RULES OF INFERENCE

Section 1.6

SECTION SUMMARY

- Valid Arguments
- Rules of Inference for Propositional Logic
- Using Rules of Inference to Build Arguments
- Rules of Inference for Quantified Statements
- Building Arguments for Quantified Statements

REVISITING THE SOCRATES EXAMPLE

- We have the two premises:
  - “All men are mortal.”
  - “Socrates is a man.”
- And the conclusion:
  - “Socrates is mortal.”
- How do we get the conclusion from the premises?

THE ARGUMENT

- We can express the premises (above the line) and the conclusion (below the line) in predicate logic as an argument:

$$\begin{array}{ll} & \forall x(Man(x) \rightarrow Mortal(x)) \\ & Man(Socrates) \\ \hline \therefore & Mortal(Socrates) \end{array}$$

- We will see shortly that this is a valid argument.

VALID ARGUMENTS

- An *argument* in propositional logic is a sequence of propositions. **All but** the final proposition are called ***premises***（前提）. The last statement is the ***conclusion***.
- The argument is valid if the premises imply the conclusion. An ***argument form*** **(论证形式)** is an argument that is valid no matter what propositions are substituted into its propositional variables.
- If the premises are $p_1, p_2, \ldots, p_n$ and the conclusion is q then

  $(p_1 \land p_2 \land \ldots \land p_n) \to q$ is a tautology.

- Inference rules are simple valid argument forms that will be used to construct more complex valid argument forms.

RULES OF INFERENCE FOR PROPOSITIONAL LOGIC: MODUS PONENS (假言推理）

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Corresponding Tautology:

$(p \wedge (p \to q)) \to q$

Example:

Let p be "It is snowing."

Let q be "I will study discrete math."

"If it is snowing, then I will study discrete math."

"It is snowing."

"Therefore , I will study discrete math."

MODUS TOLLENS (取拒式)

$$\begin{array}{c} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Corresponding Tautology:

$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

Example:
Let p be "It is snowing."
Let q be "I will study discrete math."

"If it is snowing, then I will study discrete math."
"I will not study discrete math."

"Therefore , it is not snowing."

HYPOTHETICAL SYLLOGISM (假言三段)

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Corresponding Tautology:
$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

Example:
Let p be “It snows.”
Let q be “I will study discrete math.”
Let r be “I will get an A.”

“If it snows, then I will study discrete math.”
“If I study discrete math, I will get an A.”

“Therefore , If it snows, I will get an A.”

DISJUNCTIVE SYLLOGISM (析取三段)

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Corresponding Tautology:
$(\neg p \wedge (p \vee q)) \to q$

Example:
Let p be "I will study discrete math."
Let q be "I will study English literature."

"I will study discrete math or I will study English literature."
"I will not study discrete math."

"Therefore , I will study English literature."

ADDITION (附加)

$$\frac{p}{\therefore p \vee q}$$

Corresponding Tautology:

$p \rightarrow (p \vee q)$

Example:
Let p be "I will study discrete math."
Let q be "I will visit Las Vegas."

"I will study discrete math."

"Therefore, I will study discrete math or I will visit Las Vegas."

SIMPLIFICATION (化简)

$$\frac{p \land q}{\therefore\ q}$$

Corresponding Tautology:

$(p \land q) \rightarrow p$

Example:
Let p be "I will study discrete math."
Let q be "I will study English literature."

"I will study discrete math and English literature"

"Therefore, I will study discrete math."

CONJUNCTION (合取)

$$\frac{\begin{array}{l} p \\ q \end{array}}{\therefore p \wedge q}$$

Corresponding Tautology:

$((p) \wedge (q)) \rightarrow (p \wedge q)$

Example:

Let p be "I will study discrete math."

Let q be "I will study English literature."

"I will study discrete math."

"I will study English literature."

"Therefore, I will study discrete math and I will study English literature."

RESOLUTION (消解)

Resolution plays an important role in AI and is used in Prolog.

$$\begin{array}{l} \neg p \vee r \\ p \vee q \\ \hline \therefore q \vee r \end{array}$$

Corresponding Tautology:

$((\neg p \vee r) \wedge (p \vee q)) \rightarrow (q \vee r)$

Example:

Let p be "I will study discrete math."
Let r be "I will study English literature."
Let q be "I will study databases."

"I will not study discrete math or I will study English literature."
"I will study discrete math or I will study databases."

"Therefore, I will study databases or I will study English literature."

USING THE RULES OF INFERENCE TO BUILD VALID ARGUMENTS

- A *valid argument* is a sequence of statements. Each statement is either a premise or follows from previous statements by rules of inference. The last statement is called conclusion.
- A valid argument takes the following form:

$$
\begin{array}{l}
S_1 \\
S_2 \\
. \\
. \\
. \\
S_n \\
\\
\therefore\ C
\end{array}
$$

HANDLING QUANTIFIED STATEMENTS

- Valid arguments for quantified statements are a sequence of statements. Each statement is either a premise or follows from previous statements by rules of inference which include:
  - Rules of Inference for Propositional Logic
  - Rules of Inference for Quantified Statements
- The rules of inference for quantified statements are introduced in the next several slides.

UNIVERSAL INSTANTIATION (UI)

$$\frac{\forall x P(x)}{\therefore P(c)}$$

Example:

Our domain consists of all dogs and Fido is a dog.

“All dogs are cuddly.”

“Therefore, Fido is cuddly.”

UNIVERSAL GENERALIZATION (UG)

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$$

Used often implicitly in Mathematical Proofs.

EXISTENTIAL INSTANTIATION (EI)

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$$

Example:

"There is someone who got an A in the course."
"Let's call her *a* and say that *a* got an A"

EXISTENTIAL GENERALIZATION (EG)

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$$

Example:

“Michelle got an A in the class.”
“Therefore, someone got an A in the class.”

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REVIEW

KEY TERMS

Proposition	Propositional variable	Truth value
Negation	Logical operators	
Compound proposition		Truth table
Disjunction	Conjunction	Exclusive or
Implication	Converse	Contrapositive
Inverse	Biconditional	Tautology
Contingency	Logically equivalent	Predicate
Propositional function	Domain of discourse	Existential quantifier
Universal quantifier	Free variable	Bound variable
Scope of a quantifier	Argument	Argument form
Premise	Conclusion	Valid argument form
Rule of inference	Fallacy	

RESULTS

- Logical equivalences
- De Morgan's Law
- De Morgan's Law for quantifiers
- Rules of inference for propositional logic
- Rules of inference for quantified statements

THANK YOU!