

Übungen zur Vorlesung Numerische Mathematik, WS 2014/15
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Inhalt...

Aufgabe 3

a)

b)

c)

Aufgabe 4

Gegeben:

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Nach Vorlesung gilt: $\boxed{\text{cond}_{\|\cdot\|}(A) = \|A\| * \|A^{-1}\|}$

Wir berechnen also A^{-1} mithilfe des Gauss-Jordan-Verfahren und erhalten:

$$(A|I) = \left(\begin{array}{cc|cc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 1 \end{array} \right) \Rightarrow (I|A^{-1}) = \left(\begin{array}{cc|cc} 1 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{array} \right)$$

Daraus folgt:

Spaltensummennorm:

$$\begin{aligned} \|A\|_1 &= \max \left\{ \left| \frac{1}{\sqrt{2}} \right| + \left| -\frac{1}{\sqrt{2}} \right|, \left| \frac{1}{\sqrt{2}} \right| + \left| \frac{1}{\sqrt{2}} \right| \right\} = \frac{2}{\sqrt{2}} \\ \|A^{-1}\|_1 &= \max \left\{ \left| \frac{\sqrt{2}}{2} \right| + \left| \frac{\sqrt{2}}{2} \right|, \left| -\frac{\sqrt{2}}{2} \right| + \left| \frac{\sqrt{2}}{2} \right| \right\} = \sqrt{2} \\ \text{cond}_{\|\cdot\|_1}(A) &= \|A\|_1 * \|A^{-1}\|_1 = \frac{2}{\sqrt{2}} * \sqrt{2} = 2 \end{aligned}$$

euklidische Norm:

$$\begin{aligned} \|A\|_2 &= \sqrt{\left| \frac{1}{\sqrt{2}} \right|^2 + \left| -\frac{1}{\sqrt{2}} \right|^2 + \left| \frac{1}{\sqrt{2}} \right|^2 + \left| \frac{1}{\sqrt{2}} \right|^2} = \sqrt{2} \\ \|A^{-1}\|_2 &= \sqrt{\left| \frac{\sqrt{2}}{2} \right|^2 + \left| \frac{\sqrt{2}}{2} \right|^2 + \left| -\frac{\sqrt{2}}{2} \right|^2 + \left| \frac{\sqrt{2}}{2} \right|^2} = \sqrt{2} \\ \text{cond}_{\|\cdot\|_2}(A) &= \|A\|_2 * \|A^{-1}\|_2 = \sqrt{2} * \sqrt{2} = 2 \end{aligned}$$

Zeilensummennorm:

$$\begin{aligned} \|A\|_\infty &= \max \left\{ \left| \frac{1}{\sqrt{2}} \right| + \left| \frac{1}{\sqrt{2}} \right|, \left| -\frac{1}{\sqrt{2}} \right| + \left| \frac{1}{\sqrt{2}} \right| \right\} = \frac{2}{\sqrt{2}} \\ \|A^{-1}\|_\infty &= \max \left\{ \left| \frac{\sqrt{2}}{2} \right| + \left| -\frac{\sqrt{2}}{2} \right|, \left| \frac{\sqrt{2}}{2} \right| + \left| \frac{\sqrt{2}}{2} \right| \right\} = \sqrt{2} \\ \text{cond}_{\|\cdot\|_\infty}(A) &= \|A\|_\infty * \|A^{-1}\|_\infty = \frac{2}{\sqrt{2}} * \sqrt{2} = 2 \end{aligned}$$

Gegeben:

$$B = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Analog zu oben berechnen wir B^{-1} :

$$(B|I) = \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \Rightarrow (I|B^{-1}) = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right)$$

Daraus folgt:

Spaltensummennorm:

$$\|B\|_1 = \max \{|1| + |0| + |0|, |2| + |1| + |1|, |0| + |0| + |1|\} = 4$$

$$\|B^{-1}\|_1 = \max \{|1| + |0| + |0|, |-2| + |1| + |-1|, |0| + |0| + |1|\} = 4$$

$$\text{cond}_{\|\cdot\|_1}(B) = \|B\|_1 * \|B^{-1}\|_1 = 4 * 4 = 16$$

euklidische Norm:

$$\|B\|_2 = \sqrt{|1|^2 + |0|^2 + |0|^2 + |2|^2 + |1|^2 + |1|^2 + |0|^2 + |0|^2 + |1|^2} = \sqrt{8}$$

$$\|B^{-1}\|_2 = \sqrt{|1|^2 + |0|^2 + |0|^2 + |-2|^2 + |1|^2 + |-1|^2 + |0|^2 + |0|^2 + |1|^2} = \sqrt{8}$$

$$\text{cond}_{\|\cdot\|_2}(B) = \|B\|_2 * \|B^{-1}\|_2 = \sqrt{8} * \sqrt{8} = 8$$

Zeilensummennorm:

$$\|B\|_\infty = \max \{|1| + |2| + |0|, |0| + |1| + |0|, |0| + |1| + |1|\} = 3$$

$$\|B^{-1}\|_\infty = \max \{|1| + |-2| + |0|, |0| + |1| + |0|, |0| + |-1| + |1|\} = 3$$

$$\text{cond}_{\|\cdot\|_\infty}(B) = \|B\|_\infty * \|B^{-1}\|_\infty = 3 * 3 = 9$$

Aufgabe 5

Aufgabe 6