# Components of a time series

1. Time series components

* Trend
* Seasonal
* Residual

1. Trend

* A long-term increase or decrease of the baseline of the time series
* Not necessarily linear

1. Seasonality

* A fixed periodic pattern in the time series
* Patterns relate to seasonal factors such as time of the year or day of week
* E.g.: Black Friday, Christmas, School holidays, etc.

1. Residual

* Everything leftover after subtracting the other components
* Useful to examine to determine how much structure of the underlying time series is captured in the trend and seasonality
* Large residuals -> there is still information to be extract through the trend or seasonal component or our methods for estimating the trend and seasonal component could be improved

1. Bringing it back together

* Either sum or multiply trend, seasonal, residual components together to recover the original series

1. Why is decomposition useful?

* EDA: to answer questions such as: ‘what was the impact of an ad campaign once we account for seasonality?’
* Pre-processing: useful for identifying outliers and can be used to impute outliers and missing data
* Feature engineering: derive features from the components to use as inputs in ML models
* Forecasting: forecast the components and aggregate to produce the final forecast

1. Decomposition is not always possible

* Not all time series can be easily broken down into components

# White noise

1. Motivation

* We will want to show how tools we introduce later (e.g., lag plots, correlation functions) behave for time series with various properties
* Two properties already covered in the course so far are trend and seasonality
* In this lecture we discuss a new property that can characterize the behavior of a time series – white noise

1. White noise

* Generate a time series, , by taking a random sample from any distribution of our choice (e.g. normal distribution with 0 mean and unit variance N(0, 1) ) at each time step t

A close-up of a math equation

Description automatically generated

A graph of white noise

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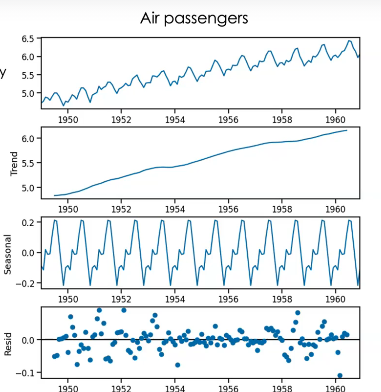
* Each value of is identically and independently distributed. There is no correlation between 2 points in time
* White noise
* If we sample from N(0, 1) -> Gaussian white noise
* Common to see this written as
* White noise has no predictive info in past values as there is no correlation at any 2 points in time
* It is useful to see how the tools we introduce later in the course behave when a time series is just white noise or contains white noise
* An important application of white noise is in the analysis of the residuals of a forecast or time series decomposition
* Residuals – difference between the actual and estimated values obtained through a forecast or time series decomposition

1. White noise applied to residuals

* Its common to think of time series as having a **non-random component** (e.g., trend, seasonality, autoregressive) and a **noise component**

Where is modeled as white noise to represent random effects for example from:

* Imperfect sensors
* Random fluctuations in the underlying process
* A forecast or time series decomposition, can be thought of as estimating the non-random component x(t)
* Assume we estimate perfectly, , then the residual would be equal to white noise:
* If the residuals of our forecast or decomposition look like white noise it means there is no more predictive information to be extracted from the data
* Some simple checks to see if your residuals are like white noise:
* Mean is zero
* Mean and variance don’t change over time – no trend or seasonality in residuals
* No autocorrelation – no correlation between a time point and any other in the past



* This decomposition produces residuals which:
* Have 0 mean
* The mean doesn’t change in time
* The variance does change in time
* This means that there is still some non white noise component left in the time series which could be caused by:
  + Imperfect extraction of trend and seasonality
  + An additional component exists which is not caught be trend or seasonality (e.g., an autoregressive component)

1. Summary

* A white noise time series is one generated by repeatedly sampling from any distribution where each sample is independent
* White noise has no correlation between any 2 time points – past values cannot be used to predict the future
* If the residuals of a decomposition or forecast look like white noise – there is no more info left to extract and we did a good job

# Additive and multiplicative decompositions

1. Time series decomposition

* Aim: decompose a time series into components
* Choice / assumption:
* How to choose?
* Domain knowledge around data generating process
* Looking at certain behaviors of the time series

1. Multiplicative decomposition – e.g.

* T-shirt sales of a growing online clothing brand
* Visitors to website increasing over time ->
* The product has seasonal demand. Each customer has higher prob, p(t), of purchasing during summer than winter

Where:

1. Multiplicative decomposition

A graph of a graph

Description automatically generated with medium confidence

* Magnitude of the seasonal fluctuations is proportional to the level of the time series

1. Additive decomposition - e.g.

* Air pollution on a particular road, pollution(t)
* There is some background level of air pollution, B(t) – long term trend
* Traffic from vehicles can add air pollution, T(t) – daily seasonality
* Traffic is busier during peak hours, hence, there is a daily seasonality
* So we expect:

1. Additive decomposition

A diagram of a graph

Description automatically generated with medium confidence

* The magnitude of the seasonal fluctuations does not change with the level of the time series

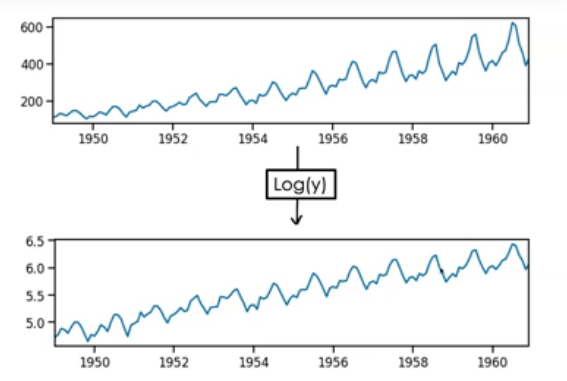
1. Summary

* Time series can be decomposed into a multiplicative or additive decomposition/model
* If the seasonal variation is proportional to the level of the time series -> multiplicative decomposition

# Log transform

1. Log transform

* The transform
* The log transform can be used to stabilize the variance of a time series
* Some forecasting models perform better if the variance of the time series does not increase with time (e.g., ARIMA)
* If the time series goes to 0, y = 0, then is undefined
* Can overcome by adjusting the transform to



1. Converting multiplicative to additive

* A multiplicative decomposition can be converted into an additive decomposition by taking the log of the target y(t)
* Take the log of y(t) for the multiplicative case:
* **If the underlying time series is multiplicative then log of series is additive**
* Provides intuition as to why log transform can stabilize the variance
* This technique is useful because some time series decomposition method (e.g., STL decompositions) only handle the additive case
* So we require log transforming the data first before using certain forecasting and decomposition methods

1. Summary

* The log transform can be used to stabilize the variance of a time series
* The log transform can convert a multiplicative time series to an additive one

# Box Cox transform

1. Motivation

* Some forecasting and decomposition methods perform better if the variance of the time series does not change with the level of the time series (e.g., ARIMA)
* The log transform does not always stabilize the variance of a time series – depends on the time series

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1. What kind of transforms are useful?

* What kind of transform of our time series, could be useful in stabilizing the variance? Depends on the time series
* To stabilize the variance we want the transformation to remove the interaction between the trend and seasonality or noise term so we can write them additively
* In our toy time series we had:
* We can think of this as:
* A better transform would be using the square root

A graph of a toy

Description automatically generated with medium confidence

* An example of what is called a power transform. We raised the original variable to some power
* A more general way of writing power transforms are
* Sometimes a power transform can be better at stabilizing the variance than a log transform – depends on the time series
* The Box Cox transform combines both a log transform and a power transform

1. Box Cox transform

* Defined as:
* Different values of – different kinds of transform

A table with mathematical equations

Description automatically generated

* In practice, is typically set between -5 and 5
* y must be positive, if the data has any negative values then it can be transformed to be positive by adding a constant beforehand to the whole time series
* A good value of makes the variance the same size across the time series
* How do we pick a good value for ?
* Try different values, plot the data and check visually that the variance is nearly constant

A graph of a plane

Description automatically generated with medium confidence

* Use a method that automatically selects that optimizes on some criteria
* Maximum likelihood (MLE) – pick that makes transformed data look the most normally distributed
* Guerro: Picks that tries to make the variance constant across the time series

A graph of a graph of a method

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1. Box Cox implementation in Scipy

A screenshot of a computer program

Description automatically generated

1. Box Cox implementation in sktime – can specify method as Guerrero and seasonal period

A screenshot of a computer

Description automatically generated

1. Summary

* Forecasting and decomposition methods sometimes work better if the variance is stable across the whole time series
* Log and power transforms can help stabilize the variance across the time series
* A Box Cox transform combines the log and power transform into a single method with a parameter
* The best is the one that makes the variance stable across the time series
* Multiple methods exist to automatically select the best . However, they don’t always agree and manual sense checking is advised

# Box Cox transform: Guerrero method

1. Box Cox recap
2. Coefficient of variation

* The coef of variation is a **scaled measure of variability** of a dataset
* Coef of variation:
* Allows us to **compare the variability across datasets** on different scales
* E.g.,

A table with numbers and text

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1. Guerrero method

* Original paper: “Time-series analysis supported by Power Transformations”
* We want to pick so that the variance of is constant
* Guerrero showed that this requirement implies that:
* Try a range of and check for which value is the most constant over time
* In practice, we have one observation at each t. How do we calculate ?
* Calculate at intervals!
* Split the time series into H evenly sized buckets (subseries), labeled by h
* Compute mean and standard deviation within each subseries
* Compute for each subseries
* How do we measure how constant is across the time series?
* Use the coef of variation of across all subseries,
* If is low -> is more constant across the time series
* Compute at multiple values of between -5 and 5
* Pick which minimizes
* This value creates a time series where is the most constant across time
* Which implies that it’s the best to use to cause the variance of to be constant
* Main parameter is the number of subseries
* If the data has seasonality -> split the subseries by the seasonal period (e.g., 1 subseries for each year if monthly data)
* If no seasonality -> then split the time series into consecutive groups of size 2 to minimize loss of info caused by grouping

1. Why use the Guerrero method?

* Makes no assumptions about the distribution of data
* Directly tries to stabilize the variance across the time series
* More relevant for our time series tasks (i.e., forecasting and decomposition)

1. Implementation in sktime

A screenshot of a computer code

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1. Summary

* Guerrero method selects that makes the variance of constant by minimizing the coef of variation