LUCRARE DE CONTROL

Subiectul A

R 1

16. 11. 2020

of 1p

-9,5

③ POP MIHAI

GRUPA : 215

① Det. mulțimea pct. limită ale șirului:

3

$$x_n = \left(\frac{n+3}{n+1}\right)^{n \cdot \cos\frac{n\pi}{2}}$$

② Fie $\sum_{n=0}^{\infty} x_n$ o serie convergentă cu termeni poz. Atunci seria

2,5

$\sum_{n=0}^{\infty} x_n^2$ este convergentă

③ Calc. derivata de ordinul $n \in \mathbb{N}$ a funcției

-3

$$f(x) = \frac{x^2}{e^{x+1}}$$

① $x_n = \left(\frac{n+3}{n+1}\right)^{n \cdot \cos\frac{n\pi}{2}}$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty}\left(\frac{n+3}{n+1}\right)^{n\cdot\cos\frac{n\pi}{2}} = \lim_{n\to\infty}\left(1+\frac{n+3}{n+1}-1\right)^{n\cdot\cos\frac{n\pi}{2}} =$$

$$= \lim_{n\to\infty}\left(1+\frac{\cancel{n}+3-\cancel{n}-1}{n+1}\right)^{n\cdot\cos\frac{n\pi}{2}} = \lim_{n\to\infty}\left(1+\frac{2}{n+1}\right)^{n\cdot\cos\frac{n\pi}{2}} =$$

$$= \lim_{n\to\infty}\underbrace{\left(\left(1+\frac{2}{n+1}\right)^{\frac{n+1}{2}}\right)}_{\to e}{}^{\frac{2}{n+1}\cdot n\cdot\cos\frac{n\pi}{2}} = e^{\lim\limits_{n\to\infty}\frac{2n}{n+1}\cdot\cos\frac{n\pi}{2}}$$

$$\lim_{n\to\infty} n\cdot\cos\frac{n\pi}{2} = \begin{cases} \lim\limits_{n\to\infty} n\cdot 0 = 0, & n = 2k+1 \\ \lim\limits_{n\to\infty} n\cdot(+1) = +\infty, & n = 4k \\ \lim\limits_{n\to\infty} n\cdot(-1) = -\infty, & n = 4k+2 \end{cases}, \quad k \in \mathbb{Z}$$

n	-2	-1	0	1	2	3	4
$\cos\frac{n\pi}{2}$	-1	0	1	0	-1	0	1

I. $n = 2k+1$

$$\lim_{n\to\infty}\left(1+\frac{2}{n+1}\right)^{n\cdot\cos\frac{n\pi}{2}} = 1^0 = 1$$

II. $n = 4k$

$$\lim_{n\to\infty}\left(1+\frac{2}{n+1}\right)^{n\cdot\cos\frac{n\pi}{2}} = e^{\lim_{n\to\infty}\frac{2n}{n+1}\cdot\cos\frac{n\pi}{2}} = e^{2\cdot\lim_{n\to\infty}\frac{\cancel{n}}{\cancel{n}(1+\frac{1}{n})}\cdot\cos\frac{n\pi}{2}} =$$

$$= e^{2\cdot\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\cdot\cos\frac{n\pi}{2}} = e^{2\cdot 1\cdot 1} = e^2$$

III. $n = 4k+2$

$$\lim_{n\to\infty}\left(1+\frac{2}{n+1}\right)^{n\cdot\cos\frac{n\pi}{2}} = e^{2\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\cdot\cos\frac{n\pi}{2}} = e^{2\cdot 1\cdot(-1)} = e^{-2} = \frac{1}{e^2}$$

$$\Rightarrow LIM(x_n) = \left\{1, e^2, \frac{1}{e^2}\right\}$$

(2) $\sum_{n=0}^{\infty} x_n^2$ - convergentă ?

$$\sum_{n=0}^{\infty} x_n \text{ - convergentă } \Leftarrow \lim_{n\to\infty} x_n = 0 \overset{C.C.}{\Longrightarrow}$$

$$\lim_{n\to\infty}\frac{x_n^2}{x_n} = \lim_{n\to\infty} x_n = 0 < +\infty \Rightarrow$$

$$\Rightarrow x_n^2 \text{ - convergentă } \Rightarrow \sum_{n=0}^{\infty} x_n^2 \text{ este convergentă}$$

(2)

③ $f(x)=\frac{x^2}{e^{x+1}} \Rightarrow f'(x)=\frac{2x\cdot e^{x+1}-x^2\cdot(e^{x+1})\cdot(x+1)'}{(e^{x+1})^2}=$

$=\frac{\cancel{e^{x+1}}\cdot(2x-x^2)}{(e^{x+1})^{\cancel{2}}}=\frac{2x-x^2}{e^{x+1}}$

$f''(x)=\frac{(2-2x)\cdot e^{x+1}-(2x-x^2)\cdot(e^{x+1})}{(e^{x+1})^2}=\frac{\cancel{e^{x+1}}\cdot(2-4x+x^2)}{(e^{x+1})^{\cancel{2}}}=\frac{2-4x+x^2}{e^{x+1}}$

$f'''(x)=\frac{(-4+2x)(e^{x+1})-(2-4x+x^2)\cdot e^{x+1}}{(e^{x+1})^2}=\frac{-6+6x-x^2}{e^{x+1}}$

$f^{(4)}(x)=\frac{(6-2x)\cdot e^{x+1}-(-6+6x-x^2)\cdot e^{x+1}}{(e^{x+1})^2}=\frac{12-8x+x^2}{e^{x+1}}$

...

$f^{(n)}(x)=\frac{(-1)^{n+1}(-n(n-1)+2nx-x^2)}{e^{x+1}}$

I Etapa de verificare

$P(1): f(1)=\frac{1}{e^{1+1}}=\frac{1}{e^2} \Rightarrow f'(1)=\frac{2-1}{e^2}=\frac{1}{e^2}$

II Etapa de demonstrare

Presupunem $p(n)$: "A" și dem. $p(n+1)$ ca fiind "A"

$p(n): f^{(n)}(x)=\frac{(-1)^{n+1}(-n(n-1)+2nx-x^2)}{e^{x+1}}$ "A"

$p(n+1): f^{(n+1)}(x)=(f^{(n)}(x))'=\frac{(-1)^{n+1}((2n-2x)-(-n(n-1)+2nx-x^2))}{e^{x+1}}=$

$=\frac{(-1)^{n+2}(-n(n+1)+2(n+1)\cdot x-x^2)}{e^{x+1}} \Rightarrow p(n+1)$ "A"