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Q1.

Y (Rpu)	X (year)	X^2	XY
61.2	2004	4016016	122644.8
58.3	2008	4032064	117066.4
67.1	2009	4036081	134803.9
69.2	2010	4040100	139092
68.9	2011	4044121	138557
83.5	2012	4048144	168002
89.1	2013	4052169	179358
80.0	2014	4056196	161120
92.3	2015	4060225	185984.5
93.0	2016	4064256	187488
97.0	2017	4068289	195649
859.6	22,129	44517661	1729765.6

Let the hypothesis be $Y = h_w(X) = w_0 + w_1 X$

using formula $\quad w_0 = \dfrac{BC - AD}{mC - A^2} \qquad\qquad w_1 = \dfrac{AB - mD}{A^2 - mC}$

$A = \sum\limits^{m} x_i = 22129$

$B = \sum\limits^{m} Y_i = 859.6$

$C = \sum\limits^{m} x_i^2 = 44517661$

$D = \sum\limits^{m} x_i Y_i = 1729765.6$

$w_0 = \dfrac{859.6 \times 44517661 - 22129 \times 1729765.1}{44517661 \times 11 - 22129 \times 22129}$

$\quad = -6520.32$

$w_1 = \dfrac{859.6 \times 22129 - 1729765.1 \times 11}{22129 \times 22129 - 44517661 \times 11}$

$\quad = 3.28$

$\therefore \ Y = -6520.32 + 3.28X$

$\quad = -6520.32 + 3.28 \times 2019$

$\quad = 102.$

Error: $J = \frac{1}{2m} \mathcal{E}(h_w(x) - Y)^2$

$$= \frac{1}{2 \times 11} | w_0 + w, Y - x)^2$$

$$= \frac{1}{2 \times 11} \{ |532 - 61.2)^2 + |65.94 - 58.3)^2 + |9.22 - 67.1)^2 + |725 - 68.9)^2$$

$$+ |75.78 - 335)^2 + |7906 - 89.1)^2 + |82.3 - 80)^2 + |85.6 - 92.3)^2$$

$$|89 - 92.3)^2 + (92.18 - 93)^2 + (95.46 - 97)^2$$

$$J = \frac{1}{2 \times 11} | 64 + 58.37 + 4.5 + 12.96 + 22.27 + 100.8 + 5.29 + 44.89$$

$$+ 10.69 + 0.67 + 2.37)$$

$$\boxed{= 14.855}$$

Q2.

X	Y	$X - \bar{X}$	$Y - \bar{Y}$	$(X - \bar{X})^2$	$(Y - \bar{Y})^2$	$(X - \bar{X})(Y - \bar{Y})$
75	82	-4.8	0.1	23.04	0.01	-0.48
80	78	0.2	-3.9	0.04	15.21	0.78
93	86	13.2	4.1	174.24	16.81	54.12
65	72	-14.8	-9.9	219.04	98.01	146.52
87	91	7.2	9.1	51.84	82.81	65.52
71	80	-8.8	-1.9	77.44	3.61	16.72
98	95	18.2	13.1	331.24	171.61	238.42
68	72	-11.8	-9.9	139.24	98.01	116.82
84	89	4.2	7.1	17.64	50.41	29.82
77	74	-2.8	-7.9	7.84	62.41	22.12
				1041.6	598.9	688.8

$\bar{X} = 79.8$

$\bar{Y} = 81.9$

a) $Y = w_0 + w_1 x$

$$w_1 = \frac{\varepsilon(x-\bar{x})(y-\bar{y})}{\varepsilon(y-\bar{x})^2}$$

$$= \frac{688.8}{1041.6} = 0.66$$

$w_0 = \bar{Y} - w_1\bar{x} = 29.12$

$$\boxed{\therefore \ Y = 0.66x + 29.12}$$

b) $X = w_0 + w_1 Y$

$$w_1 = \frac{\varepsilon(x-\bar{x})(y-\bar{y})}{\varepsilon(y-\bar{y})^2}$$

$$= \frac{688.8}{598.8} = 1.150$$

$w_0 = \bar{x} - w_1\bar{y} = -14.38$

$$\boxed{\therefore \ X = 1.15x - 14.38}$$

c) Here we will use eq ⓐ

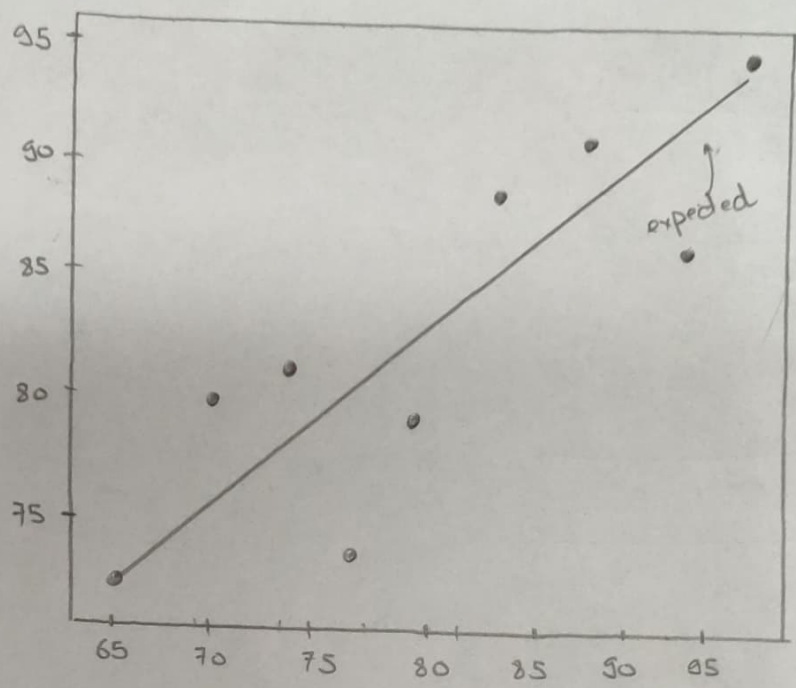$Y = 0.66 \times 96 + 29.12$

$= \underline{\underline{92.48}}$.
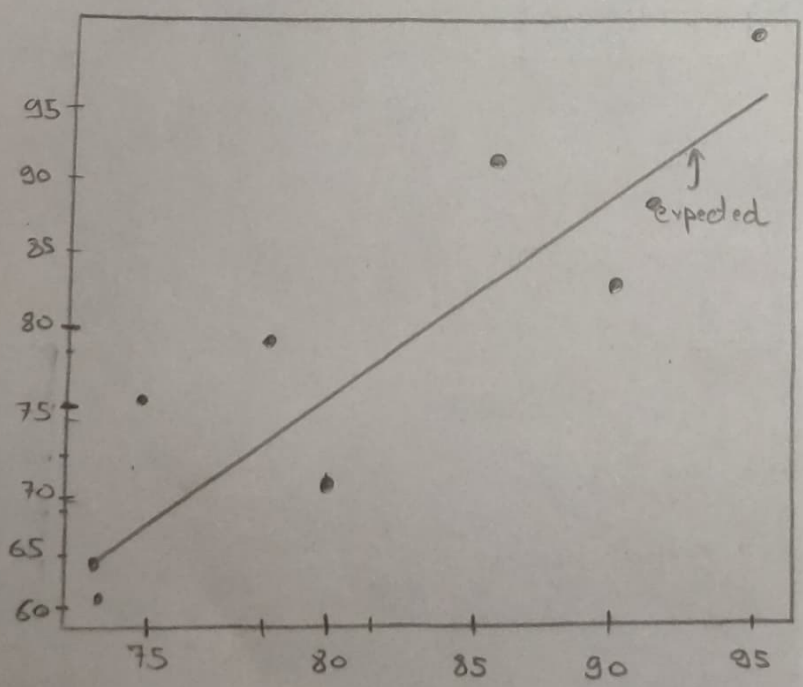
d) Here we will use eq ⓑ

$X = 1.15Y - 14.38$

$= 1.15 \times 95 - 14.38$

$= \underline{94.87}$.

e.

Mean square error = 14.2815

Mean square error = 24.9

Thus 1) 1st line will give more accurate results.
2) Both lines are independent and cannot be derived from other

Q3.

a) $PV^n = c$

$\ln P + n \ln V = \ln c$

$\ln P - n \ln \frac{1}{V} = \ln c$

$\ln P = \ln c + n \ln(1/V)$

$Y = W_0 + W_1 X \qquad \left[\text{let } Y = \ln P \ \& \ X = \ln(1/V) \right]$

$A = \sum_{i=1}^{m} X_i = \sum \ln(1/V) = -26.929$

$B = \sum_{i=1}^{m} Y_i = \sum \ln p = 20.253$

$C = \sum^{m} X_i^2 = \sum \left(\ln \frac{1}{V} \right)^2 = 120.450$

$D = \sum^{m} X_i Y_i = -\sum \ln V \times \ln p = 89.52$

$W_0 = \dfrac{BC - AD}{mC - A^2}$, $\qquad W_1 = \dfrac{AB - mD}{A^2 - mC}$

$W_0 = 9.67 \qquad\qquad\qquad W_1 = 1.40$

$\therefore c = e^{9.67} \qquad\qquad\qquad \therefore n = 1.40.$

$= 15929.3$

(b) As solved in previous question $PV^{1.40} = 15929.3$

(c) $P(100)^{1.40} = 15929.31$

$P = 24.819$

Q4. $Y = w_0 + w_1 x + w_2 x^2$

This linear hypothesis has 3 variable, thus there will be 3 equations.

$$\sum_{}^{m} x_i^2 Y_i = w_0 \sum_{}^{} x_i^2 + w_1 \sum_{}^{m} x_i^3 + w_2 \sum_{}^{m} x_i^4$$

$$\sum_{}^{m} x_i Y_i = w_0 m + w_1 \sum_{}^{m} x_i + w_2 \sum_{}^{m} x_i^2$$

$$\sum_{}^{m} Y_i = w_0 m + w_1 \sum_{}^{7} x_i + w_2 \sum_{}^{} x_i^2$$

X	Y	X^2	X^3	X^4	XY	$X^2 Y$
0	2.4	0	0	0	0	0
1	2.1	1	1	1	2.1	2.1
2	3.2	4	8	16	6.4	12.8
3	5.6	9	27	81	16.8	50.4
4	9.3	16	64	256	37.2	148.8
5	14.6	25	125	625	73	365
6	21.9	36	216	1296	131.4	788.4
21	59.1	91	441	2275	266.9	1367.5

∴ Forming equations.

$$7a + 21b + 91c = 59.1 \quad —①$$
$$91a + 441b + 2275c = 1367.5 \quad —②$$
$$21a + 91b + 441c = 266.9 \quad —③$$

on solving these

$$Y = 2.5095 - 1.2x + 0.733 x^2$$