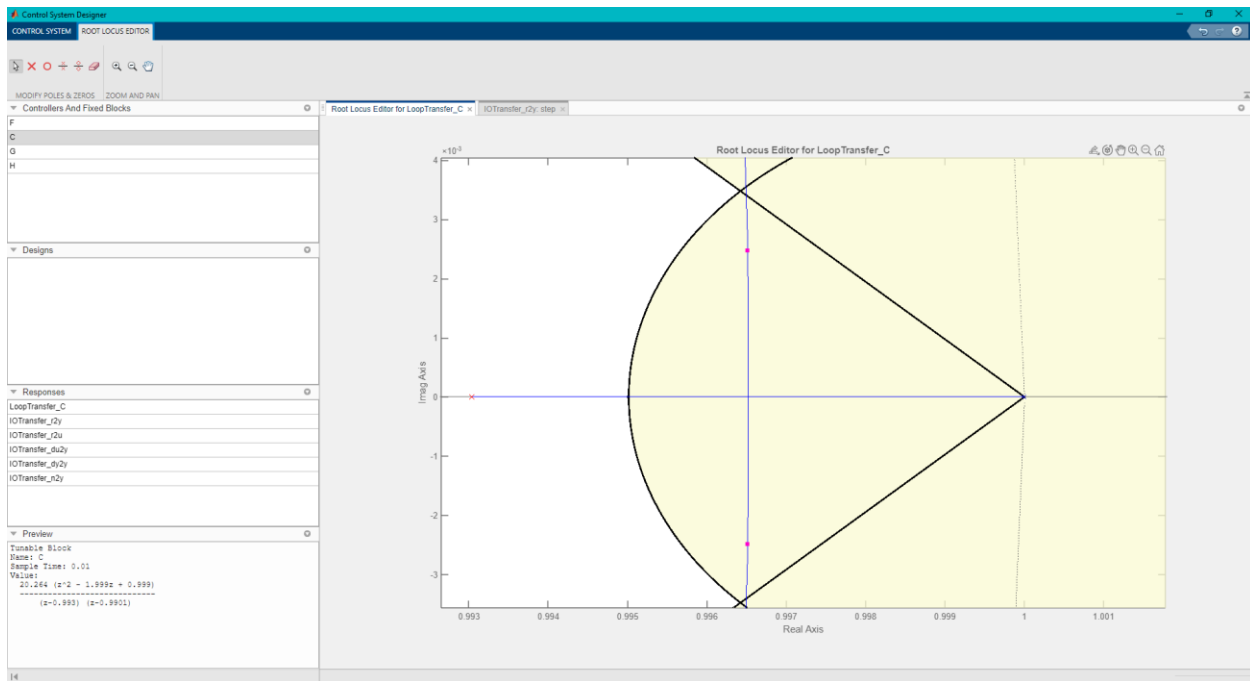
DESIGN REGION



STEP RESPONSE



ROOT-LOCUS AFTER COMPENSATION



3. Repeat in question #2 using a design by emulation.

WE WILL BE USING $T_s=0.01$ sec to design a controller using EMULATION method:

By substituting $S \rightarrow \frac{z-1}{0.01z}$ This is the formula for backward transformation.

$$G(s) = \frac{0.009s + 0.009}{s^3 + 0.1s^2 + s - 4.256 \times 10^{-19}}$$

$$G(z) = \frac{0.00005z^3 - 0.000045z^2}{50.01z^3 - 150.015z^2 + 150.005z - 50}$$

To enter the new $G(z)$ into MATLAB, we use the following code:

```
num = [0.00005 -0.000045 0 0];
den = [50.01 -150.015 150.005 -50];
G = tf(num, den, 1, 'variable', 'z')
sisotool(G)
```

POLES & ZEROS

Zeros:

Zero at $z_1 = 0$

Zero at $z_2 = 0$

Zero at $z_2 = 0.9000$

Poles:

Pole at $p_3 = 1$ (real pole)

Pole at $p_2 = 0.9999 + 0.0100i$ (complex conjugate pole)

Pole at $p_1 = 0.9999 - 0.0100i$ (complex conjugate pole)

STABILITY RANGE

$0 < K_p < 0.22309$ (CRITICALLY STABLE)

Controller zeros:

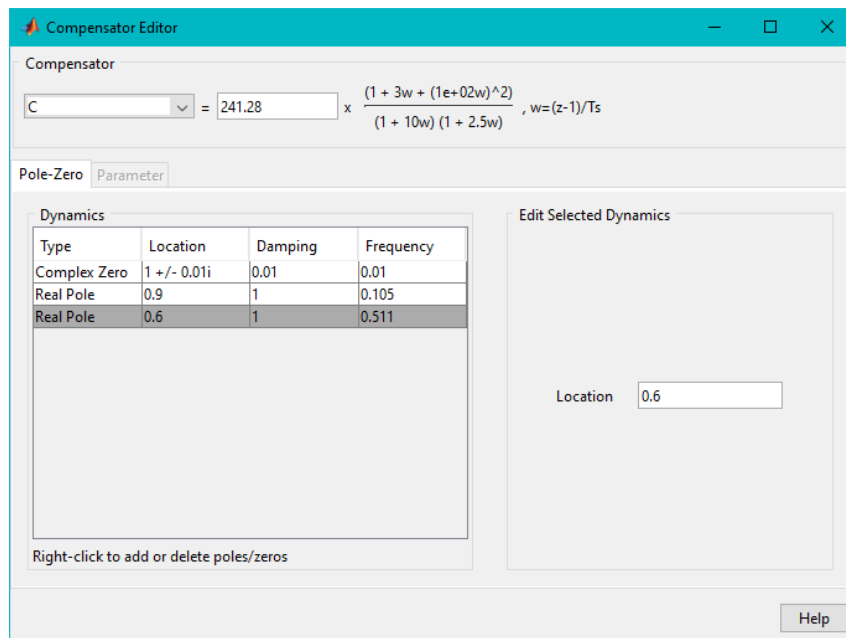
Zero at $z = 0.99945 + j 0.0099757$ (complex conjugate pole)

Zero at $z = 0.99945 - j 0.0099757$ (complex conjugate pole)

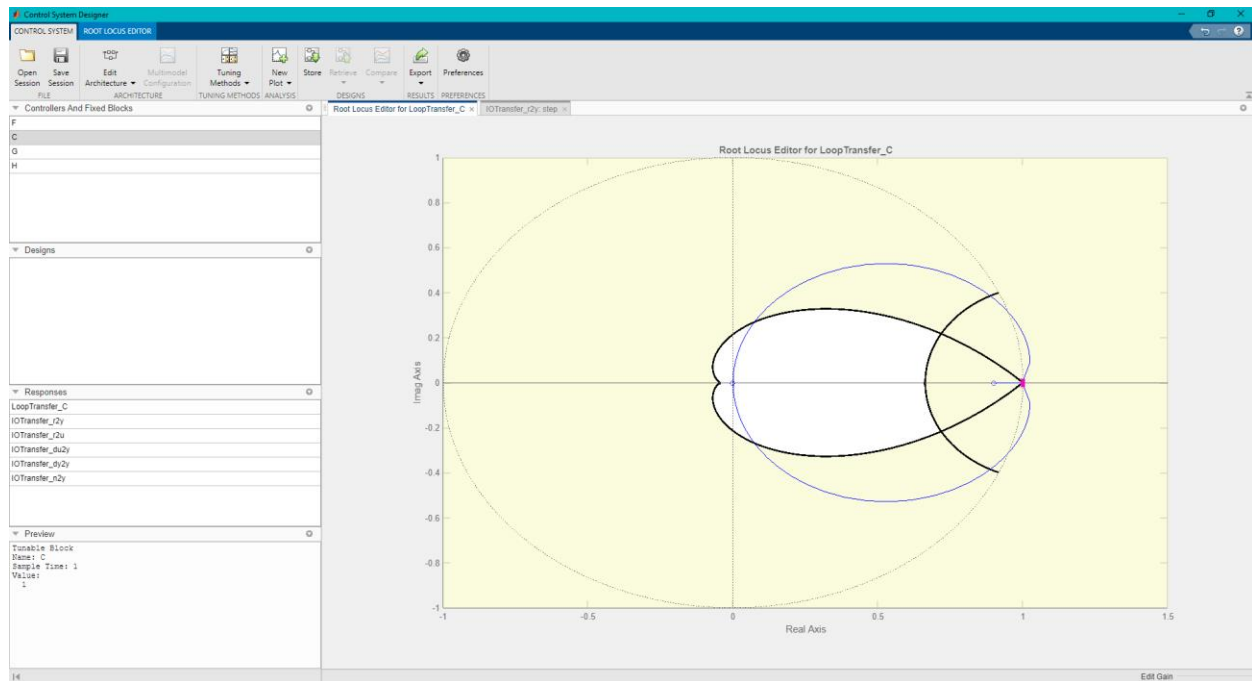
Controller Poles:

Pole at $z = 0.99005$ (real pole)

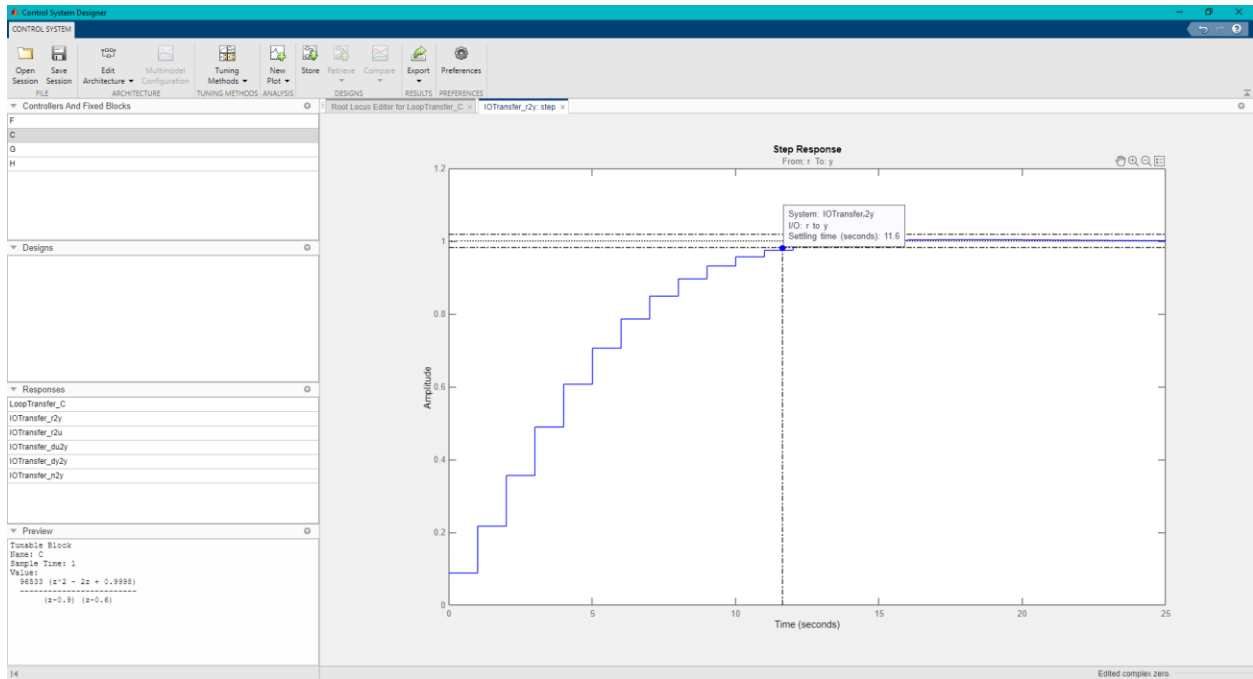
Pole at $z = 0.99304$ (real pole)



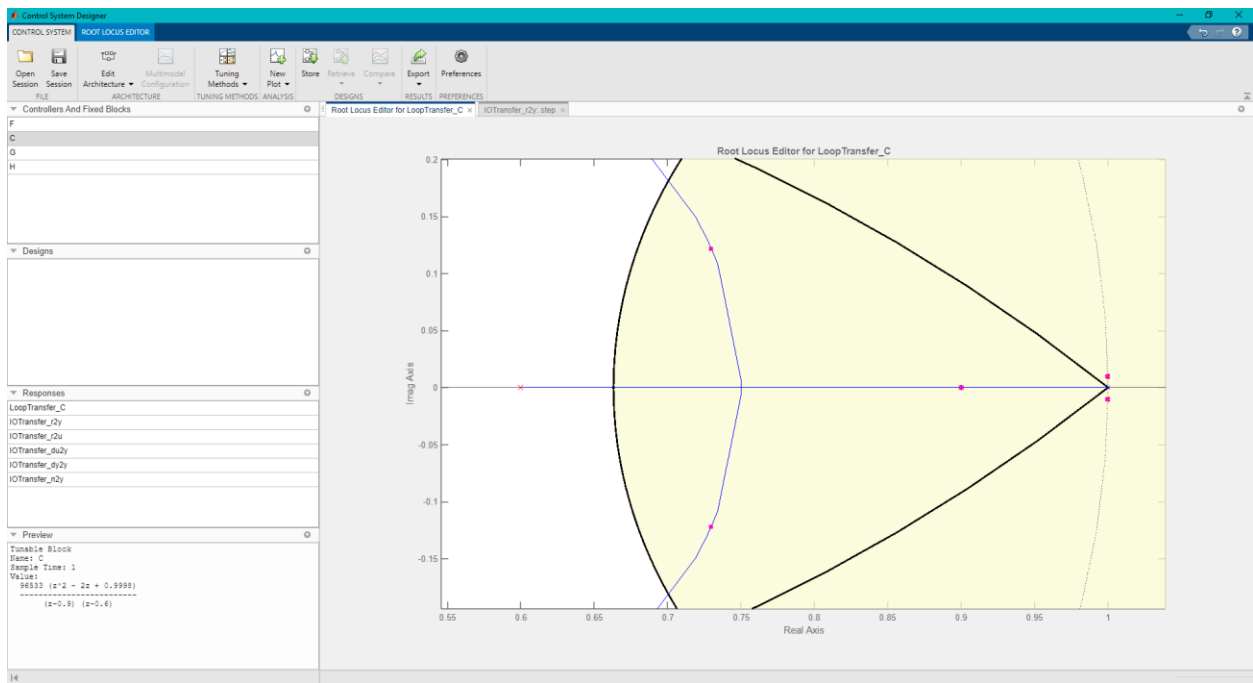
DESIGN REGION



STEP RESPONSE

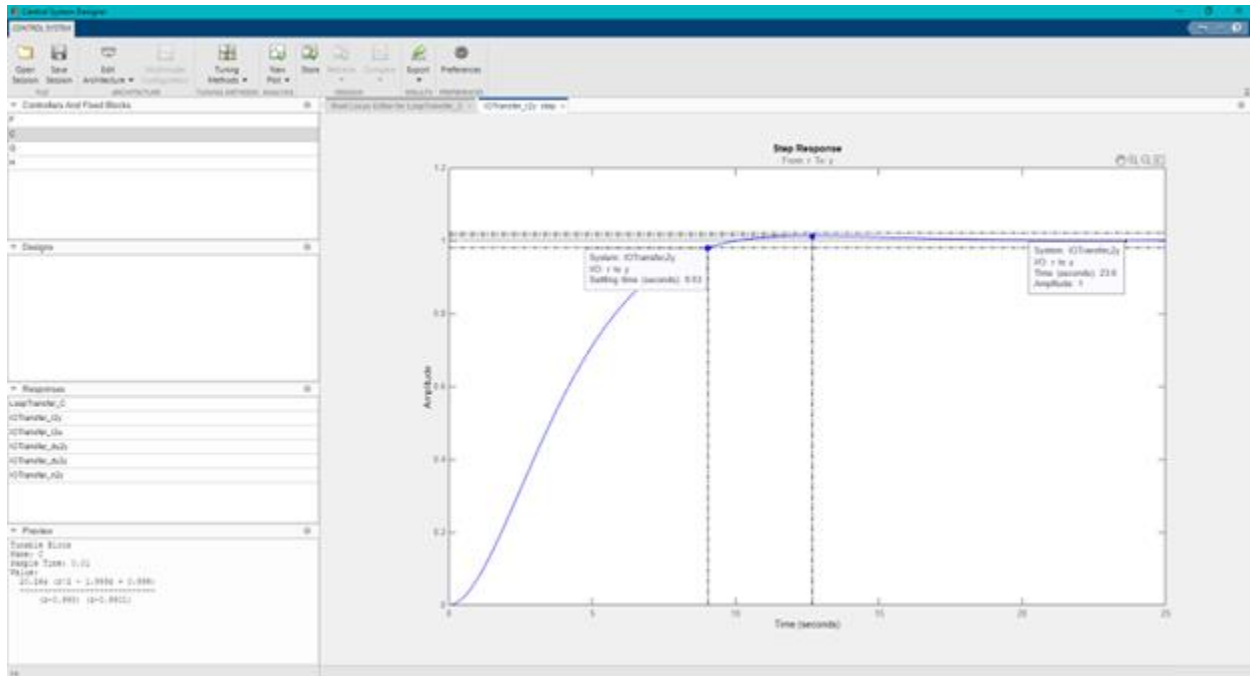


ROOT-LOCUS AFTER COMPENSATION

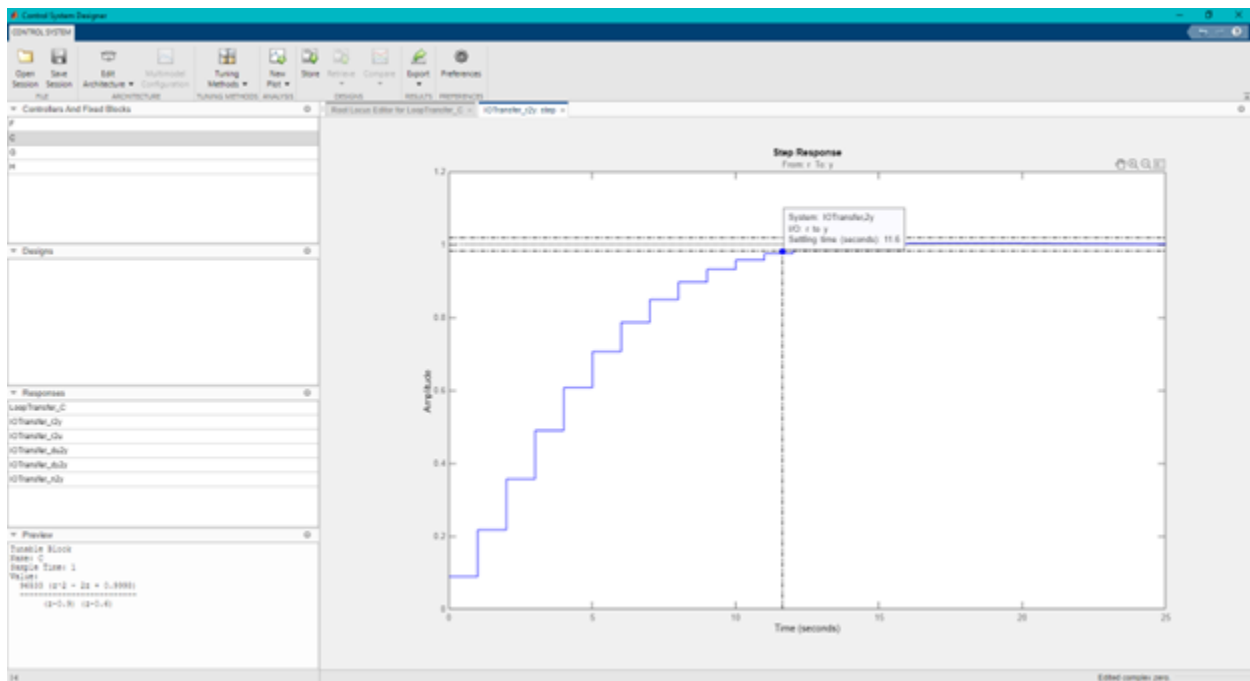


- Compare the results due to the design by emulation to those obtained by the direct design. Comment on the results.

RESULTS FROM DIRECT DIGITAL DESIGN



RESULTS FROM EMULATION



COMMENT

One advantage of DDC is that it provides real-time control of the process being controlled, which can be important in applications where rapid response times are necessary. Additionally, DDC allows for more precise control of the process, as the controller can continuously monitor and adjust the process based on real-time data.

Emulation techniques, on the other hand, have the advantage of being more flexible and less expensive than DDC. They can be used to simulate a wide range of processes and control algorithms, allowing designers to test and refine their designs without the need for expensive hardware. Emulation can also be used to quickly evaluate different control strategies and to optimize the performance of existing control systems.

Continuous State Space Representation

1. Using the continuous system dynamics given, develop and Simulink model for open loop system. Draw the system states (Call this representation “rep A”).

System state space representation (repA)

SS =

A =

	x1	x2	x3
x1	0	1	-1
x2	-0.9	-0.09	0.09
x3	0.1	0.01	-0.01

B =

	u1
x1	0
x2	0.9
x3	0

C =

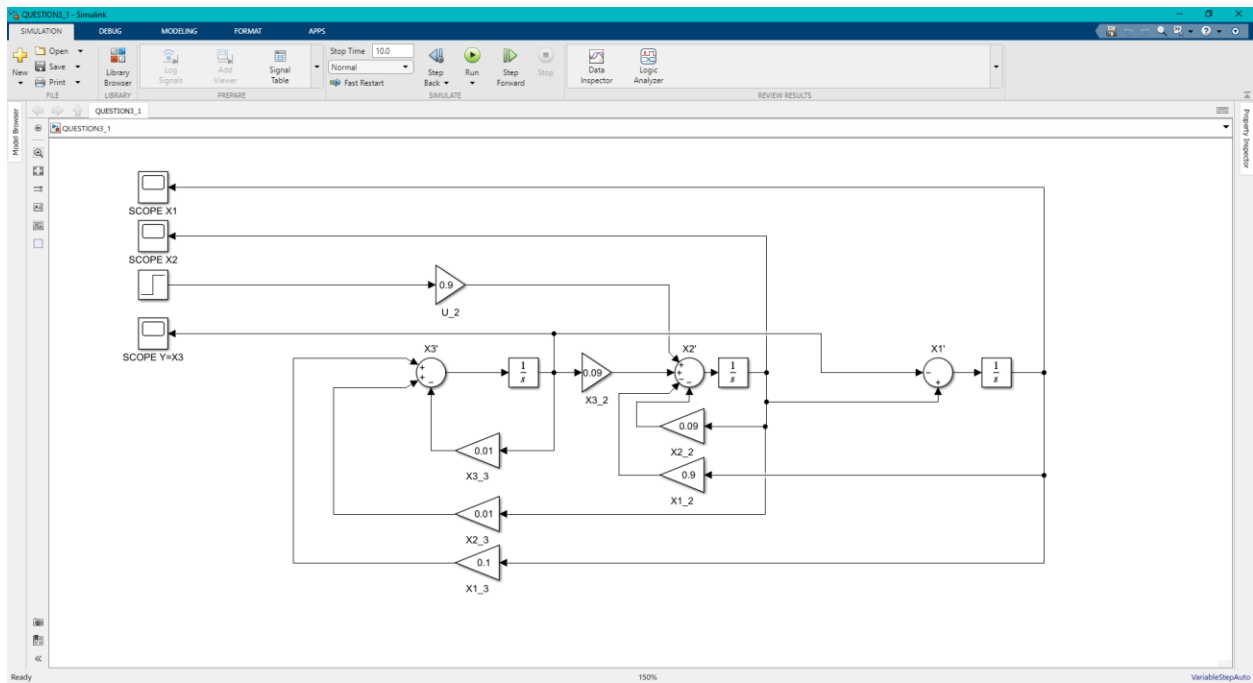
	x1	x2	x3
y1	0	0	1

D =

	u1
y1	0

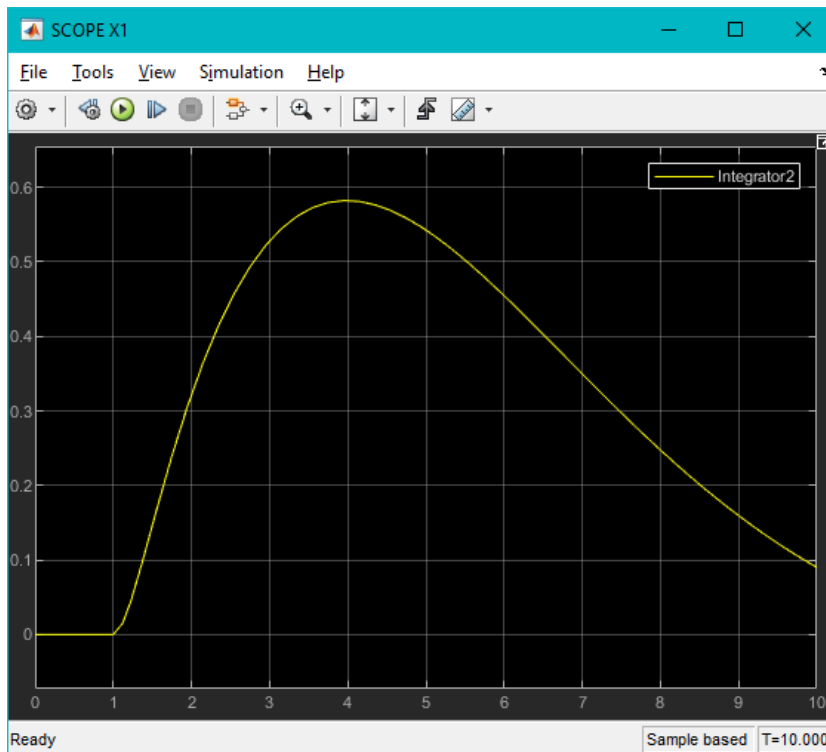
Continuous-time state-space model.

Simulink model

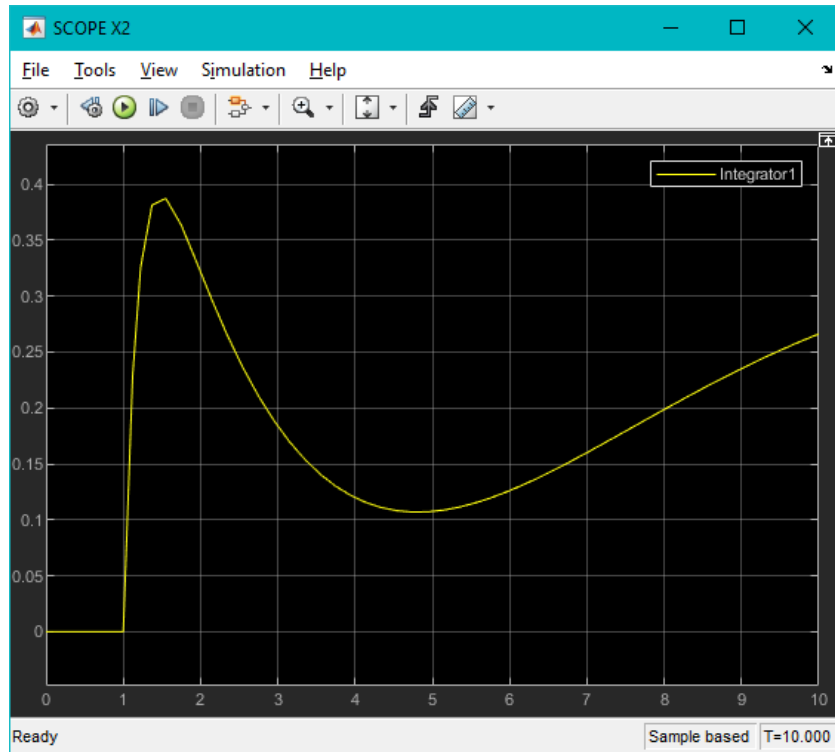


SYSTEM STATES

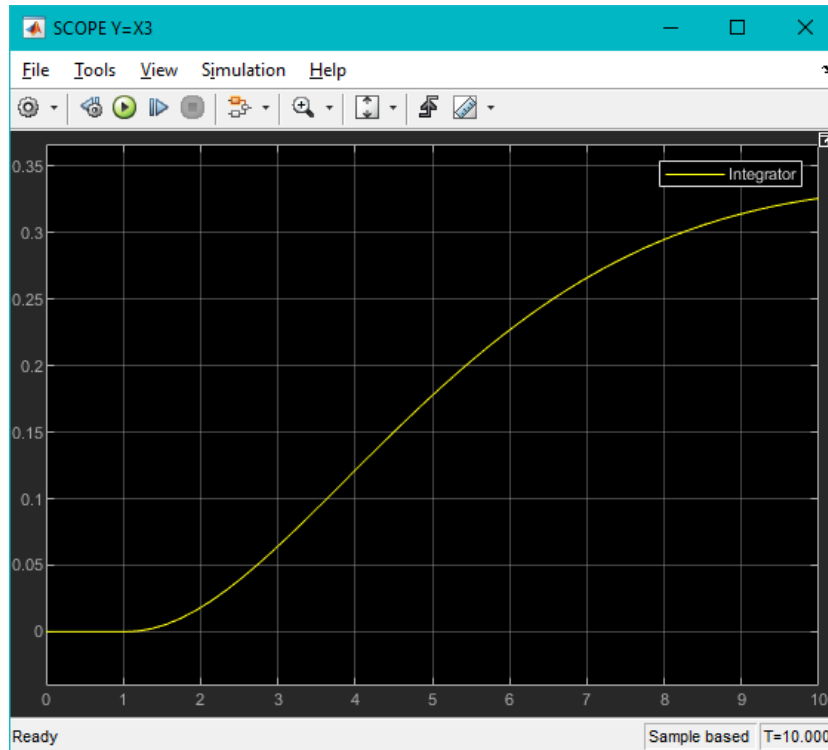
X1



X2



$$Y=X^3$$



- Find appropriate state feedback gain to place the poles of the system in suitable places (You can use the same requirements as mentioned in the transfer function approach).

DESIRED POLES

`p =`

```
-0.3557 + 0.1590i  -0.3557 - 0.1590i  -5.0000 + 0.0000i
```

CODE

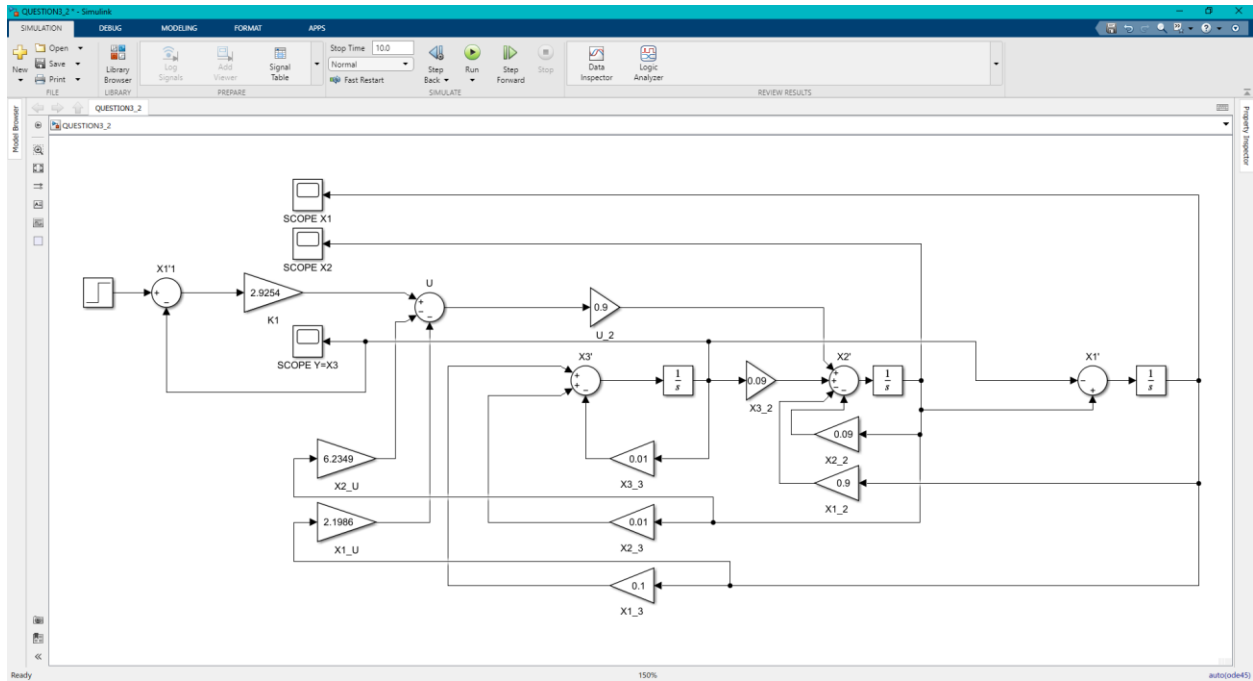
```
>> place(Anew, B, p)
```

`ans =`

```
2.9254    6.2349    2.1986
```

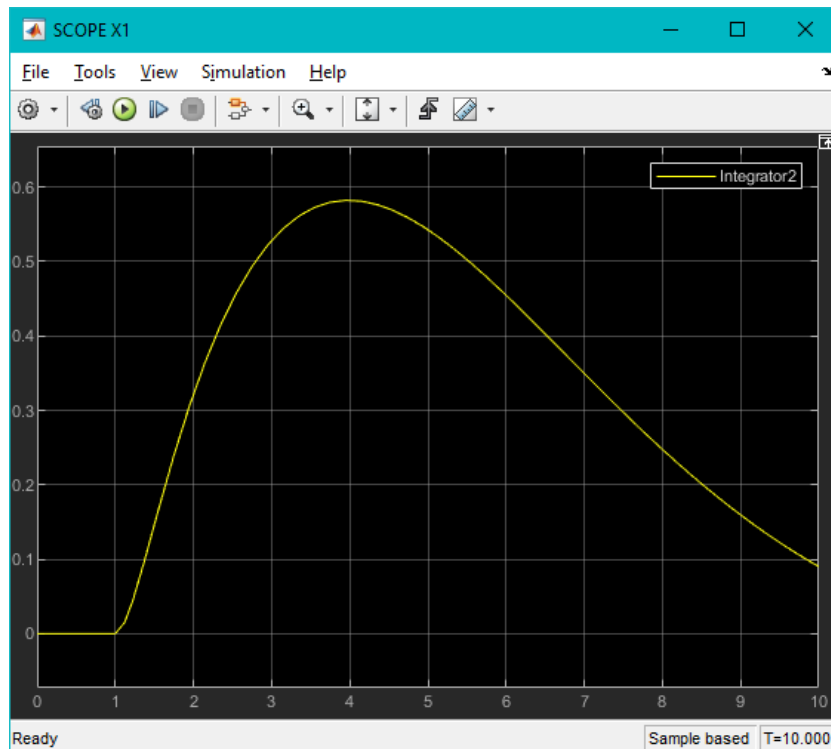

3. Implement the feedback signals using the Simulink. Draw the states and output versus time. Comment on the results.

SIMULINK MODEL WITH STATE FEEDBACK

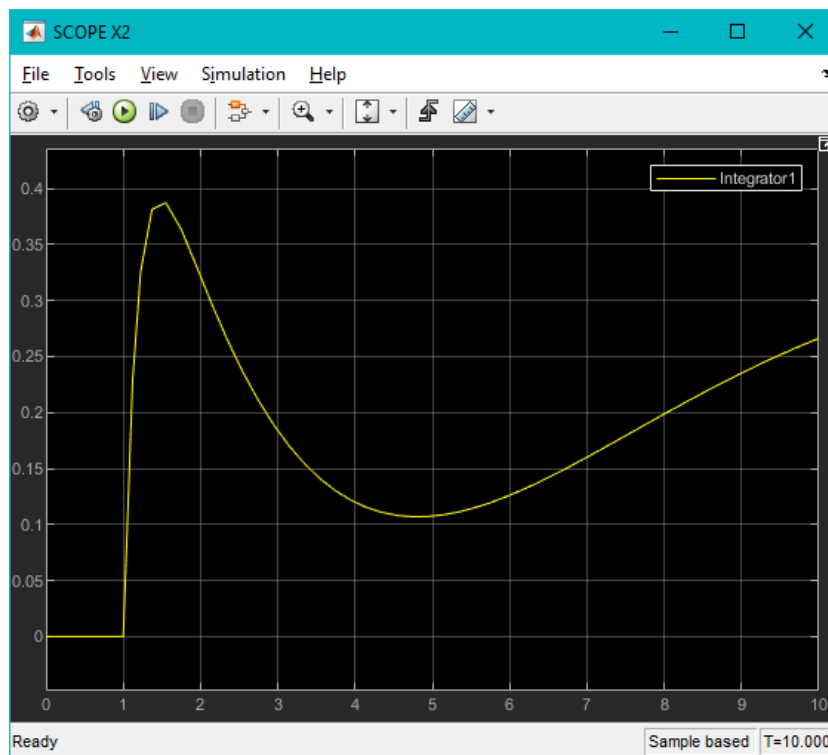


SYSTEM STATES

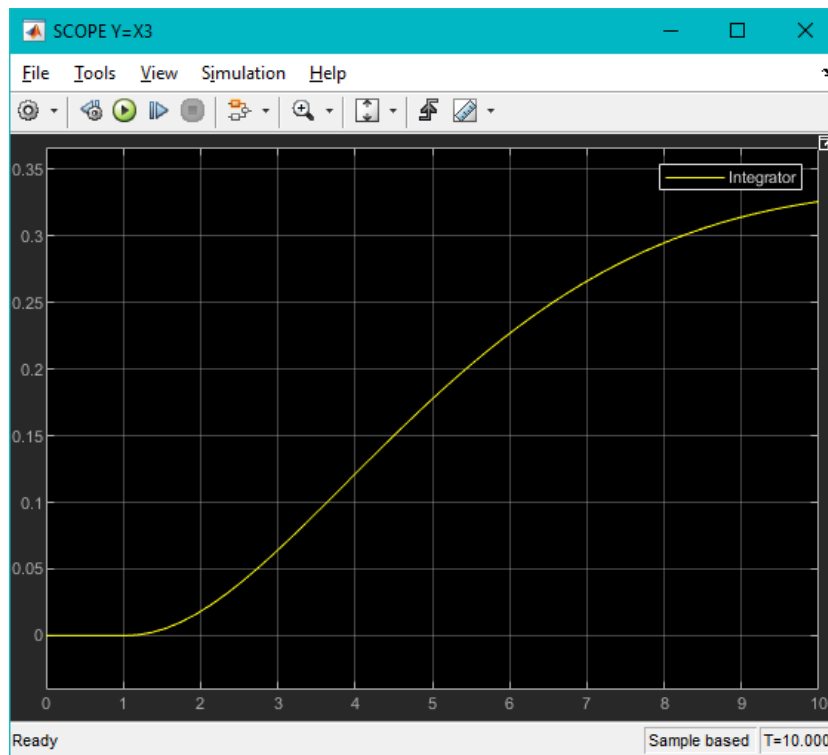
x_1



X2



Y=X3



COMMENT

When we used pole placement method to change the system feedback, we obtained better system characteristics in terms of MOS% and Natural frequency.

Discrete State Space Representation

1. Choose a suitable sampling period and find the discrete form for “rep A” (Call this representation “rep B”). Write the state space representation in the controllable canonical form (Call this representation “rep C”). Draw the system states for both representations for open loop case. Comment on the results.

DISCRETIZATION

```
>> c2d(SS, 0.5)
```

```
ans =
```

```
A =
```

	x1	x2	x3
x1	0.8796	0.4676	-0.4676
x2	-0.4209	0.8495	0.1505
x3	0.04676	0.01672	0.9833

```
B =
```

	u1
x1	0.1084
x2	0.4238
x3	0.002913

```
C =
```

	x1	x2	x3
y1	0	0	1

```
D =
```

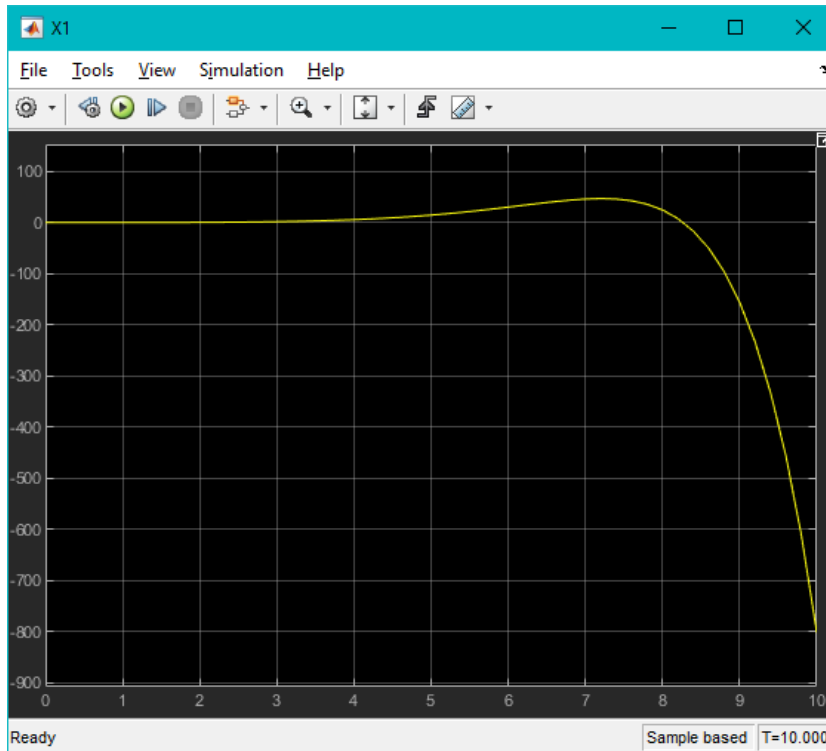
	u1
y1	0

```
Sample time: 0.5 seconds
```

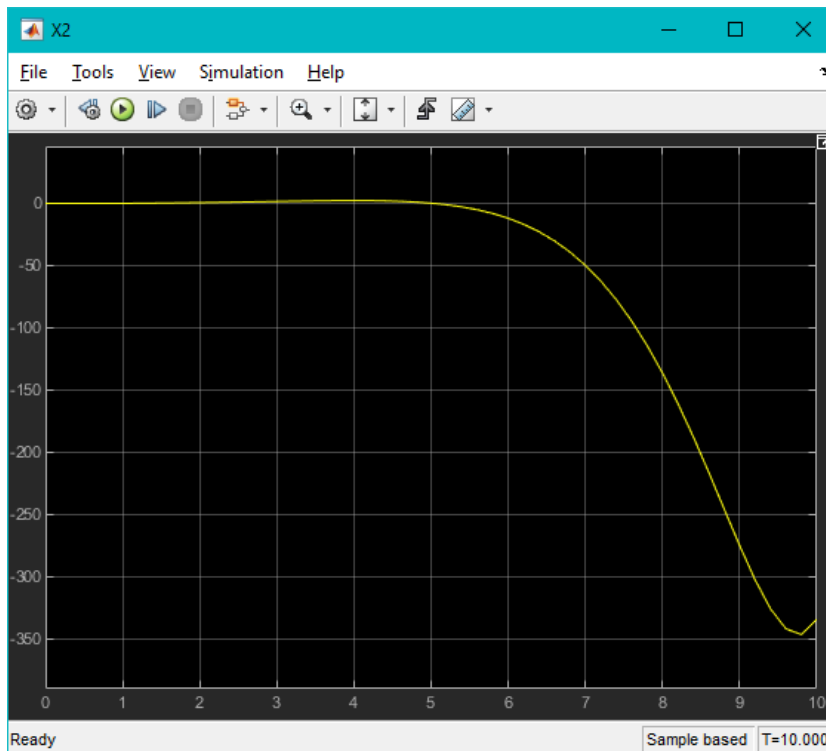
```
Discrete-time state-space model.
```

SYSTEM STATES

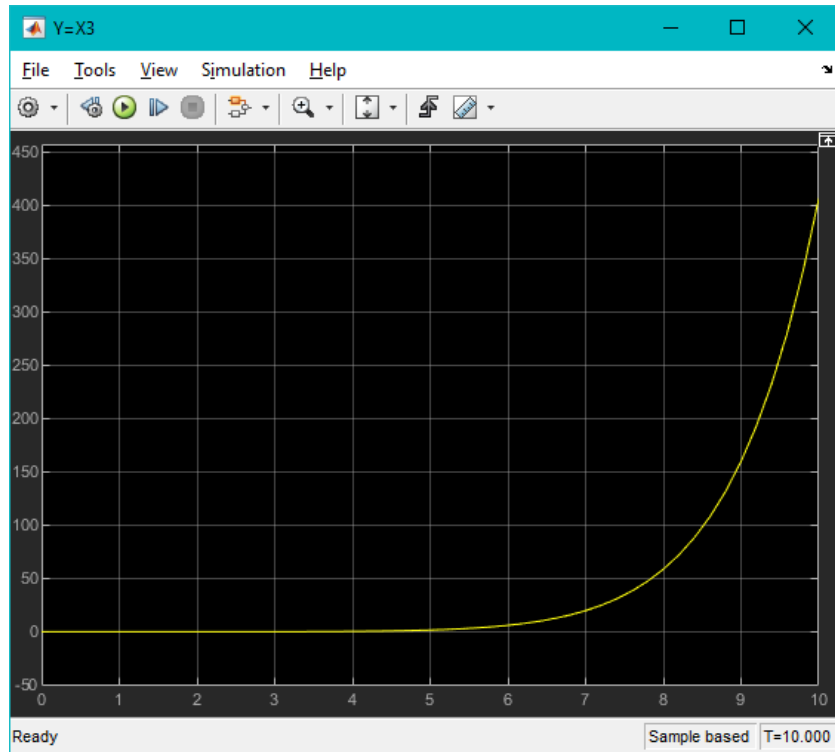
X1



X2



$$X^3=Y$$



CONTROLLABLE CANONICAL FORM

SS_CCF_disc =

A =

	x1	x2	x3
x1	0	1	0
x2	0	0	1
x3	0.9512	-2.664	-2.712

B =

	u1
x1	0
x2	0
x3	1

C =

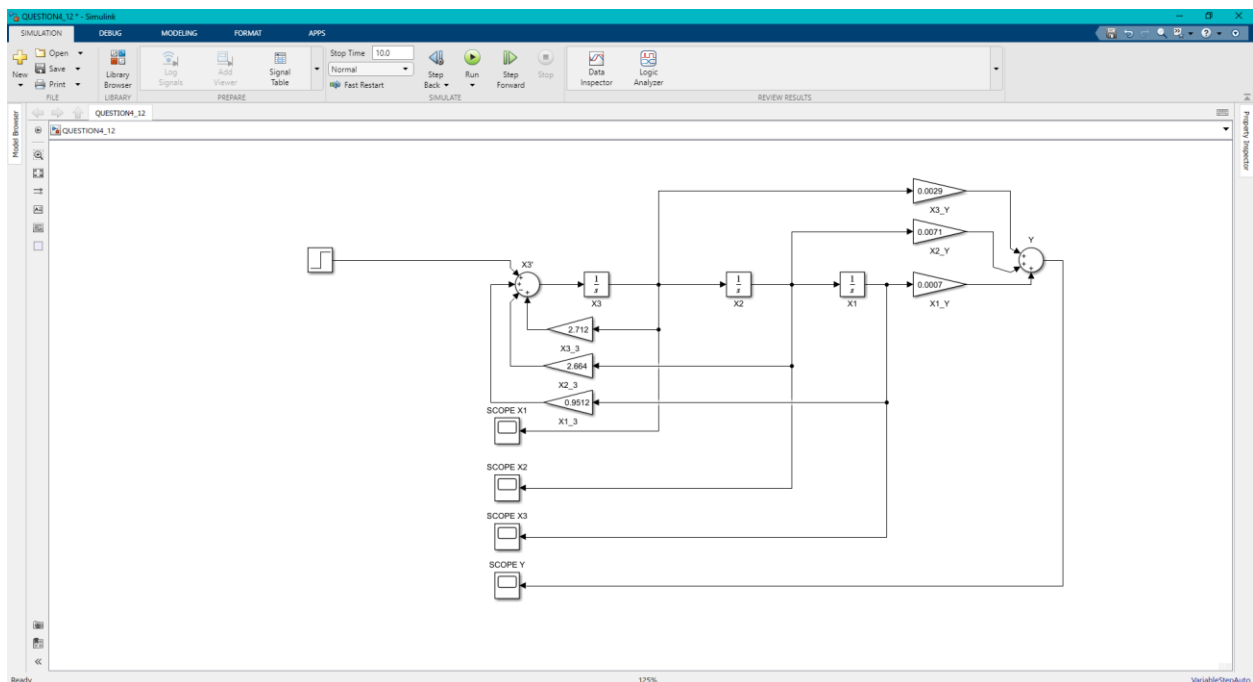
	x1	x2	x3
y1	0.0007	0.0071	0.0029

D =

	u1
y1	0

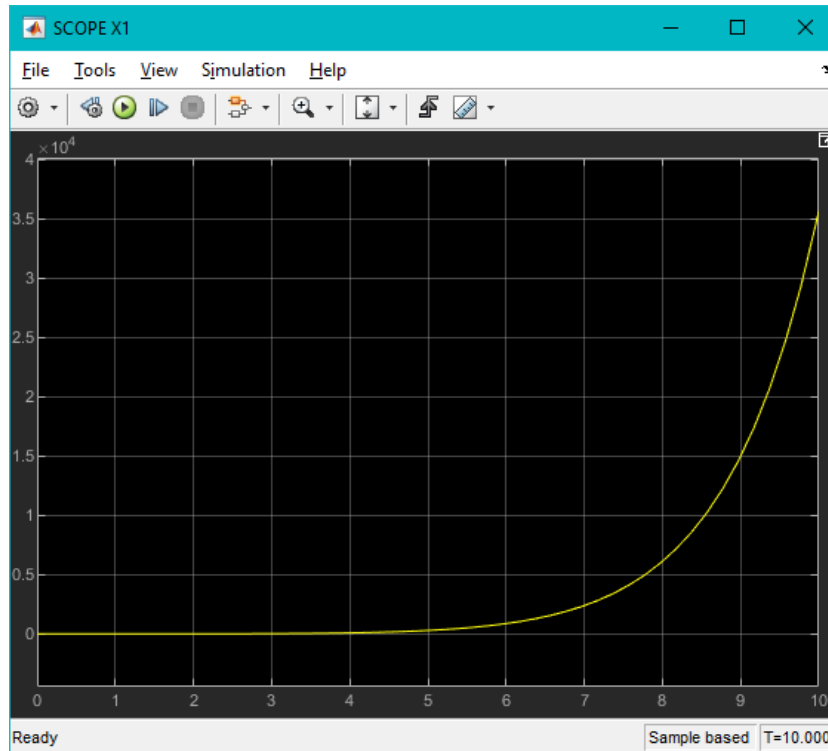
Continuous-time state-space model.

SIMULINK MODEL

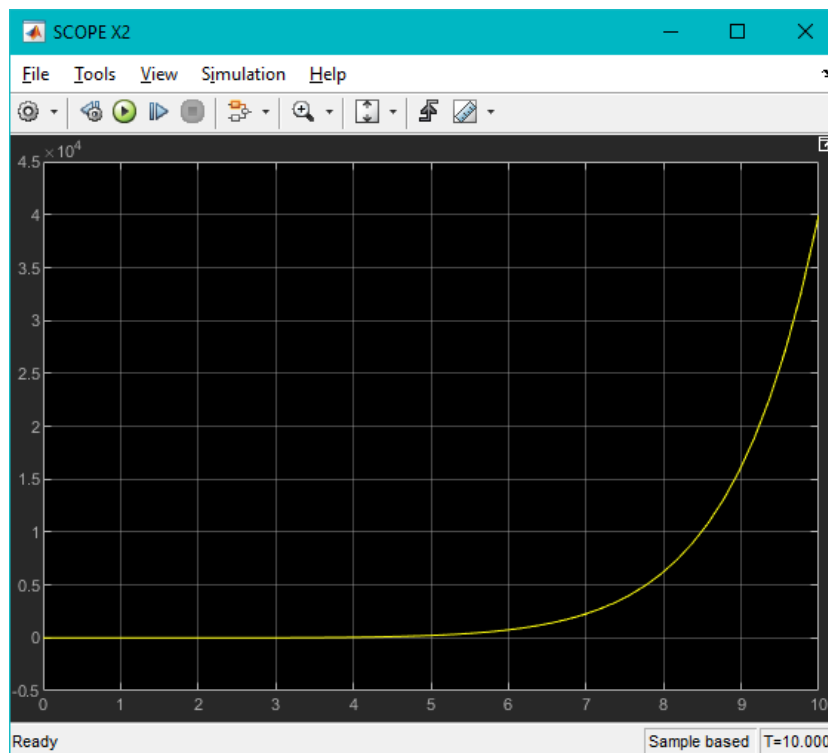


SYSTEM STATES

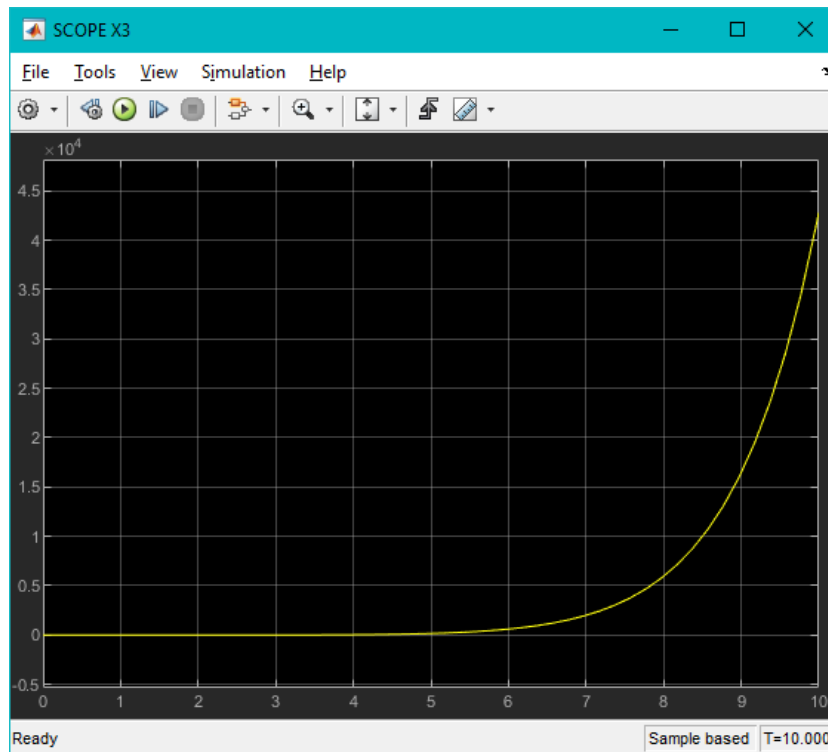
X1



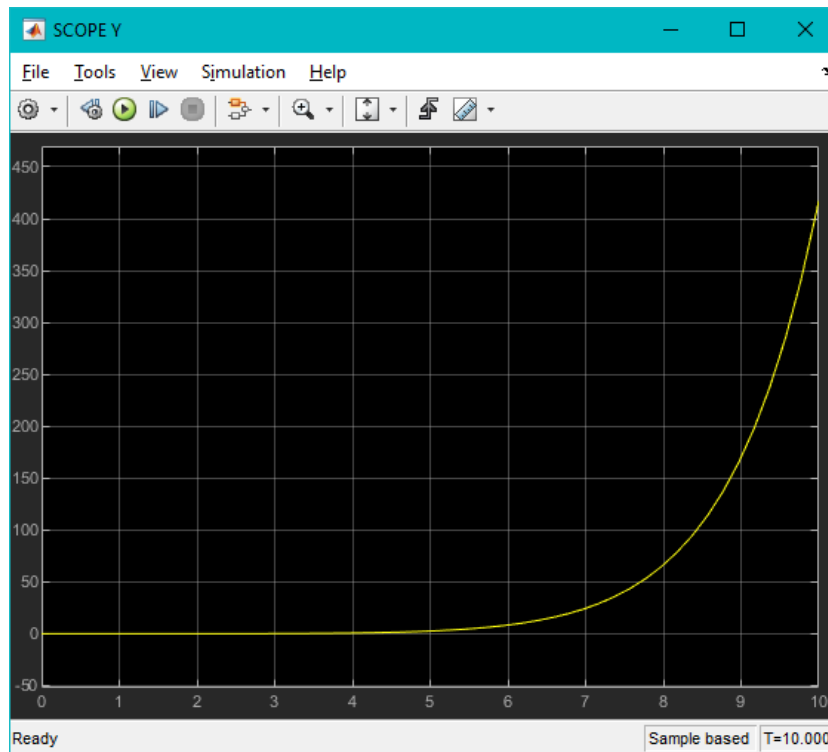
X2



X3



Y



COMMENT

We notice that for both systems repB and repA, no matter how we chose a different sampling period, the response is unstable.

2. Design state feedback vector for “rep B” to achieve same transient response specifications as before and use reference manipulation gain to achieve zero steady state error. Implement your controller on the continuous-time model (rep A). Draw the states and output versus time. Comment on the results.

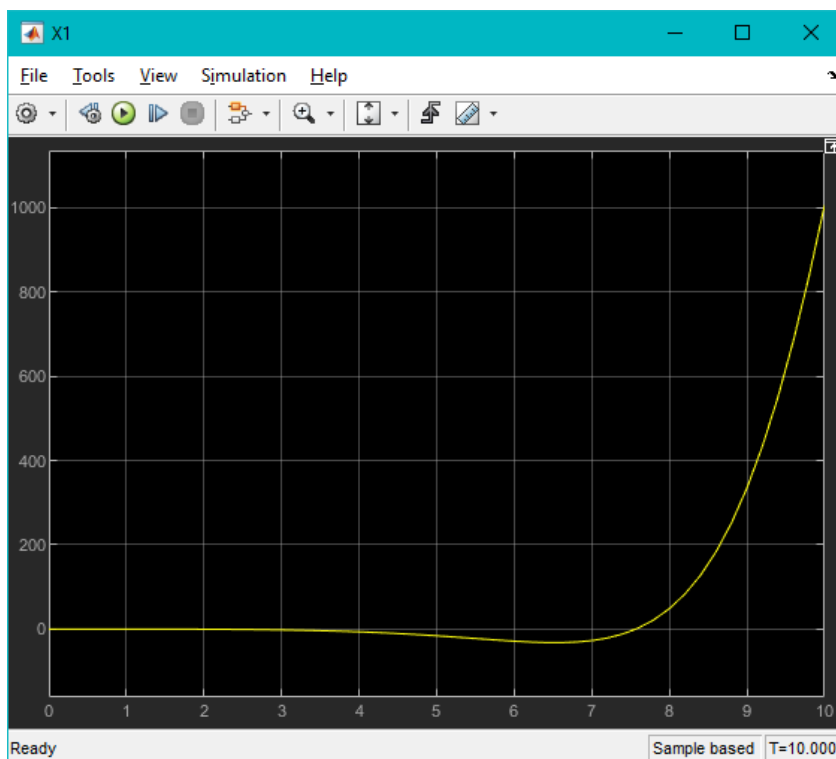
repB

POLE PLACEMENT

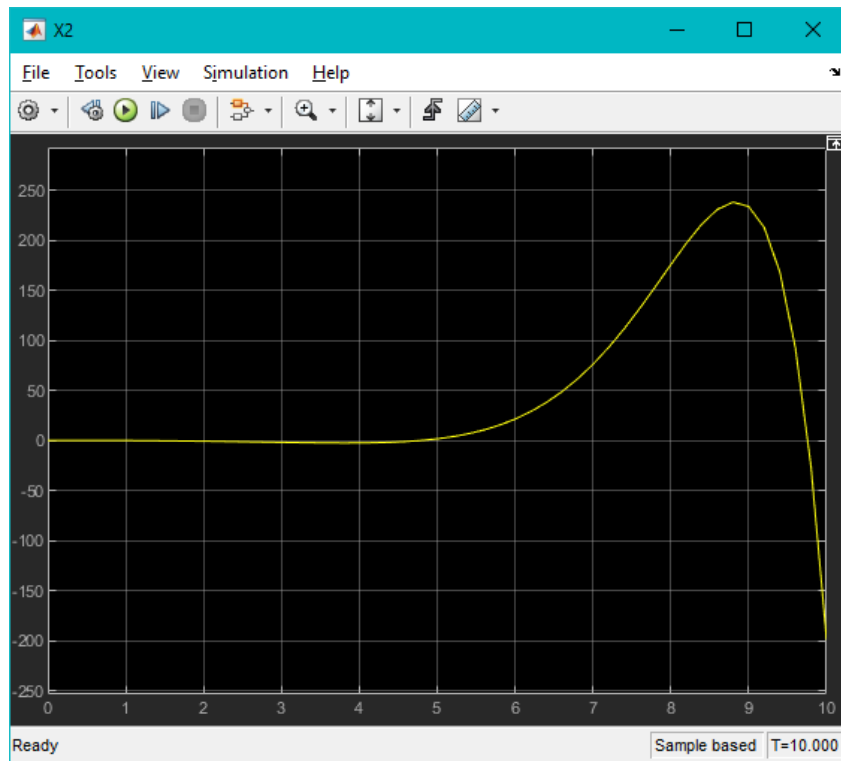
```
>> p_disc=[0.997+0.00248i 0.997-0.00248i 0.997];  
place(A_disc, B_disc, p_disc)  
  
ans =  
|  
-1.1110 -0.3758 0.3758
```

RESPONSE

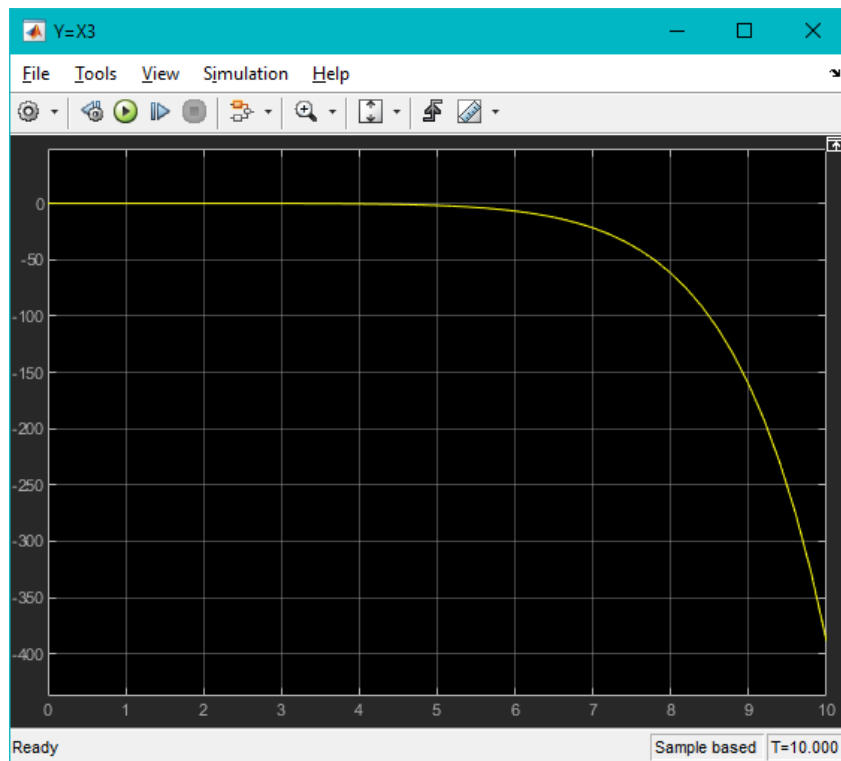
X1



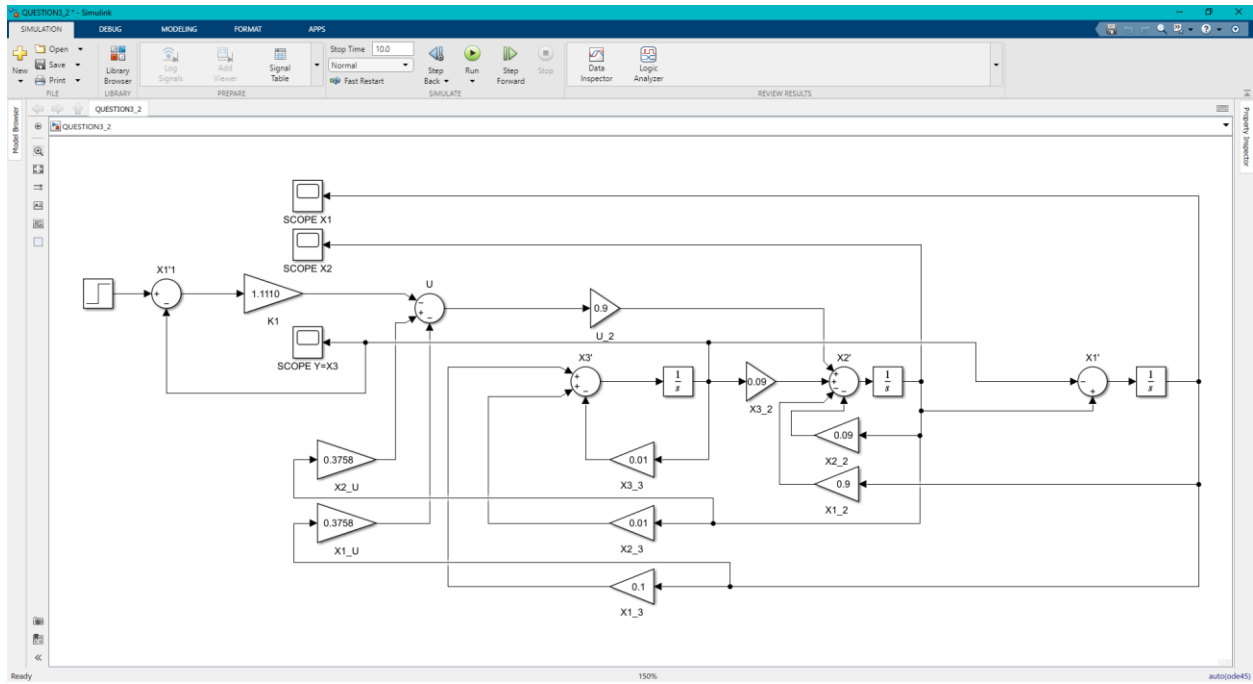
X2



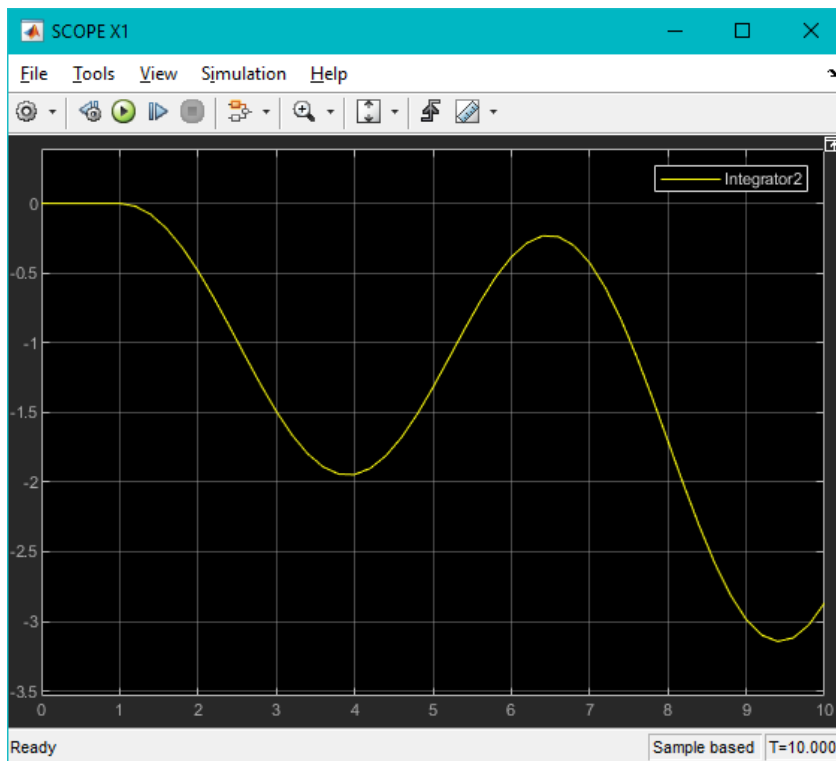
X3=Y



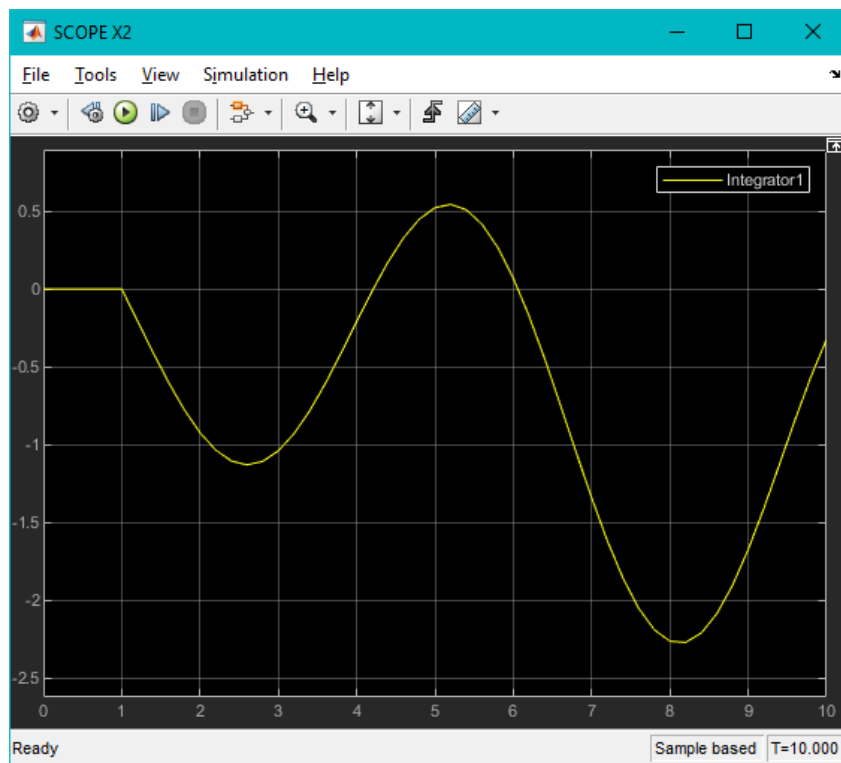
repA MODEL



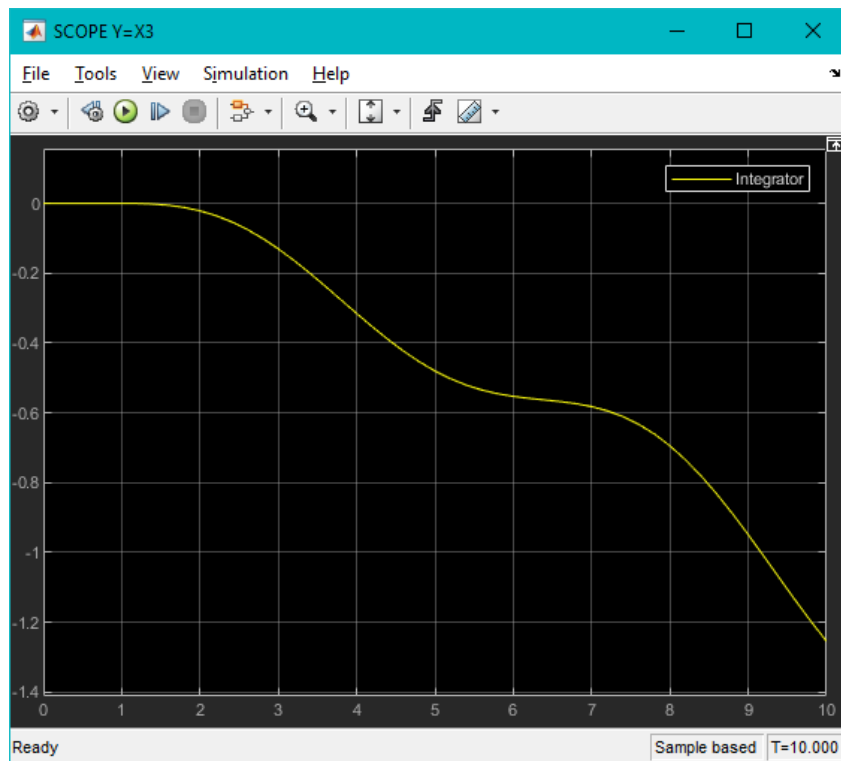
RESPONSE X1



X2



Y=X3

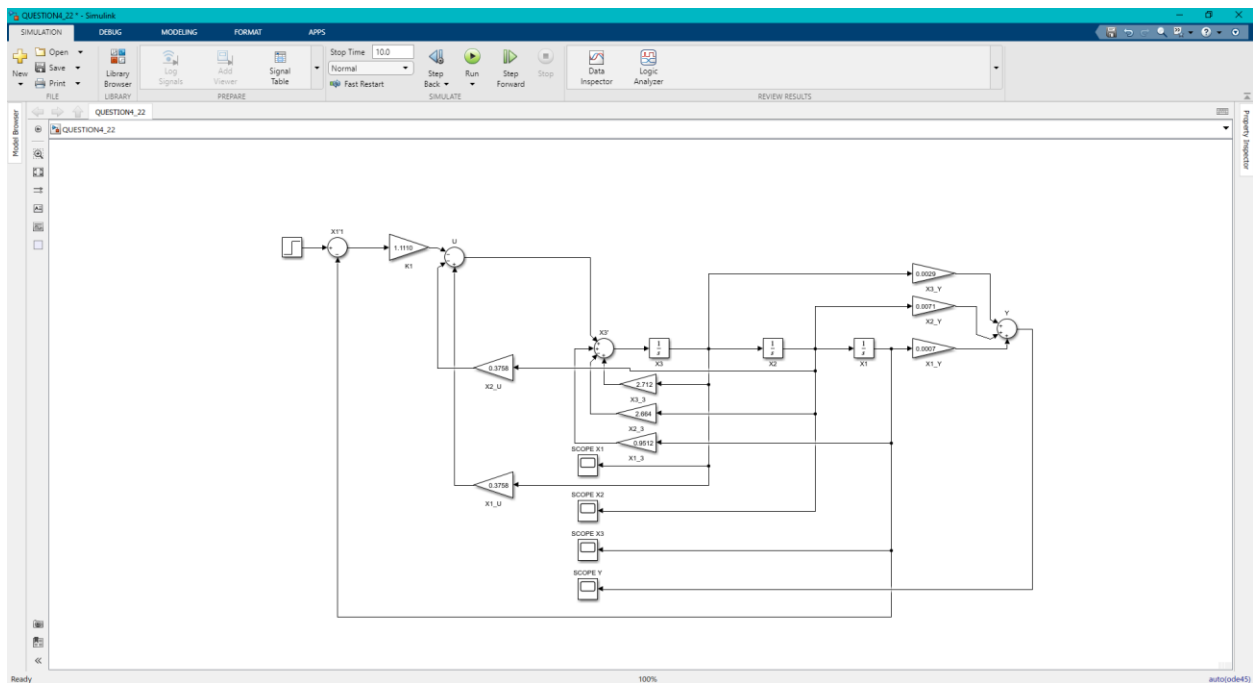


Unfortunately we were unable to reach a stable system after using state feedback gain, it was unknown why the system could not be stabilized: whether it was an unstabilizable system or there was an error in design. We assume that there is an error in the Simulink model which prevented the system from reaching stability.

3. Implement same controller designed on (2) using the states measurements from (rep C). Draw the system states, and comment on the results.

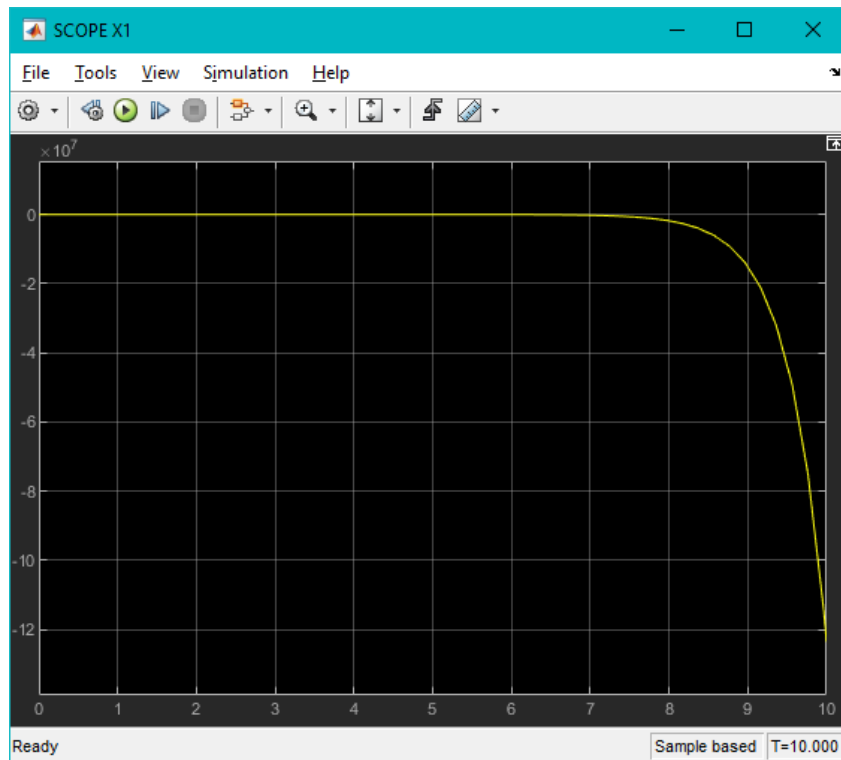
repC

MODEL

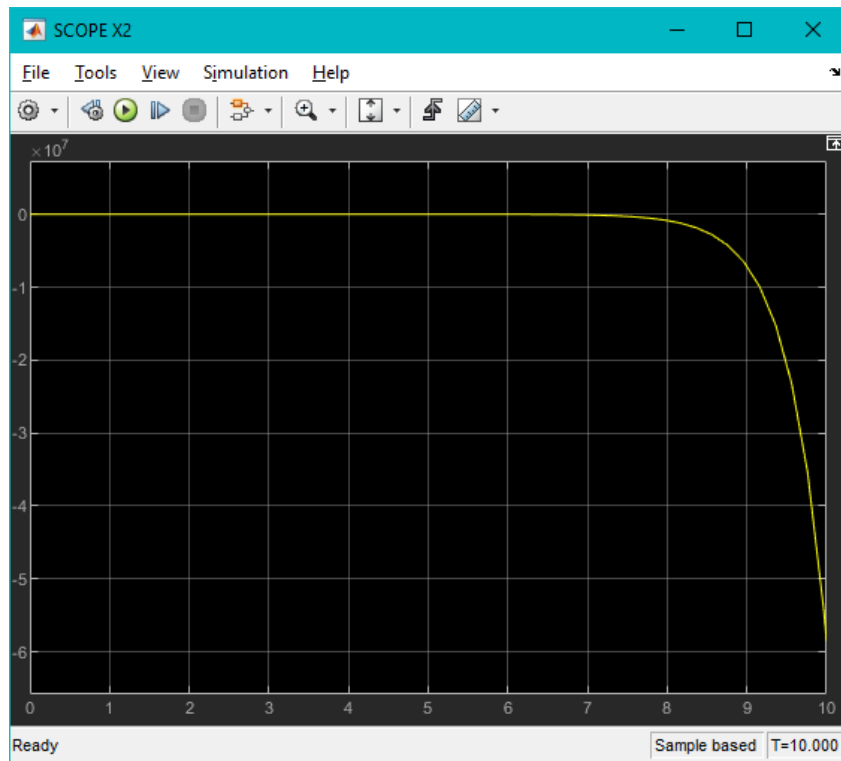


POLE PLACEMENT RESPONSE

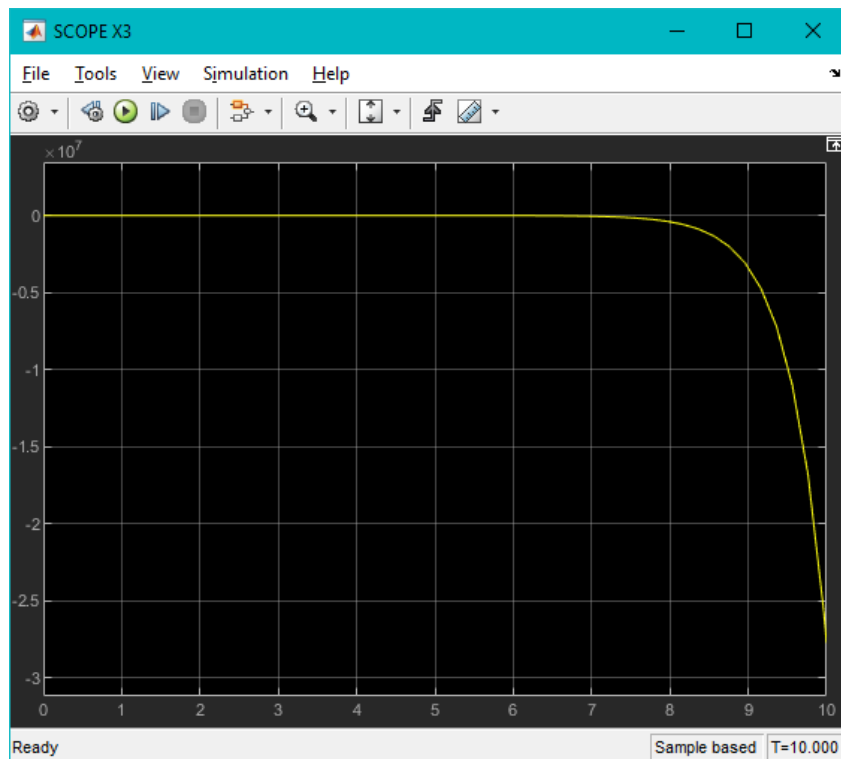
X1



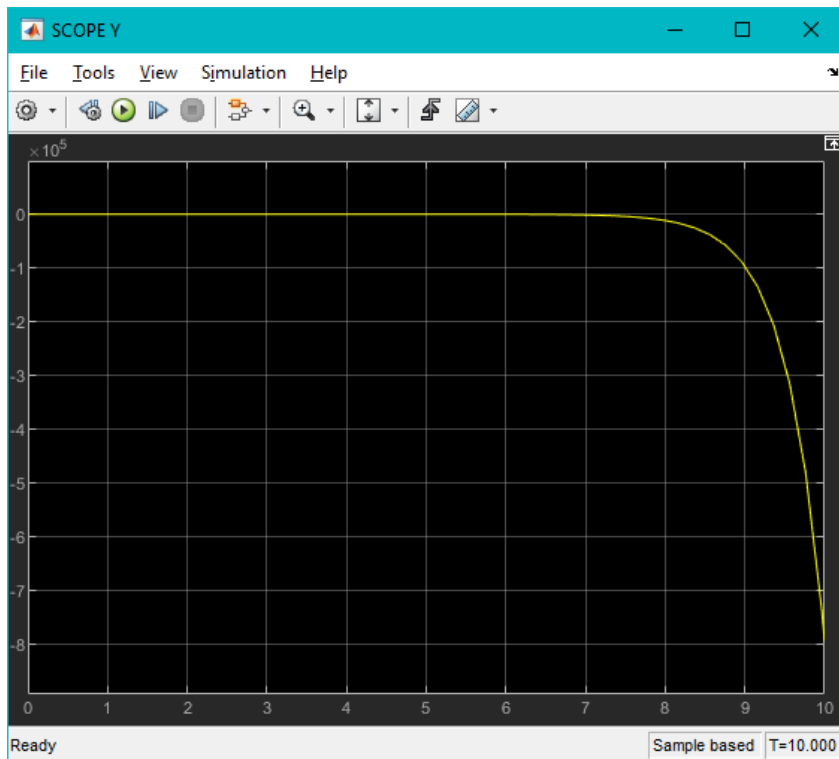
X2



X3



Y



COMMENT

Again, the state feedback gain could not stabilize the controllable canonical form of the system.

4. Using rep A and the controller designed in 2, assuming that the only measurement available is the output ω_2 , design an appropriate observer for the system and implement the state feedback. Draw the system states (starting from initial condition $x = [3 \ 5 \ 0]^T$ and the estimated states. Comment on the results.
5. For the controller in 4, if the sampling interval is changed to double its value, what will be the effect of this change on the feedback system response with the same state feedback gain? Study this effect and draw the system states.