PROPOSITIONAL LOGIC

Section 1.1

SECTION SUMMARY

- Propositions
- Connectives
  - Negation
  - Conjunction
  - Disjunction
  - Implication; contrapositive, inverse, converse
  - Biconditional
- Truth Tables

PROPOSITIONS

- **A *proposition* is a declarative sentence that is either true or false.**
- Examples of propositions:
  a) The Moon is made of green cheese.
  b) Beijing is the capital of China.
  c) Toronto is the capital of Canada.
  d) $1 + 0 = 1$
  e) $0 + 0 = 2$
- Examples that are not propositions:
  a) Sit down!
  b) What time is it?
  c) $x + 1 = 2$
  d) $x + y = z$

PROPOSITIONS

Example: Are the following sentences propositions? If they are propositions, determine whether they are true or false.

1. $1 + 2 = 3$ or $2 + 3 = 5$

2. $x + 4 > 9$

3. Take two apples.

4. Today is Wednesday.

PROPOSITIONAL LOGIC

- Constructing Propositions
  - Propositional Variables: p, q, r, s, …
  - The proposition that is always true is denoted by **T** and the proposition that is always false is denoted by **F**.
  - Compound Propositions; constructed from logical connectives and other propositions
    - Negation $\neg$
    - Conjunction $\wedge$
    - Disjunction $\vee$
    - Implication $\rightarrow$
    - Biconditional $\leftrightarrow$

COMPOUND PROPOSITIONS: NEGATION 否定

- The *negation* of a proposition p is denoted by $\neg p$ and has this truth table:

p	$\neg p$
T	F
F	T

CONJUNCTION 合取

- The *conjunction* of propositions p and q is denoted by $p \wedge q$ and has this truth table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

DISJUNCTION 析取

- The *disjunction* of propositions p and q is denoted by $p \vee q$ and has this truth table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

THE CONNECTIVE OR IN ENGLISH

- In English “or” has two distinct meanings.
  - “Inclusive Or” (同或) - In the sentence “Students who have taken CS202 or Math120 may take this class,” we assume that students need to have taken one of the prerequisites, but may have taken both. This is the meaning of disjunction. For $p \vee q$ to be true, either one or both of p and q must be true.
  - “Exclusive Or” (异或) - When reading the sentence “Soup or salad comes with this entrée,” we do not expect to be able to get both soup and salad. This is the meaning of Exclusive Or (Xor). In $p \oplus q$, one of p and q must be true, but not both. The truth table for $\oplus$ is:

p	q	$p \oplus q$
T	**T**	**F**
T	F	T
F	T	T
F	F	F

IMPLICATION 蕴含

- If p and q are propositions, then $p \rightarrow q$ is a *conditional statement* or *implication* which is read as "if p, then q" and has this truth table:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- **Example**: If p denotes "I am at home." and q denotes "It is raining." then $p \rightarrow q$ denotes "If I am at home then it is raining."
- In $p \rightarrow q$, p is the *hypothesis* (*antecedent* or *premise*) and q is the *conclusion* (or *consequence*).

UNDERSTANDING IMPLICATION

- In $p \to q$ there does not need to be any connection between the antecedent or the consequent. The "meaning" of $p \to q$ depends only on the truth values of p and q.
- These implications are perfectly fine, but would not be used in ordinary English.
  - "If the moon is made of green cheese, then I have more money than Bill Gates. "
  - "If the moon is made of green cheese then I'm on welfare."
  - "If 1 + 1 = 3, then your grandma wears combat boots."
- One way to view the logical conditional is to think of an obligation or contract.
  - "If I am elected, then I will lower taxes."

DIFFERENT WAYS OF EXPRESSING $P \to Q$

- **if** p, **then** q
- **if** p, q
- q **unless** $\neg p$
- q **if** p
- q **follows from** p
- q **is necessary for** p
- p **implies** q
- p **only if** q
- q **whenever** p
- q **when** p
- p **is sufficient for** q

- **a necessary condition for** p **is** q
- **a sufficient condition for** q **is** p

More exercises on Page 5.

CONVERSE, CONTRAPOSITIVE, AND INVERSE 逆、逆否和反

- From $p \to q$ we can form new conditional statements .
  - $q \to p$ is the **converse** of $p \to q$
  - $\neg q \to \neg p$ is the **contrapositive** of $p \to q$
  - $\neg p \to \neg q$ is the **inverse** of $p \to q$

Example: Find the converse, inverse, and contrapositive of "It raining is a sufficient condition for my not going to town."

Solution:

converse: If I do not go to town, then it is raining.

inverse: If it is not raining, then I will go to town.

contrapositive: If I go to town, then it is not raining.

BICONDITIONAL 双蕴含

- If p and q are propositions, then we can form the *biconditional* proposition $p \leftrightarrow q$, read as "p if and only if q." The biconditional $p \leftrightarrow q$ denotes the proposition with this truth table:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- If p denotes "I am at home." and q denotes "It is raining." then $p \leftrightarrow q$ denotes "I am at home if and only if it is raining."

EXPRESSING THE BICONDITIONAL

- Some alternative ways "p if and only if q" is expressed in English:
  - p **is necessary and sufficient for** q
  - **if** p **then** q **, and conversely**
  - p **iff** q

PRECEDENCE OF LOGICAL CONNECTIVES

Connective	Precedence
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

Example: $p \vee q \rightarrow \neg r$
is equivalent to $(p \vee q) \rightarrow \neg r$
If the intended meaning is $p \vee (q \rightarrow \neg r)$
then parentheses must be used.

17 THANK YOU !