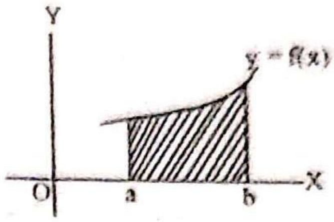
Unit 15

Area of Plane Regions

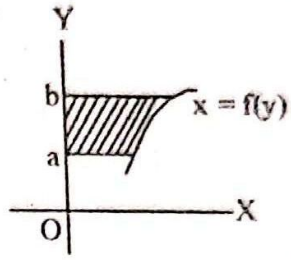
Some important formula

The area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ is given by

$$\int_a^b f(x)\,dx$$



The area bounded by the curve $x = f(y)$, the y-axis and the abscissas at $y = a$ and $y = b$ is given by $\int_a^b x\,dy$



The area bounded by two curves $y = f(x)$ and $y = \phi(x)$ and the two ordinates $x = a$ and $x = b$ is given by

$$\int_a^b [f(x) - \phi(x)]\,dx$$

Exercise - 15

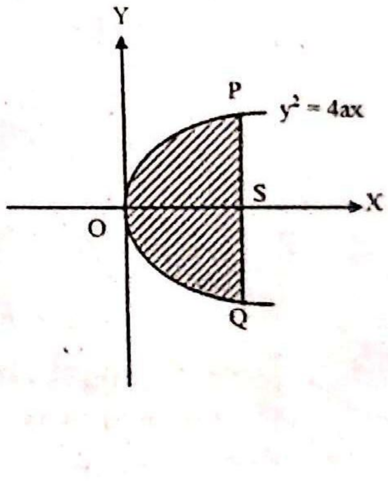
Find the following areas

Bounded by the parabola $y^2 = 4ax$ and its latus rectum. 1

Soln: Let S (a, 0) be a focus and O be the vertex of parabola $y^2 = 4ax$.

Let PQ be the latus rectum and its equation is $x = a$.

Required area = 2 × area of the portion OSP



$$= 2\int_0^a y\,dx$$

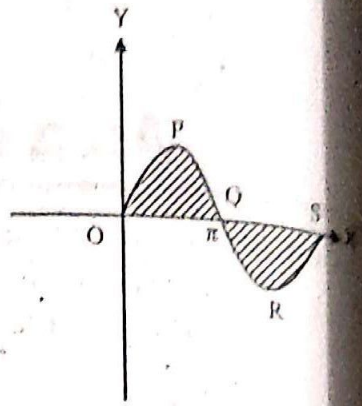
$$= 2\int_0^a \sqrt{4ax}\,dx$$

$$= 2.2\sqrt{a}\int_0^a x^{1/2}\,dx = 4\sqrt{a}\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^a$$

$$= 4\sqrt{a}\,.\,\frac{2}{3}[a^{3/2} - 0] = \frac{8}{3}\sqrt{a}\,.\,a^{3/2} = \frac{8}{3}a^2 \text{ Ans.}$$

(b) **bounded by the curve $y = \sin x$ and the axis between $x = 0$ to $x = 2\pi$.**

Soln: Required area $= 2\int_0^{\pi} y\,dx$



$$= 2\int_0^{\pi} \sin x\,dx$$

$$= 2\left[-\cos x\right]_0^{\pi}$$

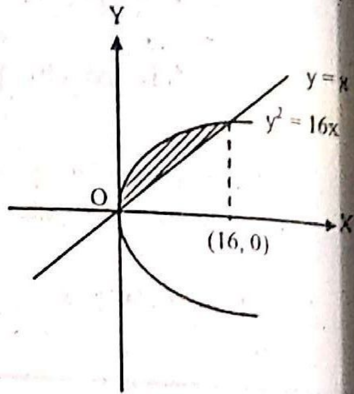
$$= 2\left[-\cos \pi + \cos 0\right]$$

$$= 2(1+1) = 4 \text{ Ans.}$$

(c) **enclosed by the parabola $y^2 = 16x$ and the line $y = x$.**

Soln: Solving $y^2 = 16x$ and $y = x$, we get

$x = 0, y = 0$ and $x = 16, y = 16$



The required area $= \int_0^{16} (y_1 - y_2)\,dx$

$$= \int_0^{16} \left(\sqrt{16x} - x\right)dx$$

$$= 4\int_0^{16} x^{1/2}\,dx - \int_0^{16} x\,dx$$

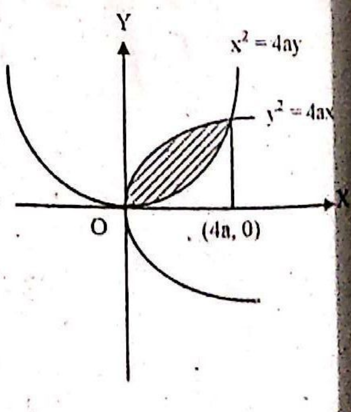
$$= 4\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^{16} - \left[\frac{x^2}{2}\right]_0^{16}$$

$$= \frac{4\times 2}{3}\left[(16)^{3/2} - 0\right] - \frac{1}{2}\left[16^2 - 0\right]$$

$$= \frac{8}{3}.4^3 - \frac{1}{2}256 = \frac{512}{3} - 128$$

$$= \frac{128}{3} \text{ Ans.}$$

(d) **enclosed by the parabolas**
$y^2 = 4ax$ and $x^2 = 4ay$.



Soln: Given two equation of parabolas are

$y^2 = 4ax$ .... (i)

and $x^2 = 4ay$ .... (ii)

solving (i) and (ii), we get

$x = 0, y = 0$ and $x = 4a, y = 4a$

Therefore, the point of intersection of the two parabolas are $(0, 0)$ and $(4a, 4a)$

$\therefore$ the area bounded by the parabolas is the area between the curves

The loop of the curve $ay^2 = x^2(a-x)$ is $\frac{8a^2}{15}$

Soln: The given curve $ay^2 = x^2(a-x)$ is symmetrical about x–axis and passes through (0, 0) and (a, 0)

The loop is formed between $x = 0$ and $x = a$.

If A be the area of loop, then

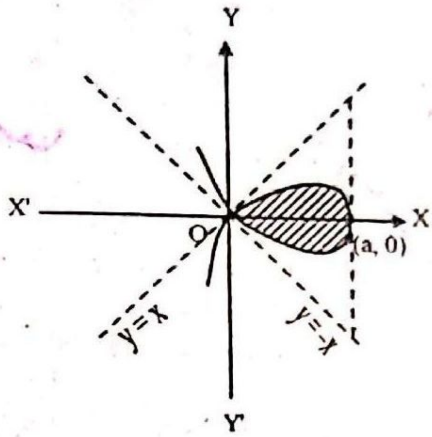
A = 2 area formed above x-axis.

$$\text{or, } A = 2\int_0^a y\,dx = 2\int_0^a \frac{x}{\sqrt{a}}\sqrt{a-x}\,dx \quad [\because ay^2 = x^2(a-x)]$$

Put $a - x = t^2$

$\therefore$ $-dx = 2t\,dt$

When $x = 0, t = \sqrt{a}$ and when $x = a, t = 0$



$\therefore$ Required area

$$A = -\frac{2}{\sqrt{a}}\int_{\sqrt{a}}^{0}(a-t^2).\,t.\,2t\,dt$$

$$= -\frac{4}{\sqrt{a}}\int_{\sqrt{a}}^{0}(at^2 - t^4)\,dt$$

$$= -\frac{4}{\sqrt{a}}\left[\frac{at^3}{3} - \frac{t^5}{5}\right]_{\sqrt{a}}^{0}$$

$$= -\frac{4}{\sqrt{a}}\left[-\frac{a(\sqrt{a})^3}{3} + \frac{(\sqrt{a})^5}{5}\right]$$

$$= -\frac{4}{\sqrt{a}}\left[\frac{-a^2\sqrt{a}}{3} + \frac{a^2\sqrt{a}}{5}\right] = -\frac{4}{\sqrt{a}}\left[\frac{-5a^2\sqrt{a} + 3a^2\sqrt{a}}{15}\right]$$

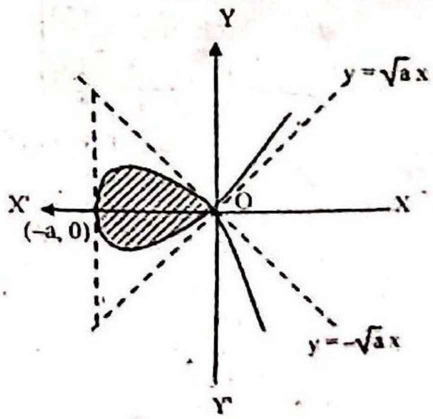
$$= \frac{4}{\sqrt{a}}\cdot\frac{2a^2\sqrt{a}}{15} = \frac{8a^2}{15} \text{ Ans.}$$

1.(a) **Find the area of a loop of the curve $y^2 = x^2(a+x)$** [B.E. 2013]

Soln: The given curve $y^2 = x^2(a+x)$ is symmetrical about x–axis and passes through the points (–a, 0) and (0, 0). $y = \pm\sqrt{a}\,x$ are equations of tangent at (0, 0). The loop is formed between –a to 0.

If A be the area of the loop, then

A = 2 × the area formed above x–axis



$$= 2\int_{-a}^{0} y\,dx$$

$$= 2\int_{-a}^{0} x\sqrt{x+a}\,dx$$

Put $x + a = t^2$ $\therefore$ $dx = 2t\,dt$

when $x = -a$, $t = 0$ and when $x = 0$, $t = \sqrt{a}$

$$\therefore \quad A = 2\int_0^{\sqrt{a}} (t^2 - a)\,.\,t\,2t\,dt$$

$$= 4\int_0^{\sqrt{a}} (t^4 - at^2)\,dt$$

$$= 4\left[\frac{t^5}{5} - \frac{at^3}{3}\right]_0^{\sqrt{a}}$$

$$= 4\left[\frac{(\sqrt{a})^5}{5} - \frac{a(\sqrt{a})^3}{3}\right] = 4\left[\frac{3a^2\sqrt{a} - 5a^2\sqrt{a}}{15}\right]$$

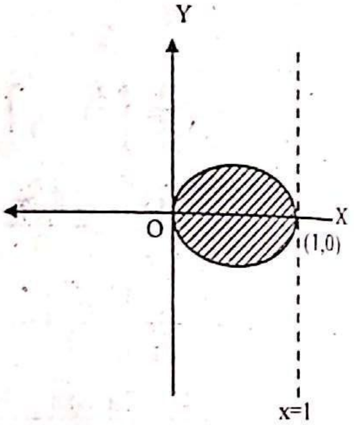
$$= \frac{-8}{15}a^2\sqrt{a} = \frac{8}{15}a^{5/2} \text{ (Numerically) Ans.}$$

(b) $y^2 = x(x-1)^2$ **[B.E. 2013]**

Soln: The given curve $y^2 = x(x-1)^2$ is symmetrical about x–axis and passes through (0, 0) and (1, 0). $x = 0$ and $x = 1$ respectively area equation of tangent at (0, 0) and (1, 0) respectively.

It A be the area of the whole loop, then

A $= 2 \times$ area formed above x–axis.



$$= 2\int_0^1 y\,dx = 2\int_0^1 \sqrt{x}\,(x-1)\,dx$$

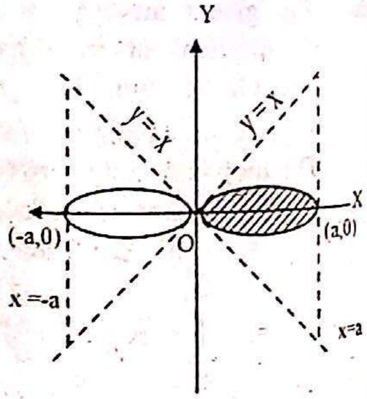
$$= 2\int_0^1 (x^{3/2} - x^{1/2})\,dx$$

$$= 2\left[\frac{x^{5/2}}{5/2} - \frac{x^{3/2}}{3/2}\right]_0^1 = 4\left[\frac{1}{5} - \frac{1}{3} - 0\right]$$

$$= 4\left(\frac{3-5}{15}\right) = \frac{-8}{15} = \frac{8}{15} \text{ (Numerically) Ans.}$$

4. Find the area of a loop of the curve $a^2y^2 = a^2x^2 - x^4$.

Soln: The curve $a^2y^2 = a^2x^2 - x^4$ is symmetrical about both the axis and passes through $(\pm a, 0)$ and (0, 0). Equation of tangent to the curve at origin are $x = \pm y$ and at $(-a, 0)$ and $(a, 0)$ respectively are $x = -a$ and $x = a$.



The area A of a loop is given by

A $= 2 \times$ area above x–axis.

$$= 2\int_0^a y\,dx$$

$$= 2\int_0^a \frac{x\sqrt{a^2 - x^2}}{a}\,dx$$

Put $a^2 - x^2 = t^2$ $\quad\therefore\ -2x\,dx = 2t\,dt$

When $x = 0$, $t = a$ and when $x = a$, $\quad t = 0$

$$\therefore\ A = -2\int_a^0 \frac{t^2}{a}\,dt = -\frac{2}{a}\left[\frac{t^3}{3}\right]_a^0$$

$$= -\frac{2}{3a}[0 - a^3] = \frac{2a^2}{3}\ \text{Ans.}$$

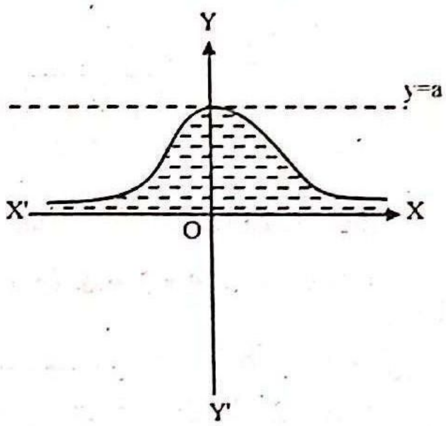
[Note : since there are two loops, the whole area bounded by the curve is 2 . $\frac{2a^2}{3} = \frac{4a^2}{3}$]

Find the area included between the curve $x^2y = a^2(a - y)$ and the x–axis.

Sol: The given curve is symmetrical about y–axis and passes through the point (0, a). The line $y = a$ is the tangent to the curve at (0, a) from the given curve $y = \frac{a^3}{a^2 + x^2}$ and when $x \to \infty$, $y \to 0$. So, $y = 0$ is the asymptote to the curve.

x–will be real only for $y > a$.

The required area A of the curve included between the curve and x–axis is given by



$$A = 2\int_0^a x\,dy$$

$$= 2\int_0^a a\sqrt{\frac{a-y}{y}}\,dy$$

$$= 2a\int_0^a \sqrt{\frac{a-y}{y}}\,dy$$

Put $y = a\sin^2\theta$ $\quad\therefore\ dy = 2a\sin\theta\cos\theta\,d\theta$

When $y = 0$, $\theta = 0$ and when $y = a$, $\theta = \frac{\pi}{2}$.

$$\therefore\ A = 2a\int_0^{\pi/2}\sqrt{\frac{a - a\sin^2\theta}{a\sin^2\theta}}\,2a\sin\theta\cos\theta\,d\theta$$

$$= 4a^2\int_0^{\pi/2}\frac{\cos\theta}{\sin\theta}\cdot\sin\theta\cos\theta\,d\theta$$

$$= 4a^2\int_0^{\pi/2}\cos^2\theta\,d\theta$$

$$= 4a^2\int_0^{\pi/2}\frac{1+\cos 2\theta}{2}\,d\theta$$

$$= 2a^2\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= 2a^2\left[\frac{\pi}{2} + \frac{\sin \pi}{2} - 0\right] = \pi a^2 \text{Ans.}$$

6. Find the area between each of the following curves and its asymptotes:

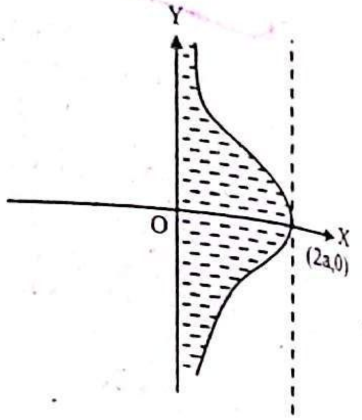
(a) $ay^2 = 4a^2(2a - x)$

Soln: The curve $ay^2 = 4a^2(2a - x)$ is symmetrical about x–axis and passes through the point $(2a, 0)$. For y to be real $0 < x \le 2a$ and $x = 2a$ is the tangent at $(2a, 0)$.

From the curve,

$$y^2 = \frac{4a^2(2a - x)}{x}$$

When $x \to 0$, $y \to \pm\infty$, so $x = 0$ is the asymptote to the given curve.



The required area A between the curve and its asymptote is given by

$$A = 2\int_0^{2a} y\,dx$$

$$= 2\int_0^{2a}\sqrt{\frac{4a^2(2a-x)}{x}}\,dx$$

Put $x = 2a\sin^2\theta$ $\quad\therefore\ dx = 4a\sin\theta\cos\theta\,d\theta$

When $x = 0$, $\theta = 0$ and when $x = 2a$, $\theta = \frac{\pi}{2}$.

$$\therefore A = 2\int_0^{\pi/2} 2a\sqrt{\frac{2a - 2a\sin^2\theta}{2a\sin^2\theta}}\ 4a\sin\theta\cos\theta\,d\theta$$

$$= 16a^2\int_0^{\pi/2}\frac{\cos\theta}{\sin\theta}\sin\theta\cos\theta\,d\theta$$

$$= 16a^2\int_0^{\pi/2}\cos^2\theta\,d\theta = 16a^2\int_0^{\pi/2}\frac{1+\cos 2\theta}{2}\,d\theta$$

$$= \frac{16a^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2} = 8a^2\left[\frac{\pi}{2} + \frac{\sin \pi}{2}\right]$$

$$= 4\pi a^2 \text{ Ans.}$$

(b) $y^2(a - x) = x^3$

Soln: The given curve $y^2(a - x) = x^3$ is symmetrical about x–axis and passes through the point $(0, 0)$.

For y to be real, $0 \le x < 2a$.

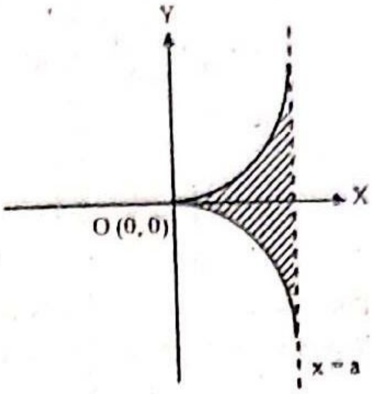
From the curve $y^2 = \frac{x^3}{a - x}$, we have when $x \to a$, $y \to \pm\infty$.

So, $x = a$ is the asymptote to the curve $y = 0$ is the tangent to the curve at $(0, 0)$.

The area A between the curve and its asymptote is given by

$A = 2 \times$ area formed above x–axis

$$= 2\int_0^a y\,dx$$



$$= 2\int_0^a \sqrt{\frac{x^3}{a-x}}\,dx$$

$$= 2\int_0^a x\sqrt{\frac{x}{a-x}}\,dx$$

Put $x = a\sin^2\theta$ $\quad \therefore\ dx = 2a\sin\theta\cos\theta\,d\theta$

When $x = 0$, $\theta = 0$ and when $x = a$, $\theta = \frac{\pi}{2}$.

$$\therefore A = 2\int_0^{\pi/2} a\sin^2\theta\sqrt{\frac{a\sin^2\theta}{a - a\sin^2\theta}}\,.\,2a\sin\theta\cos\theta\,d\theta.$$

$$= 2\,.\,2a^2\int_0^{\pi/2}\sin^2\theta\,.\,\frac{\sin\theta}{\cos\theta}\,.\,\sin\theta\cos\theta\,d\theta$$

$$= 4a^2\int_0^{\pi/2}\sin^4\theta\,.\,d\theta$$

properties

$$= 4a^2\sqrt{\pi}\,\frac{\Gamma\left(\frac{4+1}{2}\right)}{2\Gamma\left(\frac{4+2}{2}\right)} \qquad \left[\because \int_0^{\pi/2}\sin^n x\,dx = \frac{\sqrt{\pi}\,\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{n+2}{2}\right)}\right]$$

$$= 4a^2\frac{.\,\Gamma\left(\frac{5}{2}\right)}{2\Gamma(3)} = 4a^2\frac{\sqrt{\pi}\,.\,\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}}{2\times 2\times 1}$$

$$= \frac{3}{4}\pi a^2 \text{ Ans.}$$

(c) $a^2x^2 = y^2(a^2 - x^2)$

Sol: The given curve is symmetrical about both the axes and passes through the origin. $y = \pm x$ are equations of tangent at the origin.

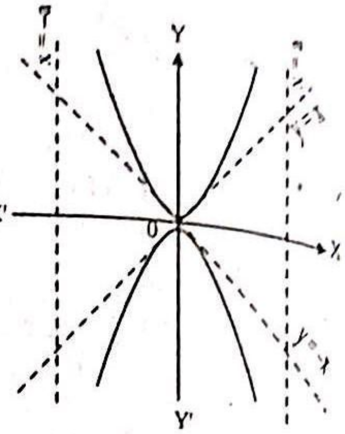
From the curve $y^2 = \frac{a^2x^2}{a^2 - x^2}$

When $x \to \pm a$, $y \to \pm\infty$, so $x = a$ and $x = -a$ are asymptotes to the given curve.

The required area A between the curve and its asymptote is given by

$$A = 4\int_0^a y\,dx$$

$$= 4\int_0^a \sqrt{\frac{a^2x^2}{a^2-x^2}}\,dx$$



$$= 4a\int_0^a \frac{x}{\sqrt{a^2-x^2}}\,dx$$

Put $a^2 - x^2 = t$ $\quad\therefore -2x\,dx = dt$

When $x = 0$, $t = a^2$ and when $x = a$, $t = 0$

$$\therefore \quad A = -4a \cdot \frac{1}{2}\int_{a^2}^{0} \frac{dt}{\sqrt{t}}$$

$$= -2a\int_{a^2}^{0} t^{-/12}\,dt \quad = 2a\int_{a^2}^{0} t^{-1/2}\,dt$$

$$= 2a\left[\frac{t^{1/2}}{\frac{1}{2}}\right]_0^{a^2} \quad = 4a\left[\sqrt{a^2} - 0\right]$$

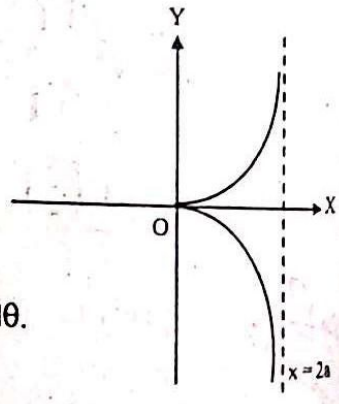
$$= 4a \cdot a \; = 4a^2 \text{ Ans.}$$

(d) $y^2(2a - x) = x^3$

Soln: The given curve is symmetrical about x–axis and passes through origin. $y = 0$ is the equation of tangent at the origin and $x = 2a$ is asymptote to the curve. The curve exists only for $0 \le x < 2a$. The required area between the curve and its asymptote is given by

$$A = 2\int_0^{2a} y\,dx$$

$$= 2\int_0^{2a} \frac{x\sqrt{x}}{\sqrt{2a-x}}\,dx$$



Put $x = a\sin^2\theta$ $\qquad\therefore\; dx = 2a\sin\theta\cos\theta\,d\theta.$

$$\therefore \quad A = 2\int_0^{\pi/2} \frac{2a\sin^2\theta\sqrt{2a\sin^2\theta}}{\sqrt{2a - 2a\sin^2\theta}} \cdot 4a\sin\theta\cos\theta\,d\theta$$

$$= 16a^2\int_0^{\pi/2} \frac{\sin^2\theta \cdot \sin\theta}{\cos\theta} \cdot \sin\theta\cos\theta\,d\theta$$

$$= \frac{2}{2} \int_0^{\pi} a^2 (1 + \cos \theta)^2 \, d\theta$$

$$= a^2 \int_0^{\pi} 4 \cos^4 \frac{\theta}{2} \, d\theta$$

Put $\frac{\theta}{2} = t$, $d\theta = 2dt$

When $\theta = 0$, $t = 0$ and when $\theta = \pi$, $t = \frac{\pi}{2}$

$$\therefore \ A = 4a^2 \int_0^{\pi/2} \cos^4 t \, 2dt$$

$$= 8a^2 \int_0^{\pi/2} \cos^4 t \, dt$$

$$= 8a^2 \frac{\sqrt{\pi}\, \Gamma\left(\frac{4+1}{2}\right)}{2\Gamma\left(\frac{4+2}{2}\right)} = \frac{8a^2 \sqrt{\pi}\, \Gamma\left(\frac{5}{2}\right)}{2\Gamma(3)}$$

$$= \frac{8a^2 \sqrt{\pi} \cdot \frac{3}{2} \cdot \frac{1}{2} \sqrt{\pi}}{2 \times 2 \times 1} = \frac{3}{2} \pi a^2 \text{ Ans.}$$

11. **Find the whole area of the curve $(x^2 + y^2)^2 = a^2 (x^2 - y^2)$**

Soln: The given curve is $(x^2 + y^2)^2 = a^2 (x^2 - y^2)$

To change polar form, put $x = r \cos \theta$ and $y = r \sin \theta$, then the equation becomes

$(r^2 \cos^2\theta + r^2 \sin^2\theta)^2 = a^2 (r^2 \cos^2\theta - r^2 \sin^2\theta)$

or, $r^4 (\cos^2\theta + \sin^2\theta)^2 = a^2 r^2 (\cos^2\theta - \sin^2\theta)$

or, $r^2 = a^2 \cos 2\theta$

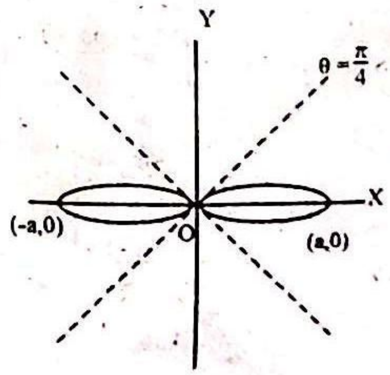
The curve is symmetrical about the initial line

When $\theta = 0$, $r = \pm a$, so it meets

x-axis at $(-a, 0)$ and $(a, 0)$

$r = 0 \Rightarrow a^2 \cos 2\theta = 0 \Rightarrow 0 \cos 2\theta = \cos \frac{\pi}{2}$

$\therefore \ \theta = \frac{\pi}{4}$ is the tangent at pole



$\therefore$ Required area of one loop

$$= 2 \int_0^{\pi/4} \frac{1}{2} r^2 \, d\theta$$

$$= \frac{2}{2}\int_0^{\pi/4} a^2 \cos 2\theta \, d\theta$$

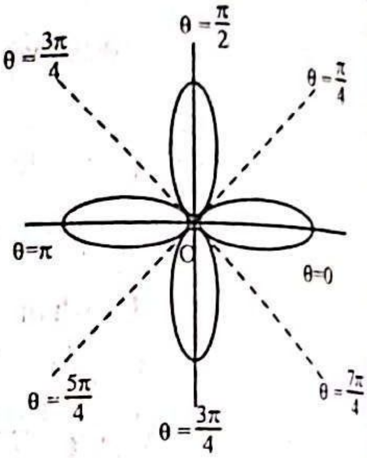
$$= a^2\left[\frac{\sin 2\theta}{2}\right]_0^{\pi/4}$$

$$= \frac{a^2}{2}\left(\sin 2 . \frac{\pi}{4} - \sin 0\right) = \frac{a^2}{2} \text{ Ans}$$

12. Find the area of whole region bounded by the curve

(a) $r = a \cos 2\theta$

Soln: Required area of whole region



$$A = 8\int_0^{\pi/4} \frac{1}{2} r^2 \, d\theta$$

$$= 4\int_0^{\pi/4} a^2 \cos^2 2\theta \, d\theta$$

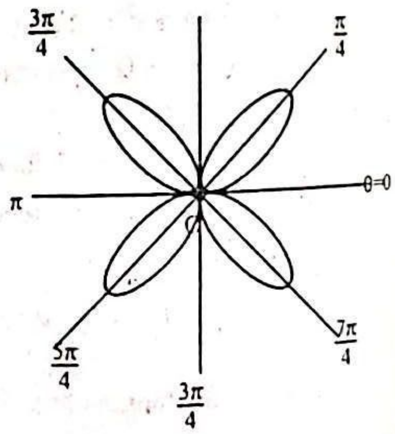
$$= 4a^2 \int_0^{\pi/4} \frac{1 + \cos 4\theta}{2} \, d\theta$$

$$= 2a^2\left[\theta + \frac{\sin 4\theta}{4}\right]_0^{\pi/4}$$

$$= 2a^2\left[\frac{\pi}{4} + \frac{\sin \pi}{4}\right] = \frac{\pi a^2}{2} \text{ Ans.}$$

(b) $r = a \sin 2\theta$

Soln: Whole region bounded by the given curve is



$$= 4\int_0^{\pi/2} \frac{1}{2} r^2 \, d\theta$$

$$= 2\int_0^{\pi/2} a \sin^2 2\theta \, d\theta$$

$$= 2a^2 \int_0^{\pi/2} \frac{1 - \cos 4\theta}{2} \, d\theta$$

$$= a^2\left[q - \frac{\sin 4\theta}{4}\right]_0^{\pi/4}$$

$$= a^2\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4}\right]$$

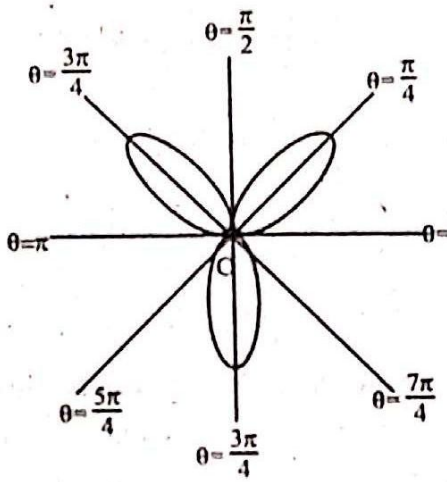
$$= \frac{\pi a^2}{2} \text{ Ans.}$$

$r = a \sin 3\theta$

Soln: The curve consists of three equal loops, one loop is obtained when θ increases from 0 to 60° whole region bounded by the curve is

$$= 3\int_0^{\pi/3} \frac{1}{2} r^2 \, d\theta$$

$$= \frac{3}{2}\int_0^{\pi/3} a^2 \sin^2 3\theta \, d\theta$$



Put $3\theta = t$ $\therefore 3d\theta = dt$

When $\theta = 0$, $t = 0$ and

when $\theta = \frac{\pi}{3}$, $t = \frac{\pi}{2}$

$$\therefore \text{ Required area } = \frac{3a^2}{2}\int_0^{\pi/2} \sin^2 t \, . \frac{dt}{3}$$

$$= \frac{a^2}{2}\int_0^{\pi/2} \frac{1 - \cos 2t}{2} \, dt$$

$$= \frac{a^2}{4}\left[t - \frac{\sin 2t}{2}\right]_0^{\pi/2} = \frac{a^2}{4}\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{\pi a^2}{8} \text{ Ans.}$$

13. **Find the area common to the circle $r = a$ and the cardioid $r = a(1 + \cos\theta)$**

Soln: Given equation are

$r = a$ .... (i)

and $r = a(1 + \cos\theta)$ .... (ii)

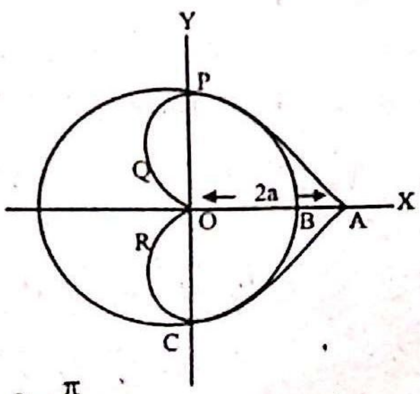
Solving (i) and (ii), we get

$a = a(1 + \cos\theta)$

or, $\cos\theta = \theta$

or, $\cos\theta = \cos\frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{2}$



$\because$ The two curves intersect at P where $\theta = \frac{\pi}{2}$

The area A common to the curves (i) and (ii) is given by

A = area ORCBPQO

= 2 (Area OBP + Ara OPQ)

$$= 2\left[\frac{1}{2}\int_0^{\pi} a^2 d\theta + \frac{1}{2}\int_{\pi/2}^{\pi} a^2 (1 + \cos\theta)^2 d\theta\right]$$

$$=\frac{a^2\pi}{2}+a^2\int_{\pi/2}^{\pi}\left[1+2\cos\theta+\frac{1+\cos 2\theta}{2}\right]d\theta$$

$$=\frac{a^2\pi}{2}+a^2\left[\frac{3}{2}\theta+2\sin\theta+\frac{\sin 2\theta}{4}\right]_{\pi/2}^{\pi}$$

$$=\frac{a^2\pi}{2}+a^2\left[\frac{3\pi}{2}+2\sin\pi+\frac{\sin 2\pi}{4}-\frac{3\pi}{4}-2\sin\frac{\pi}{2}-\frac{\sin\pi}{4}\right]$$

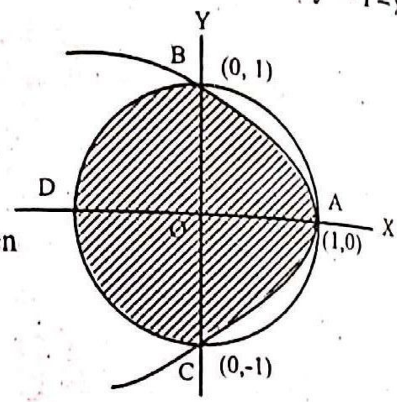
$$=\frac{a^2\pi}{2}+\frac{3a^2\pi}{2}+0+0-\frac{3\pi a^2}{4}-2a^2$$

$$=\frac{5a^2\pi}{4}-2a^2=a^2\left(\frac{5\pi}{4}-2\right)\text{ Ans.}$$

14. Find the area common to the circle. $x^2+y^2=1$ and the parabola $y^2=1-x$.

Soln: The given curves are $x^2+y^2=1$ .... (i)

and $y^2=1-x$ .... (ii)



Solving (i) and (ii),

we get $(0, \pm 1)$ and $(1, 0)$ as the point of intersection of these two curves.

If A be the area common to both curve, then

A = 2 Area OAB + 2 Area OBD

$$=2\int_0^1\sqrt{1-x}\,dx+2\int_0^1\sqrt{1-x^2}\,dx$$

$$=I_1+I_2$$

$$I_1=2\int_0^1\sqrt{(1-x)}\,dx=\left[\frac{2(1-x)^{3/2}}{-\frac{3}{2}}\right]_0^1=-\frac{4}{3}[0-1]=\frac{4}{3}$$

Put $x=\sin\theta$, then $dx=\cos\theta\, d\theta$.

When $x=0$, $\theta=0$ and when $x=1$, $\theta=\frac{\pi}{2}$

$$\therefore I_2=\frac{1}{2}\int_0^{\pi/2}\sqrt{1-\sin^2 q}\,\cos\theta\, d\theta$$

$$=\frac{1}{2}\int_0^{\pi/2}\cos^2\theta\, d\theta=2\int_0^{\pi/2}\frac{1+\cos 2\theta}{2}\,d\theta$$

$$=\left[q+\frac{\sin 2\theta}{2}\right]_0^{\pi/2}=\frac{\pi}{2}$$

$\therefore$ Required area $=\frac{4}{3}+\frac{\pi}{2}$ Ans.