CSC520 - Artificial Intelligence

Lecture 11

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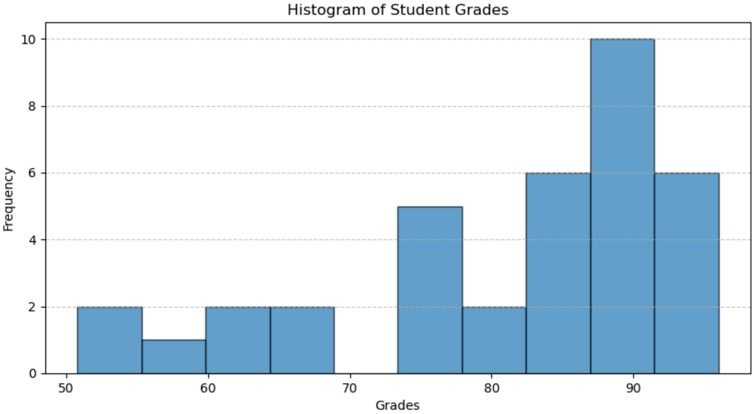
Feb 13, 2025

Agenda

- Knowledge-based agents
- Propositional logic

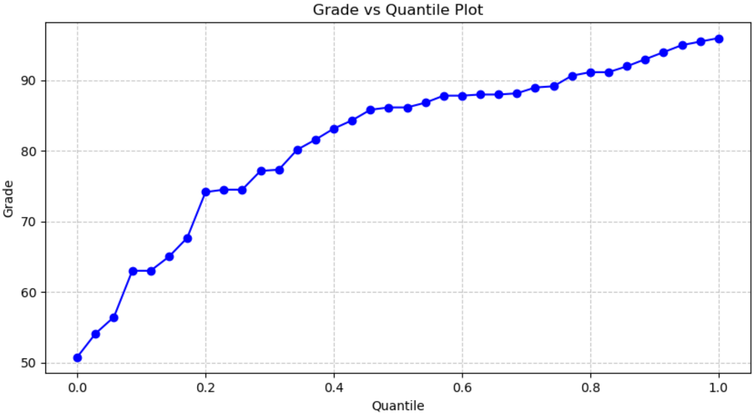
Assignment 1 Histogram

520 Grades Histogram.png



Assignment 1 Quantiles

520 Grades Quantiles.png



Knowledge-based Agent

- Knowledge-base is a set of sentences in a formal language
- Sentences represent some assertions about the world
- Agent can add sentences (using TELL) to its knowledge-base and query its knowledge-base (using ASK)
  - TELL: If there is a high chance of rain, carry an umbrella
  - TELL: If it is cloudy, there is a high chance of rain
  - TELL: It is cloudy
  - ASK: Should I carry an umbrella?
- Declarative approach for building an agent
  - TELL it what it needs to know
- In contrast, procedural approach encodes behaviors in code

Knowledge-based Agent

function KB-AGENT(*percept*) **return** an action

persistent: KB, a knowledge base, t, a time counter, initially 0

TELL(KB, MAKE-PERCEPT-SENTENCE($percept, t$))

$action \leftarrow$ ASK(KB, MAKE-ACTION-QUERY(t)

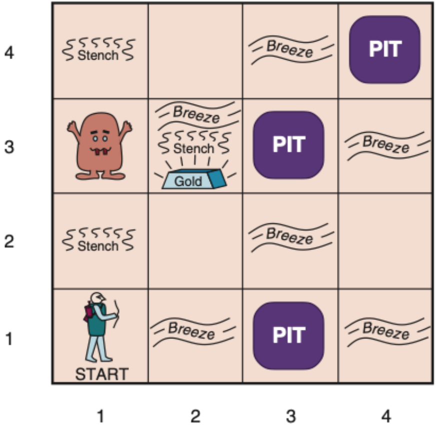
TELL(KB, MAKE-ACTION-SENTENCE($action, t$))

$t \leftarrow t + 1$

return *action*

Wumpus World

- Performance measure
  - Gold +1000, death -1000
  - -1 per step, -10 for arrow
- Environment
  - 4x4 grid of rooms
  - Squares adjacent to wumpus are smelly and adjacent to pit are breezy
  - Glitter if gold in the square
  - Agent dies if pit or wumpus
  - Shooting kills wumpus
- Actuators
  - Forward, turn left, turn right, grab, shoot, climb
- Sensors
  - Stench, breeze, glitter, bump, scream



Wumpus Environment

- Fully observable? No, partially observable due to local perception
- Deterministic? Yes, since outcomes are specified
- Episodic? No, sequential at the level of actions
- Static? Yes, wumpus and pits don't move
- Discrete? Yes
- Single-agent? Yes, wumpus is part of the env

Exploring the Wumpus World

- KB initially contains the rules of the environment
- $[1,1]$: Percept $[none, none, none, none, none]$, $[1,2]$ and $[2,1]$ are safe
- $[2,1]$: Percept $[none, breeze, none, none, none]$, pit in $[2,2]$ or $[3,1]$

Exploring the Wumpus World

- $[1, 2]$: Percept $[\mathit{stench}, \mathit{none}, \mathit{none}, \mathit{none}, \mathit{none}]$, wumpus can be in $[1, 1]$, $[1, 3]$ or $[2, 2]$
- Wumpus not in $[1, 1]$
- Wumpus not in $[2, 2]$ since no stench in $[2, 1]$
- Wumpus is in $[1, 3]$
- $[2, 2]$ is safe since no breeze in $[1, 2]$
- pit in $[3, 1]$; move to safe cell $[2, 2]$

Exploring the Wumpus World

- [2, 2]: Percept [*none*, *none*, *none*, *none*, *none*], [2, 3] and [3, 2] are safe; agent moves to [2, 3]
- [2, 3]: Percept [*stench*, *breeze*, *glitter*, *none*, *none*]
- Pick up gold!
- Pit in [3, 3] or [2, 4]

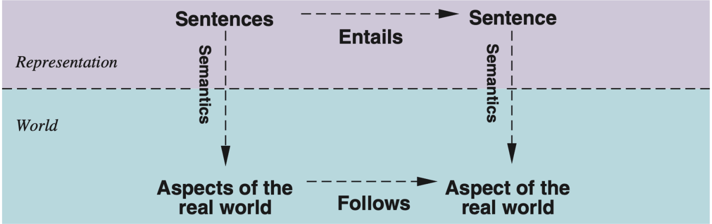
Logic in General

- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the meaning of sentences
  - Define truth of a sentence in a "possible world" (model)
- E.g., consider the language of arithmetic
- Syntax
  - $x + 2 \geq y$ is a sentence
  - $x + y >$ is not a sentence
- Semantics
  - $x + 2 \geq y$ is *true* iff $x + 2$ is no less than y
  - $x + 2 \geq y$ is *true* in a *world* where $x = 7, y = 1$
  - $x + 2 \geq y$ is *false* in a *world* where $x = 0, y = 6$

Entailment

- Entailment means that one thing follows from another
- $KB \models \alpha$
- Knowledge base KB entails a sentence α if and only if α is true in *all worlds* where KB is true
- E.g., $x + y = 4$ entails $x = 4 - y$

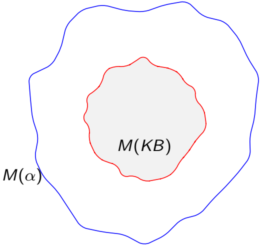
Entailment



If *KB* is true in the real world, then any sentence α derived from *KB* by a *sound inference procedure* is also true in the real world

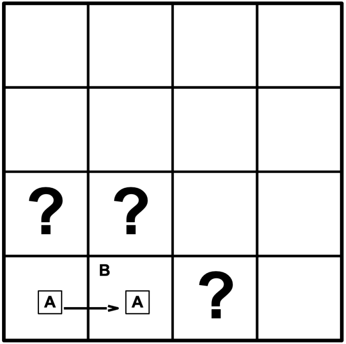
Models

- Model is a formally structured world with respect to which truth can be evaluated
- m is a model of a sentence α if α is *true* in m
  - Suppose α is the sentence $x + 2 \geq y$
  - Then, one model m_1 of α is a world where $x = 7, y = 1$
  - Another model m_2 of α is a world where $x = 4, y = 2$
- $M(\alpha)$ is a set of all models of α
  - For the sentence $x + 2 \geq y$: $M(\alpha) = \{m_1, m_2, \ldots\}$
- Then $KB \models \alpha$ iff $M(KB) \subseteq M(\alpha)$

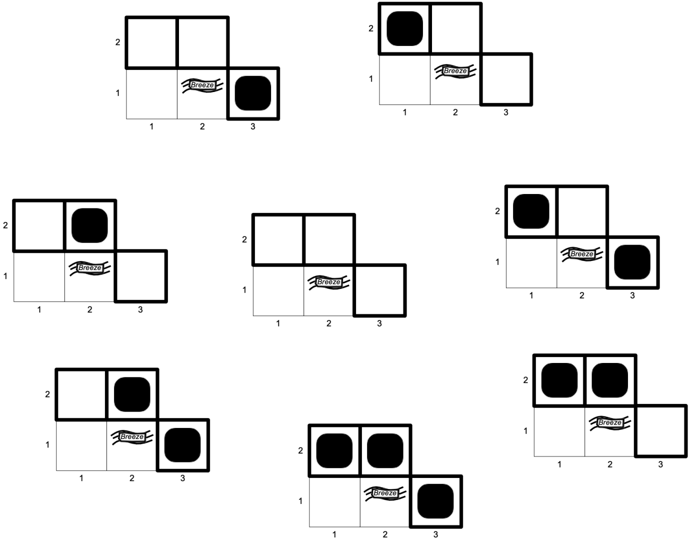


Entailment in Wumpus World

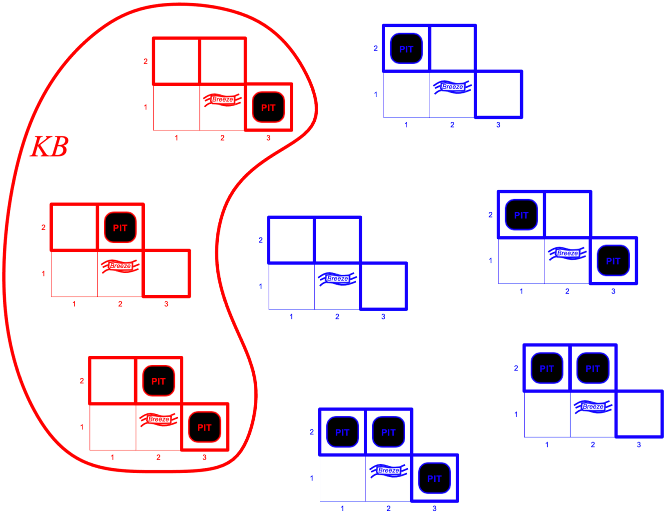
- Consider the situation after detecting nothing in $[1, 1]$ followed by detecting breeze in $[2, 1]$
- Possible models for ?s assuming only pits
- 3 Boolean variables, $2^3 = 8$ possible models



Wumpus Models

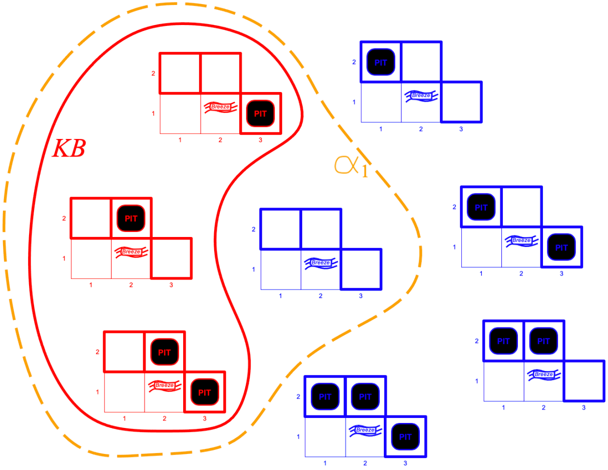


Wumpus Models



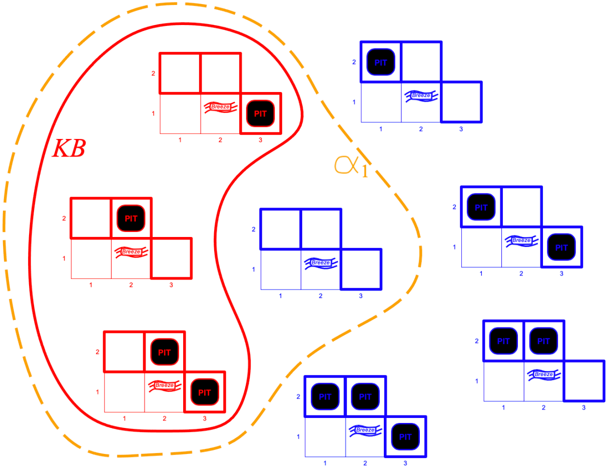
KB = wumpus world rules + observations

Wumpus Models



KB = wumpus world rules + observations
$\alpha_1 = [1, 2]$ is safe; $KB \models \alpha_1$ proved by model checking.

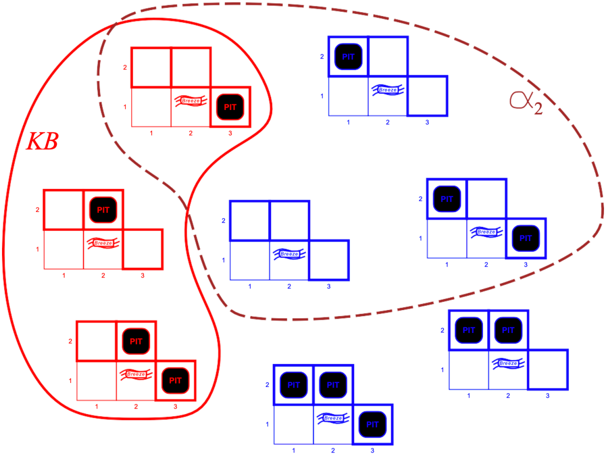
Wumpus Models



$$KB = \text{wumpus world rules} + \text{observations}$$

$\alpha_1 = [1,2]$ is safe; $KB \models \alpha_1$ proved by model checking. $M(KB) \subset M(\alpha_1)$

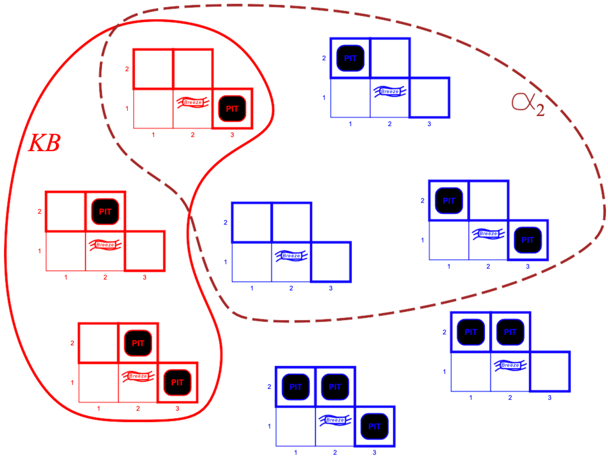
Wumpus Models



$$KB = \text{wumpus world rules} + \text{observations}$$

$\alpha_2 = [2,2]$ is safe; Does KB entail α_2?

Wumpus Models



KB = wumpus world rules + observations

$\alpha_2 = [2, 2]$ is safe; Does KB entail α_2? $M(KB) \not\subset M(\alpha_2)$

Inference Procedures

- $KB \vdash_i \alpha$: Sentence α can be derived from KB by procedure i
- Soundness: Procedure i is sound if it derives only entailed sentences
  - When $KB \vdash_i \alpha$ it is also true that $KB \models \alpha$
- Completeness: Procedure i is complete if it can derive any sentence that is entailed
  - Whenever $KB \models \alpha$ it is also true that $KB \vdash_i \alpha$

Propositional Logic

- Propositional logic is the simplest logic
- Proposition symbols e.g. P, Q, etc. are sentences
- Negation: If S is a sentence, $\neg S$ is a sentence
- Conjunction: If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence
- Disjunction: If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence
- Implication: If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence
- Biconditional: If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence

Propositional Logic Semantics

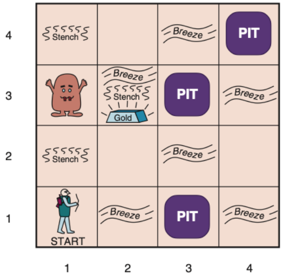
- Model specifies true or false value for each proposition
  - Consider three propositions: $P_{1,2}, P_{2,2}, P_{3,1}$
  - A possible model is: $m_1 = \{P_{1,2} = \mathit{false}, P_{2,2} = \mathit{false}, P_{3,1} = \mathit{true}\}$
- Rules for evaluating truth with respect to a model m
  - $\neg S$ is true iff S is false
  - $S_1 \wedge S_2$ is true iff S_1 is true and S_2 is true
  - $S_1 \vee S_2$ is true iff S_1 is true or S_2 is true
  - $S_1 \Rightarrow S_2$ is true iff S_1 is false or S_2 is true
  - $S_1 \Leftrightarrow S_2$ is true iff $S_1 \Rightarrow S_2$ is true and $S_2 \Rightarrow S_1$ is true
- Simple recursive process evaluates an arbitrary sentence
  - $\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1})$

Truth Tables for Connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Wumpus World Sentences

- Let $P_{i,j}$ be *true* if there is a pit in $[i, j]$
- Let $B_{i,j}$ be *true* if there is breeze in $[i, j]$
- There is no pit in $[1, 1]$
  $R_1 : \neg P_{1,1}$
- Pits cause breeze in adjacent squares
  $R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$
  $R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$
- There is no breeze in $[1, 1]$
  $R_4 : \neg B_{1,1}$
- There is breeze in $[2, 1]$
  $R_5 : B_{2,1}$



Model Checking Wumpus World Sentences

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

KB

- $R_1 : \neg P_{1,1}$
- $R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
- $R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
- $R_4 : \neg B_{1,1}$
- $R_5 : B_{2,1}$

- Does $KB \models \alpha$ where $\alpha = \neg P_{1,2}$?
- Does $KB \models \alpha$ where $\alpha = \neg P_{2,2}$?

Inference by Enumeration

function TT-ENTAILS(KB, α) true or false

  inputs: KB, a knowledge base, α, a sentence

  symbols $\leftarrow$ a list of proposition symbols in KB and α

  return TT-CHECK-ALL($KB, \alpha, symbols, \{\}$)

function TT-CHECK-ALL($KB, \alpha, symbols, model$) true or false

  if EMPTY($symbols$) **then**

    if PL-TRUE($KB, model$) **then return** PL-TRUE($\alpha, model$)

    else return true

  else

    $P \leftarrow$ FIRST($symbols$)

    $rest \leftarrow$ REST($symbols$)

    return TT-CHECK-ALL($KB, \alpha, rest, model \cup \{P = true\}$)

      and TT-CHECK-ALL($KB, \alpha, rest, model \cup \{P = false\}$)

Class Exercise

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

KB

- $R_1 : \neg P_{1,1}$
- $R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
- $R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
- $R_4 : \neg B_{1,1}$
- $R_5 : B_{2,1}$

- Does $KB \models \alpha$ where $\alpha = \neg P_{3,1}$? Explain your answer.