北京航空航天大学

2019－2020 学年 第二学期期末

《离散数学》

考 试 A 卷

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考试日期: 2020 年 6 月 6 日

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班号 _____ 学号_____ 姓名_____ 成绩_____

《离散数学》期末考试卷

Note：1、Examination duration: 180 minutes.

2、Please check additional notes for online open-book examination.

Questions：1-40, 2 points each; 41, 8 points. 42, 10 points.

1. Find a proposition with three variables *p*, *q*, and *r* that is true when *p* and *r* are true and *q* is false, and false otherwise.

2. (a) Find a proposition with the given truth table.

p	*q*	?
T	T	F
T	F	F
F	T	T
F	F	F

(b) Find a proposition using only *p*, *q*, ¬, and the connective ∨ that has this truth table.

3. Explain why the negation of "Al and Bill are absent" is not "Al and Bill are present".

Use the following to answer questions 4-5:
In the questions below write the statement in the form "If ..., then ...."

4. To get a good grade it is necessary that you study.
5. You need to be registered in order to check out library books.

Use the following to answer questions 6-7:
In the questions below suppose the variable x represents students and y represents courses, and:

$M(y)$: y is a math course $F(x)$: x is a freshman
$B(x)$: x is a full-time student $T(x, y)$: x is taking y.

Write the statement in good English without using variables in your answers.

6. $\forall x \exists y\, T(x, y)$.
7. $\forall x \exists y\, [(B(x) \wedge F(x)) \rightarrow (M(y) \wedge T(x, y))]$.

Use the following to answer questions 8-9:
In the questions below suppose the variable x represents students and y represents courses, and:
$U(y)$: y is an upper-level course $M(y)$: y is a math course $F(x)$: x is a freshman
$B(x)$: x is a full-time student $T(x, y)$: student x is taking course y.
Write the statement using these predicates and any needed quantifiers.

8. All students are freshmen.
9. No math course is upper-level.

Use the following to answer questions 10-11:
In the questions below assume that the universe for x is all people and the universe for y is the set of all movies. Write the English statement using the following predicates and any needed quantifiers:
$S(x, y)$: x saw y $L(x, y)$: x liked y $A(y)$: y won an award $C(y)$: y is a comedy.

10. No one liked every movie he has seen.
11. Ben has never seen a movie that won an award.

12. Write the contrapositive, converse, and inverse of the following: You sleep late if it is Saturday.

Use the following to answer questions 13-14:
Use a Venn diagram to determine which relationship, $\subseteq$, $=$, $\supseteq$, is true for the pair of sets.

13. $A \cup B$, $A \cup (B - A)$.
14. $(A - C) - (B - C)$, $A - B$.

Use the following to answer questions 15-16:
In the questions below determine whether the given set is the power set of some set. If the set is a power set, give the set of which it is a power set.

15. $\{\varnothing,\{a,\varnothing\}\}$.
16. $\{\varnothing,\{a\},\{\varnothing\},\{a,\varnothing\}\}$.

Use the following to answer questions 17-18:
In the questions below determine whether the set is finite or infinite. If the set is finite, find its size.

17. $\{x \mid x \in \mathbf{Z} \text{ and } x^2 < 10\}$.
18. $A \times B$, where $A = \{1,2,3,4,5\}$ and $B = \{1,2,3\}$.

Use the following to answer questions 19-20:
In the questions below suppose $g : A \to B$ and $f : B \to C$ where $A = B = C = \{1,2,3,4\}$, $g = \{(1,4),(2,1),(3,1),(4,2)\}$ and $f = \{(1,3),(2,2),(3,4),(4,2)\}$.

19. Find $f \circ g$.
20. Find $g \circ (g \circ g)$.

Use the following to answer questions 21-22:
In the questions below determine whether the binary relation is: (1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive.

21. The relation R on $\{1,2,3,\ldots\}$ where aRb means $a \mid b$.
22. The relation R on $\{a, b, c\}$ where $R = \{(a, a),(b, b),(c, c),(a, b),(a, c),(c, b)\}$.

Use the following to answer questions 23-24:
In the questions below suppose R and S are relations on $\{a, b, c, d\}$, where $R = \{(a, b),(a, d),(b, c),(c, c),(d, a)\}$ and $S = \{(a, c),(b, d),(d, a)\}$.

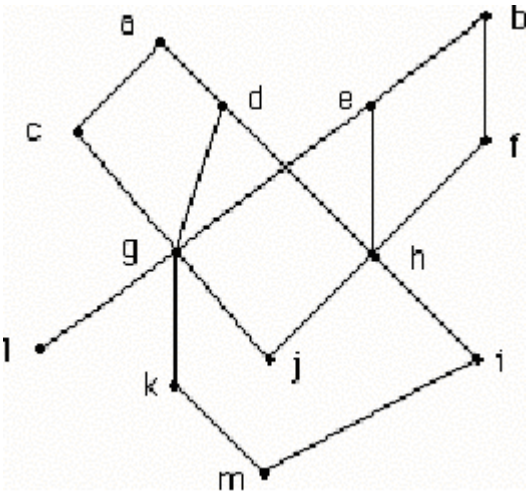
23. Construct R^2.
24. Construct $R \circ S$.

25. Find the transitive closure of R on $\{a, b, c, d\}$ where $R = \{(a, a),(b, a),(b, c),(c, a),(c, c),(c, d),(d, a),(d, c)\}$.

26. On the island of knights and knaves you encounter two people. *A* and *B*. Person *A* says, "*B* is a knave." Person *B* says, "At least one of us is a knight." Determine whether each person is a knight or a knave. Why?

27. What is the rule of inference used in the following:
If it snows today, the university will be closed. The university will not be closed today. Therefore, it did not snow today.

28. The diagram shown is the Hasse diagram for a partially ordered set. Referring to this diagram:
(a) List the maximal elements
(b) List the minimal elements
(c) Find all upper bounds for *f*, *g*
(d) Find all lower bounds for *d*, *f*
(e) Find lub({*g*, *j*, *m*})
(f) Find glb({*d*, *e*})
(g) Find the greatest element
(h) Find the least element



Use the following to answer question 29:
In the questions below either give an example or prove that there are none.

29. A simple digraph with indegrees 0,1,2,2 and outdegrees 0,1,1,3.

30. There are two assumptions:
(1) “Logic is difficult or not many students like logic.”
(2) “If mathematics is easy, then logic is not difficult.”
By translating these assumptions into statements involving propositional variables and logical connectives, determine whether each of the following are valid conclusions of these assumptions:
(a) That mathematics is not easy or logic is difficult. (2 points)
(b) That if not many students like logic, then either mathematics is not easy or logic is not difficult. (2 points)

31. If $R = \{(1,2),(1,4),(2,3),(3,1),(4,2)\}$, find the symmetric closure of R.

32. Draw all nonisomorphic trees with 5 vertices.

33. Prove that $(q \wedge (p \rightarrow \neg q)) \rightarrow \neg p$ is a tautology using propositional equivalence and the laws of logic.

34. Prove that $p \rightarrow q$ and its converse are not logically equivalent.

35. Determine whether $(p \rightarrow q) \wedge (\neg p \rightarrow q) \equiv q$.

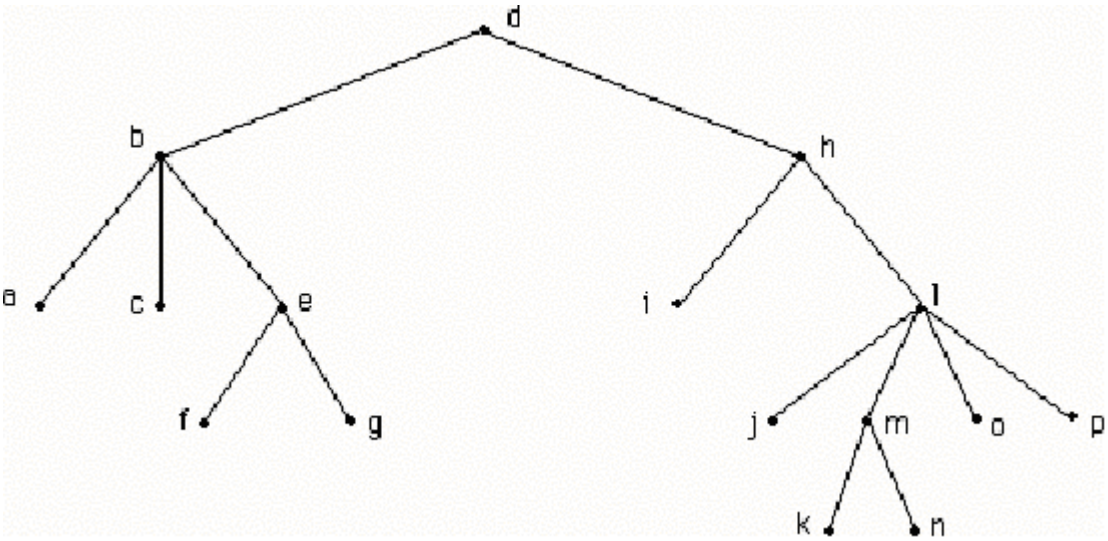
36. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving a proof using logical equivalence.

37. Suppose $f : \mathbf{N} \rightarrow \mathbf{N}$ has the rule $f(n) = 4n^2 + 1$. Determine whether f is onto $\mathbf{N}$.

38. Suppose $g : A \rightarrow B$ and $f : B \rightarrow C$ where $A = \{1,2,3,4\}$, $B = \{a, b, c\}$, $C = \{2,7,10\}$, and f and g are defined by $g = \{(1,b),(2,a),(3,a),(4,b)\}$ and $f = \{(a,10),(b,7),(c,2)\}$. Find f^{-1}.

39. Suppose $\boldsymbol{T}$ is a full m-ary tree with l leaves. Prove that $\boldsymbol{T}$ has $(l - 1)/(m - 1)$ internal vertices.

Use the following to answer question 40:
In the questions below refer to the given tree.



40. Find the inorder traversal.

41. Show the premises "All movies produced by John Sayles are wonderful." and "John Sayles produced a movie about coal miners." imply the conclusion "There is a wonderful movie about coal miners." (8 points)

42. The construction of a concept map through *hierarchical organization* (等级结构), *progressive differentiation* (渐进差别), *and integrative reconciliation* (综合贯通) shapes an individual's ability to assimilate and integrate knowledge. Please draw a concept map about graphs that contains at least 3 levels and 20 concepts (key terms) with top down tree structure. (10 points)