

LECTURE II

LOGIC PROGRAMMING

PROLOG EXAMPLES

1. PROLOG responds to a PROGRAMMER QUERY

$$\exists x_1 \dots x_n P_1(\) \wedge P_2(\) \wedge \dots \wedge P_m(\) \quad (a)$$

where arities of P_i are n_i , & x_1, \dots, x_n are all free variables in all P_i .

2. PROLOG uses "program.pl" clauses (data clauses) to satisfy a goal (b)
3. A goal can be satisfied immediately
4. If not then the first program clause is invoked which head (modulo substitution - with UA) matches the goal under consideration.
5. If goal cannot be satisfied backtracking is initiated.
6. Backtracking reviews what was done attempting to re-satisfy the goals by finding an alternative way to

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satisfy them.

7. If one is not content with an answer to the QUERY one can resume backtracking - hitting ";"
8. Matching clauses atoms - UA & substitution is used.

RECALL:

- (i) isnumber(zero).
- (ii) isnumber(S(X)) :- isnumber(X)

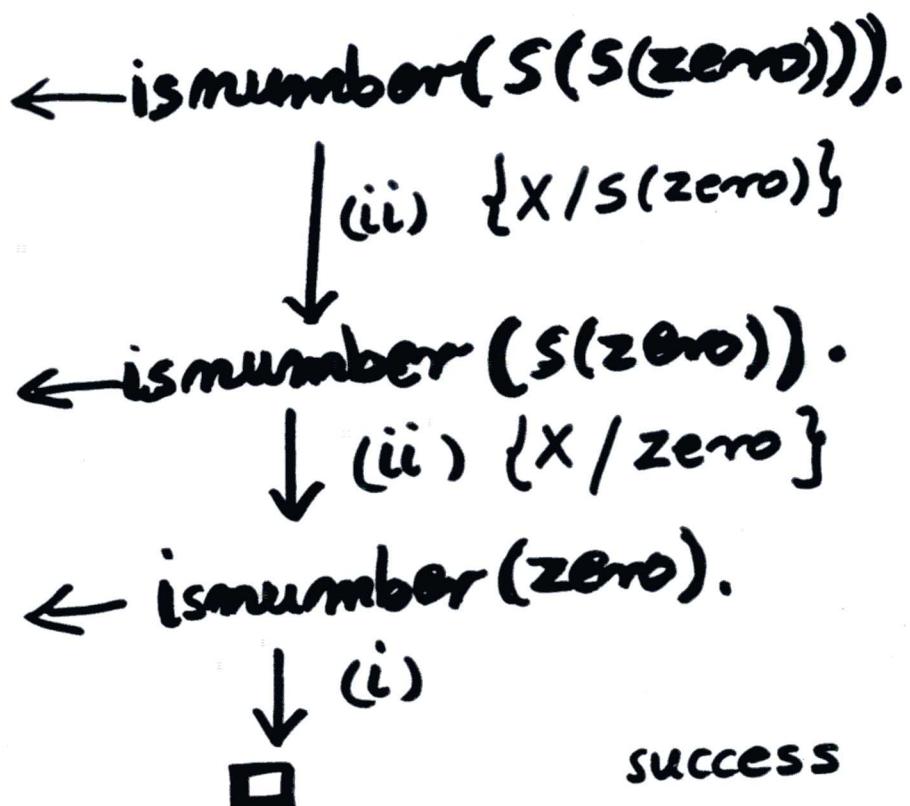
Query:

>> isnumber(zero):
yes

>> isnumber(s(s(zero))). (*)
yes.

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SLD-tree for $\Delta \models \text{QUERY } (*)$ collapses
to single branch (a trunk).

$$Q = \exists x \text{ isnumber}(x)$$

Again

$\gg \text{isnumber}(x).$ $(**)$
 $x = \text{zero};$ success 1

$x = S(\text{zero});$ success 2

$x = S(S(\text{zero}))$ RTN success 3
 yes

Once we find 1st success ";" converts this
to failure and prompts PROLOG to resume
searching SLD-tree

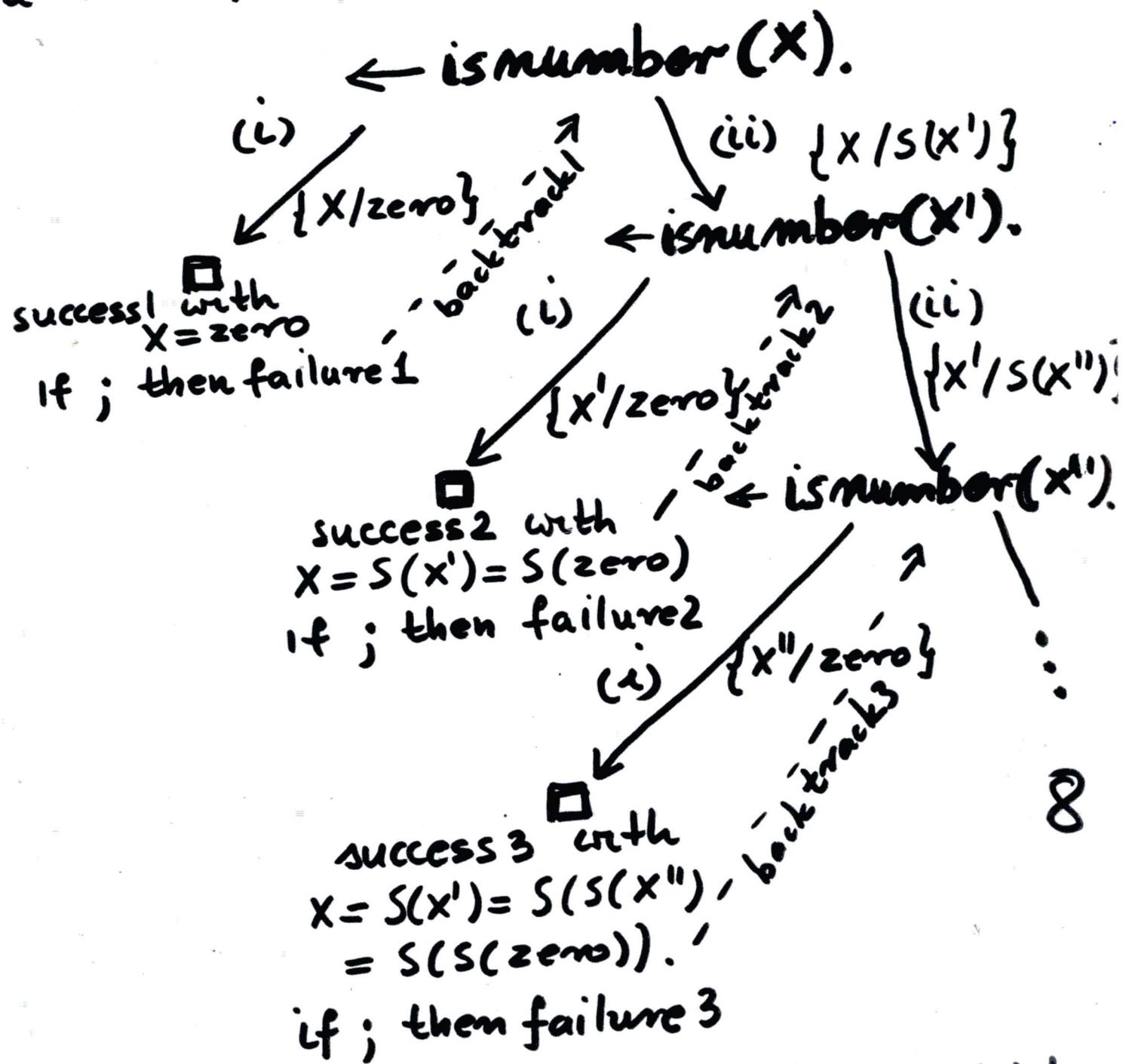
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success 1 becomes failure 1 & PROLOG back-tracks

to the first moment, where there is
a choice.

SLD-tree for $A \Leftarrow (*)$



Note: SLD tree grows from left to right.
The branch on right-hand side is ∞ .

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If we now change the order of clause
in Δ to $\tilde{\Delta}$:

- (j) $\text{isnumber}(S(X)) :- \text{isnumber}(X).$
- (jj) $\text{isnumber}(\text{zero}).$

Of course $\Delta = \tilde{\Delta}.$

$\gg \text{isnumber}(\text{zero}).$

yes

$\gg \text{isnumber}(S(\text{zero})).$

yes

The same responses - as expected
since $\Delta = \tilde{\Delta}.$

But

$\gg \text{isnumber}(X).$ (8)

out of local stack

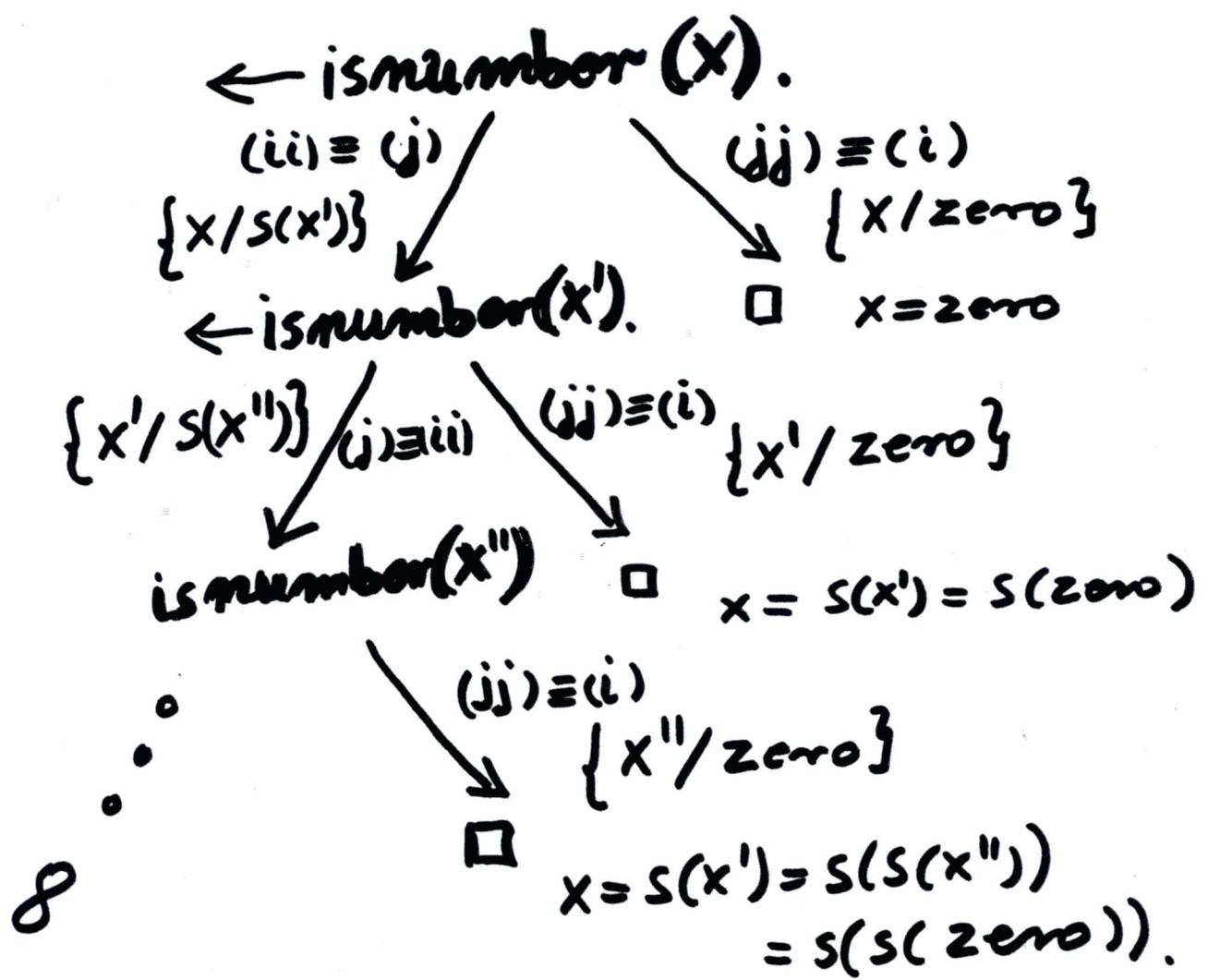
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Order of boundary conditions matters!

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SLD tree for $\tilde{a} \Leftarrow (*)$



The infinite branch is flipped to the left-hand side for program \tilde{a} .

Since PROLOG applies depth-first search

from left to right we first get entangled in ∞ -branch before reaching success leaf.

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Boundary conditions should be put first.

Example 1:

- (i) plus(zero, X, X) :- isnumber(X).
- (ii) plus(s(X), Y, s(Z)) :- plus(X, Y, Z).

$$" 0 + X = X "$$

$$" \text{if } X + Y = Z \Rightarrow (X+1) + Y = Z + 1 "$$

>> plus(s(zero), s(zero), s(s(zero))). (a)

>> plus(s(zero), s(zero), X). Yes (b)

$$" 1 + 1 = ? "$$

$$X = s(s(zero))$$

>> plus(s(zero), X, s(s(zero))).

$$" 1 + X = 2 " ?$$

$$X = s(zero).$$

>> plus(X, Y, s(s(zero))).

$$" X + Y = 2 "$$

$$X = zero$$

$$Y = s(s(zero));$$

$$X = s(zero)$$

$$Y = s(zero);$$

$$X = s(s(zero))$$

$$Y = zero;$$

NO.

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SLD tree for a & a)

$\leftarrow \text{plus}(\text{s(zero)}, \text{s(zero)}, \text{s(s(zero))}).$

\downarrow only with (ii)
 $\{x/\text{zero}, Y/\text{s(zero)}, Z/\text{s(s(zero))}\}$

$\leftarrow \text{plus}(\text{zero}, \text{s(zero)}, \text{s(zero)}).$

\downarrow only with (i)
 $\{x'/\text{s(zero)}\}$

$\leftarrow \text{isnumber}(\text{s(zero)}).$

\downarrow only with (ii)
 $\{x''/\text{zero}\}$

$\leftarrow \text{isnumber}(\text{zero}).$

\downarrow only with (i)
 Yes.

(b)

Note that PROLOG does not report the substitution for which success took place as a) does not involve a free variable.

SLD-tree here is also a single trunk.

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SLD tree for $A \Leftarrow b$

$\Leftarrow \text{plus}(\text{s(zero)}, \text{s(zero}), X).$

only with (ii)
 $\downarrow \{x'/\text{zero}, Y'/\text{s(zero)}, X/\text{s}(Z')\}$

$\Leftarrow \text{plus}(\text{zero}, \text{s(zero)}, Z').$

only with (i)
 $\downarrow \{X''/\text{s(zero)}, Z'/\text{s(zero)}\}$

$\Leftarrow \text{isnumber}(\text{s(zero)}).$

only with (ii) $\{X''/\text{zero}\}$
 \downarrow

$\Leftarrow \text{isnumber}(\text{zero}).$

only with (i)
 \downarrow

□ success

$x \rightarrow \text{s}(Z') \rightarrow \text{s}(\text{s}(\text{zero}))$

$x = \text{s}(\text{s}(\text{zero})).$

If you prompt here Prolog with ;
 for further search (as SLD tree
 yields a unique here branch). the answer
 is No.

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It is characteristic for logic programming that there are many equivalent solutions.

But we must be careful in Prolog:

- order of clauses matters
- order of atoms in program clause matters

even (zero).

(i)

even (X) :- even ($S(S(X))$). (ii)

>> even ($S(S(zero))$).

"is 2 even?"

"zero is even"
"if X is even
then $X-2$ even"

out of local stack!

\leftarrow even ($S(S(zero))$).

\downarrow only with (ii)
 $\{X / S(S(zero))\}$

\leftarrow even ($S(S(S(S(zero))))$).



; goes to ∞ .

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Note that source of
"out of local stack"
[execution aborted]

is

- either repeating current goal..
- or compounding composition of function : ie. $f(f(\dots))$

Another warning :

- (i) $\text{badplus}(\text{zero}, X, X) :- \text{isnumber}(X).$
- (ii) $\text{badplus}(X, S(Y), S(Z)) :- \text{badplus}(X, Y, Z).$
" $0 + X = X$ "
" if $X + Y = Z$ then $X + (Y + 1) = Z + 1$ "

(i) & (ii) look fine but we cannot add
 $S(\text{zero}) + \text{zero}$!

so one pair of correct adding is lost!

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There is in-built arithmetic
in PROLOG:

The predicate:

$N \text{ is } X \equiv \text{is}(N, X).$

assign the value (a numeric one)
to variable X.

There are other arithmetic operators:

"+", "-", "*", "/", and "mod".

So >> X is 3+2.

X=5.

>> X is X+3.

Error in arithmetic
expression

no

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>> $X \text{ is } 17 * 23.$

$$X = 391$$

Also non-logical way of comparison operators:

$X > Y$ $X = ::= Y$ is equal

$X < Y$ $X = \backslash = Y$ $X \neq Y$

$X \geq Y$

$X \leq Y$

>> $X = 3.$

$$X = 3$$

Symbolic computation introduced before

- mainly to show recursive nature of logic programming
- arithmetic is done to do practical computation in PROLOG

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We define a predicate which allows transition from symbolic arithmetic to a real one:

e.g.

testing

$\text{multiply}(X, Y, Z)$.

for $X = S(S(S(S(S(\text{zero}))))))$
 $\overset{\text{|||}}{\text{"5"}}$

$Y = S(S(S(S(S(S(S(S(\text{zero}))))))))$
 $\overset{\text{|||}}{\text{"7"}}$

If PROLOG gives symbolic correct result

$Z = \underbrace{S(\dots S(\text{zero})\dots)}_{35 \text{ times}}$
 $\overset{\text{|||}}{\text{"35"}}$

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one has to count number of composed functors.

AWKWARD !

So we can define a predicate:

(i) show(zero, 0).

(ii) show(s(X), N) :- show(X, M),
N is M+1.

Now

>> show(X, 5), show(Y, 7),
multiply(X, Y, Z), show(Z, N).

1. first numeric 5 is assigned
to symbolic "5" i.e.:

$$X = s(s(s(s(s(zero)))))$$

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2. then numeric 7 is assigned to symbolic "7" i.e.:

$$Y = s(s(s(s(s(s(s(zero)))))))$$

3. We pass both X & Y with symbolic values to predicate multiply which returns Z symbolically expressed as "35"

$$Z = \underbrace{s(s\dots(zero)\dots)}_{35\text{-times}}$$

4. Then Z is passed to show & this predicate retranslate Z (symbolically) into N as 35.

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>> show(X,2).

a)

X = s(s(zero)). < correct answer

" ; " < if PROLOG is prompted
for further searching

↓

"out of local stack" !!!
incorrect answer

>> show(s(s(zero)), N).

b)

N = 2.

" ; "

↑

No < correct answer

So most of the predicates
in PROLOG may not operate
symmetrically!