Solution to cs224 assignment2(written)

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February 24, 2019

1 Notations

1. d : Vector dimension
2. n : Vocabulary size
3. $\mathbf{U} \in \mathbb{R}^{d \times n}$: The colums of $\mathbf{U}$ are all the 'outside' vectors $\mathbf{u}_w \in \mathbb{R}^{d \times 1}$
4. $\mathbf{V} \in \mathbb{R}^{d \times n}$: The columms of $\mathbf{V}$ are all the 'center' vector $\mathbf{v}_w \in \mathbb{R}^{d \times 1}$
5. $\mathbf{y}, \hat{\mathbf{y}}$: The true and predicted distribution

$$\mathbf{z} = \mathbf{U}^{\mathsf{T}} \mathbf{v}_c \in \mathbb{R}^{n \times 1}$$
$$\hat{\mathbf{y}} = softmax(\mathbf{z}) \in \mathbb{R}^{n \times 1}$$
$$J_{naive\_softmax} = CE(\mathbf{y}, \hat{\mathbf{y}})$$

$$\delta = \frac{\partial J_{native\_softmax}}{\partial \mathbf{z}} = (\hat{\mathbf{y}} - \mathbf{y})^{\mathsf{T}} \in \mathbb{R}^{1 \times n} \tag{1}$$

2 Answers

1. Answer to 1-(b)
$$\frac{\partial J_{native\_softmax}}{\partial \mathbf{v}_c} = (\hat{\mathbf{y}} - \mathbf{y})^{\mathsf{T}} \mathbf{U}^{\mathsf{T}} \in \mathbb{R}^{1 \times d} \tag{2}$$
2. Answer to 1-(c)
$$\frac{\partial J_{native\_softmax}}{\partial \mathbf{U}} = \delta \frac{\partial \mathbf{z}}{\partial \mathbf{U}} = (\delta^{\mathsf{T}} \mathbf{v}_c^{\mathsf{T}})^{\mathsf{T}} = \mathbf{v}_c \delta = \mathbf{v}_c (\hat{\mathbf{y}} - \mathbf{y})^{\mathsf{T}} \in \mathbb{R}^{d \times n} \tag{3}$$
3. Answer to 1-(d)
$$\sigma'(\mathbf{x}) = \sigma(\mathbf{x}) \circ (1 - \sigma(\mathbf{x})) \tag{4}$$
4. Answer to 1-(e)
$$\begin{aligned} \frac{\partial J_{neg\_sample}}{\partial \mathbf{v}_c} &= -\frac{\sigma'(\mathbf{u}_o^{\mathsf{T}} \mathbf{v}_c)}{\sigma(\mathbf{u}_o^{\mathsf{T}} \mathbf{v}_c)} \frac{\partial (\mathbf{u}_o^{\mathsf{T}} \mathbf{v}_c)}{\partial \mathbf{v}_c} - \sum_{k=1}^{K} \frac{\sigma'(-\mathbf{u}_k^{\mathsf{T}} \mathbf{v}_c)}{\sigma(-\mathbf{u}_k^{\mathsf{T}} \mathbf{v}_c)} \frac{\partial (-\mathbf{u}_k^{\mathsf{T}} \mathbf{v}_c)}{\partial \mathbf{v}_c} \\ &= -(1 - \sigma(\mathbf{u}_o^{\mathsf{T}} \mathbf{v}_c)) \mathbf{u}_o^{\mathsf{T}} + \sum_{k=1}^{K} (1 - \sigma(-\mathbf{u}_k^{\mathsf{T}} \mathbf{v}_c)) \mathbf{u}_k^{\mathsf{T}} \end{aligned} \tag{5}$$

$$\frac{\partial J_{neg\_sample}}{\partial \mathbf{u}_o} = -\left(1 - \sigma(\mathbf{u}_o^\top \mathbf{v}_c)\right)\mathbf{v}_c^\top \tag{6}$$

$$\frac{\partial J_{neg\_sample}}{\partial \mathbf{u}_k} = (1 - \sigma(-\mathbf{u}_k^\top \mathbf{v}_c))\mathbf{v}_c^\top \tag{7}$$

Computing of $J_{naive\_softmax}$ needs the inner product between $\mathbf{v}_c$ and all n vocabulary vectors, while J_{neg_sample} only $k+1$ vectors.

5. Answer to 1-(f)

$$\frac{\partial J_{skip\_gram}(\mathbf{v}_c, w_{t-m}, ..., w_{t+m}, \mathbf{U})}{\partial \mathbf{U}} = \sum_{-m \leq j \leq m, j \neq 0} \frac{\partial J(\mathbf{v}_c, w_{t+j}, \mathbf{U})}{\partial \mathbf{U}} \tag{8}$$

$$\frac{\partial J_{skip\_gram}(\mathbf{v}_c, w_{t-m}, ..., w_{t+m}, \mathbf{U})}{\partial \mathbf{v}_c} = \sum_{-m \leq j \leq m, j \neq 0} \frac{\partial J(\mathbf{v}_c, w_{t+j}, \mathbf{U})}{\partial \mathbf{v}_c} \tag{9}$$

$$\frac{\partial J_{skip\_gram}(\mathbf{v}_c, w_{t-m}, ..., w_{t+m}, \mathbf{U})}{\partial \mathbf{v}_w} = 0 \quad when \quad w \neq c \tag{10}$$