3.2.43

Să se determine rangul listelor de vectori din $\mathbb{R}^4$:

(1) $[\underbrace{[0,1,3,2]}_{u_1}, \underbrace{[1,0,5,1]}_{u_2}, \underbrace{[-1,0,1,1]}_{u_3}, \underbrace{[3,-1,-3,-4]}_{u_4}, \underbrace{[2,0,1,-1]}_{u_5}]^t$

$$A=\begin{pmatrix} 0 & 1 & -1 & 3 & 2 \\ 1 & 0 & 0 & -1 & 0 \\ 3 & 5 & 1 & -3 & 1 \\ 2 & 1 & 1 & -4 & -1 \end{pmatrix}$$

$d_p = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0 \Rightarrow$ hạng $(A) = 2$

$d_c = \begin{vmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 3 & 5 & 1 \end{vmatrix} = -5-1 = -6 \neq 0 \Rightarrow$ hạng $(A) = 3$

$d_c = \begin{vmatrix} 0 & 1 & -1 & 2 \\ 1 & 0 & 0 & 0 \\ 3 & 5 & 1 & 1 \\ 2 & 1 & 1 & -1 \end{vmatrix} = 1 \cdot (-1)^3 \cdot \begin{vmatrix} 1 & -1 & 2 \\ 5 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} =$

$= -(-\cancel{1}+\cancel{10}-\cancel{1}-\cancel{2}-\cancel{5}-\cancel{1}) = 0$

$d_c = \begin{vmatrix} 0 & 1 & 3 \\ 1 & 0 & -1 \\ 3 & 5 & -3 \end{vmatrix} = 15 - \cancel{3} + \cancel{3} = 15 \neq 0$

$d_c = \begin{vmatrix} 0 & 1 & 3 & 2 \\ 1 & 0 & -1 & 0 \\ 3 & 5 & -3 & 1 \\ 2 & 1 & -4 & -1 \end{vmatrix} \overset{C_1 = C_1 + C_3}{=\!=} \begin{vmatrix} 3 & 1 & 3 & 2 \\ 0 & 0 & -1 & 0 \\ 0 & 5 & -3 & 1 \\ -2 & 1 & -4 & -1 \end{vmatrix} =$

$= (-1)(-1)^5 \begin{vmatrix} 3 & 1 & 2 \\ 0 & 5 & 1 \\ -2 & 1 & -1 \end{vmatrix} = -15 - 2 + 20 - 3 = 0$

$\Rightarrow$ deci hạng $(A) = 3$

2) $v = [(1,2,3,0), (0,1,-1,1), (3,7,8,1), (1,3,2,1)]^t$

pivot

$A = \begin{pmatrix} 1 & 0 & 3 & 1 \\ 2 & 1 & 7 & 3 \\ 3 & -1 & 8 & 2 \\ 0 & 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \sim$

$\sim \begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow$ hạng $(A) = 2$

(3) $v = \{[1,2,-1,2], [2,3,0,-1], [2,4,0,6], [1,2,1,4], [3,6,-1,-1], [1,3,-1,0]\}^t$

$$A = \begin{pmatrix} 1 & 2 & 2 & 1 & 3 & 1 \\ 2 & 3 & 4 & 2 & 6 & 3 \\ -1 & 0 & 0 & 1 & -1 & -1 \\ 2 & -1 & 6 & 4 & -1 & 0 \end{pmatrix}$$

$$d_p = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1 \neq 0 \Rightarrow \text{rang}(A) = 2$$

$$d_c = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ -1 & 0 & 0 \end{vmatrix} = -8 + 6 = -2 \neq 0 \Rightarrow \text{rang}(A) = 3$$

$$d_c = \begin{vmatrix} 1 & 2 & 2 & 1 \\ 2 & 3 & 4 & 2 \\ -1 & 0 & 0 & 1 \\ 2 & -1 & 6 & 4 \end{vmatrix} \overset{C_1 = C_1 + C_4}{=\!=} \begin{vmatrix} 2 & 2 & 2 & 1 \\ 4 & 3 & 4 & 2 \\ 0 & 0 & 0 & 1 \\ 6 & -1 & 6 & 4 \end{vmatrix} =$$

$$= 1 \cdot (-1)^7 \begin{vmatrix} 2 & 2 & 2 \\ 4 & 3 & 4 \\ 6 & -1 & 6 \end{vmatrix} = -2 \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 6 & -1 & 6 \end{vmatrix} = -2 \cdot (\cancel{18} + 24 - \cancel{4} - \cancel{18} + \ldots$$
$$- 24) = 0$$

$$d_c = \begin{vmatrix} 1 & 2 & 2 & 3 \\ 2 & 3 & 4 & 6 \\ -1 & 0 & 0 & -1 \\ 2 & -1 & 6 & -1 \end{vmatrix} \underset{C_4 = C_4 - C_1}{=\!=} \begin{vmatrix} 1 & 2 & 2 & 2 \\ 2 & 3 & 4 & 4 \\ -1 & 0 & 0 & 0 \\ 2 & -1 & 6 & -3 \end{vmatrix} = (-1) \cdot (-1)^5 \begin{vmatrix} 2 & 2 & \ldots \\ 3 & 4 & 4 \\ -1 & 6 & \ldots \end{vmatrix}$$

$$= 2 \cdot \begin{vmatrix} 1 & 1 & 1 \\ 3 & 4 & 4 \\ -1 & 6 & -3 \end{vmatrix} = 2 \cdot (-12 + 18 - \cancel{4} + \cancel{4} + 9 - 24)$$
$$= 2 \cdot (-9) = -18 \Rightarrow \text{rang}(A) = 4$$