CS460: Intro to Database Systems

Class 12: Tree-Structured Indexing

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https://midas.bu.edu/classes/CS460/

Tree-structured indexing

Intro & B^+-Tree

Insert into a B^+-Tree

Delete from a B^+-Tree

Prefix Key Compression & Bulk Loading

Introduction

Recall: 3 alternatives for data entries k*:

- Data record with key value **k**
- <**k**, rid of data record with search key value **k**>
- <**k**, list of rids of data records with search key **k**>

Choice is orthogonal to the *indexing technique* used to locate data entries k*.

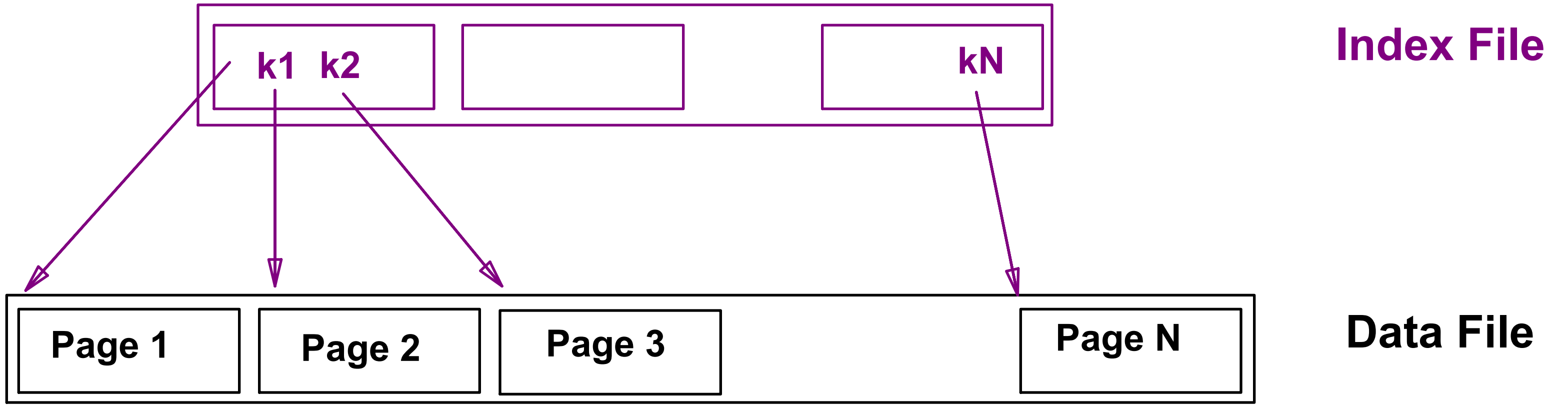
Tree-structured indexing techniques support both *range searches* and *equality searches*.

Range Searches

“Find all students with gpa > 3.0”

- If data is in sorted file, do binary search to find first such student, then scan to find others.
- Cost of maintaining sorted file + performing binary search in a database can be quite high. Q: Why???

Simple idea: Create an “index” file.



☛ *Can do binary search on a (smaller) index file!*

B+ Tree: The Most Widely-Used Index

Insert/delete at $log_F(N)$ cost; keep tree *height-balanced.*

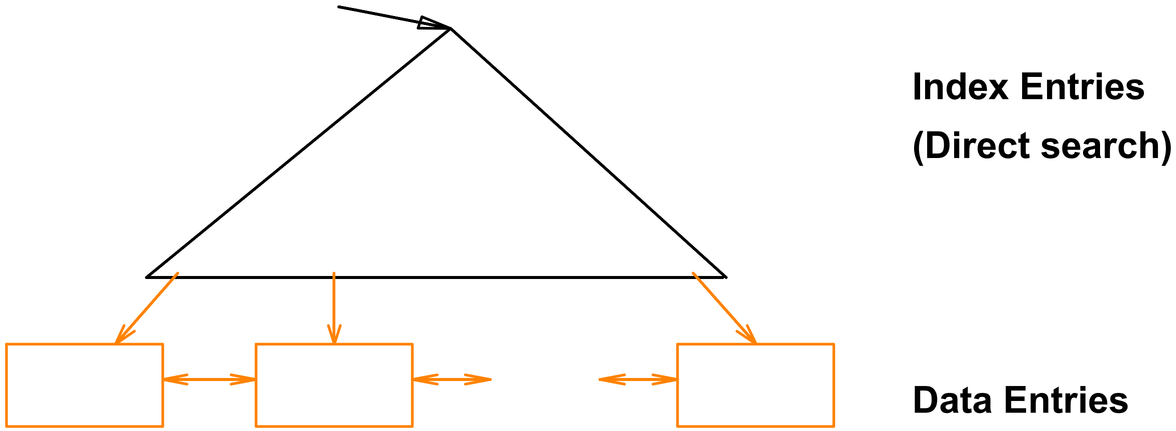
(F = fanout, N = # leaf pages)

Minimum 50% occupancy (except for root).

Each node contains $d \leq m \leq 2d$ entries. "d" is called the *order* of the tree.

Supports equality and range-searches efficiently.

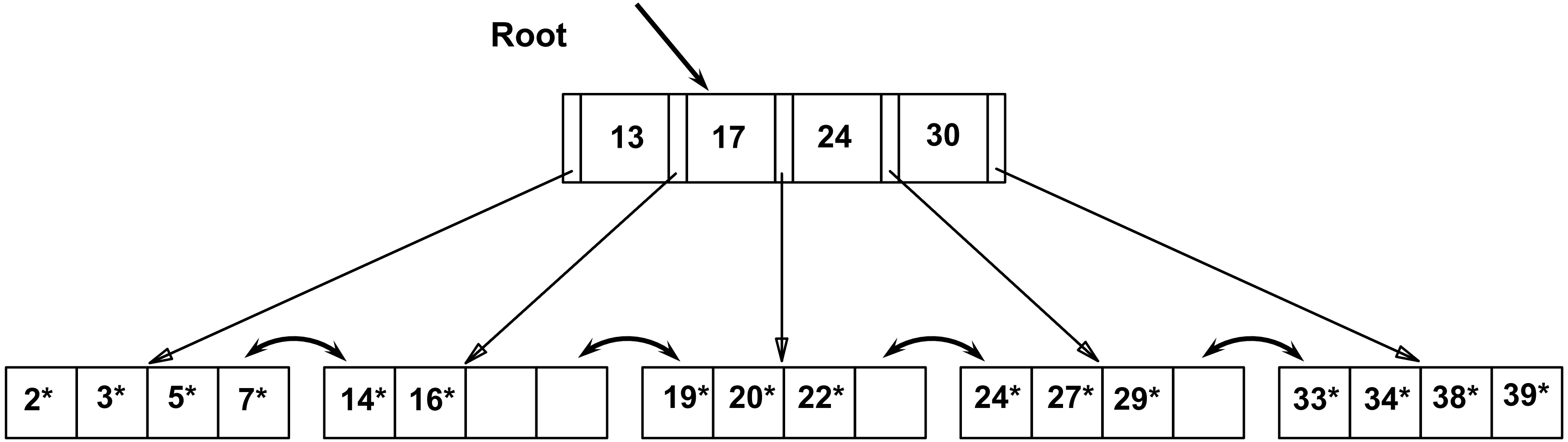
All searches go from root to leaves, in a dynamic structure.



Example B+ Tree

Search begins at root, and key comparisons direct it to a leaf.

Search for 5*, 15*, all data entries >= 24* ...



☛ *Based on the search for 15*, we <u>know</u> it is not in the tree!*

B+ Trees in Practice (cool facts!)

Typical order: 100. Typical fill-factor: 67%.

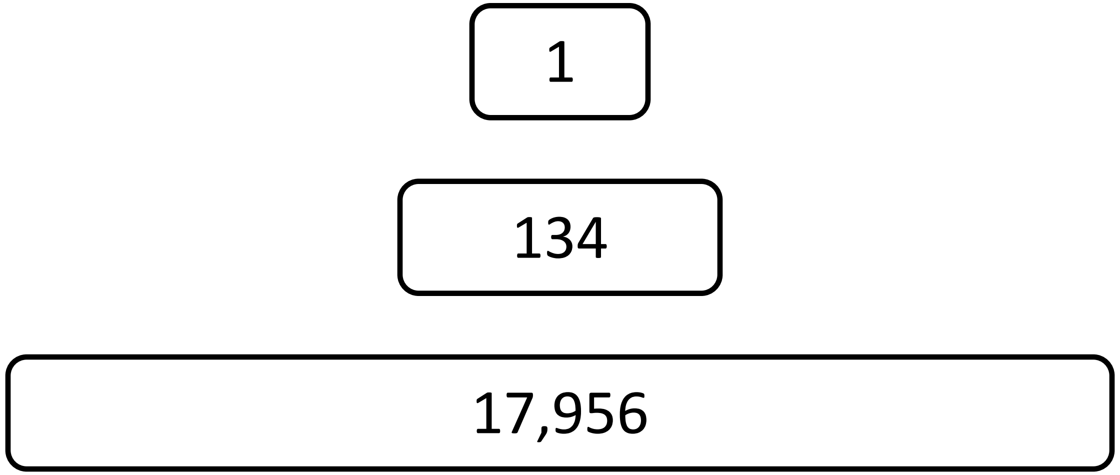
- average fanout = $2 \cdot 100 \cdot 0.67 = 134$

Typical capacities:

- Height 4: 133^4 = 312,900,721 entries
- Height 3: 133^3 = 2,406,104 entries

Can often hold top levels in buffer pool:

- Level 1 = 1 page = 8 KB
- Level 2 = 134 pages = 1 MB
- Level 3 = 17,956 pages = 140 MB



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Inserting a Data Entry into a B+ Tree

Find correct leaf *L.*

Put data entry onto *L*.

- If *L* has enough space, *done*!
- Else, must ***split*** *L (into L and a new node L2)*
  - Redistribute entries evenly, **copy up** middle key.
  - Insert index entry pointing to *L2* **into parent** of *L*.

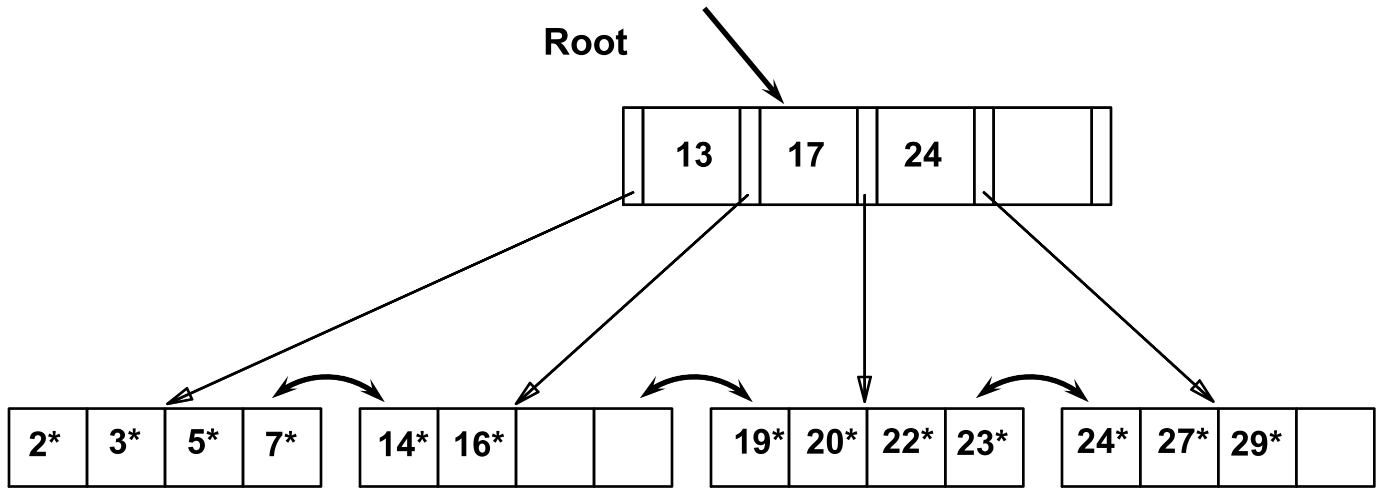
This can happen recursively

- To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)

Splits "grow" tree; root split increases height.

- Tree growth: gets *wider* or *one level taller at top.*

Example B+ Tree - Inserting 8*



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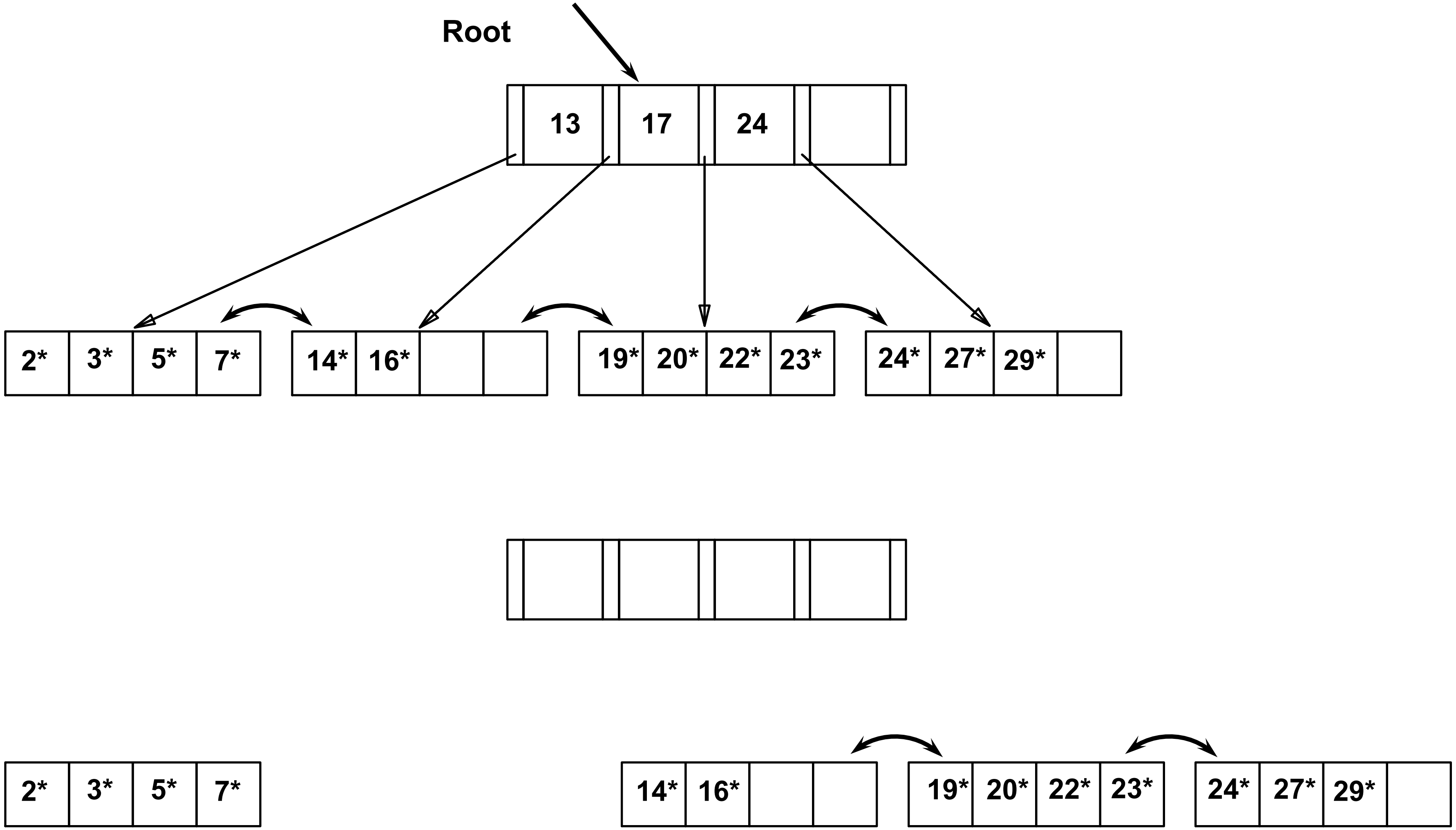

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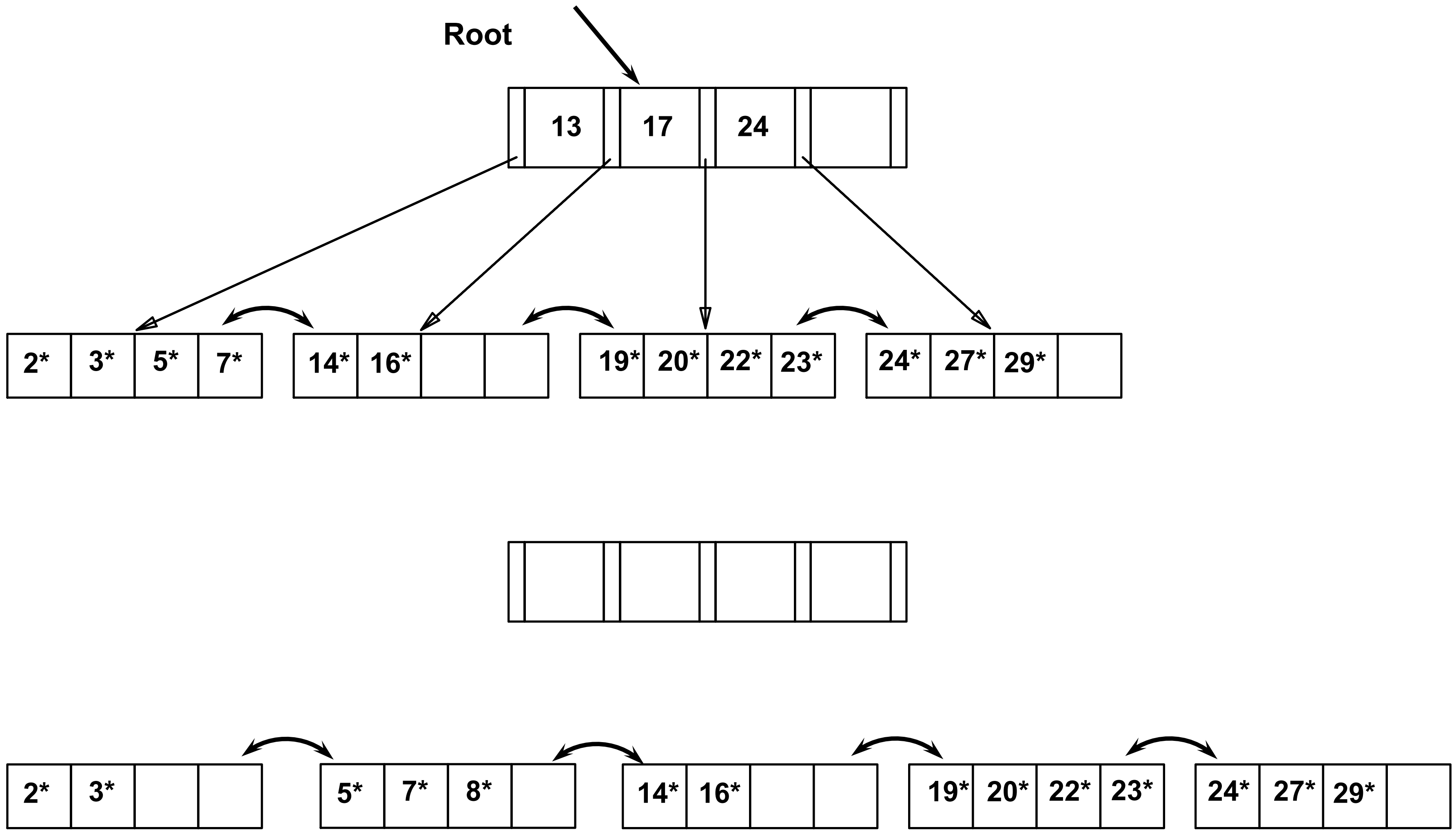

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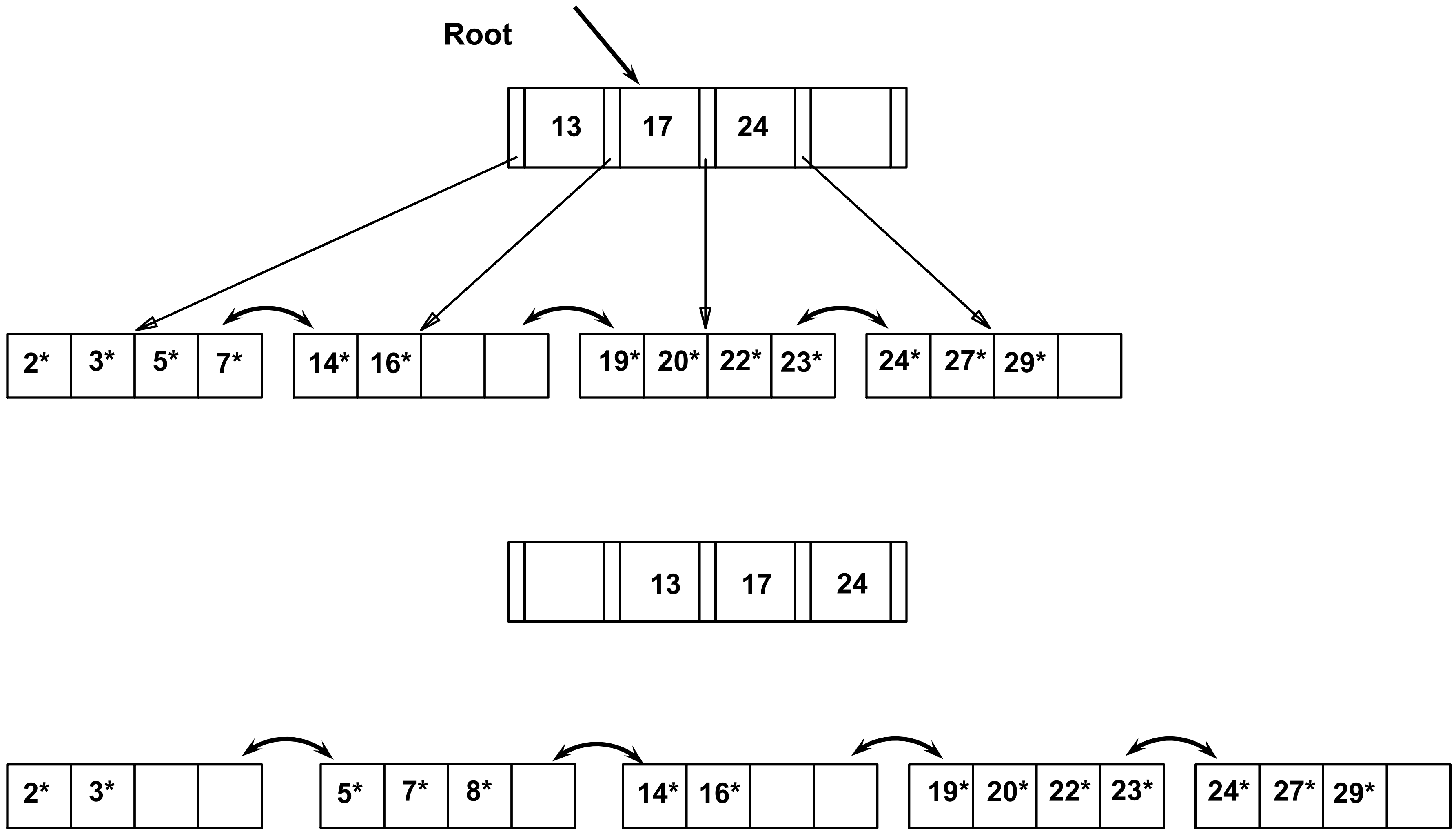

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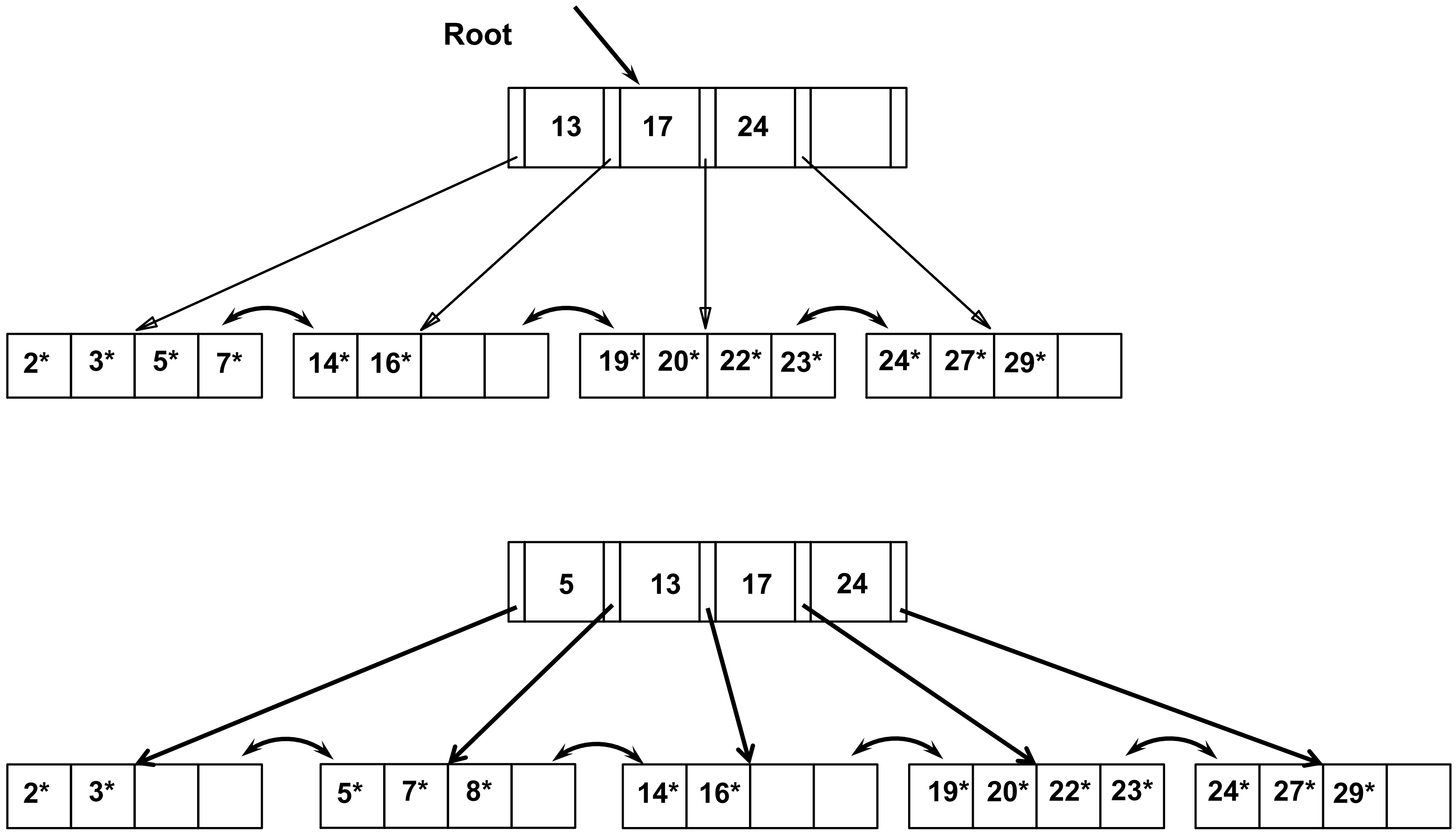
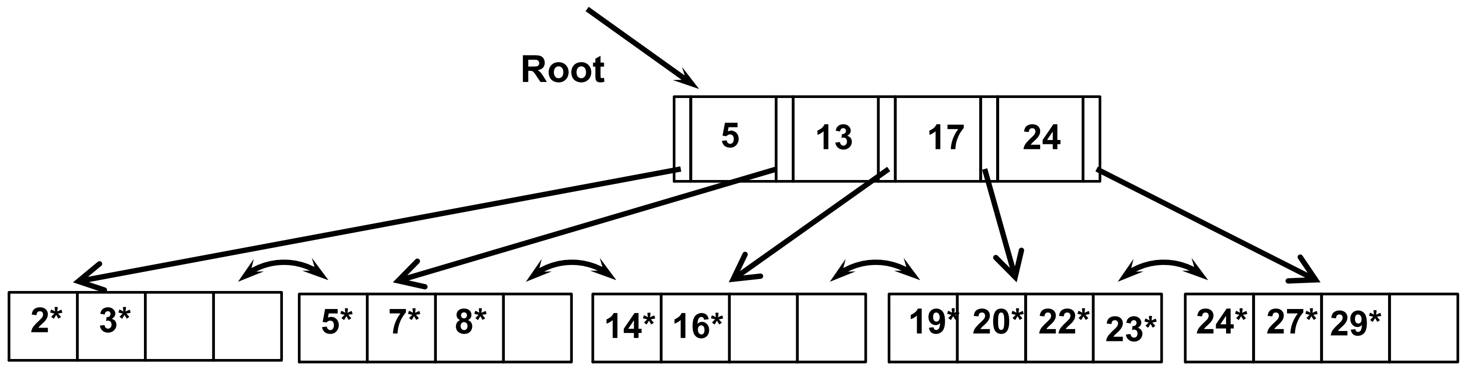
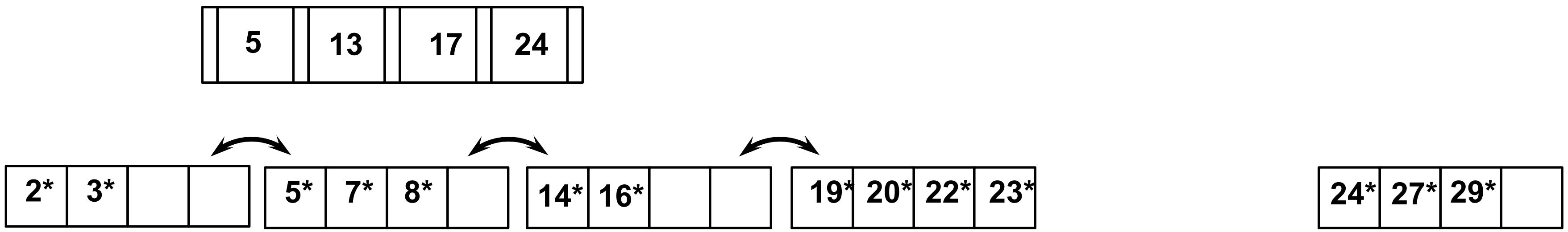


Example B+ Tree - Inserting 21*





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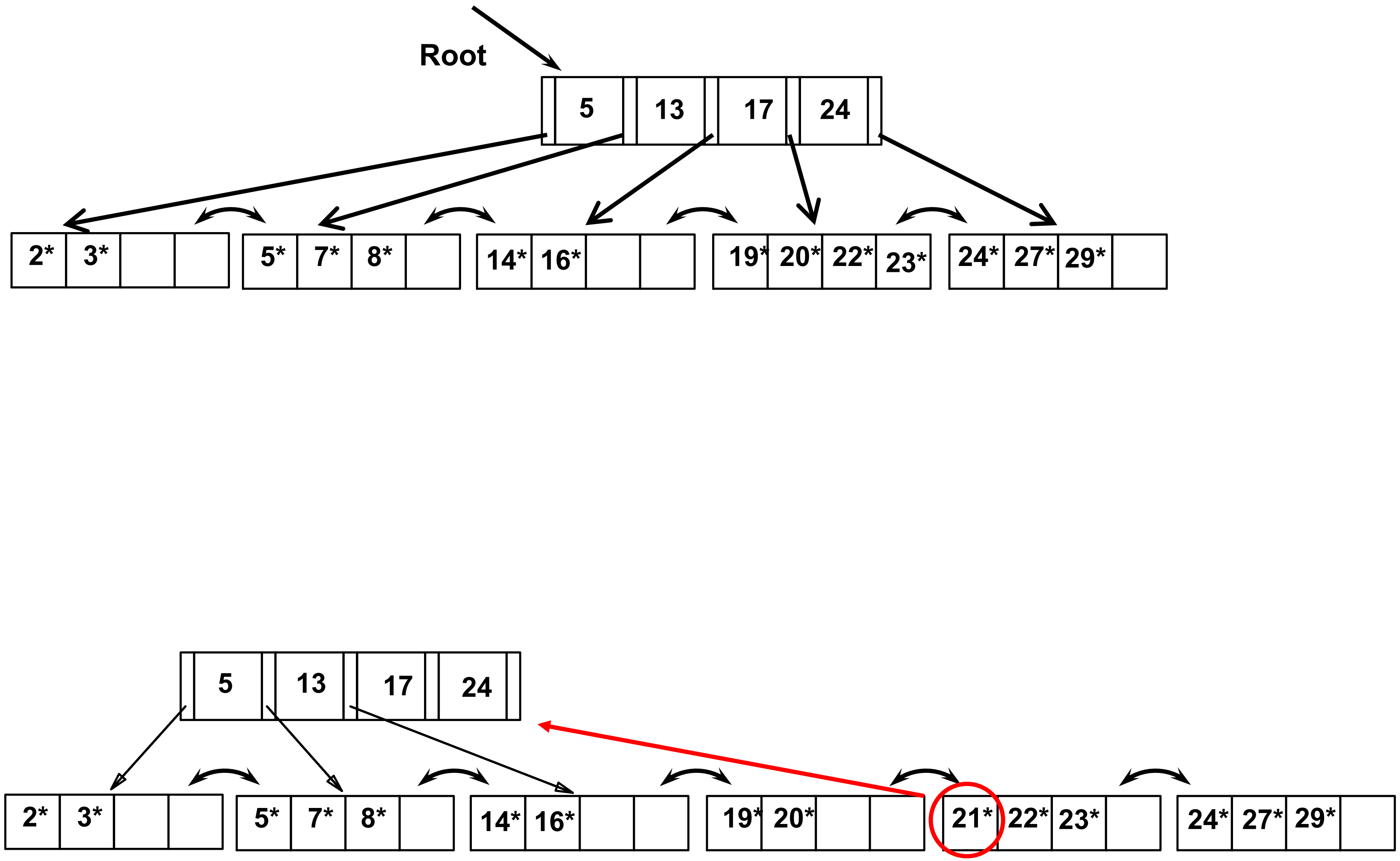

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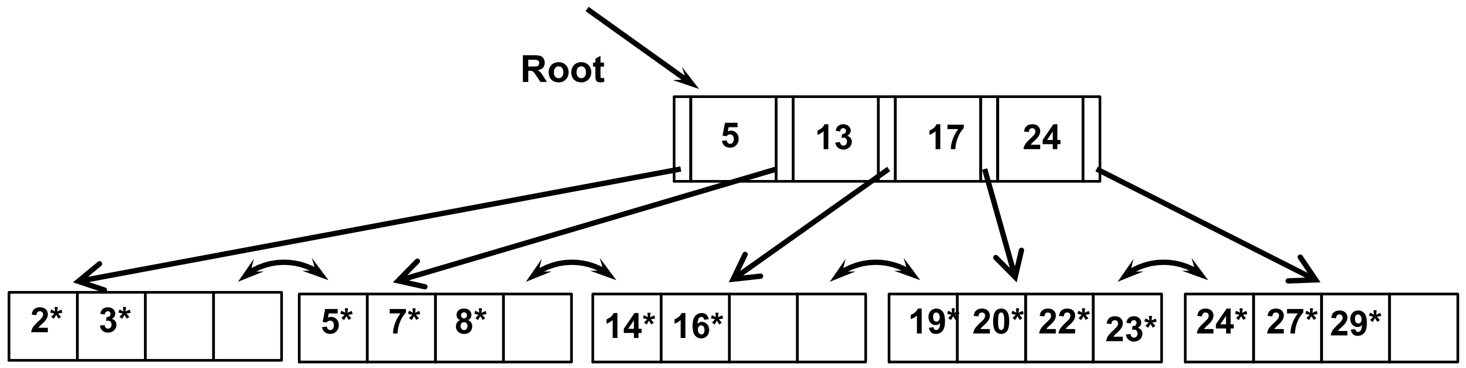

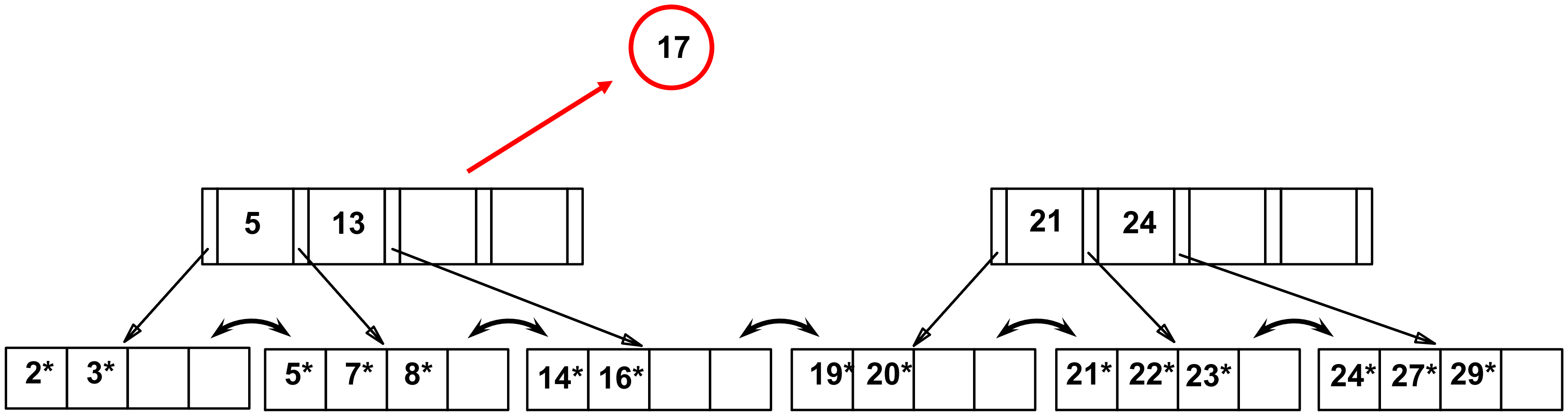

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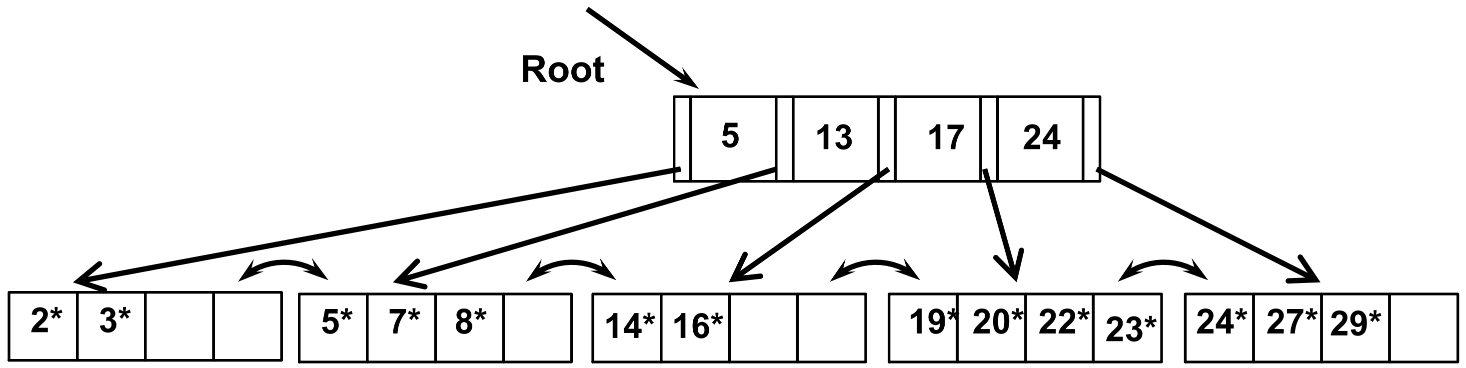
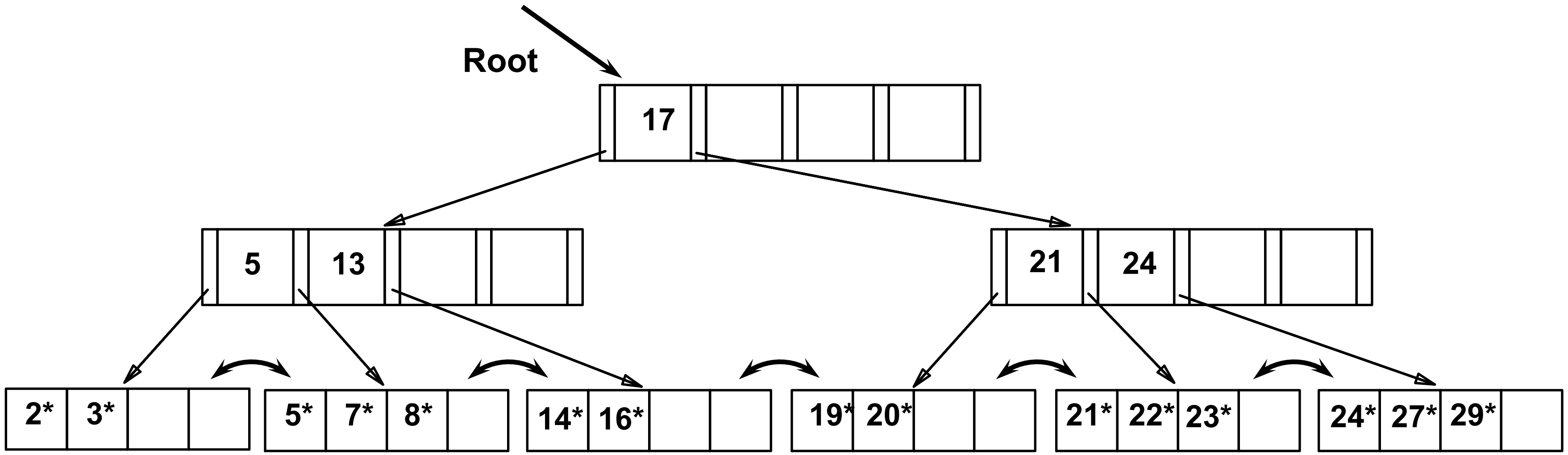
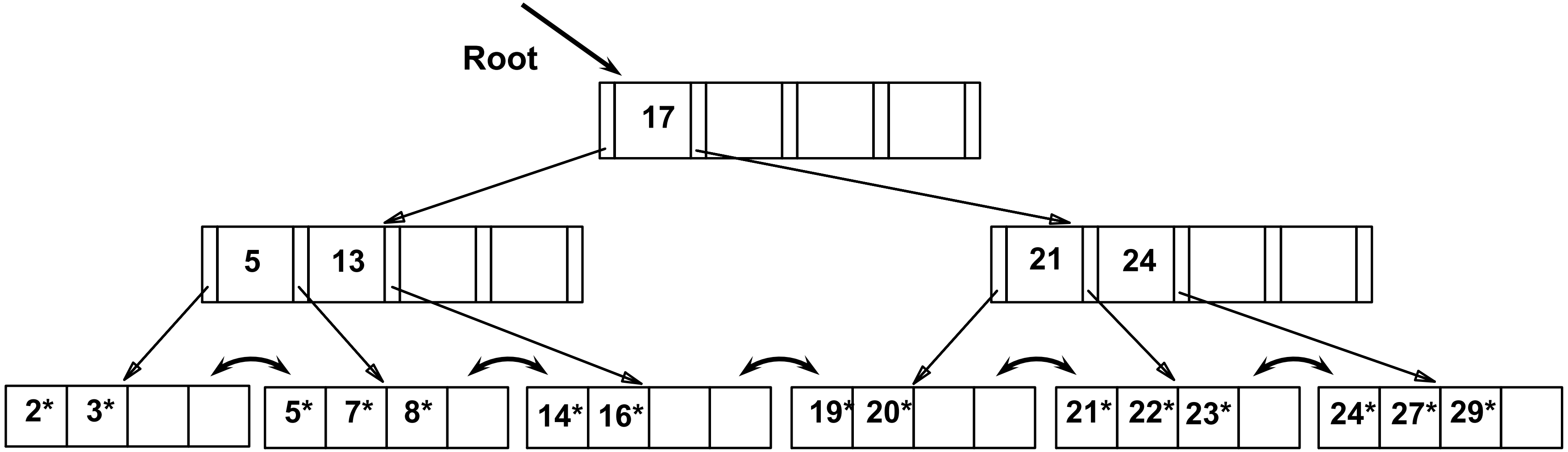




Example B+ Tree



Notice that root was split, leading to increase in height.

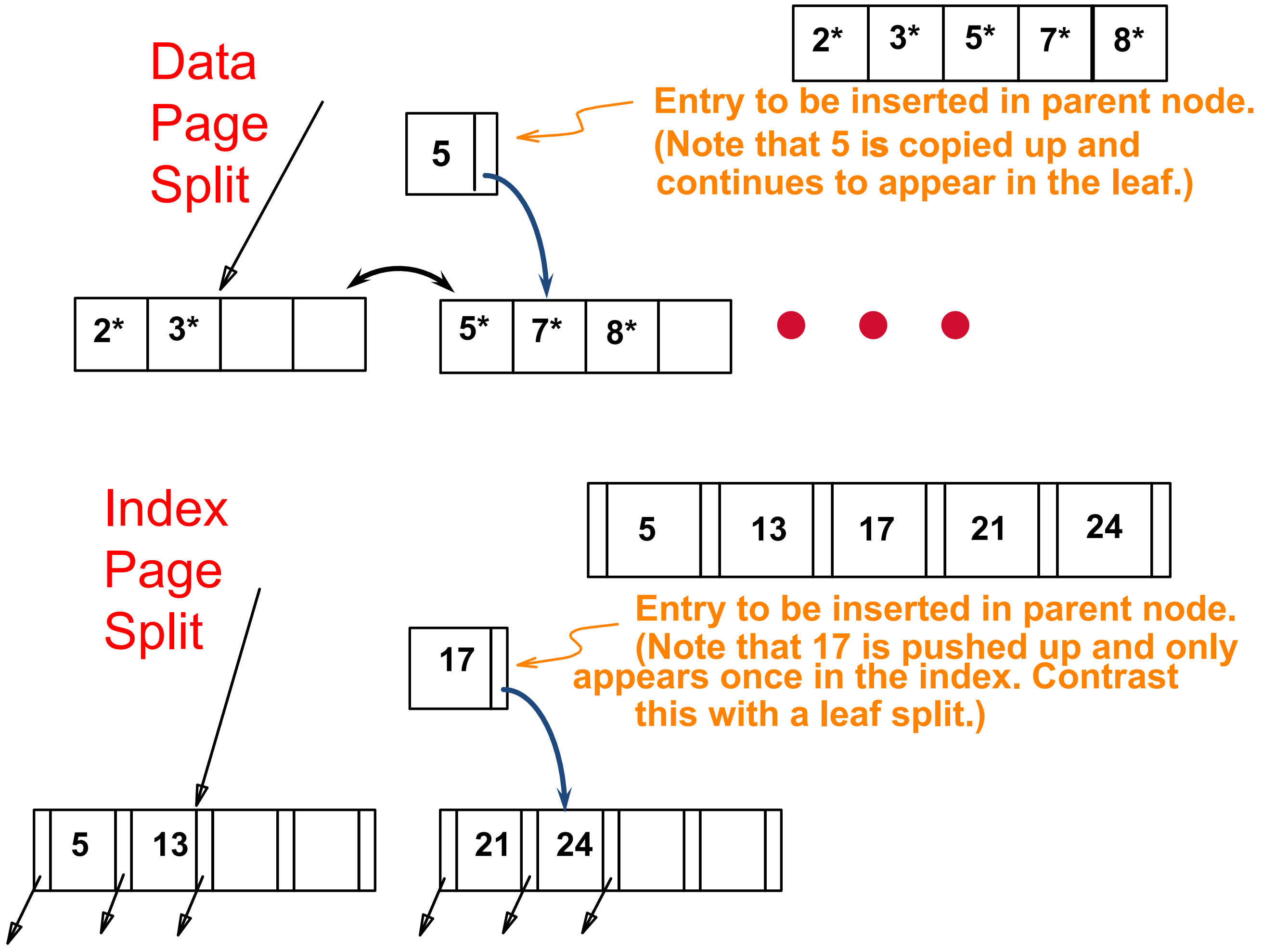
In this example, we can avoid split by re-distributing entries; however, this is usually not done in practice.

Example: Data vs. Index Page Split

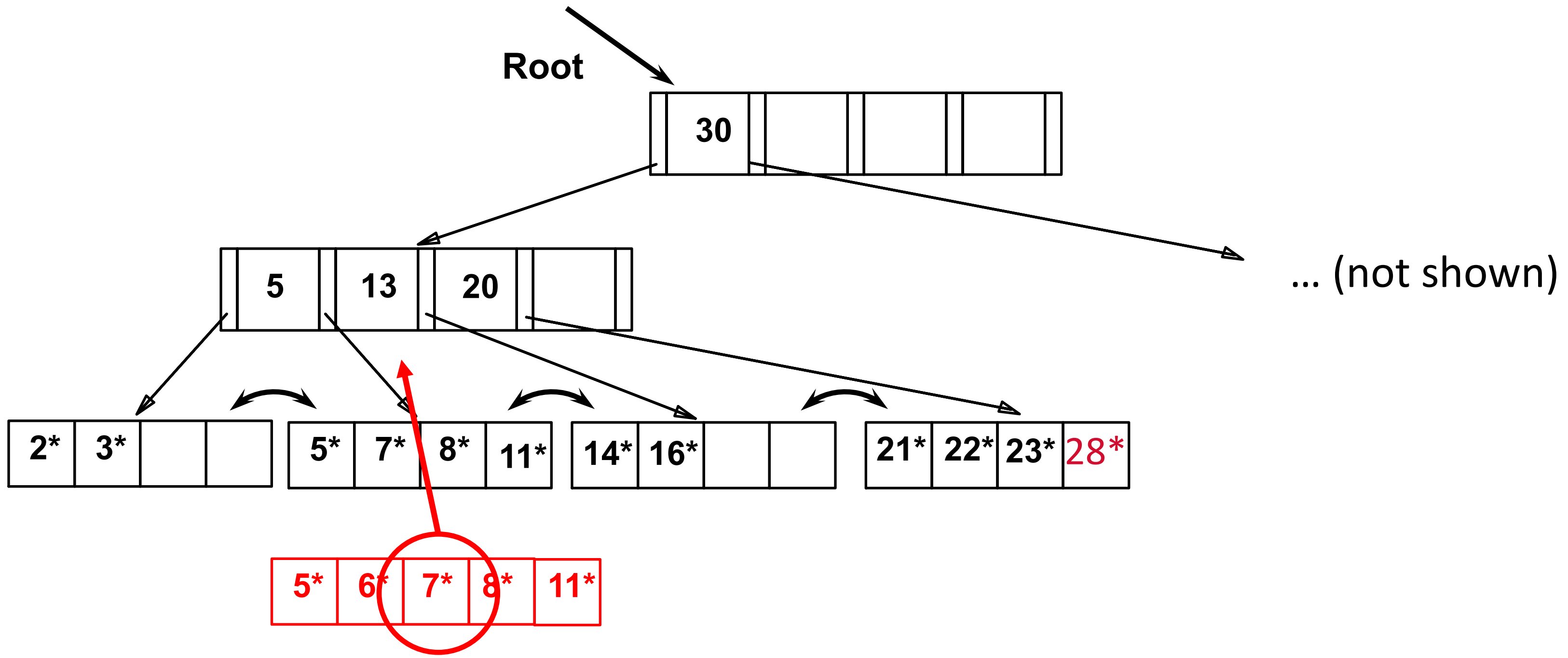
minimum occupancy is guaranteed in both leaf and index page splits

copy-up for data page splits

push-up for index page split



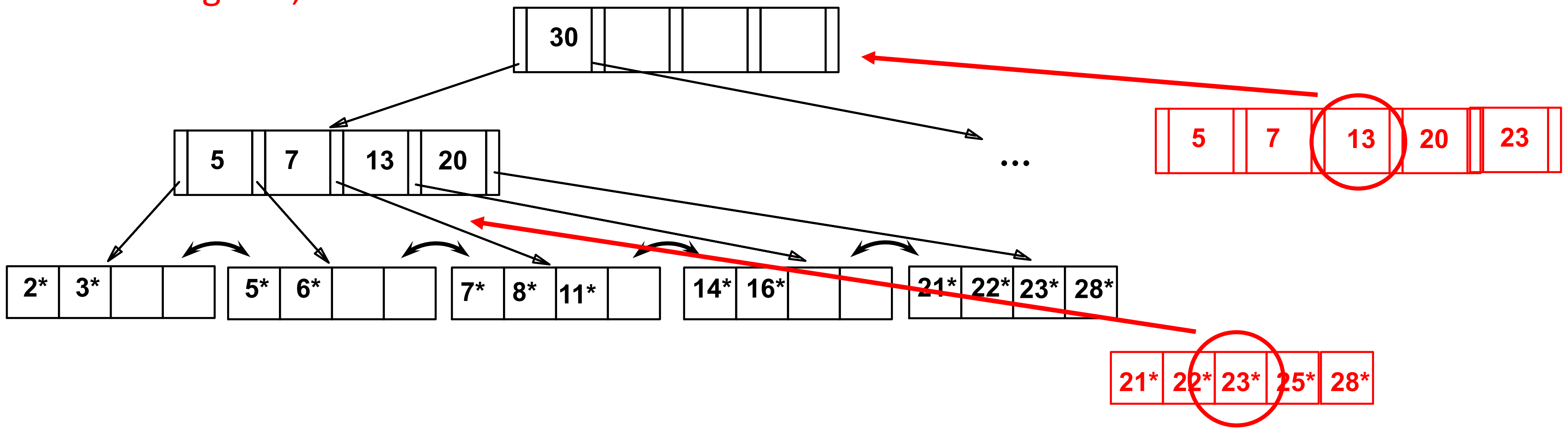
Now you try…



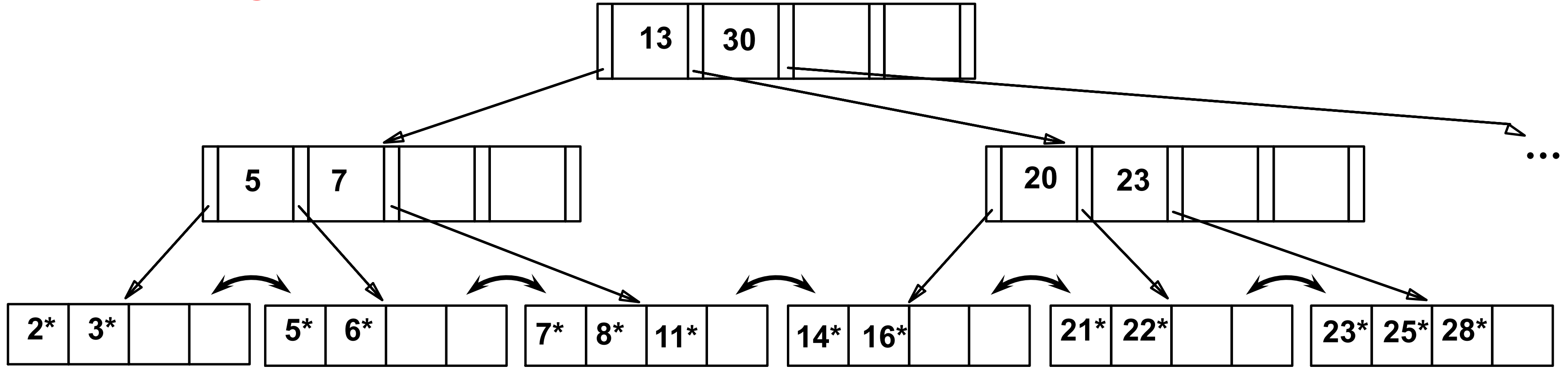
Insert the following data entries (in order): 28*, 6*, 25*

Answer…

After inserting 28*, 6*



After inserting 25*



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Deleting a Data Entry from a B+ Tree

Start at root, find leaf *L* where entry belongs.

Remove the entry.

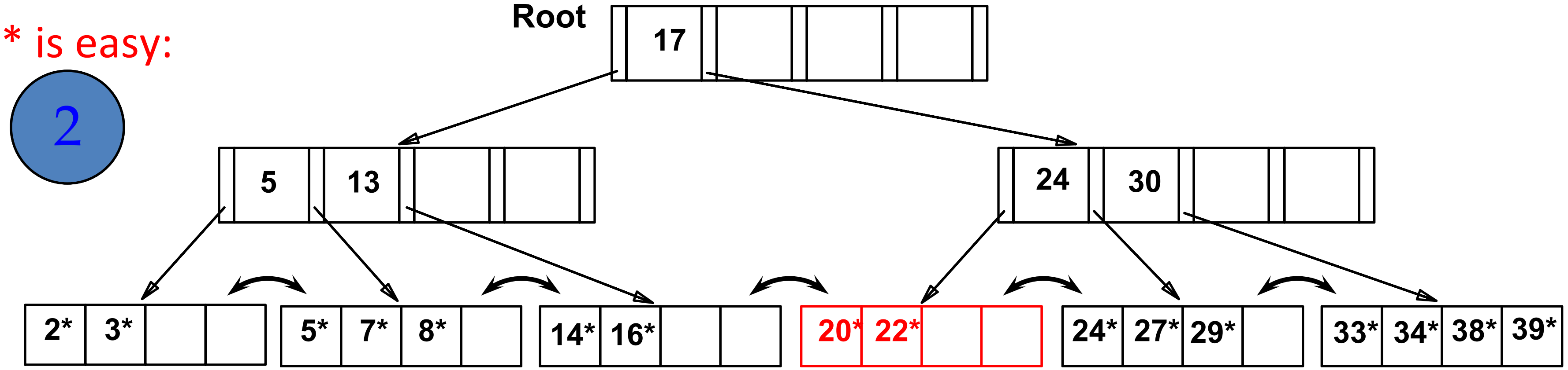
- If L is at least half-full, *done!*
- If L has only **d-1** entries,
  - Try to re-distribute, borrowing from *sibling* *(adjacent node with same parent as L)*.
  - If re-distribution fails, *merge* *L* and sibling.

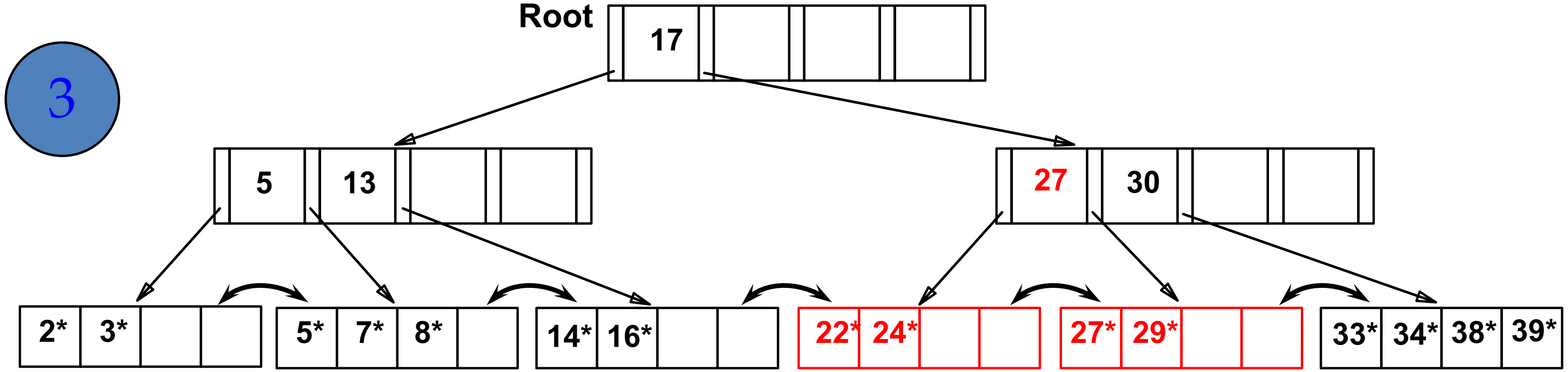
If merge occurred, must delete entry (pointing to *L* or sibling) from parent of *L*.

Merge could propagate to root, decreasing height.

Example: Delete 19* & 20*

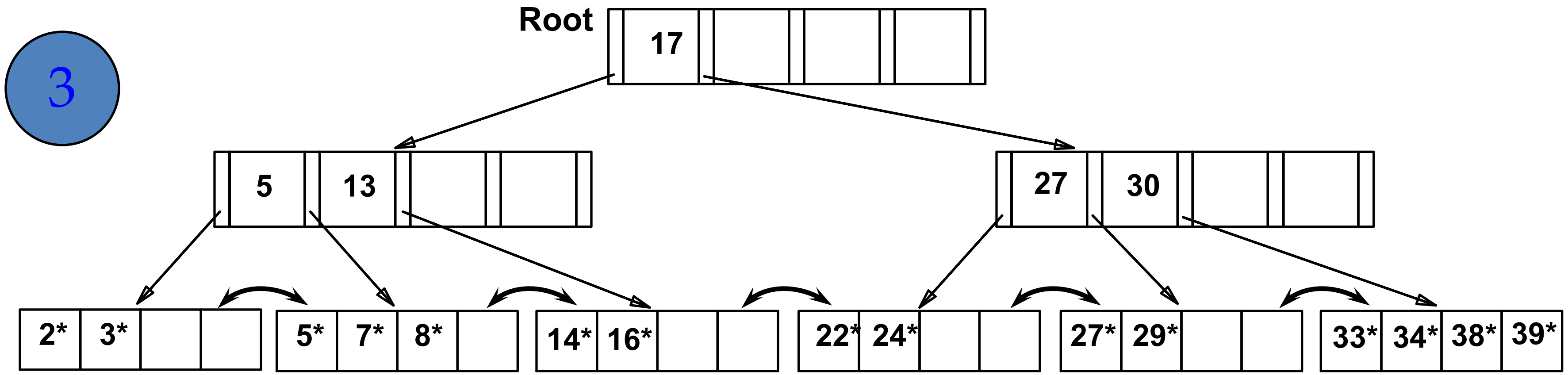
Deleting 19* is easy:

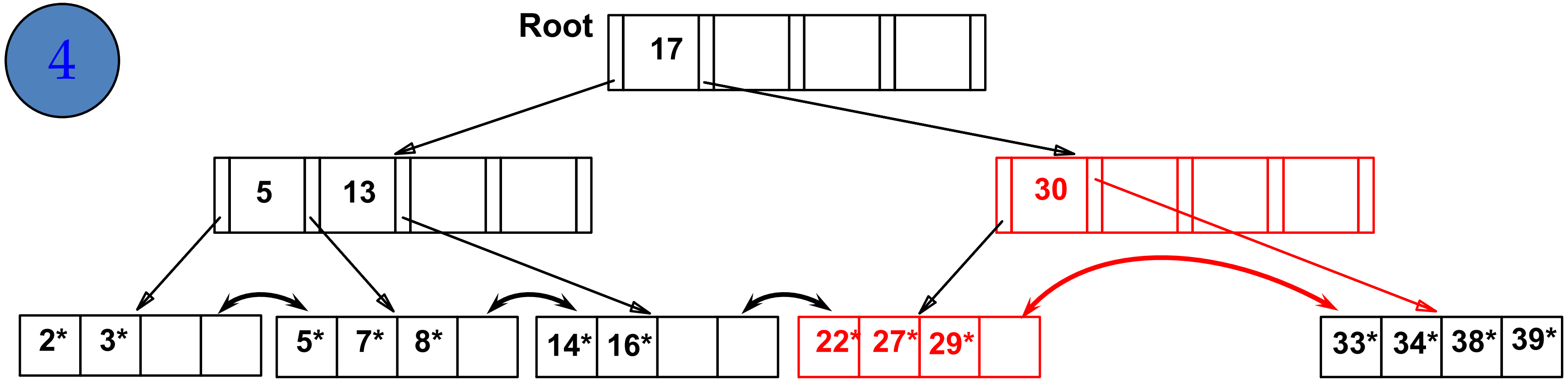




Deleting 20* is done with re-distribution. Notice how middle key is *copied up*.

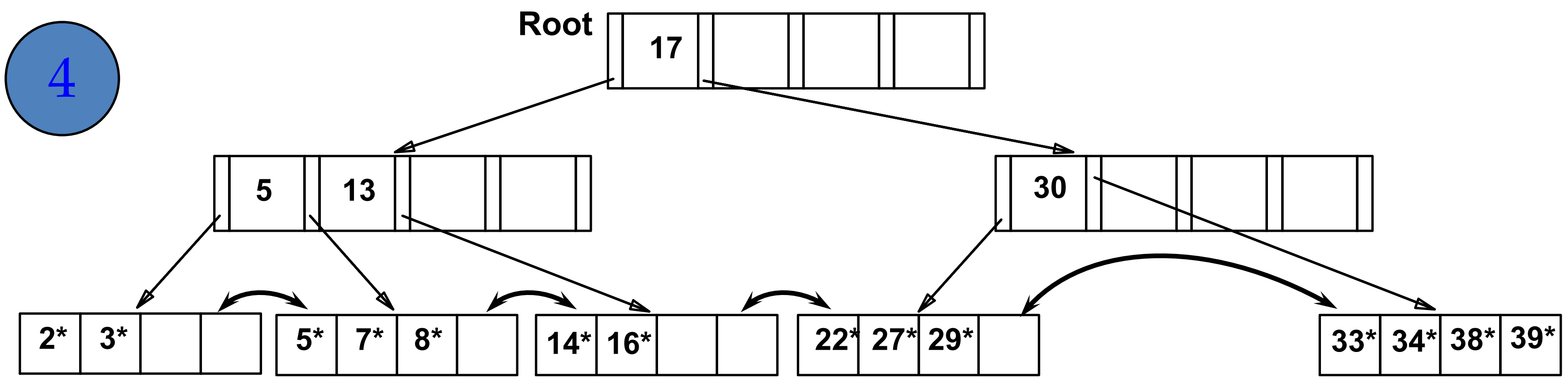
... and then deleting 24*

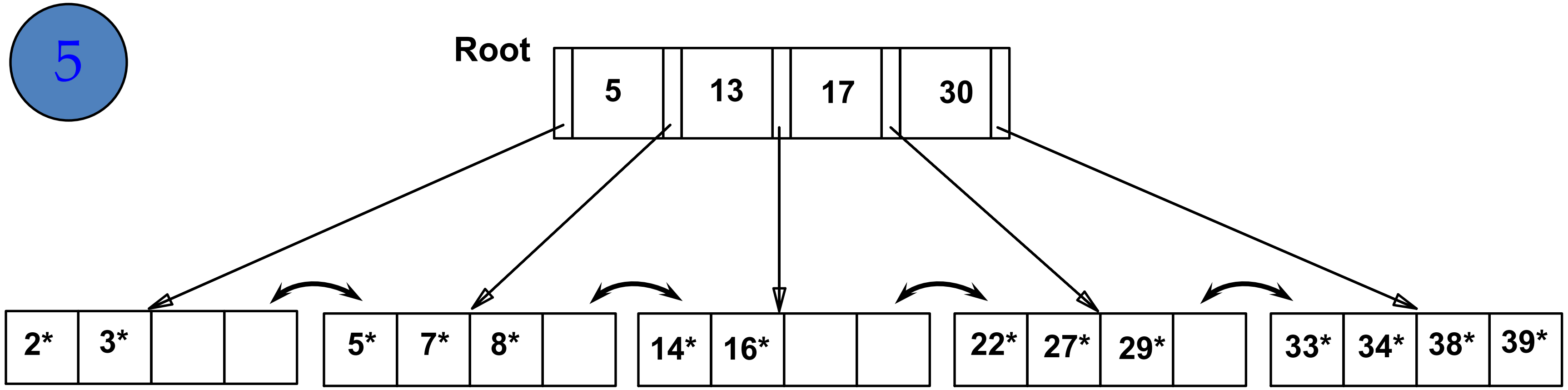




Must merge leaves ... **but are we done??**

... merge non-leaf nodes, shrink tree



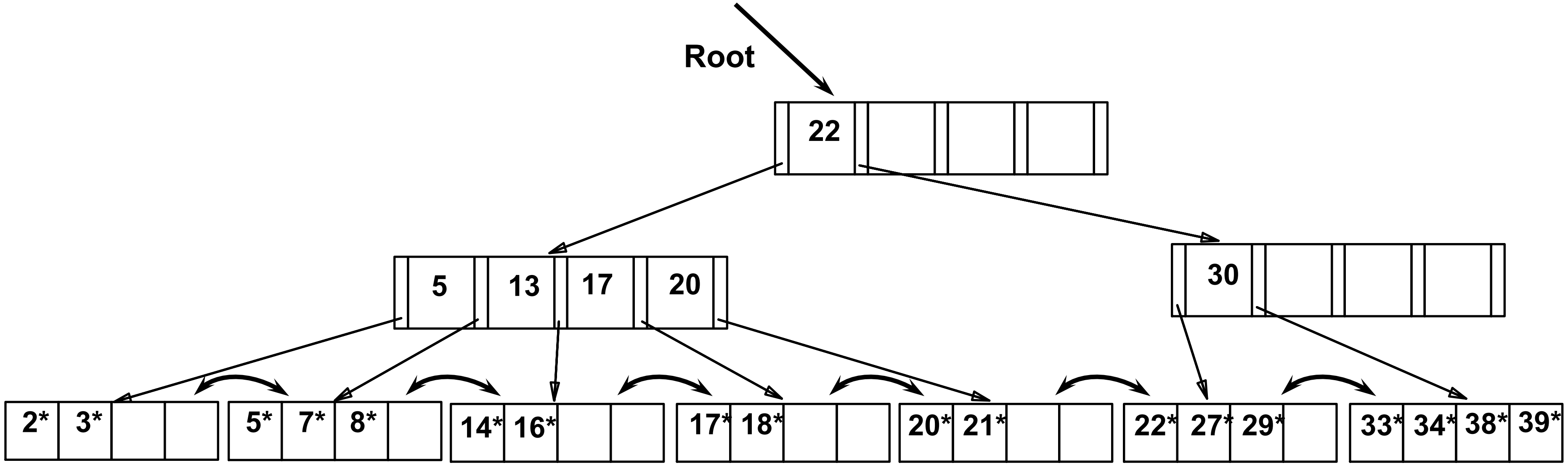


Example of non-leaf re-distribution

Tree is shown below *during deletion* of 24*.

What could be a possible initial tree? 

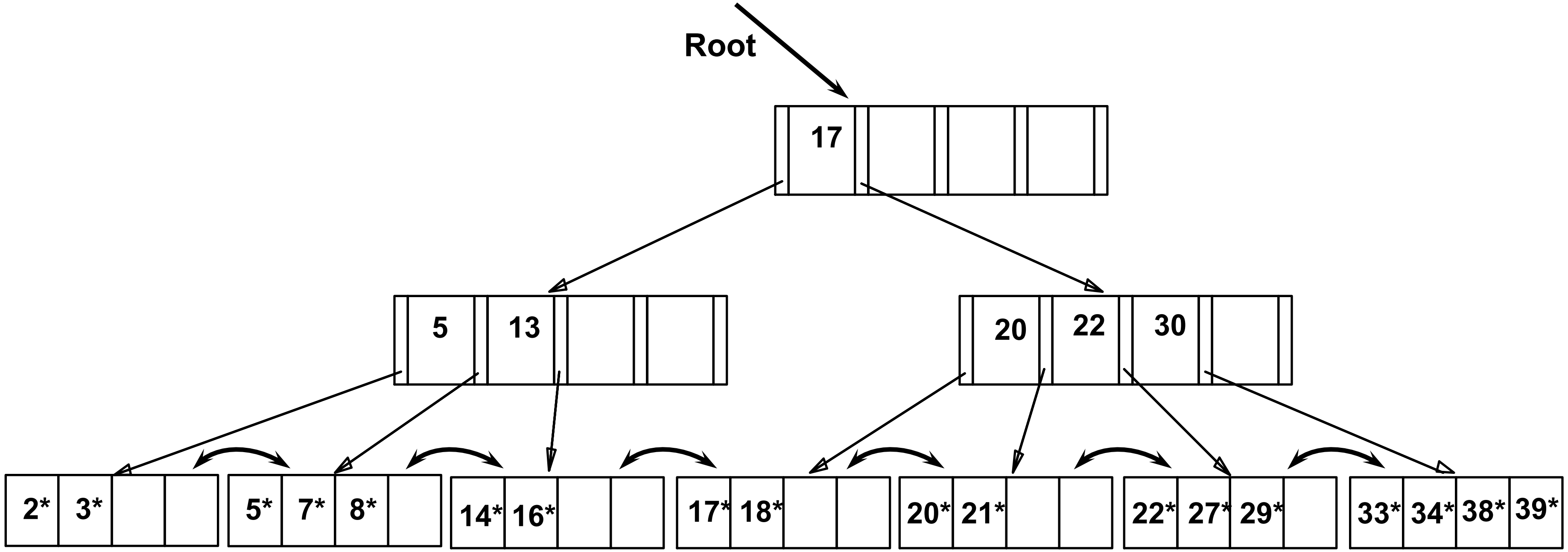
In contrast to previous example, can re-distribute entry from left child of root to right child.



After Re-distribution

Intuitively, entries are re-distributed by *“pushing through”* the splitting entry in the parent node.

it suffices to re-distribute index entry with key 20; we havere-distributed 17 as well for illustration

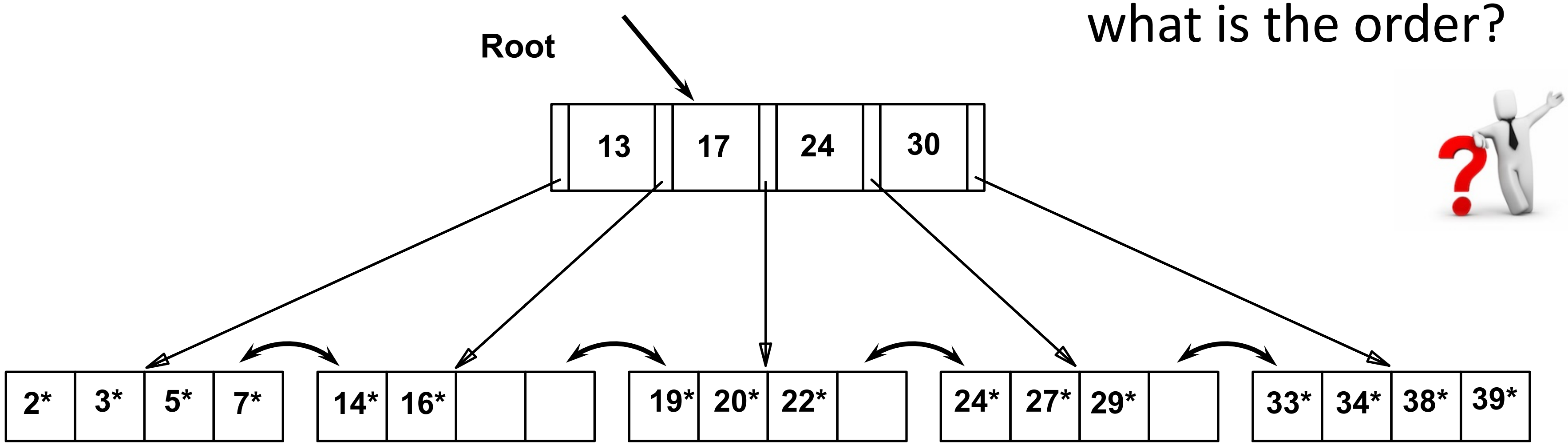


Reminders

begin at root, compare keys to reach the leaf

“order” *d* means d to 2*d elements

what is the order?



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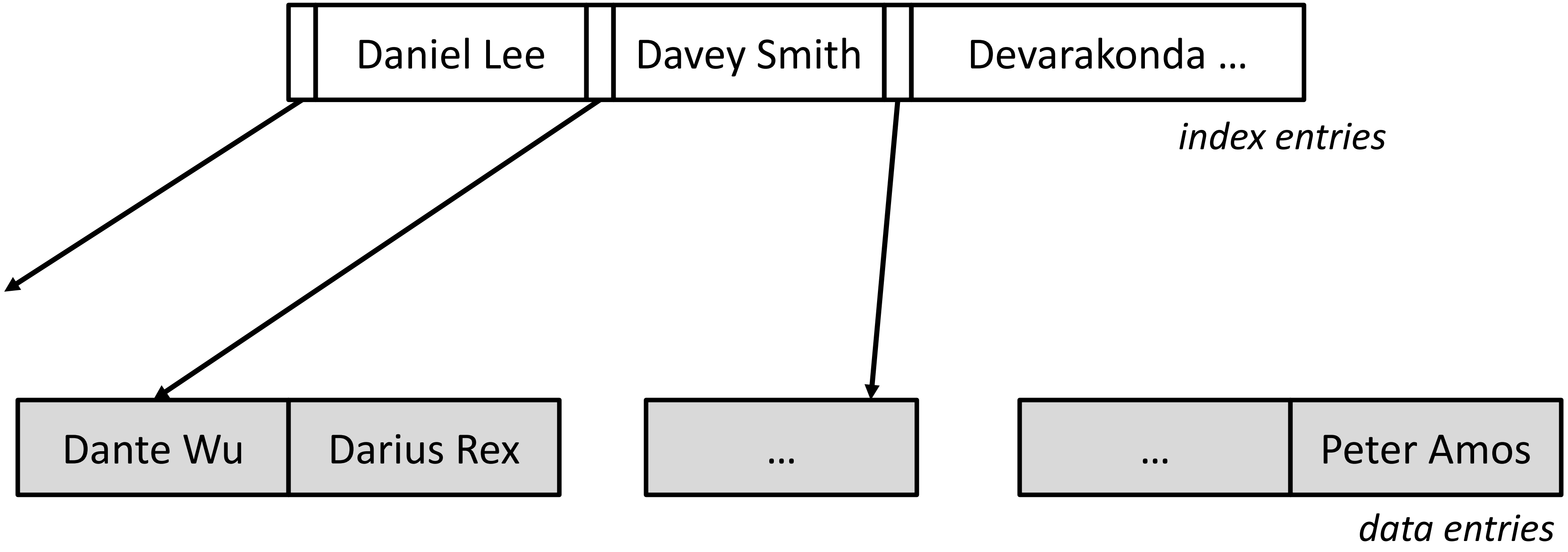
Prefix Key Compression

we want to increase fan-out

why?

key values in index entries (internal nodes) are used to "direct traffic"



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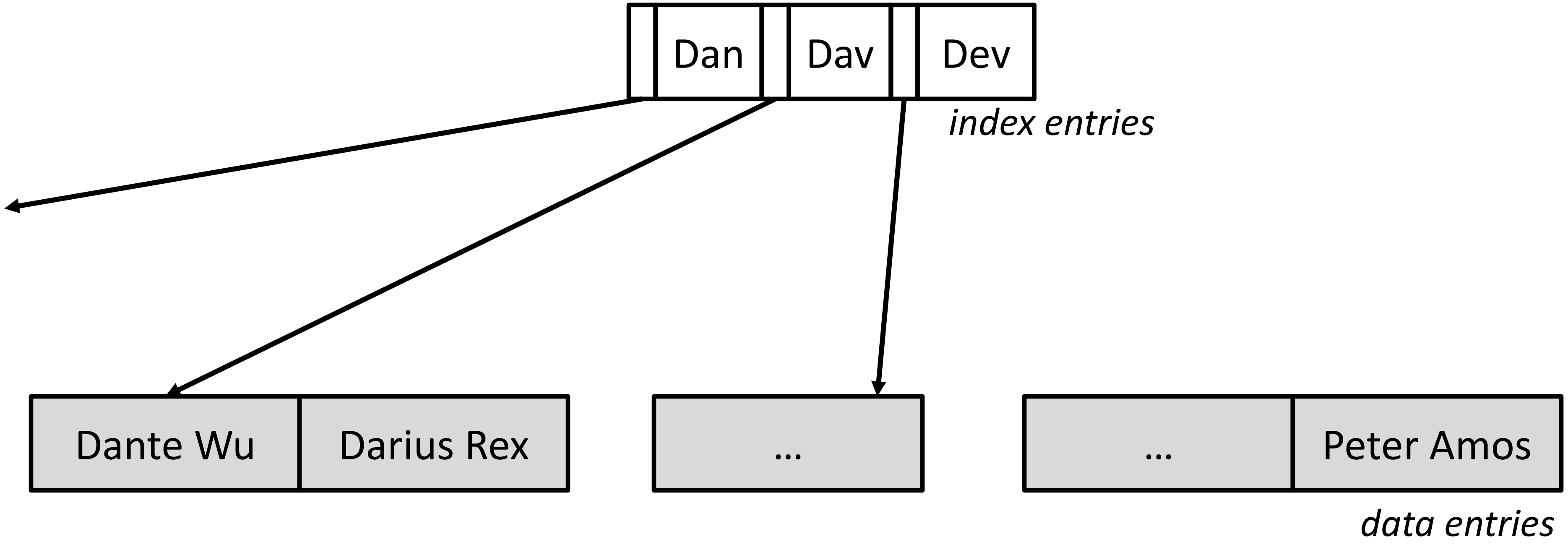

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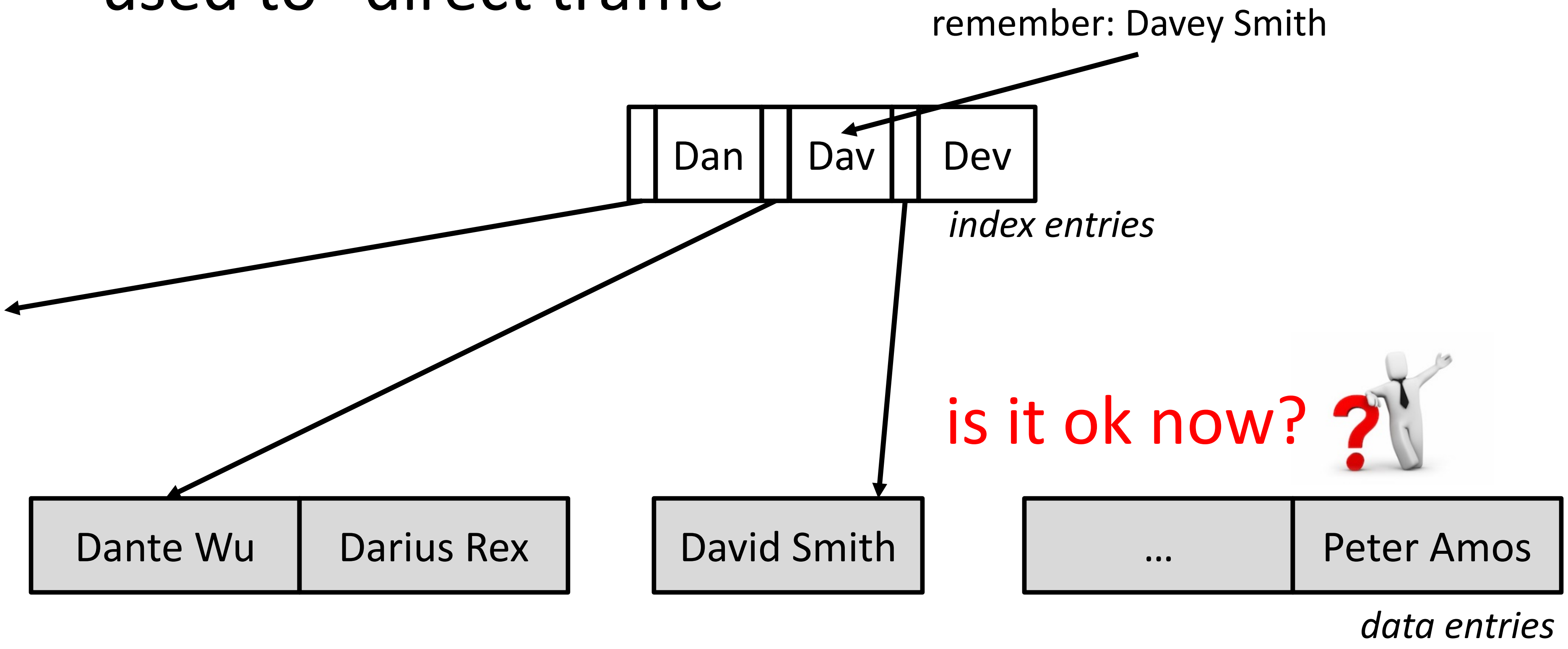

is it ok now?

Prefix Key Compression

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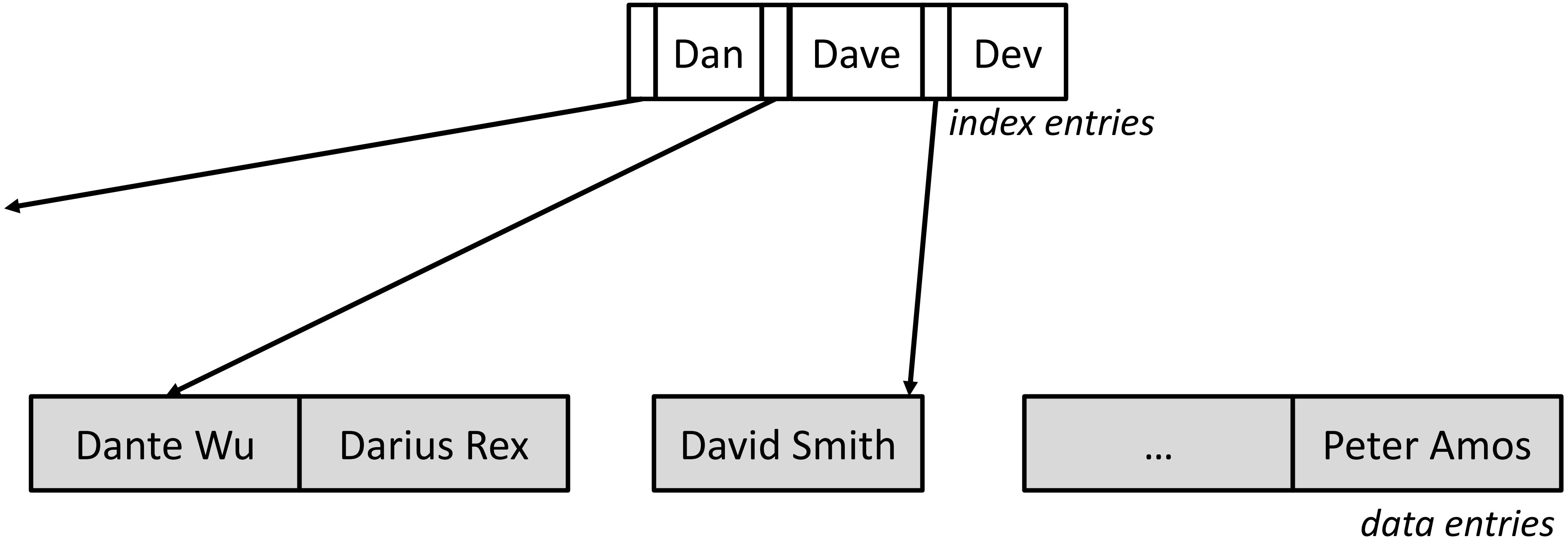


Prefix Key Compression

we want to increase fan-out

keys in index entries (internal nodes) are used to “direct traffic”

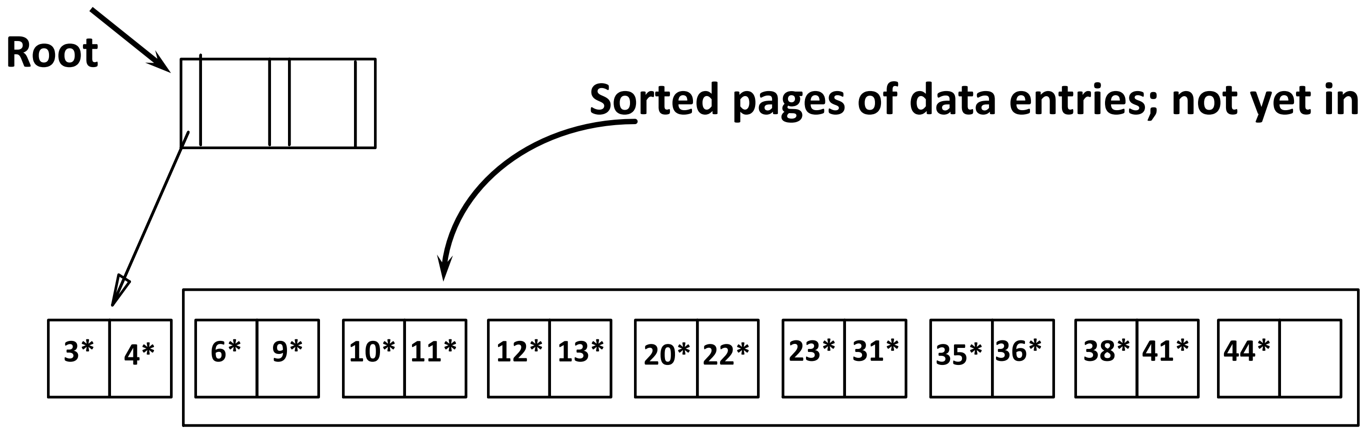
insert/delete must be suitably modified

Bulk Loading of a B+ Tree

If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.

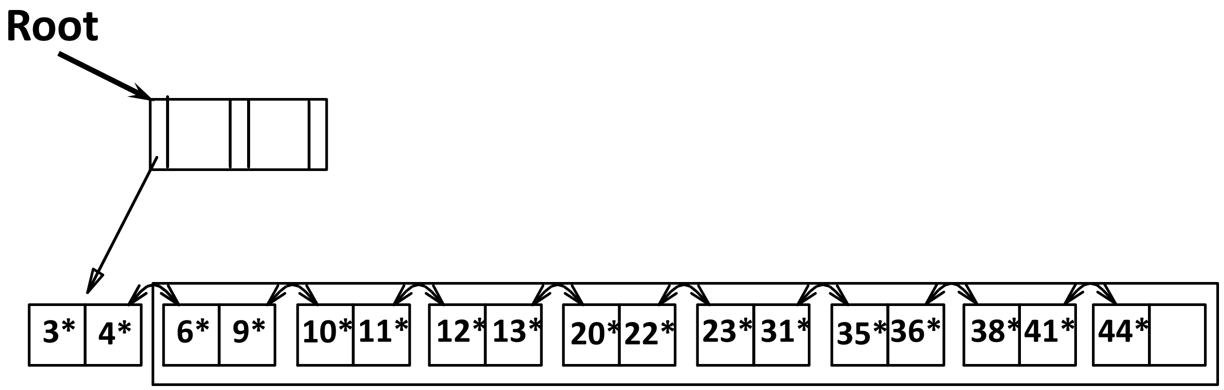
Bulk Loading can be done much more efficiently.

Initialization: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.



Bulk Loading (Contd.)

where to insert: into right-most index page just above leaf level
what to insert: the left-most value of the new leaf



what to do when full? when this fills up, splits node
(if needed split may go up right-most path to the root)

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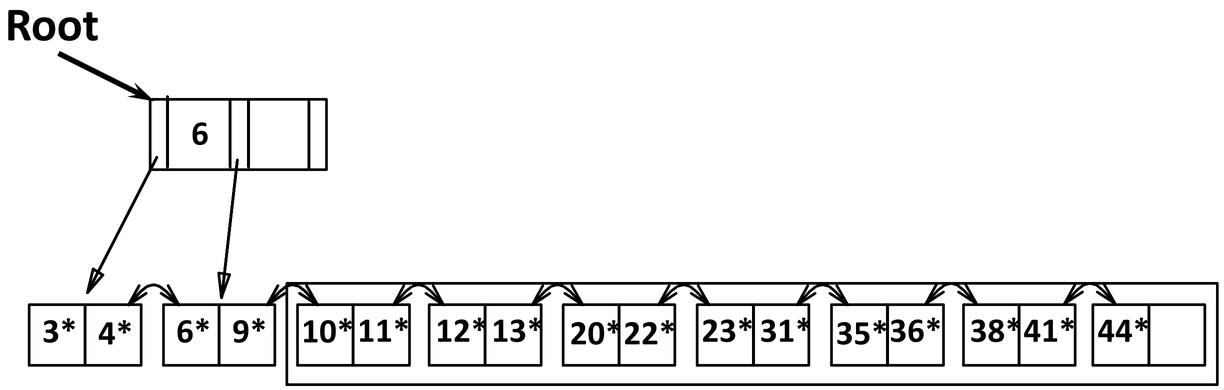

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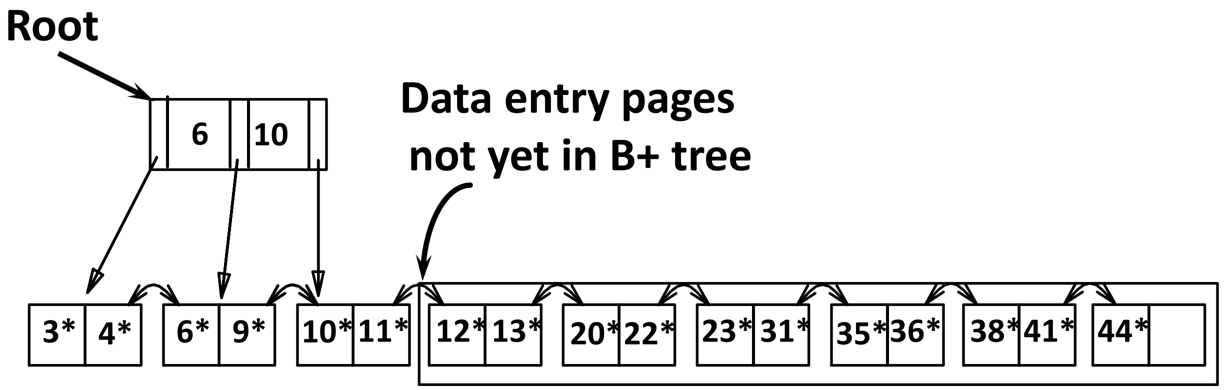

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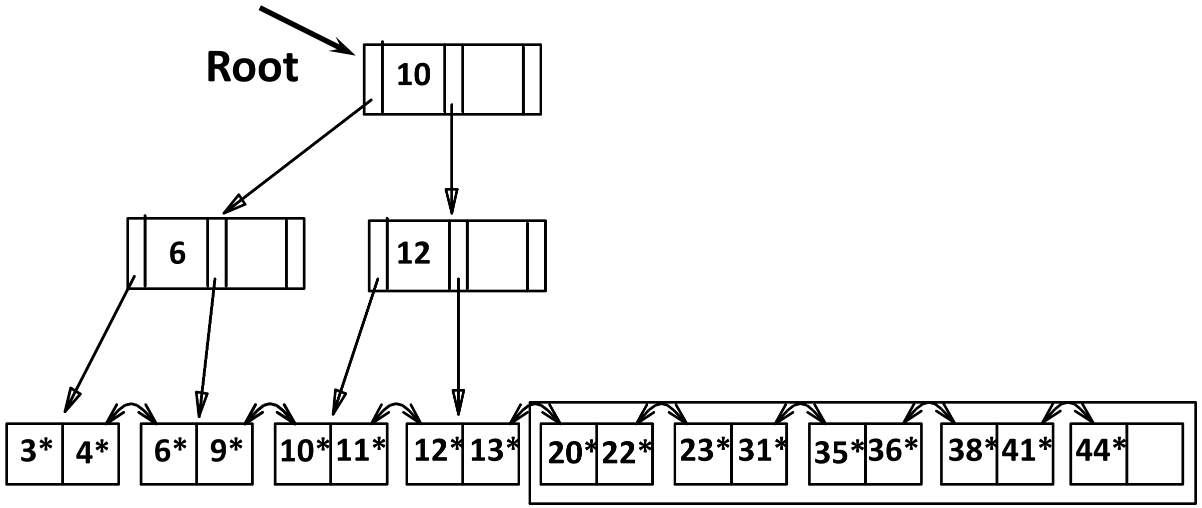

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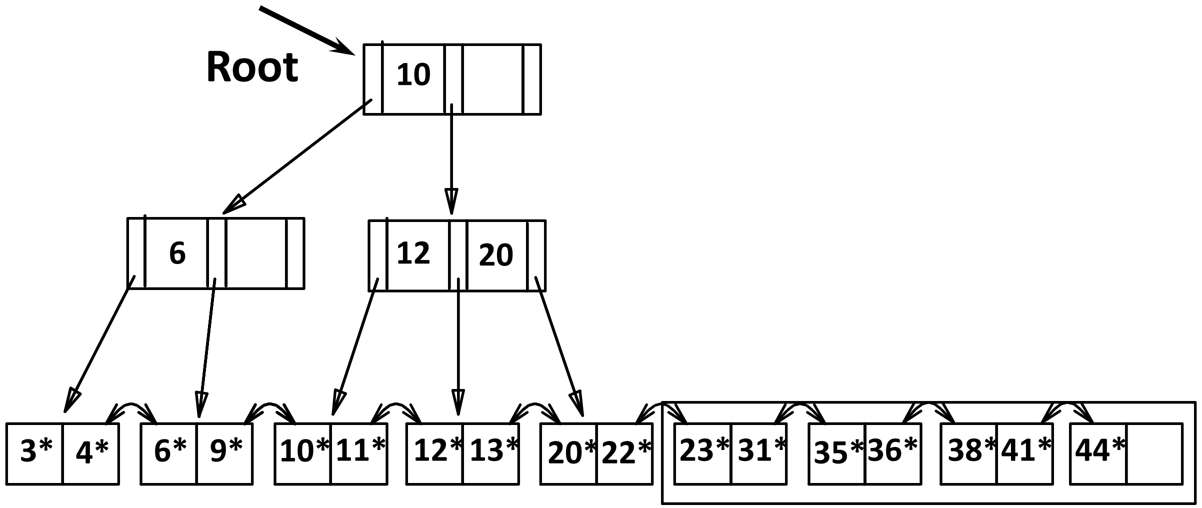

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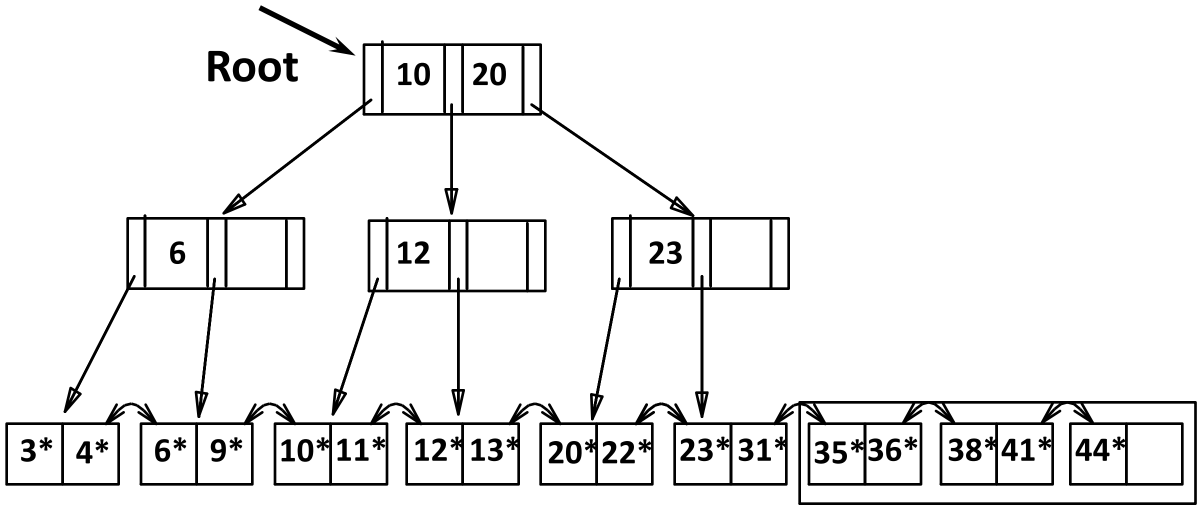

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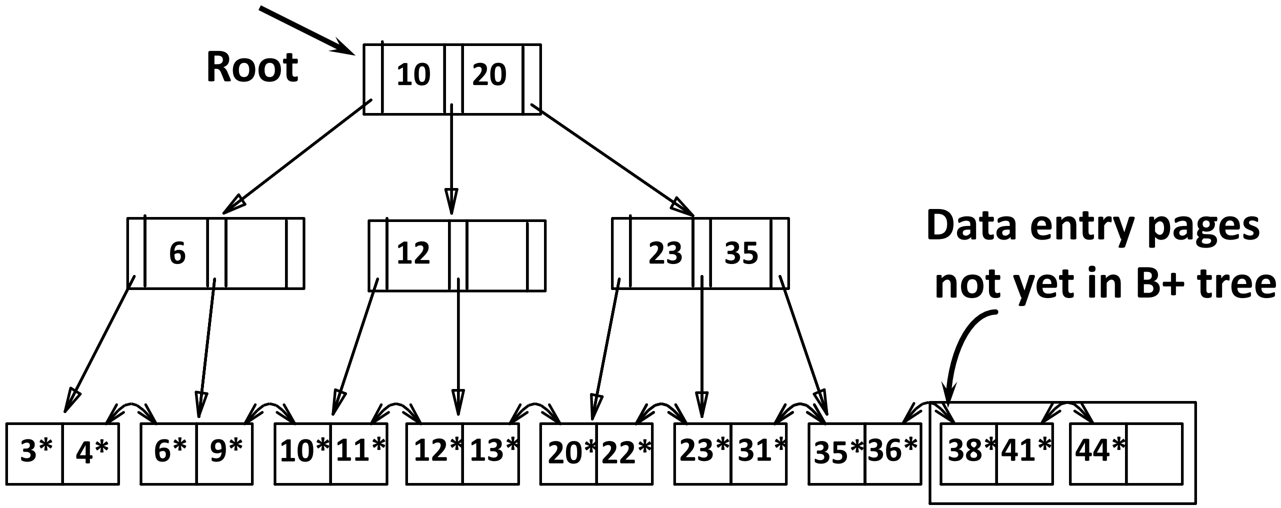

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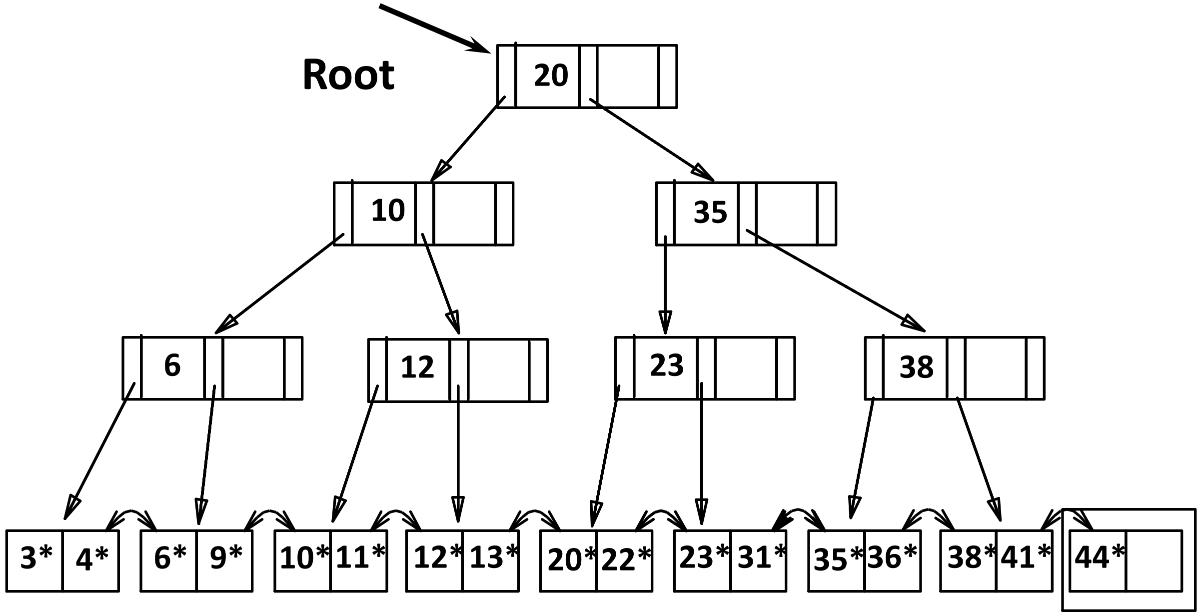

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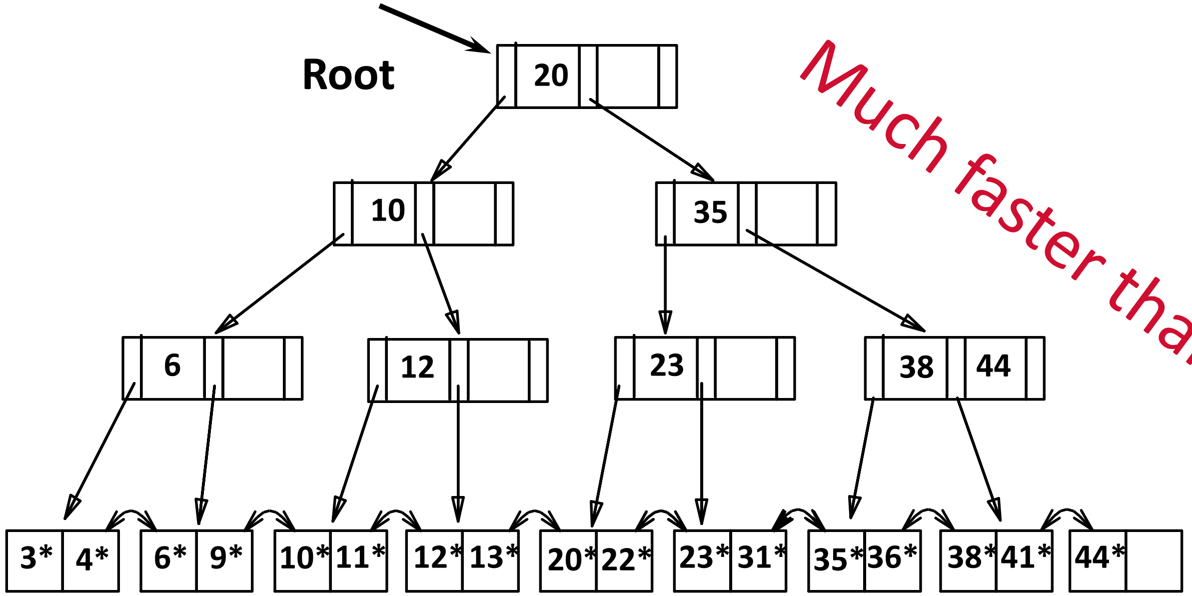

Much faster than inserts!!!

what to do when full? when this fills up, splits node
(if needed split may go up right-most path to the root)

Summary of Loading Options

Option 1: multiple inserts.

- Slow.
- Does not give sequential storage of leaves.

Option 2: *Bulk Loading*

- Fewer I/Os during build.
- Leaves will be stored sequentially (and linked, of course).
- Can control “fill factor” on pages.

A Note on "Order"

Order (d) concept replaced by physical space criterion in practice ("*at least half-full*").

- Index pages can typically hold many more entries than leaf pages.
- Variable sized records and search keys mean different nodes will contain different numbers of entries.
- Even with fixed length fields, multiple records with the same search key value (*duplicates*) can lead to variable-sized data entries (if we use Alternative (3)).

Many real systems are even sloppier than this --- only reclaim space when a page is *completely* empty.

Summary

Tree-structured indexes are ideal for range-searches, also good for equality searches.

B+ tree is a dynamic structure.

- Inserts/deletes leave tree height-balanced; $log_F(N)$ cost.
- High fanout (F) means depth rarely more than 3 or 4.
- Almost always better than maintaining a sorted file.
- Typically, 67% occupancy on average.
- If data entries are data records, splits can change rids!

B+ Trees

"It could be said that the world's information is at our fingertips because of B-trees"

Goetz Graefe

Google (prev. Microsoft, HP Fellow)

ACM Software System Award