



Metrics for Multiclass Classification

Model output and classification

Class 0	Class 1	Class 2
0.8	0.15	0.05
0.2	0.3	0.5
0.2	0.6	0.2
0.1	0.8	0.1

The models output an array with probabilities for each class.

Model output and classification

Class 0	Class 1	Class 2		Pred
0.8	0.15	0.05	➔	0
0.2	0.3	0.5		2
0.2	0.6	0.2		1
0.1	0.8	0.1		1

The prediction is the class with the highest probability

Confusion Matrix

		Predicted			
		0	1	2	3
Real / Actual	0	Correct	Incorrect	Incorrect	Incorrect
	1	Incorrect	Correct	Incorrect	Incorrect
	2	Incorrect	Incorrect	Correct	Incorrect
	3	Incorrect	Incorrect	Incorrect	Correct

- **In the diagonal:** correctly classified observations
- **Sum of diagonal:** # of correctly classified observations
- **Outside diagonal:** incorrectly classified observations
- **Sum of matrix:** total # of observations

- We can't speak of positive and negative class any more.
- Now we talk about classes, referencing each class.
- One or more classes can be the minority class or classes.

Accuracy in multiclass

		Predicted			
Real / Actual		0	1	2	3
	0	5	1	1	1
	1	0	4	1	2
	2	0	1	2	0
	3	1	0	0	1

$$\text{Accuracy} = \frac{\text{Number of correct predictions}}{\text{Total number of predictions}}$$

- Correctly classified = 12
- Total observations = 20
- Accuracy = $12 / 20 * 100 = 60\%$

Precision and recall

		Predicted			
		0	1	2	3
Real / Actual	0	5	1	1	1
	1	0	4	1	2
	2	0	1	2	0
	3	1	0	0	1

- True Positive Rate (Recall or Sensitivity)
✓ $TP\ rate = TP / (TP + FN)$
- Positive Predictive Value (Precision)
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	0	1	2	3
0	5	1	1	1
1	0	4	1	2
2	0	1	2	0
3	1	0	0	1

Recall: from the total class, how many correctly classified?

$$\text{Class 0} = 5 / (5 + 1 + 1 + 1) = 5 / 8 = 0.625$$

$$\text{Class 1} = 4 / (4 + 1 + 2) = 4 / 7 = 0.571$$

$$\text{Class 2} = 2 / (2 + 1) = 2 / 3 = 0.667$$

$$\text{Class 3} = 1 / (1 + 1) = 1 / 2 = 0.5$$

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$$\text{Class 0} = 5 / (5 + 1) = 5 / 6 = 0.833$$

$$\text{Class 1} = 4 / (4 + 1 + 1) = 4 / 6 = 0.667$$

$$\text{Class 2} = 2 / (2 + 1 + 1) = 2 / 4 = 0.5$$

$$\text{Class 3} = 1 / (1 + 2 + 1) = 1 / 4 = 0.25$$

Precision: from the total predicted class, how many were true?

Precision and recall

- True Positive Rate (Recall or Sensitivity)
✓ TP rate = $TP / (TP + FN)$
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How do we make sense of all these values?

Precision: from the total predicted class, how many were true?

Macro: take the average

Real / Actual	Predicted			
	0	1	2	3
0	5	1	1	1
1	0	4	1	2
2	0	1	2	0
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Mean Recall = 0.59

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Mean Precision = **0.563**

Precision: from the total predicted class, how many were true?

Weight the average

Take the mean of each metric weighted by the support

Real / Actual	Predicted				Recall	Support
	0	1	2	3		
	0	5	1	1		
	1	0	4	1		
	2	0	1	2		
	3	1	0	0		
					Class 0 = 0.625	8
					Class 1 = 0.571	7
					Class 2 = 0.667	3
					Class 3 = 0.5	2



$$\begin{aligned}\text{Weighted Recall} &= \\ & (0.625 \cdot 8 + 0.571 \cdot 7 + 0.667 \cdot 3 + 0.5 \cdot 2) \\ & (8 + 7 + 3 + 2) = \\ & \mathbf{0.599}\end{aligned}$$

$$\text{Class 0} = 5 / (5 + 1) = 5 / 6 = 0.833$$

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Precision: from the total predicted class, how many were true?

Weight the average

Take the mean of each metric weighted by the support

Real / Actual	Predicted				Recall	Support
	0	1	2	3		
	0	5	1	1		
	1	0	4	1		
	2	0	1	2		
	3	1	0	0		
					Class 0 = 0.625	8
					Class 1 = 0.571	7
					Class 2 = 0.667	3
					Class 3 = 0.5	2



$$\begin{aligned}\text{Weighted Recall} &= \\ & (0.625 \cdot 8 + 0.571 \cdot 7 + 0.667 \cdot 3 + 0.5 \cdot 2) \\ & (8 + 7 + 3 + 2) = \\ & \mathbf{0.599}\end{aligned}$$

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$$\begin{aligned}\text{Weighted Precision} &= \\ & (0.833 \cdot 8 + 0.667 \cdot 7 + 0.5 \cdot 3 + 0.25 \cdot 2) \\ & (8 + 7 + 3 + 2) = \\ & \mathbf{0.665}\end{aligned}$$

Precision: from the total predicted class, how many were true?

Micro:

Real / Actual	Predicted			
	0	1	2	3
0	5	1	1	1
1	0	4	1	2
2	0	1	2	0
3	1	0	0	1

- Micro-averaging uses a 1 vs rest approach
- Considers all TP, TN and FP together

Micro:

		Predicted			
Real / Actual		0	1	2	3
	0	5	1	1	1
	1	0	4	1	2
	2	0	1	2	0
	3	1	0	0	1

- True Positive Rate (Recall or Sensitivity)
✓ TP rate = $TP / (TP + FN)$
- Positive Predictive Value (Precision)
✓ PP value = $TP / (TP + FP)$

- $TP = 5 + 4 + 2 + 1 = 11$
- $FP = 1 + 1 + 1 + 1 + 1 + 1 + 2 = 8$
- $FN = 1 + 1 + 1 + 1 + 2 + 1 + 1 = 8$
- $Recall = 11 / (11 + 8) = 0.57$
- $Precision = 11 / (11 + 8) = 0.57$

THANK YOU

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