Übungen zur Vorlesung Numerische Mathematik, WS 2014/15
Blatt 02 zum 27.10.2014

von	Janina Geiser	Mat Nr. 6420269
	Michael Hufschmidt	Mat.Nr. 6436122
	Farina Ohm	Mat Nr. 6314051
	Annika Seidel	Mat Nr. 6420536

Inhalt...

Aufgabe 3

a)

b)

c)

Aufgabe 4

Gegeben:

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Nach Vorlesung gilt: $cond_{||.||}(A) = ||A|| * ||A^{-1}||$

Wir berechnen also A^{-1} mithilfe des Gauss-Jordan-Verfahren und erhalten:

$$(A|I) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & |1 & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & |0 & 1 \end{pmatrix} \Rightarrow (I|A^{-1}) = \begin{pmatrix} 1 & 0 & |\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 1 & |\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

Daraus folgt:
Spaltensummennorm:

$$||A||_1 = max\{|\frac{1}{\sqrt{2}}| + |-\frac{1}{\sqrt{2}}|, |\frac{1}{\sqrt{2}}| + |\frac{1}{\sqrt{2}}|\} = \frac{2}{\sqrt{2}}$$
$$||A^{-1}||_1 = max\{|\frac{\sqrt{2}}{2}| + |\frac{\sqrt{2}}{2}|, |-\frac{\sqrt{2}}{2}| + |\frac{\sqrt{2}}{2}|\} = \sqrt{2}$$
$$cond_{||.||_1}(A) = ||A|_1 * ||A^{-1}||_1 = \frac{2}{\sqrt{2}} * \sqrt{2} = 2$$

euklidische Norm:

$$||A||_2 = \sqrt{|\frac{1}{\sqrt{2}}|^2 + |-\frac{1}{\sqrt{2}}|^2 + |\frac{1}{\sqrt{2}}|^2 + |\frac{1}{\sqrt{2}}|^2} = \sqrt{2}$$
$$||A^{-1}||_2 = \sqrt{|\frac{\sqrt{2}}{2}|^2 + |\frac{\sqrt{2}}{2}|^2 + |-\frac{\sqrt{2}}{2}|^2 + |\frac{\sqrt{2}}{2}|^2} = \sqrt{2}$$
$$cond_{||.||_2}(A) = ||A|_2 * ||A^{-1}||_2 = \sqrt{2} * \sqrt{2} = 2$$

Zeilensummennorm:

$$||A||_\infty = max\{|\frac{1}{\sqrt{2}}| + |\frac{1}{\sqrt{2}}|, |-\frac{1}{\sqrt{2}}| + |\frac{1}{\sqrt{2}}|\} = \frac{2}{\sqrt{2}}$$
$$||A^{-1}||_\infty = max\{|\frac{\sqrt{2}}{2}| + |-\frac{\sqrt{2}}{2}|, |\frac{\sqrt{2}}{2}| + |\frac{\sqrt{2}}{2}|\} = \sqrt{2}$$
$$cond_{||.||_\infty}(A) = ||A|_\infty * ||A^{-1}||_\infty = \frac{2}{\sqrt{2}} * \sqrt{2} = 2$$

$$B = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Aufgabe 5

Aufgabe 6