Übungen zur Vorlesung Numerische Mathematik, WS 2014/15
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Inhalt...

Aufgabe 3

a)

b)

c)

Aufgabe 4

Gegeben:

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Nach Vorlesung gilt: $\boxed{\mathbf{cond}_{||.||}(\mathbf{A}) = ||\mathbf{A}|| * ||\mathbf{A^{-1}}||}$

Wir berechnen also A^{-1} mithilfe des Gauss-Jordan-Verfahren und erhalten:

$$(A|I) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & |1 & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & |0 & 1 \end{pmatrix} \Rightarrow (I|A^{-1}) = \begin{pmatrix} 1 & 0 & |\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 1 & |\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

Daraus folgt:
Spaltensummennorm:

$$||A||_1 = max\{|\frac{1}{\sqrt{2}}| + |-\frac{1}{\sqrt{2}}|, |\frac{1}{\sqrt{2}}| + |\frac{1}{\sqrt{2}}|\} = \frac{2}{\sqrt{2}}$$
$$||A^{-1}||_1 = max\{|\frac{\sqrt{2}}{2}| + |\frac{\sqrt{2}}{2}|, |-\frac{\sqrt{2}}{2}| + |\frac{\sqrt{2}}{2}|\} = \sqrt{2}$$
$$cond_{||.||_1}(A) = ||A|_1 * ||A^{-1}||_1 = \frac{2}{\sqrt{2}} * \sqrt{2} = 2$$

euklidische Norm:

$$||A||_2 = \sqrt{|\frac{1}{\sqrt{2}}|^2 + |-\frac{1}{\sqrt{2}}|^2 + |\frac{1}{\sqrt{2}}|^2 + |\frac{1}{\sqrt{2}}|^2} = \sqrt{2}$$
$$||A^{-1}||_2 = \sqrt{|\frac{\sqrt{2}}{2}|^2 + |\frac{\sqrt{2}}{2}|^2 + |-\frac{\sqrt{2}}{2}|^2 + |\frac{\sqrt{2}}{2}|^2} = \sqrt{2}$$
$$cond_{||.||_2}(A) = ||A|_2 * ||A^{-1}||_2 = \sqrt{2} * \sqrt{2} = 2$$

Zeilensummennorm:

$$||A||_\infty = max\{|\frac{1}{\sqrt{2}}| + |\frac{1}{\sqrt{2}}|, |-\frac{1}{\sqrt{2}}| + |\frac{1}{\sqrt{2}}|\} = \frac{2}{\sqrt{2}}$$
$$||A^{-1}||_\infty = max\{|\frac{\sqrt{2}}{2}| + |-\frac{\sqrt{2}}{2}|, |\frac{\sqrt{2}}{2}| + |\frac{\sqrt{2}}{2}|\} = \sqrt{2}$$
$$cond_{||.||_\infty}(A) = ||A|_\infty * ||A^{-1}||_\infty = \frac{2}{\sqrt{2}} * \sqrt{2} = 2$$

Gegeben:

$$B = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Analog zu oben berechnen wir B^{-1}:

$$(B|I) = \begin{pmatrix} 1 & 2 & 0 & |1 & 0 & 0 \\ 0 & 1 & 0 & |0 & 1 & 0 \\ 0 & 1 & 1 & |0 & 0 & 1 \end{pmatrix} \Rightarrow (I|B^{-1}) = \begin{pmatrix} 1 & 0 & 0 & |1 & -2 & 0 \\ 0 & 1 & 0 & |0 & 1 & 0 \\ 0 & 0 & 1 & |0 & -1 & 1 \end{pmatrix}$$

Daraus folgt:
Spaltensummennorm:

$$||B||_1 = max\{|1| + |0| + |0|, |2| + |1| + |1|, |0| + |0| + |1|\} = 4$$
$$||B^{-1}||_1 = max\{|1| + |0| + |0|, |-2| + |1| + |-1|, |0| + |0| + |1|\} = 4$$
$$cond_{||.||_1}(B) = ||B|_1 * ||B^{-1}||_1 = 4 * 4 = 16$$

euklidische Norm:

$$||B||_2 = \sqrt{|1|^2 + |0|^2 + |0|^2 + |2|^2 + |1|^2 + |1|^2 + |0|^2 + |0|^2 + |1|^2} = \sqrt{8}$$
$$||B^{-1}||_2 = \sqrt{|1|^2 + |0|^2 + |0|^2 + |-2|^2 + |1|^2 + |-1|^2 + |0|^2 + |0|^2 + |1|^2} = \sqrt{8}$$
$$cond_{||.||_2}(B) = ||B|_2 * ||B^{-1}||_2 = \sqrt{8} * \sqrt{8} = 8$$

Zeilensummennorm:

$$||B||_\infty = max\{|1| + |2| + |0|, |0| + |1| + |0|, |0| + |1| + |1|\} = 3$$
$$||B^{-1}||_\infty = max\{|1| + |-2| + |0|, |0| + |1| + |0|, |0| + |-1| + |1|\} = 3$$
$$cond_{||.||_\infty}(B) = ||B|_\infty * ||B^{-1}||_\infty = 3 * 3 = 9$$

Aufgabe 5

Aufgabe 6