

# CEC 2022 Competition on Seeking Multiple Optima in Dynamic Environments

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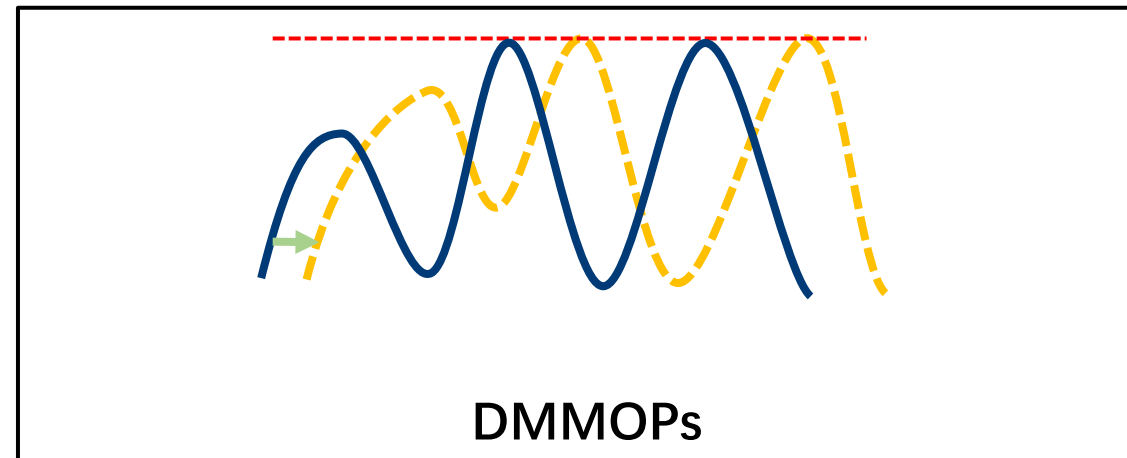
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# Outline

- ◆ **Introduction**
- ◆ Benchmarks
- ◆ Experiment Settings
- ◆ Results

- Dynamic multimodal optimization problems (DMMOPs) have the dynamic and multimodal natures\*.
- DMMOPs changes from one environment to another.
- In each environment, there exists multiple global peaks.



\* Wenjian Luo, Xin Lin, Tao Zhu, and Peilan Xu. A Clonal Selection Algorithm for Dynamic Multimodal Function Optimization. *Swarm and Evolutionary Computation*, 50, 100459, 2019

# Definitions of DMMOPs

• Problem  $f(\mathbf{x}, t)$  is a maximized DMMOP.  
•  $\mathbf{o}_{tn_t}$  represents the  $n_t$ -th optimal solution in the  $t$ -th environment.  
• Parameter  $\Omega$  defines the whole decision space.  
• Goal:  
The optimization algorithms solving DMMOPs should find all the optimal solutions in each environment.

## □ Definitions

$$\begin{pmatrix} \mathbf{o}_{11} & \mathbf{o}_{12} & \cdots & \mathbf{o}_{t1} \\ \mathbf{o}_{12} & \mathbf{o}_{22} & \cdots & \mathbf{o}_{t2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{o}_{1n_1} & \mathbf{o}_{2n_2} & \cdots & \mathbf{o}_{tn_t} \end{pmatrix} = \arg \max_{\mathbf{x} \in \Omega} f(\mathbf{x}, t)$$

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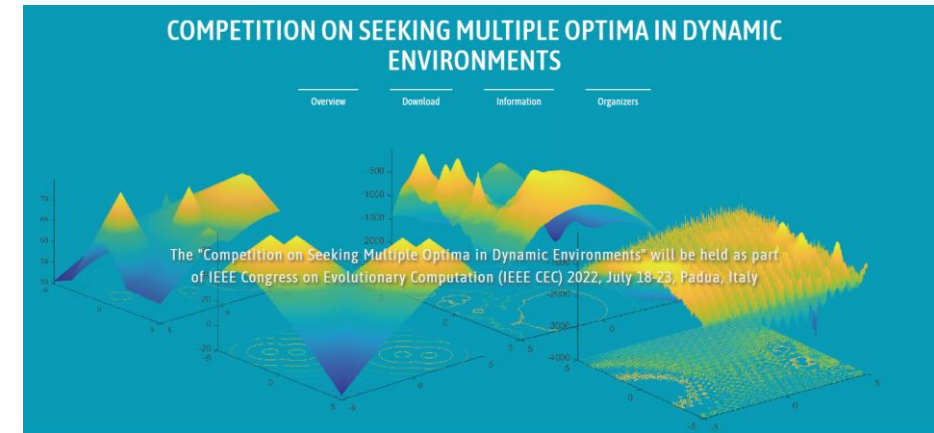
# Competition on DMMOPs

- The goal of this competition is to test the performance of various optimization algorithms for solving DMMOPs.

- The benchmark suit contains 24 problems, which are from our previous work\*.

- Website of the competition

[http://mi.hitsz.edu.cn/activities/2022dmml\\_competition.html](http://mi.hitsz.edu.cn/activities/2022dmml_competition.html)



## ORGANIZERS

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\* X. Lin, W. Luo, P. Xu, Y. Qiao, and S. Yang, PopDMMO: A general framework of population-based stochastic search algorithms for dynamic multimodal optimization, *Swarm and Evolutionary Computation*, p. 101011, 2021.

# Outline

◆ Introduction

◆ **Benchmarks**

◆ Experiment Settings

◆ Results

## □ Functions 1~4: Simple Multimodal Functions

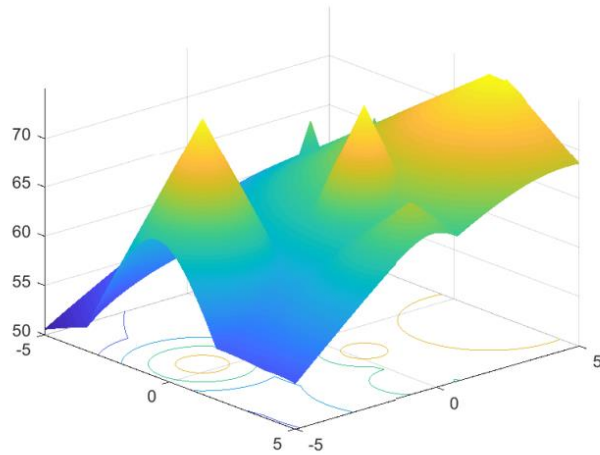
Constructed by the DF generator\*

$$f(x) = \max_{\substack{i=1,\dots,G, \\ G+1,\dots,G+L}} \left( H_i - W_i * \sqrt{\sum_{j=1}^D (x_j - X_{ij})^2} \right)$$

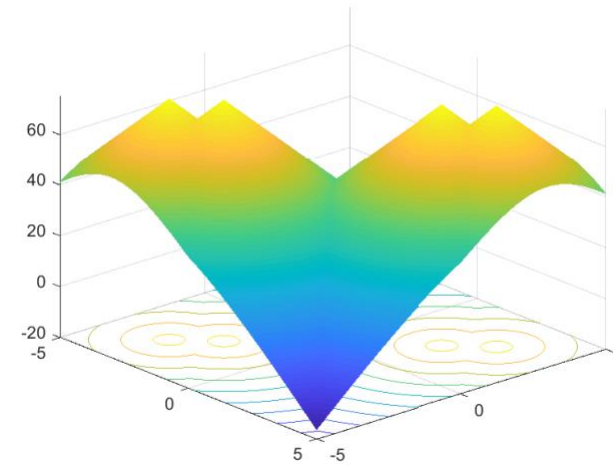
- $H_i$ ,  $W_i$  and  $X_{ij}$  represent the initial height, width and position of the  $i$ -th peak, respectively.
- $G$  and  $L$  represent the number of global peaks and the maximum number of local peaks.
- The heights of the global peaks are the same and higher than the local peaks.
- The minimum distance between peaks is no less than 0.1.

\* R. W. Morrison and K. A. De Jong, "A test problem generator for non-stationary environments," *Proceedings of the 1999 Congress on Evolutionary Computation*, 1999, pp. 2047-2053 Vol. 3.

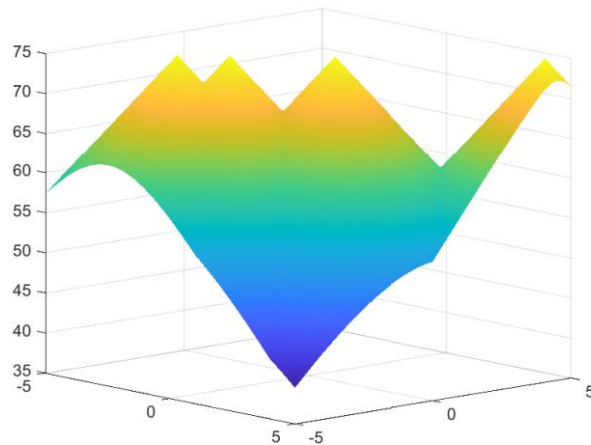
- The first four functions simulate the fitness landscapes of benchmarks in CEC2013 competitions for multimodal optimization.



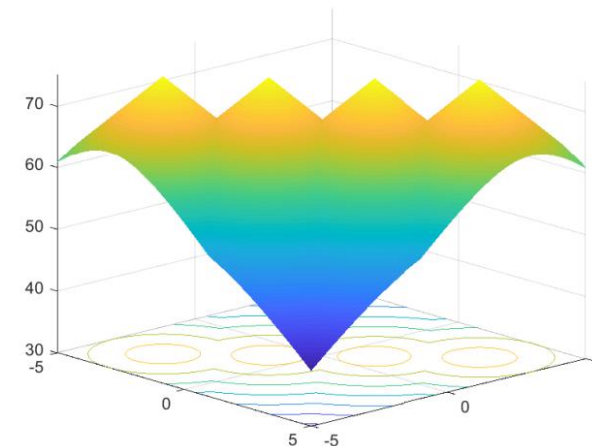
F1



F2



F3



F4



# Multimodal Functions

## □ Functions 5~8: Complex Multimodal Functions

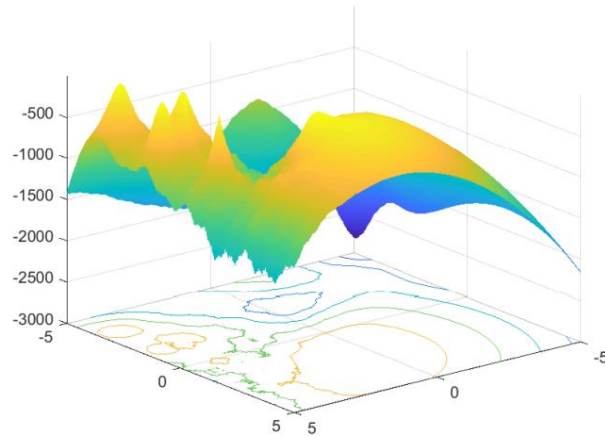
Constructed by the composition functions in CEC 2013 competition for multimodal optimization\*.

$$f(x) = - \sum_{i=1}^n \omega_i (\hat{f}_i(\frac{x - \mathbf{o}_i}{\lambda_i} * \mathbf{M}_i))$$

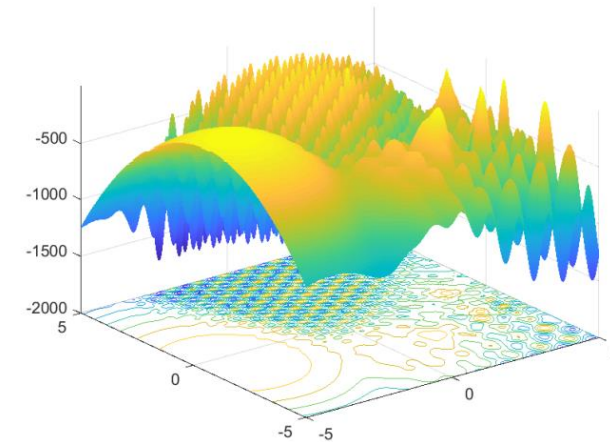
- The composition function is composed by  $n$  basic functions.
- $\omega_i$  is the weight of the  $i$ -th basic functions, and  $\hat{f}_i$  is the normalized version of  $f_i$ .
- $\mathbf{o}_i$ ,  $\lambda_i$  and  $\mathbf{M}_i$  are the global optima, scale and rotation matrix of the  $i$ -th basic function, respectively.

\* Li, X., Engelbrecht, A., & Epitropakis, M. G. Benchmark functions for CEC'2013 special session and competition on niching methods for multimodal function optimization. RMIT University, Evolutionary Computation and Machine Learning Group, Australia, Tech. Rep. 2013

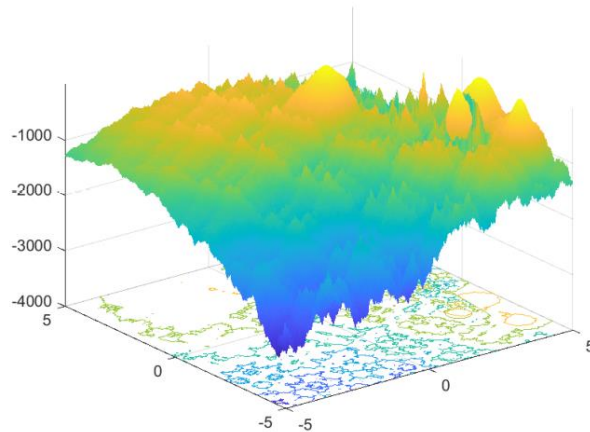
- The last four functions are directly from the composition functions in CEC2013 competitions for multimodal optimization.



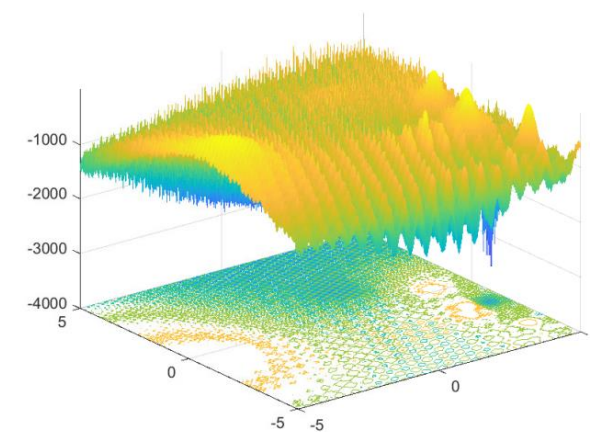
F5



F6



F7



F8

# Dynamic Change Modes

- Modes 1~6: Basic change modes from GDBG\*

- .

C1: Small step  
changes

C2: Large  
step changes

C3: Random  
changes

C4: Chaotic  
changes

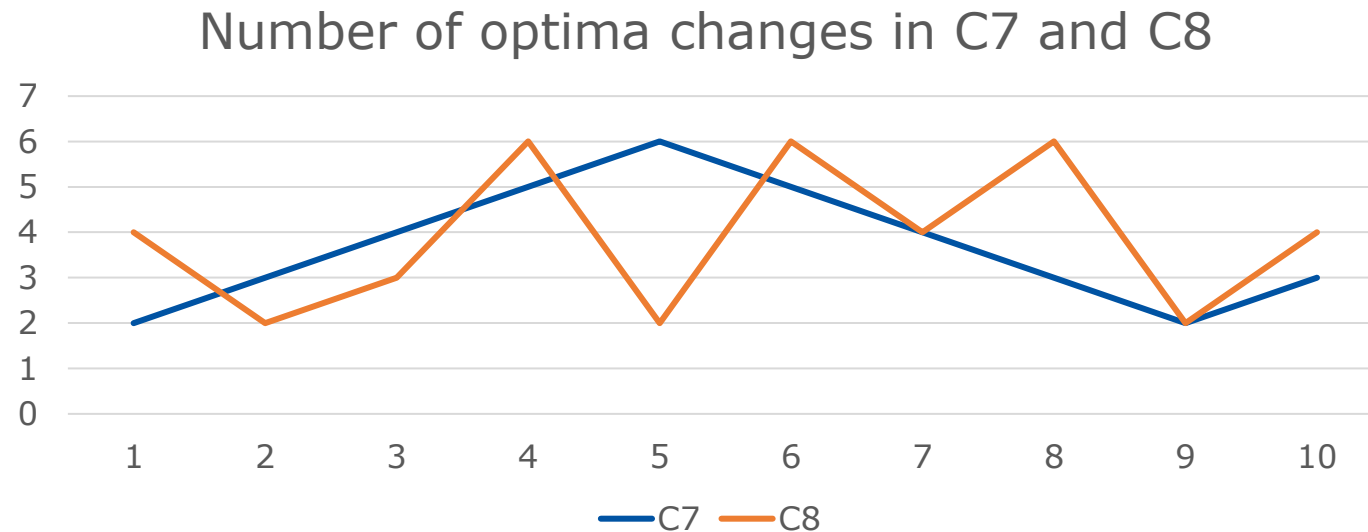
C5: Recurrent  
changes

C6: Recurrent  
with noisy  
changes

\* Li, C., Yang, S., Nguyen, T. T., Yu, E. L., Yao, X., Jin, Y., Beyer, H., Suganthan, P. N. Benchmark generator for CEC 2009 competition on dynamic optimization. Tech. Rep. 2008.

# Dynamic Change Modes

- Modes 7~8: Additional change modes about the number of optima.
  - C7 changes the number of optima linearly.
  - C8 changes the number of optima randomly.
  - Other core parameters changes according to C1.



## □ Move conditions

- The nearest distance from the current optimum to the rest optima must be larger than a predefined threshold.
- If the condition is not satisfied, the current optimum should move multiple times until satisfying the condition.

## □ Information of DMMOPs

- 60 environments for each problem
- $5000 \cdot D$  number of fitness evaluations in each environment

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# General Settings

Table 1: Details of the benchmark problems

Group	Index	Multimodal function	Dynamic Mode	Dimension
G1	P1	F1	C1	5
	P2	F2	C1	5
	P3	F3	C1	5
	P4	F4	C1	5
	P5	F5	C1	5
	P6	F6	C1	5
	P7	F7	C1	5
	P8	F8	C1	5
G2	P9	F8	C1	5
	P10	F8	C2	5
	P11	F8	C3	5
	P12	F8	C4	5
	P13	F8	C5	5
	P14	F8	C6	5
	P15	F8	C7	5
	P16	F8	C8	5
G3	P17	F1	C1	10
	P18	F2	C1	10
	P19	F3	C1	10
	P20	F4	C1	10
	P21	F5	C1	10
	P22	F6	C1	10
	P23	F7	C1	10
	P24	F8	C1	10

- The 24 benchmarks are divided into 3 groups, i.e., G1, G2 and G3.
  - G1  $\Rightarrow$  Multimodal property (F1~F8)
  - G2  $\Rightarrow$  Dynamic property (C1~C8)
  - G3  $\Rightarrow$  High dimensional property (D = 10)

## □ Parameters:

- Runs: 30. Each problem is executed 30 times.
- Frequency: each environment contains  $5000 * D$  fitness evaluations.
- Number of Environments: 60.
- Maximum Fitness evaluations: the maximum of fitness evaluations in all environments is set to  $5000 * D * 60$ .
- Environmental change condition: there is no fitness evaluation for the current environment.
- Termination condition: all the fitness evaluations are consumed.



Peak Ratio (PR) is adopted as the performance metric.

$$PR = \frac{\sum_{i=1}^{Run} \sum_{j=1}^{Env} NPF_{ij}}{\sum_{i=1}^{Run} \sum_{j=1}^{Env} Peaks_{ij}}$$

- *Run*: the total number of the independent runs, i.e., 30.
- *Env*: the number of the environment in a run
- $NPF_{ij}$ : the number of the found optimal solutions in the final population of the  $j$ -th environment and  $i$ -th run.
- $Peaks_{ij}$ : the total number of global peaks in the corresponding environment and run.

# Outline

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Name	Authors
MSA	Junfeng Tang
AMLPS-RS-CMSA-ESII	Ali Ahrari, Saber Elsayed, Ruhul Sarker, Daryl Essam, Carlos A. Coello Coello
AMPSO	Mai Peng, Hai Xia, Changhe Li
mEGA	Yuxin Liu, Chengyang Bu
NBNC-EA-SVR	Xunfeng Wu, Wu Lin, Yulong Ye
DMMDE	Xin Lin, Wenjian Luo, Peilan Xu, Yingying Qiao, Shengxiang Yang
HmSO	Qingshan Tan, Juncheng Wang, Changhe Li
ELDE	Dezheng Zhang, Hongyu Lin, Leiyu Zhang, Jing Liang, Kunjie Yu, Caitong Yue

# Results at 1e-3

Group	Problem	MSA	AMLPS-RS-CMSA-ESII	AMPSO	mEGA	NBNC-EA-SVR	DMMDE	HmSO	ELDE
G1	P1	0.445	0.696	<b>0.766</b>	0.617	0.615	0.409	0.626	0.475
	P2	0.683	0.997	<b>1.000</b>	0.932	0.543	0.834	0.996	0.224
	P3	0.739	0.954	<b>0.999</b>	0.922	0.634	0.472	0.983	0.250
	P4	0.791	0.970	<b>0.999</b>	0.944	0.627	0.512	0.997	0.452
	P5	0.243	0.547	0.555	0.352	<b>0.592</b>	0.244	0.575	0.242
	P6	0.227	<b>0.461</b>	0.264	0.247	0.276	0.249	0.255	0.236
	P7	0.020	<b>0.634</b>	0.622	0.315	0.580	0.289	0.606	0.302
	P8	0.001	<b>0.608</b>	0.448	0.040	0.372	0.193	0.277	0.002
G2	P9	0.001	<b>0.608</b>	0.448	0.040	0.372	0.193	0.277	0.002
	P10	0.000	<b>0.600</b>	0.296	0.009	0.325	0.199	0.121	0.000
	P11	0.000	<b>0.564</b>	0.270	0.010	0.310	0.186	0.105	0.001
	P12	0.000	<b>0.563</b>	0.245	0.008	0.324	0.207	0.095	0.001
	P13	0.001	<b>0.570</b>	0.321	0.014	0.336	0.194	0.149	0.054
	P14	0.001	<b>0.580</b>	0.299	0.019	0.336	0.196	0.129	0.030
	P15	0.024	<b>0.365</b>	0.212	0.043	0.228	0.118	0.121	0.042
	P16	0.012	<b>0.533</b>	0.218	0.044	0.331	0.188	0.133	0.013
G3	P17	0.014	0.505	<b>0.544</b>	0.388	0.337	0.250	0.293	0.297
	P18	0.033	<b>0.938</b>	0.930	0.915	0.526	0.495	0.493	0.279
	P19	0.077	<b>0.873</b>	0.721	0.696	0.567	0.335	0.621	0.407
	P20	0.076	<b>0.950</b>	0.781	0.921	0.537	0.336	0.659	0.489
	P21	0.122	<b>0.420</b>	0.335	0.335	0.329	0.325	0.284	0.331
	P22	0.071	<b>0.581</b>	0.257	0.249	0.272	0.232	0.209	0.246
	P23	0.000	<b>0.651</b>	0.550	0.405	0.472	0.168	0.248	0.331
	P24	0.000	<b>0.527</b>	0.381	0.046	0.390	0.152	0.133	0.099

# Results at 1e-4

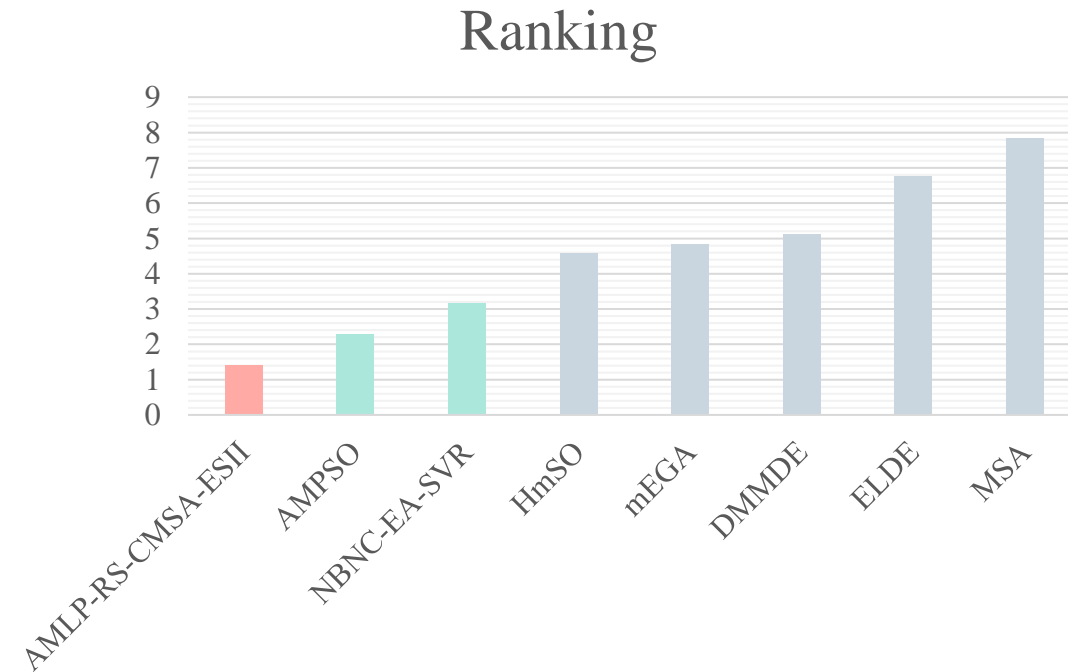
Group	Problem	MSA	AMLPS-RS-CMSA-ESII	AMPSO	mEGA	NBNC-EA-SVR	DMMDE	HmSO	ELDE
G1	P1	0.076	0.696	<b>0.765</b>	0.612	0.585	0.405	0.625	0.173
	P2	0.001	0.997	<b>1.000</b>	0.866	0.522	0.819	0.993	0.006
	P3	0.012	0.954	<b>0.999</b>	0.844	0.559	0.460	0.981	0.058
	P4	0.009	0.970	<b>1.000</b>	0.874	0.546	0.498	0.994	0.130
	P5	0.185	0.547	0.551	0.328	<b>0.588</b>	0.243	0.571	0.203
	P6	0.197	<b>0.461</b>	0.264	0.243	0.272	0.249	0.255	0.230
	P7	0.013	<b>0.634</b>	0.618	0.290	0.566	0.284	0.596	0.270
	P8	0.000	<b>0.608</b>	0.441	0.034	0.365	0.188	0.259	0.000
G2	P9	0.000	<b>0.608</b>	0.441	0.034	0.364	0.188	0.259	0.000
	P10	0.000	<b>0.600</b>	0.287	0.006	0.323	0.194	0.103	0.000
	P11	0.000	<b>0.564</b>	0.260	0.008	0.308	0.182	0.090	0.001
	P12	0.000	<b>0.563</b>	0.256	0.006	0.322	0.201	0.073	0.000
	P13	0.000	<b>0.570</b>	0.310	0.012	0.331	0.187	0.133	0.005
	P14	0.001	<b>0.580</b>	0.288	0.017	0.329	0.190	0.115	0.012
	P15	0.018	<b>0.365</b>	0.195	0.040	0.225	0.114	0.109	0.017
	P16	0.008	<b>0.533</b>	0.213	0.039	0.325	0.183	0.127	0.003
G3	P17	0.000	0.505	<b>0.543</b>	0.373	0.334	0.250	0.218	0.229
	P18	0.000	<b>0.938</b>	0.921	0.661	0.521	0.486	0.239	0.007
	P19	0.000	<b>0.873</b>	0.717	0.662	0.558	0.333	0.415	0.204
	P20	0.000	<b>0.950</b>	0.777	0.801	0.526	0.333	0.421	0.216
	P21	0.075	<b>0.420</b>	0.335	0.330	0.327	0.324	0.262	0.331
	P22	0.031	<b>0.581</b>	0.254	0.247	0.271	0.231	0.182	0.241
	P23	0.000	<b>0.651</b>	0.543	0.366	0.469	0.168	0.186	0.326
	P24	0.000	<b>0.527</b>	0.369	0.035	0.382	0.151	0.100	0.061

# Results at $1e-5$

Group	Problem	MSA	AMLPS-RS-CMSA-ESII	AMPSO	mEGA	NBNC-EA-SVR	DMMDE	HmSO	ELDE
G1	P1	0.000	0.696	<b>0.765</b>	0.560	0.543	0.403	0.622	0.018
	P2	0.000	0.997	<b>0.999</b>	0.690	0.507	0.807	0.973	0.000
	P3	0.000	0.954	<b>0.999</b>	0.739	0.521	0.449	0.973	0.020
	P4	0.000	0.970	<b>0.999</b>	0.752	0.526	0.488	0.987	0.019
	P5	0.103	0.547	0.546	0.297	<b>0.582</b>	0.241	0.564	0.144
	P6	0.148	<b>0.461</b>	0.263	0.240	0.271	0.249	0.254	0.217
	P7	0.004	<b>0.634</b>	0.606	0.261	0.549	0.279	0.565	0.158
	P8	0.000	<b>0.608</b>	0.433	0.030	0.357	0.183	0.223	0.000
G2	P9	0.000	<b>0.608</b>	0.433	0.030	0.357	0.183	0.223	0.000
	P10	0.000	<b>0.600</b>	0.274	0.005	0.320	0.191	0.081	0.000
	P11	0.000	<b>0.564</b>	0.247	0.007	0.306	0.179	0.074	0.000
	P12	0.000	<b>0.563</b>	0.267	0.004	0.320	0.198	0.053	0.000
	P13	0.000	<b>0.570</b>	0.297	0.010	0.326	0.181	0.110	0.000
	P14	0.000	<b>0.580</b>	0.275	0.014	0.324	0.183	0.096	0.004
	P15	0.010	<b>0.365</b>	0.178	0.035	0.223	0.111	0.087	0.006
	P16	0.003	<b>0.533</b>	0.205	0.034	0.323	0.177	0.118	0.000
G3	P17	0.000	0.505	<b>0.542</b>	0.167	0.334	0.250	0.143	0.184
	P18	0.000	<b>0.938</b>	0.733	0.120	0.516	0.478	0.075	0.000
	P19	0.000	<b>0.873</b>	0.712	0.299	0.540	0.331	0.223	0.015
	P20	0.000	<b>0.950</b>	0.770	0.298	0.515	0.330	0.217	0.000
	P21	0.048	<b>0.420</b>	0.335	0.327	0.325	0.324	0.233	0.325
	P22	0.016	<b>0.581</b>	0.250	0.245	0.269	0.230	0.147	0.238
	P23	0.000	<b>0.651</b>	0.534	0.326	0.468	0.167	0.117	0.280
	P24	0.000	<b>0.527</b>	0.344	0.028	0.371	0.150	0.064	0.026

# Ranking of the Algorithms at $1e-4$

Rank	Name	Score*
1	AMLPS-RS-CMSA-ESII	1.4167
2	AMPSO	2.2917
3	NBNC-EA-SVR	3.1667
4	HmSO	4.5833
5	mEGA	4.8333
6	DMMDE	5.1250
7	ELDE	6.7500
8	MSA	7.8333



\* Scores are obtained by the Friedman test. The smaller the score, the better the algorithm.

THANK YOU