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Long-range memory, distributional variation and randomness of bitcoin volatility

Salim Lahmiri [a,1], Stelios Bekiros [b,2,*], Antonio Salvi [c,3]

[a] ESCA School of Management, 7, rue Abou Youssef El Kindy, Boulevard Moulay Youssef, 20070 Casablanca, Morocco
[b] European University Institute (EUI), Department of Economics, Via delle Fontanelle 18, I-50014, Florence, Italy
[c] LUM Jean Monnet University, Faculty of Economics, 18, 70010 Casamassima, Bari, Italy

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ABSTRACT

We investigate the nonlinear patterns of volatility in seven Bitcoin markets. In particular, we explore the fractional long-range dependence in conjunction with the potential inherent stochasticity of volatility time series under four diverse distributional assumptions, i.e., Normal, Student-*t*, Generalized Error (GED), and *t*-Skewed distribution. Our empirical findings signify the existence of long-range memory in Bitcoin market volatility, irrespectively of distributional inference. The same applies to entropy measurement, which indicates a high degree of randomness in the estimated series. As Bitcoin markets are highly disordered and risky, they cannot be considered suitable for hedging purposes. Our results provide strong evidence against the efficient market hypothesis.

1. Introduction

Volatility plays a major role in risk modeling and assessment as well as in the pricing of complex financial derivative products. Therefore, studying the inherent features of the conditional variance of financial time series has received a growing interest in econophysics recently. For instance, an adaptive stochastic model was proposed to explore the internal dynamics of American and Spanish stock markets [1] or GARCH-family models were employed in predicting gold market volatility [2]. A novel set-up to forecast the parameters of constant elasticity of variance implied by American option pricing was introduced in [3], whilst in [4] the dynamics of the links between volatility and market integration as well as between persistence and integration were investigated for emerging stock markets. Furthermore, a hybrid algorithm combining grey modeling and extreme machine learning to forecast the volatility of China interbank offered rate was presented in [5], the complexity in volatility series of world major financial and commodity markets was examined in [6], while volatility predictability in exchange markets via the utilization of artificial neural networks, was proposed in [7].

Other studies attempted an exhaustive investigation of fractal scaling effects and risk preference of traders under option pricing and portfolio hedging [8], the analysis of asymmetry, leverage and persistence of shocks upon price volatility in the major markets of fertilizers [9], or the impact of leverage effects and economic policy uncertainty on future realized volatility via regime switching [10]. Overall, the study of long memory in volatility series becomes more and more influential, and ultimately of immense importance in recent econophysics literature. This particular nonlinear fractional pattern was examined in case of G7's major stock market indices [11], for high-frequency returns of the Athens composite share price index [12], in case of gold price returns during different crisis sub-periods [13], in Moroccan family business stocks [14], Chinese and U.S. stock markets [15] and for the Indian realized volatility series [16].

The primary aim of our work is to extract and detect potential long memory patterns hidden under inherent randomness within the volatility series of various Bitcoin markets under different distributional assumptions. To the best of our knowledge this crucial issue is missing from the econophysics literature on Bitcoin [17–19] and no one has examined it before. While the long range dependence of Bitcoin was examined via detrended fluctuation analysis [17] and rescaled Hurst exponent [18], yet the aforementioned direction was never pursued. More recently, chaos,

* Corresponding author.
E-mail addresses: slahmiri@esca.ma (S. Lahmiri), stelios.bekiros@eui.eu (S. Bekiros), salvi@lum.it (A. Salvi).

[1] ESCA School of Management, 7, Abou Youssef El Kindy Street, BD Moulay Youssef, Casablanca, Morocco.
[2] Department of Economics, Via delle Fontanelle 18, I-50014, Florence, Italy.
[3] Faculty of Economics, 18, 70010 Casamassima, Bari, Italy.

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randomness and multi-fractality in Bitcoin markets were examined in [19]. Empirical results showed that Hurst exponent estimated by detrended fluctuation analysis changes significantly during the first years of the existence of Bitcoin and tends to stabilize in recent times [17]. In addition, the rescaled Hurst exponent showed strong anti-persistence in Bitcoin returns [18]. In [19] it was demonstrated that Bitcoin price is chaotic during periods of low and high variability, while the level of uncertainty significantly increased during high price variability. Both prices and returns exhibit long-range correlations indicating that fat-tailed probability distributions could be the main source of multi-fractality. Other works examined the similarity between Bitcoin, gold and US dollar volatility [20] and the ability of several generalized autoregressive conditional heteroskedasticity (GARCH) models to explain Bitcoin variance [21]. It was found that GARCH modeling explained gold and American dollar variance [20], while fitted the data adequately [21].

Bitcoin markets which entail various digital currencies trading are speculative and highly volatile [19–21]. Due to the remarkable development of the Bitcoin markets in terms of trading volume through recent years, the dynamics of market volatility may exhibit some unique characteristics, such as fractality, long-range memory and randomness, require further documentation and analysis in order to better comprehend the inherent nonlinear dynamics and the forecastability of crypto currency markets. We intend to show which factors drive those nonlinear systems considering that the magnitude of long-range dependencies could destabilize them towards deterministic chaotic trajectories or high stochasticity or even hybrid behaviours.

We employ diverse specifications for the filtered error distribution of the first moment of the return time series so that the second moment (variance) is modeled under the presence of fat-tails, leptokurtosis, and skewness, which are well-documented stylized facts of financial markets. Those features are investigated for seven different Bitcoin markets for the sake of generalization of the findings. Eventually, we attempt to better grasp the speculative nature of digital currency markets and accordingly enrich the relevant econophysics literature [1–21].

The remainder of the paper is organized as follows: Section 2 outlines the introduced approaches utilized in our exhaustive investigation. We incorporate and combine GARCH-based estimation, long-range memory detection, different distributional assumptions and randomness entropic measurement. The data analysis and empirical results are exposed in Section 3. Finally, inference and conclusions are presented in Section 4.

2. Methodological approaches

In quantitative finance, volatility refers to the conditional standard deviation (or conditional variance) of the underlying asset returns. In this regard, the accurate estimation of time-varying volatility, without a doubt is important for risk evaluation, derivative pricing, hedging, trading and forecasting. In our study, we assess fractality in the volatility structure of Bitcoin markets, via the application of the fractionally integrated GARCH (FIGARCH) framework, which captures long-range memory by incorporating fractional integration in any model specification of the 2nd moment [22]. Indeed, the FIGARCH model [22] was proposed as an extension to the standard GARCH family in order to quantify long-range memory in volatility series. As opposed to the naive GARCH model, the FIGARCH process is capable of distinguishing between short memory and long memory in the underlying time series. We employ FIGARCH to estimate time-varying volatility and assess its inherent long-range dependence structure under different assumptions regarding the distribution of the filtered residuals. Hence, the normal (Gaussian), Student's *t*, generalized error distribution (GED) and *t*-skewed distribution are incorporated in our models in an attempt to reveal which one better matches the observed stylized features of Bitcoin volatility.

Consider that return series are denoted by r_t where t is the time index. Then, the popular benchmark GARCH (1,1) model for simplicity herein, is given by:

$$r_t = \mu + \varepsilon_t \tag{1}$$

with,

$$\varepsilon_t = h_t^{0.5}\eta_t \tag{2}$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} \tag{3}$$

where μ is the conditional mean, h is the conditional standard deviation (volatility), and η~N(0,1). Then, the FIGARCH (1,*d*,1) is given by:

$$h_t = \omega + \beta h_{t-1} + [1-(1-\beta L)^{-1}(1-\phi L)(1-L)^d]\varepsilon_t^2 \tag{4}$$

where d is the fractional integration parameter used to characterize long-range memory in the volatility series h_t. For the parameters, it is indicated that $0 \le d \le 1$, $\omega > 0$, ϕ, $\beta < 1$, while L stands for the lag operator. In the FIGARCH framework [22], when $0 < d < 1$ intermediate ranges of persistence are allowed. When $d = 1$ volatility shocks exhibit full integrated persistence, whilst in case of $d = 0$ volatility shocks decay under a geometric rate. In our work, all parameters of the FIGARCH model are estimated by using the standard maximum likelihood method. In addition, as mentioned previously, the FIGARCH process is estimated for four distributions of the error terms η_t: normal (Gaussian), Student-*t*, generalized error distribution (GED), and *t*-skewed distribution.

The use of the normality assumption is widely considered valid in many empirical applications for financial markets, wherein the Efficient Market Hypothesis (EMH) emerges as the dominant model. However, it has been documented that it does not fit real data and its alternative, namely the Fractal Market Hypothesis (FMH), could be more realistic. Therefore, alternative distributional assumptions are desirable and perhaps impending for estimation purposes. Specifically, the Student-*t* distribution incorporates fat-tailedness in data, the *t*-skewed distribution is able to flexibly fit leptokurtic and skewed return distributions, and the generalized error distribution (GED) poses the property of flexible symmetry and tails as well. Under each distribution assumption, a particular long memory parameter d is obtained.

Finally, we apply the well-known Shannon entropy measure [23] to quantify randomness, stochasticity whereby informational redundancy, in the investigated series. Considering the time series$\{x_t\}_{t=1}^n$, we express Shannon entropy (*SE*) as:

$$SE(x) = -\sum_{y=1}^{n} p_i \log(p_i) \tag{5}$$

where p_i is a discrete probability such that $\sum_i p_i = 1$. The Shannon entropy obtains its maximum score when all the values of the underlying time series $\{x_t\}_{t=1}^n$are equally probable. Interestingly, when it approaches log(n), $\{x_t\}_{t=1}^n$is nearly random. Conversely, the Shannon entropy reaches the minimum value when a specific x_i is guaranteed to occur, with $Prob(x_i) = 1$.

3. Data and empirical results

Daily Bitcoin price datasets for seven markets were extracted from [24] source. The seven Bitcoin markets (in US dollars) and their respective time periods are the following: BITX (15 November 2016 to 9 November 2017), CEX.IO (24 August 2015 to 9 November 2017), COINBASE (13 January 2015 to 9 November 2017), EXMO (6

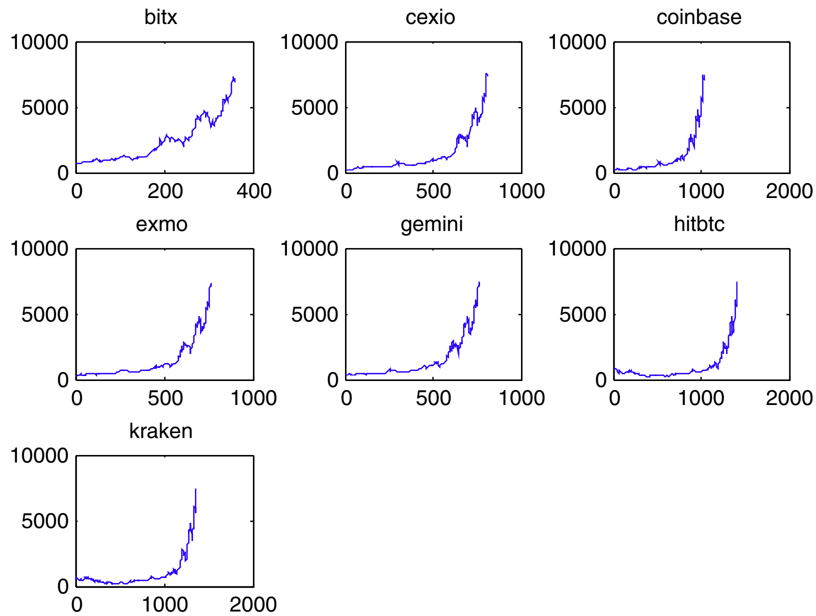

Fig. 1. Evolution of Bitcoin prices in each market. The sample size and price levels are displayed on the *x*-axis and *y*-axis respectively.

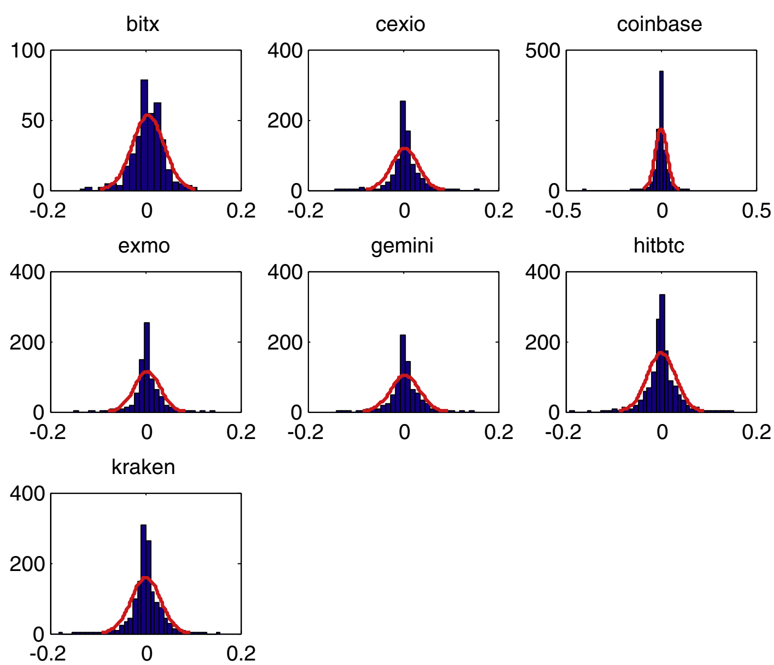

Fig. 2. Histogram of returns depicting the normal distribution fit with a red color. Returns and associated frequencies are respectively presented on the *x*-axis and *y*-axis.

October 2015 to 9 November 2017), GEMINI (8 October 2015 to 9 November 2017), HITBTC (27 December 2013 to 9 November 2017), and KRAKEN (14 February 2014 to 9 November 2017). For each one of the Bitcoin markets, the continuously compounded returns are estimated as $r_t = \log(p_t)$-$\log(p_{t-1})$, where p_t is the Bitcoin price on day *t*. For illustration purposes, the evolution of the prices is exhibited in Fig. 1, and the histogram of returns also displaying the assumed normal distribution fit, is shown in Fig. 2. It can be observed that returns are not normally distributed as indicated in Fig. 2. All return distributions are leptokurtic with statistically significant long tails. These initial observations lead directly to the implementation of alternative distributions, which could realistically describe better these data characteristics and stylized patterns. In other words, it appears extremely appealing to examine volatility dynamics under different distributional assumptions.

The estimated volatility based on the *t*-skewed distribution for the 1st moment residuals is displayed in Fig. 3. As shown, the volatility associated with HITBTC and KRAKEN is more pronounced than that of the other markets. The estimated long-range parameter *d* is presented in Fig. 4 for each market and under each distributional assumption. Parameter *d* measures the persistence of the conditional variance i.e, h_t in Eq. 4 thereby indicates the degree of memory in the underlying volatility structure. According to Fig. 4, the computed parameter *d* is between zero and one, hence volatil-

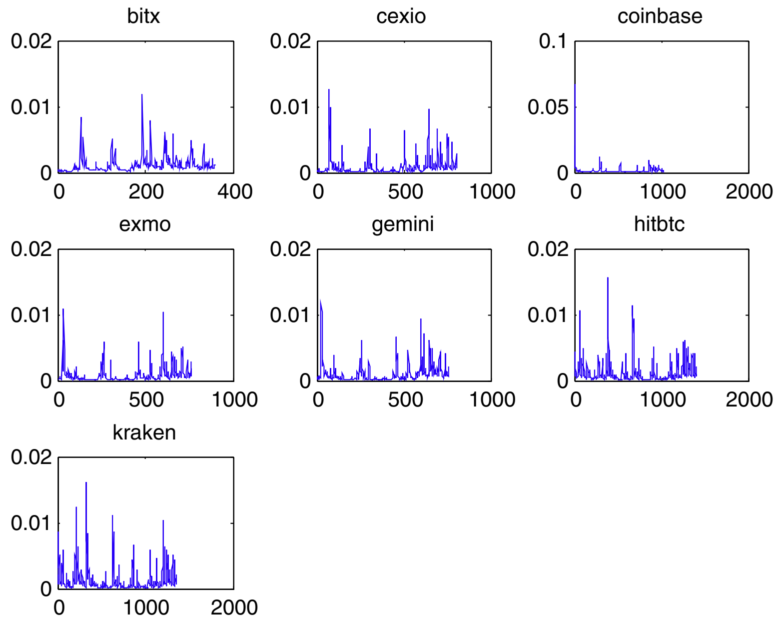

Fig. 3. Estimated volatility based on the t-skewed distribution assumption for the residuals. The sample size and the volatility values h_t are displayed on x-axis and y-axis respectively.

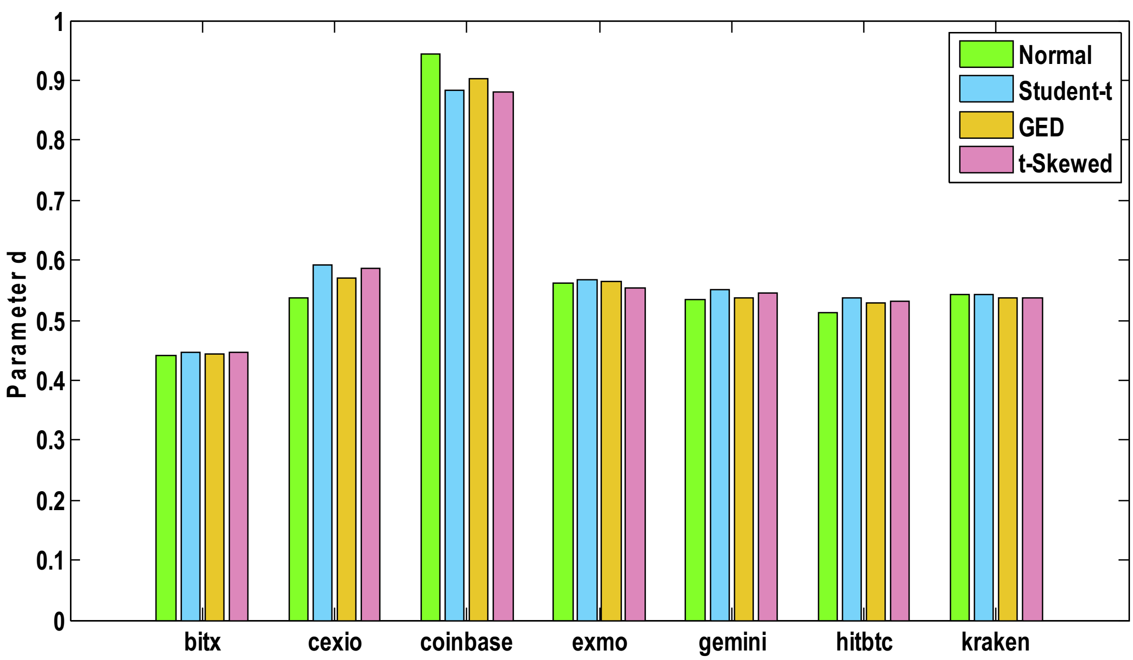

Fig. 4. Estimated long-range parameter d under each distributional assumption.

ity in all Bitcoin markets exhibits long-range dependence under all assumptions. Interestingly, we indicate that the long-range memory property in CEX.IO and COINBASE markets varies vis-à-vis the employed distributions. On the contrary, it is similar across distributions for the other Bitcoin markets. Furthermore, we obtain the score of $0.5 < d < 1$ for all Bitcoin markets and for all distribution assumptions, except for the BITX market where $0 < d < 0.5$. Evidently, the COINBASE market exhibits the highest long-range parameter d, whilst the BITX presents the lowest one.

Finally, the Shannon entropy statistic of the volatility series (h_t) for all markets and under each distribution is displayed in Fig. 5. The COINBASE and KRAKEN markets respectively exhibit the lowest and highest Shannon entropy for all assumptions. Obviously, all entropic scores exceed the double value estimated for the COINBASE market. In that, all volatilities show a significantly higher level of disorder than for the COINBASE. This could be rationalized by the fact that price evolution - and eventually volatility behaviour - in the COINBASE market, entails more information than in other markets. It is extremely interesting that the long-range memory of COINBASE volatility is the highest, whilst the entropy of the COINBASE market is the lowest.

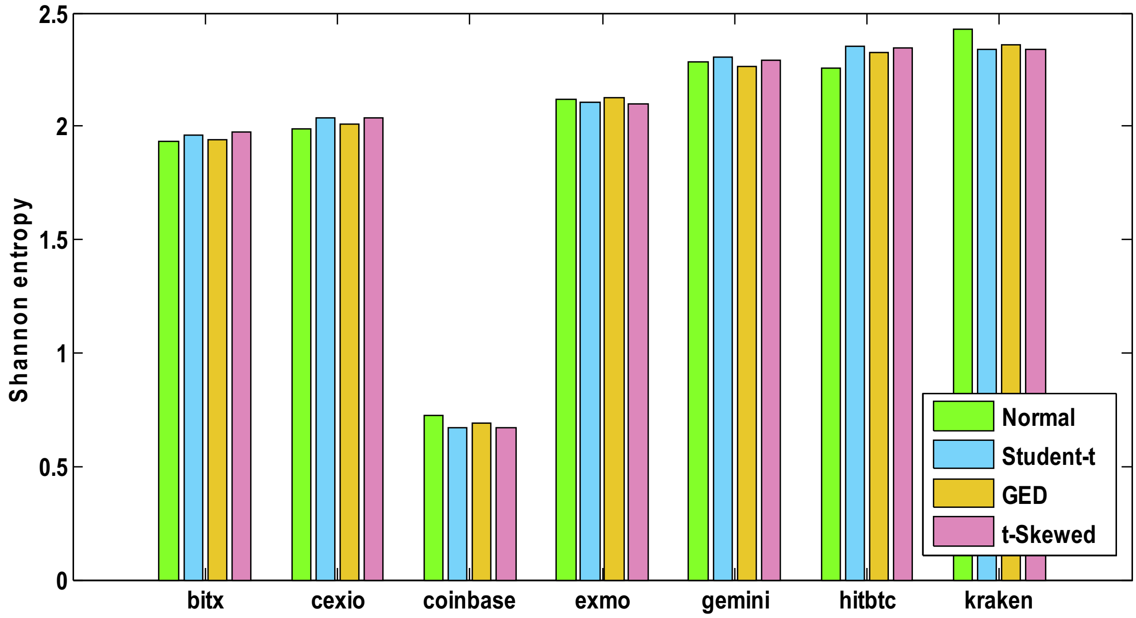

Fig. 5. Computed Shannon entropy for the volatility series of each Bitcoin market under all distributional assumptions.

4. Conclusions

This work attempts for the first time to capture the underlying nonlinear patterns of the time-varying volatility for seven major Bitcoin markets. We employ a FIGARCH-based modeling approach to gauge the long-range memory structure of the examined series, in conjunction with the measurement of the Shannon entropy so as to further assess fractality and randomness of the estimated volatilities. To achieve a realistic and accurate estimation of the volatility and its nonlinear structure, the FIGARCH model was estimated under four different distributional assumptions for the error term. Specifically, we used the Normal, Student-*t*, generalized error and *t*-skewed distribution. The empirical results revealed a significant underlying long-range memory property in all volatility series under all distributional assumptions. The presence of long range memory leads to dependencies between distant volatility trajectories of the investigated nonlinear systems, namely for all Bitcoin markets. Consequently, future volatility can be predicted via the utilization of past information i.e., volatility values. This provides strong evidence against the efficient market hypothesis.

We found that the order of integration in every other Bitcoin market is higher compared to the one of BITX, especially the order of integration for COINBASE. The long-range dependence in BITX is the lowest whilst for COINBASE the highest. Thus, predicting volatility in BITX market can be cumbersome compared to the other Bitcoin markets, and certainly harder than predicting COINBASE volatility.

Additionally, the long-range parameter of COINBASE appears to be significantly high, i.e., approximately 0.9 taking into account all four distributional assumptions, which leads to the conclusion that COINBASE volatility exhibits a mean-reverting behavioural pattern. One important key result is the presence of high entropy in all markets, except COINBASE. This implies the existence of a low degree of “organization” in all markets, while on the contrary the low entropic score for the COINBASE provides evidence of systemic order and stability.

The discussion regarding the effect of the choice of distribution assumption upon the estimated long-range memory parameter *d*, also after the visual inspection of Fig. 4, suggests that this impact is absent across Bitcoin markets. However, some exceptions are pronounced, as in the case of CEX.IO and COINBASE whereby the parameter *d* varies significantly vis-à-vis distribution selection. Similarly, our results indicate that the calculated entropy does not vary according to the choice of distribution, except only for HITBTC and KRAKEN.

Overall, we found evidence of long-range dependence in the volatility series of the investigated markets. Indeed, as most Bitcoin markets are highly disordered and risky, they cannot be considered suitable for hedging purposes. Our findings may be of utmost importance to investors, policy makers, traders and portfolio managers.

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