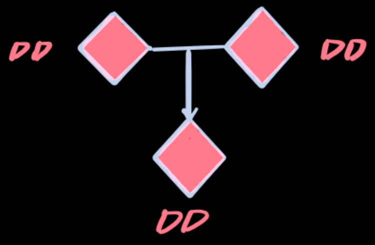
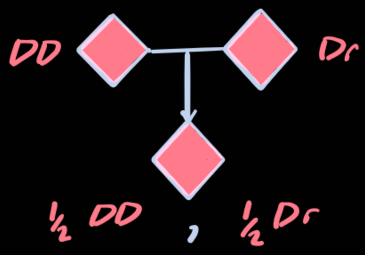
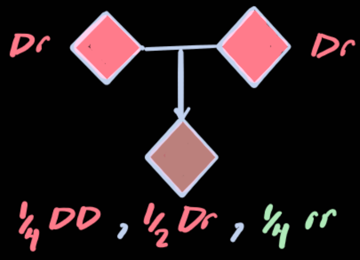
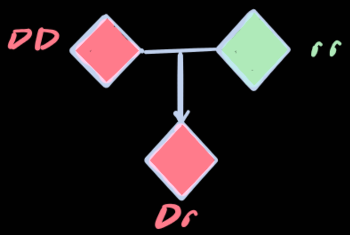
I) Autosome Dominant Inheritance

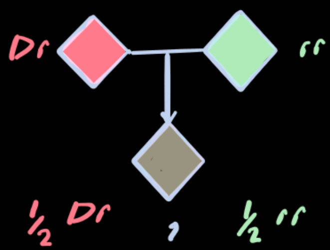
$\boxed{DD\ ,\ Dr}$ $\boxed{rr}$

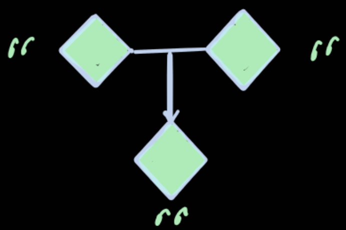


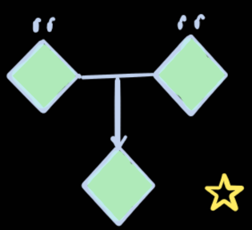


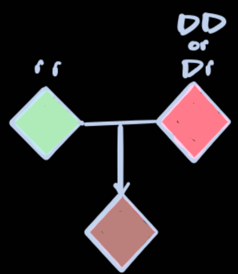


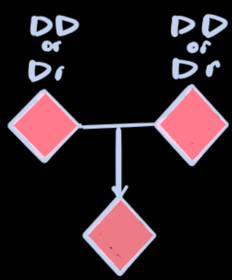










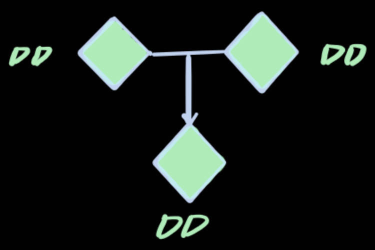


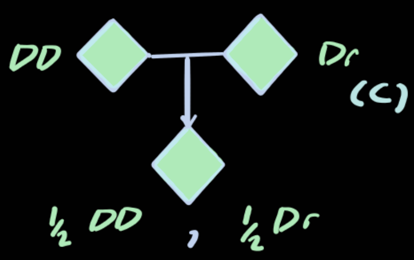
Affected:	0%	75%	93.75%
DD :	0%	0%	56.25%
Dr :	0%	75%	37.5%
rr :	100%	25%	6.25%

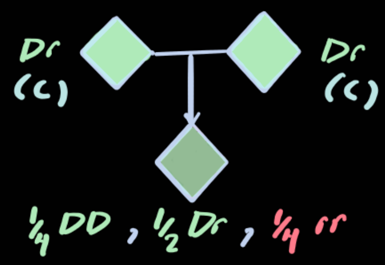
Tip: If the parents are unaffected, so do all of their children.

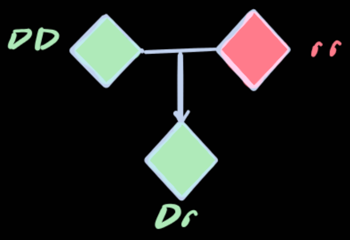
II) Autosome Recessive Inheritage

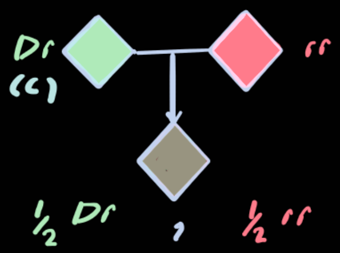
rr | DD , Dr

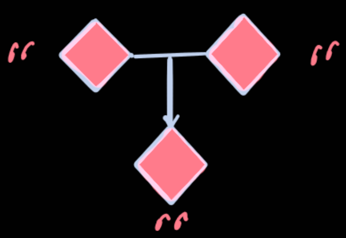


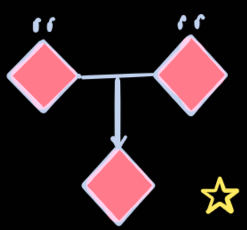


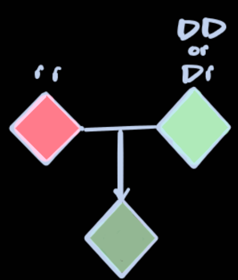


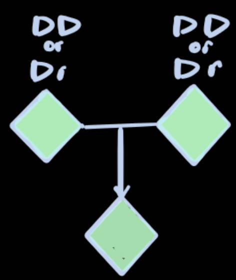












Affected:	100%	25%	6.25%
DD :	0%	0%	56.25%
(c) Dr :	0%	75%	37.5%
rr :	100%	25%	6.25%

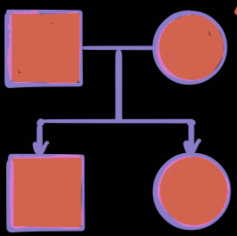
Tip: If the parents are affected, so do all their children.

II) X-Linked Dominant inheritage

X^DX^D, X^DX^r, X^DY

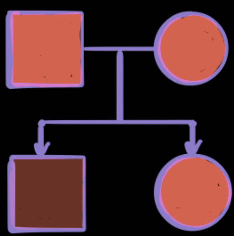
X^rX^r, X^rY

X^DY — X^DX^D



X^DY — X^DX^D

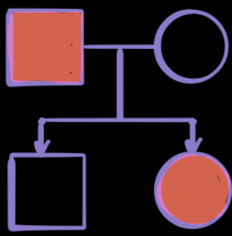
X^DY — X^DX^r



1/2 X^DY — X^DX^D 1/2

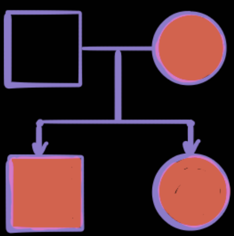
1/2 X^rY — X^DX^r 1/2

X^DY — X^rX^r



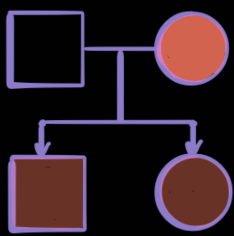
X^rY — X^DX^r

X^rY — X^DX^D



X^DY — X^DX^r

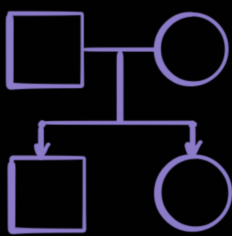
X^rY — X^DX^r



1/2 X^DY — X^DX^r 1/2

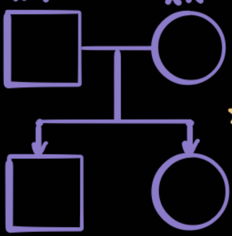
1/2 X^rY — X^rX^r 1/2

X^rY — X^rX^r

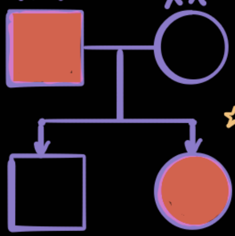


X^rY — X^rX^r

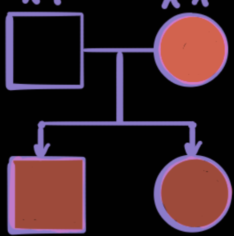
X^rY — X^rX^r



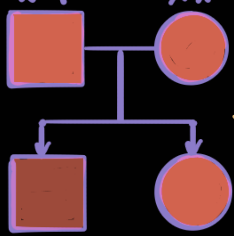
X^DY — X^rX^r



X^rY — X^DX^D or X^DX^r



X^DY — X^DX^D or X^DX^r



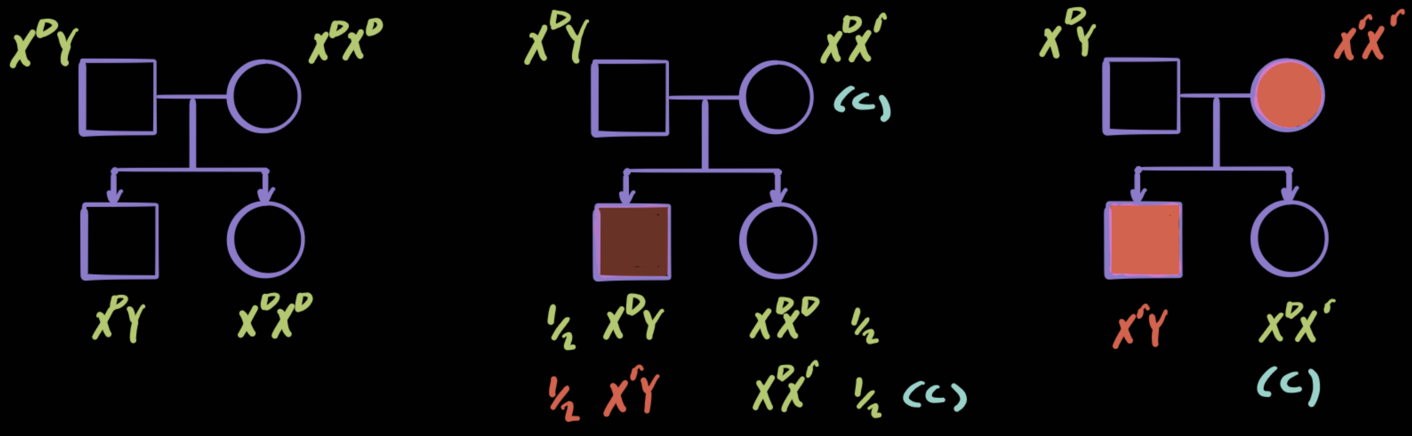
Affected:	0%	0%	0%	100%	75%	75%	75%	100%
X^DX^D:		0%		0%		0%		75%
X^DX^r:		0%		100%		75%		25%
X^rX^r:		100%		0%		25%		0%
X^DY:	0%		0%		75%		75%	
X^rY:	100%		100%		25%		25%	

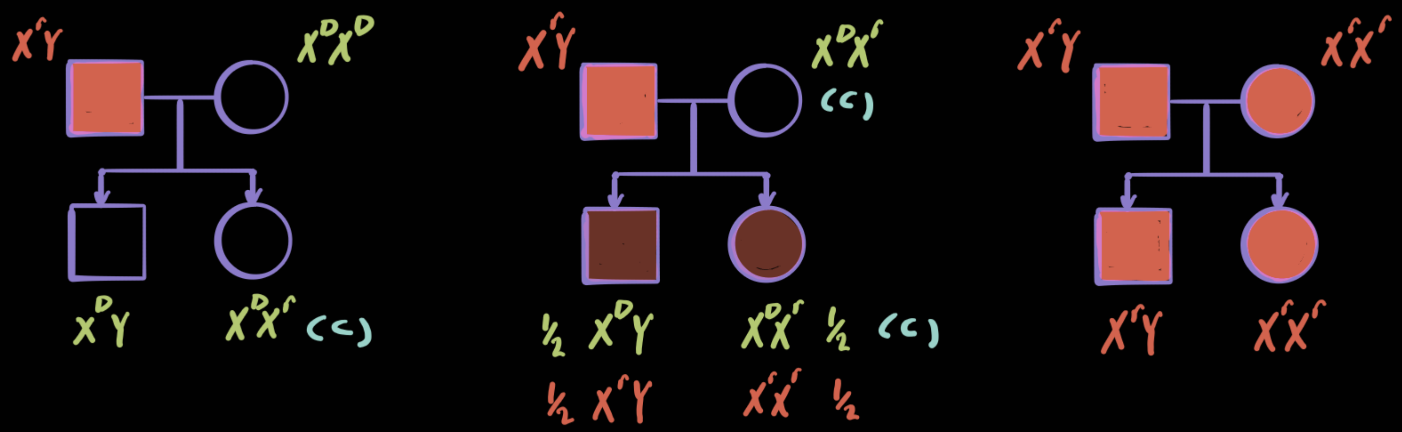
Tips {
1) If both parents are unaffected, so do all their children.
2) If the father is affected, so do all his daughters.

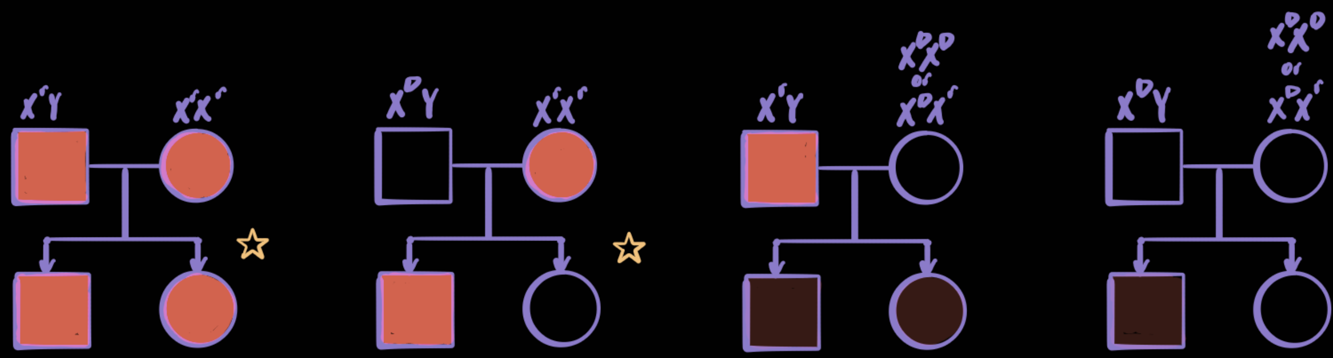
IV) X_Linked recessive inheritage

X^rX^r
X^rY

X^DX^D, X^DX^r
X^DY





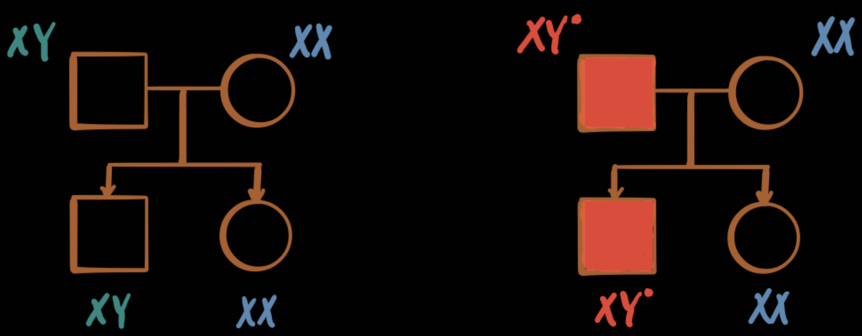


Affected:	100%	100%	100%	0%	25%	25%	25%	0%
X^DX^D:		0%		0%		0%		75%
X^DX^r:		0%		100%		75%		25% (Carrier)
X^rX^r:		100%		0%		25%		0%
X^DY :	0%		0%		75%		75%	
X^rY :	100%		100%		25%		25%	

Tips {
1) If both parents are affected , so do all their children.
2) If the mother is affected, so do all her sons.

V) Y-Linked inheritance

$\boxed{XY}$ $\boxed{XY^\circ}$



Affected :	0%	100%
XY :	100%	0%
XY° :	0%	100%

Tip: { 1) Women can't be affected.
2) If the father is affected, so do all his sons.

Using the tips, we can check which inheritance models are possible and valid for an input pedigree.

We may also check the most probable model for the given pedigree.

Next, I will give a full algorithm.

Reminder:

I A.D	II A.R	III XL.D		IV XL.R		V YL		
U.A P	A.P	U.A P	A.F	A.P	A.M	U.F	A.F	No Affected Woman!
⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	
U.A Ch	A.Ch	U.A Ch	A.D	A.Ch	A.S	U.S	A.S	
No Carrier		No Carrier				No Carrier		

Algorithm ... ★ $Fact_i = 1$ if i_{th} man/woman is affected.

★ Let us call a group of couple & their children a Subtree.

★ Since there are only 5 models, we'll try each of them individually.

★ For each woman ○, there are six possibilities: DD, Dr, rr, X^DX^D, X^DX^r, X^rX^r.

Let's name i_{th} woman's j_{th} possibility's probability "$Female\_Gene\_Prob_{i,j}$"

★ For each man □, there are seven possibilities: DD, Dr, rr, X^DY, X^rY, XY^D, XY^r.

Let's name i_{th} man's j_{th} possibility's probability "$Male\_Gene\_Prob_{i,j}$".

★ Let's call the probability of k_{th} model for i_{th} subtree "$P_{i,k}$";

Starting from the greatest parents, we may find $P_{i,k}$ based on

Female/Male_Gene_Prob s.

★ The probability for the whole tree will be $\prod_{i=1}^{n} P_{i,k}$. (n = number of subtrees)

This answer will be named "$Model\_Prob_k$".

$O(10n)$ → number of people in pedigree.

Model I) subtree num: k woman(daughter) num: i man(son) num: j **One Based!**

$\times$ rr — rr $\times$; rr $\times$ all $\times$

$$P_{k,1} = \Bigg(F\_G\_P_{m,1,3} \times M\_G\_P_{d,1,3} \times \prod^{\text{Children}} \left(\text{Not Fact}_{\text{child}}\right)$$

✓ DD — DD ✓ ; DD ✓ $\times$ all

$$+\ F\_G\_P_{m,1,1} \times M\_G\_P_{d,1,1} \times \prod^{\text{Children}} \left(\text{Fact}_{\text{child}}\right)$$

✓ DD — Dr ✓ ; DD, Dr ✓

$$+\ F\_G\_P_{m,1,1} \times M\_G\_P_{d,1,2} \times \prod^{\text{Children}} \left(\text{Fact}_{\text{child}}\right)$$

✓ Dr — DD ✓ ; DD, Dr ✓

$$+\ F\_G\_P_{m,1,2} \times M\_G\_P_{d,1,1} \times \prod^{\text{Children}} \left(\text{Fact}_{\text{child}}\right)$$

✓ Dr — rr $\times$; 1/2 Dr ✓, 1/2 rr $\times$

$$+\ F\_G\_P_{m,1,2} \times M\_G\_P_{d,1,3} \times \prod^{\text{Children}} \left(\tfrac{1}{2} \times \text{Fact}_{\text{child}} + \tfrac{1}{2} \times \left(\text{Not Fact}_{\text{child}}\right)\right)$$

$\times$ rr — Dr ✓ ; 1/2 Dr ✓, 1/2 rr $\times$

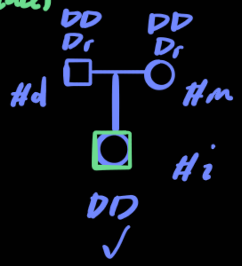
$$+\ F\_G\_P_{m,1,3} \times M\_G\_P_{d,1,2} \times \prod^{\text{Children}} \left(\tfrac{1}{2} \times \text{Fact}_{\text{child}} + \tfrac{1}{2} \times \left(\text{Not Fact}_{\text{child}}\right)\right)$$

✓ Dr — Dr ✓ ; 3/4 { 1/4 DD ✓, 1/2 Dr ✓ }, 1/4 rr $\times$

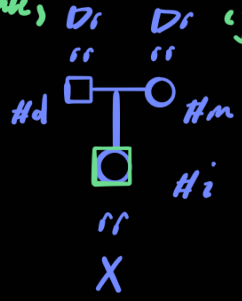
$$+\ F\_G\_P_{m,1,2} \times M\_G\_P_{d,1,2} \times \prod^{\text{Children}} \left(\tfrac{3}{4} \times \text{Fact}_{\text{child}} + \tfrac{1}{4} \times \left(\text{Not Fact}_{\text{child}}\right)\right) \Bigg)$$

This will be used for later generation as their parents!

$$\underbrace{\text{Female\_Gene\_Prob}^{(DD)}_{i,1,1\,(j)}}_{\text{(Male)}} = \underbrace{\Big(\big(\text{Female\_Gene\_Prob}_{m,1,1} + \tfrac{1}{2}\,\text{Female\_Gene\_Prob}_{m,1,2}\big) \times \big(\text{Male\_Gene\_Prob}_{d,1,1} + \tfrac{1}{2}\,\text{Male\_Gene\_Prob}_{d,1,2}\big)\Big)}_{A} \times \text{Fact}_{i(j)}$$



$$\underbrace{\text{Female\_Gene\_Prob}^{(rr)}_{i,1,3\,(j)}}_{\text{(Male)}} = \underbrace{\Big(\big(\text{Female\_Gene\_Prob}_{m,1,3} + \tfrac{1}{2}\,\text{Female\_Gene\_Prob}_{m,1,2}\big) \times \big(\text{Male\_Gene\_Prob}_{d,1,3} + \tfrac{1}{2}\,\text{Male\_Gene\_Prob}_{d,1,2}\big)\Big)}_{B} \times \big(\text{Not Fact}_{i(j)}\big)$$



$$\underbrace{\text{Female\_Gene\_Prob}^{(Dr)}_{i,1,2\,(j)}}_{\text{(Male)}} = \big(1 - (A + B)\big) \times \text{Fact}_{i(j)}$$

Dr
✓

$$\text{Mode\_Prob}_1 = \prod_i^{\text{number of subtrees}} P_{i,1}$$

End of mode #1.

Model II)   subtree num: k woman (daughter) num: i man (son) num: j One Based!

$$P_{k,2} = \Bigg(\begin{array}{c} \overset{\checkmark}{rr} \;\top\; \overset{\checkmark}{rr} \\ rr\,\checkmark \end{array} \; F\_G\_P_{m,2,3} \times M\_G\_P_{d,2,3} \times \prod^{\text{Children}} \left(Fact_{Child} \right)$$

$$+ \begin{array}{c} \overset{\times}{DD} \;\top\; \overset{\times}{DD} \\ DD\;\times \end{array} \; F\_G\_P_{m,2,1} \times M\_G\_P_{d,2,1} \times \prod^{\text{Children}} \left(Not\ Fact_{Child} \right)$$

$$+ \begin{array}{c} \overset{\times}{DD} \;\top\; \overset{\times}{Dr} \\ DD \\ Dr \\ \times \end{array} \; F\_G\_P_{m,2,1} \times M\_G\_P_{d,2,2} \times \prod^{\text{Children}} \left(Not\ Fact_{Child} \right)$$

$$+ \begin{array}{c} \overset{\times}{Dr} \;\top\; \overset{\times}{DD} \\ DD \\ Dr \\ \times \end{array} \; F\_G\_P_{m,2,2} \times M\_G\_P_{d,2,1} \times \prod^{\text{Children}} \left(Not\ Fact_{Child} \right)$$

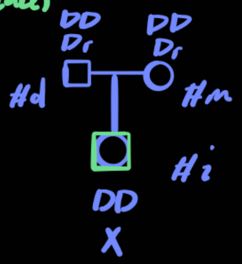
$$+ \begin{array}{c} \overset{\times}{Dr} \;\top\; \overset{\checkmark}{rr} \\ \tfrac{1}{2}\ Dr\ \times \\ \tfrac{1}{2}\ rr\ \checkmark \end{array} \; F\_G\_P_{m,2,2} \times M\_G\_P_{d,2,3} \times \prod^{\text{Children}} \left(\tfrac{1}{2} \times Fact_{Child} + \tfrac{1}{2} \times \left(Not\ Fact_{Child} \right) \right)$$

$$+ \begin{array}{c} \overset{\checkmark}{rr} \;\top\; \overset{\times}{Dr} \\ \tfrac{1}{2}\ Dr\ \times \\ \tfrac{1}{2}\ rr\ \checkmark \end{array} \; F\_G\_P_{m,2,3} \times M\_G\_P_{d,2,2} \times \prod^{\text{Children}} \left(\tfrac{1}{2} \times Fact_{Child} + \tfrac{1}{2} \times \left(Not\ Fact_{Child} \right) \right)$$

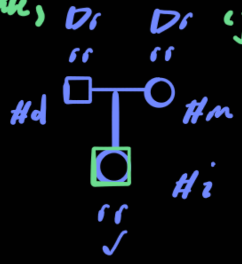
$$+ \begin{array}{c} \overset{\times}{Dr} \;\top\; \overset{\times}{Dr} \\ \tfrac{3}{4} \left\{ \begin{array}{l} \tfrac{1}{4}\ DD\ \times \\ \tfrac{1}{2}\ Dr\ \times \end{array} \right. \\ \tfrac{1}{4}\ rr\ \checkmark \end{array} \; F\_G\_P_{m,2,2} \times M\_G\_P_{d,2,2} \times \prod^{\text{Children}} \left(\tfrac{1}{4} \times Fact_{Child} + \tfrac{3}{4} \times \left(Not\ Fact_{Child} \right) \right) \Bigg)$$

This will be used for later generation as their parents!

$$\underset{\text{(Male)}}{\text{Female\_Gene\_Prob}}^{(DD)}_{\substack{i,2,1 \\ (j)}} = \underbrace{\left(\left(\text{Female\_Gene\_Prob}_{m,2,1} + \tfrac{1}{2}\,\text{Female\_Gene\_Prob}_{m,\,,2} \right) \times \left(\text{Male\_Gene\_Prob}_{d,2,1} + \tfrac{1}{2}\,\text{Male\_Gene\_Prob}_{d,\,,2} \right) \right)}_{A} \times \left(\text{Not Fact}_{\substack{i \\ (j)}} \right)$$



$$\underset{\text{(Male)}}{\text{Female\_Gene\_Prob}}^{(rr)}_{\substack{i,2,3 \\ (j)}} = \underbrace{\left(\left(\text{Female\_Gene\_Prob}_{m,2,3} + \tfrac{1}{2}\,\text{Female\_Gene\_Prob}_{m,2,2} \right) \times \left(\text{Male\_Gene\_Prob}_{d,2,3} + \tfrac{1}{2}\,\text{Male\_Gene\_Prob}_{d,2,2} \right) \right)}_{B} \times \text{Fact}_{\substack{i \\ (j)}}$$



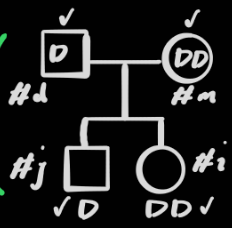
$$\underset{\text{(Male)}}{\text{Female\_Gene\_Prob}}^{(Dr)}_{\substack{i,2,2 \\ (j)}} = \left(1 - (A + B) \right) \times \left(\text{Not Fact}_{\substack{i \\ (j)}} \right)$$

Dr
X

End of mode #2.

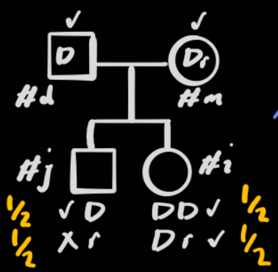
Mode III)

$P_{k,3} = \Bigg($



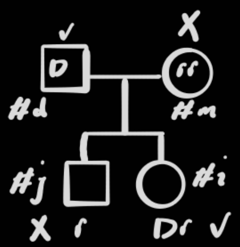
$$M.G.P_{d,3,4} \times F.G.P_{m,3,4} \times \left(\prod_i^{\text{Daughters}} Fact_i\right) \times \left(\prod_j^{\text{Sons}} Fact_j\right)$$

$+$



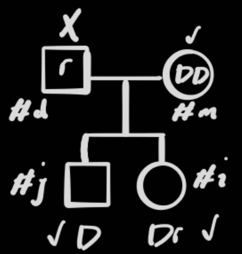
$$M.G.P_{d,3,4} \times F.G.P_{m,3,5} \times \left(\prod_i^{\text{Daughters}} Fact_i\right) \times \left(\prod_j^{\text{Sons}} \left(\tfrac{1}{2} Fact_j + \tfrac{1}{2} Not Fact_j\right)\right)$$

$+$



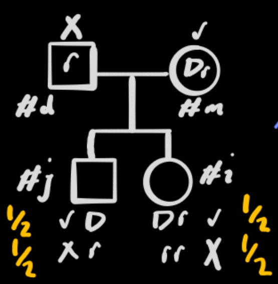
$$M.G.P_{d,3,4} \times F.G.P_{m,3,6} \times \left(\prod_i^{\text{Daughters}} Fact_i\right) \times \left(\prod_j^{\text{Sons}} Not\ Fact_j\right)$$

$+$



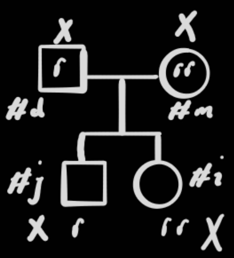
$$M.G.P_{d,3,5} \times F.G.P_{m,3,4} \times \left(\prod_i^{\text{Daughters}} Fact_i\right) \times \left(\prod_j^{\text{Sons}} Fact_j\right)$$

$+$



$$M.G.P_{d,3,5} \times F.G.P_{m,3,5} \times \left(\prod_i^{\text{Daughters}} \left(\tfrac{1}{2} Fact_i + \tfrac{1}{2} Not Fact_i\right)\right) \times \left(\prod_j^{\text{Sons}} \left(\tfrac{1}{2} Fact_j + \tfrac{1}{2} Not Fact_j\right)\right)$$

$+$



$$M.G.P_{d,3,5} \times F.G.P_{m,3,6} \times \left(\prod_i^{\text{Daughters}} Not\ Fact_i\right) \times \left(\prod_j^{\text{Sons}} Not\ Fact_j\right)\Bigg)$$

$$F\_G\_P_{i,3,4} = M.G.P_{d,3,4} \times \left(F.G.P_{m,3,4} + \tfrac{1}{2}\, F.G.P_{m,3,5}\right) \times Fact_i$$

(d) X^RY — $X^RX^R / X^RX^r \times \tfrac{1}{2}$ (m)

X^RX^R (i)

✓

$$F\_G\_P_{i,3,6} = M.G.P_{d,3,5} \times \left(F.G.P_{m,3,6} + \tfrac{1}{2}\, F.G.P_{m,3,5}\right) \times (Not\ Fact_i)$$

(d) X^rY — $X^RX^r \times \tfrac{1}{2} / X^rX^r$ (m)

X^rX^r (i)

✗

$$F\_G\_P_{i,3,5} = \left(1 - \left(F.G.P_{i,3,4} + F.G.P_{i,3,6}\right)\right) \times Fact_i$$

X^RX^r

✓

$$M\_G\_P_{j,3,4} = \left(F.G.P_{m,3,4} + \tfrac{1}{2}\, F.G.P_{m,3,5}\right) \times Fact_j$$

(d) X^rY / X^RY — $X^RX^R / X^RX^r \times \tfrac{1}{2}$ (m)

X^RY (j)

✓

$$M\_G\_P_{j,3,5} = \left(F.G.P_{m,3,6} + \tfrac{1}{2}\, F.G.P_{m,3,5}\right) \times (Not\ Fact_j)$$

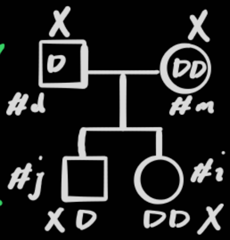
(d) X^RY / X^rY — $X^RX^r \times \tfrac{1}{2} / X^rX^r$ (m)

X^rY (j)

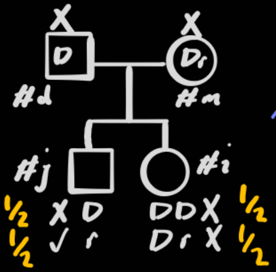
✗

End of mode #3.

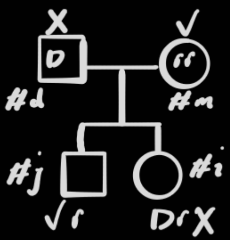
Mode IV)

$P_{k,4} = \Bigg($ 

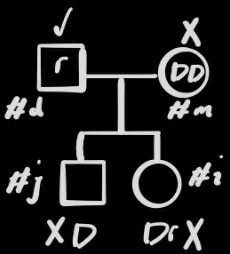
$$M.G.P_{d,4,4} \times F.G.P_{m,4,4} \times \left(\prod_i^{\text{Daughters}} \text{Not Fact}_i\right) \times \left(\prod_j^{\text{Sons}} \text{Not Fact}_j\right)$$

$+$ 

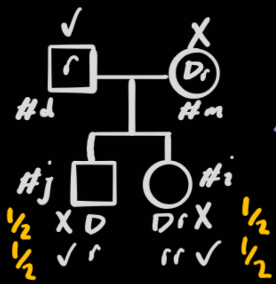
$$M.G.P_{d,4,4} \times F.G.P_{m,4,5} \times \left(\prod_i^{\text{Daughters}} \text{Not Fact}_i\right) \times \left(\prod_j^{\text{Sons}} \left(\tfrac{1}{2}\,\text{Not Fact}_j + \tfrac{1}{2}\,\text{Fact}_j\right)\right)$$

$+$ 

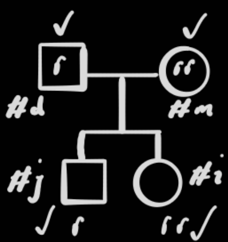
$$M.G.P_{d,4,4} \times F.G.P_{m,4,6} \times \left(\prod_i^{\text{Daughters}} \text{Not Fact}_i\right) \times \left(\prod_j^{\text{Sons}} \text{Fact}_j\right)$$

$+$ 

$$M.G.P_{d,4,5} \times F.G.P_{m,4,4} \times \left(\prod_i^{\text{Daughters}} \text{Not Fact}_i\right) \times \left(\prod_j^{\text{Sons}} \text{Not Fact}_j\right)$$

$+$ 

$$M.G.P_{d,4,5} \times F.G.P_{m,4,5} \times \left(\prod_i^{\text{Daughters}} \left(\tfrac{1}{2}\,\text{Fact}_i + \tfrac{1}{2}\,\text{Not Fact}_i\right)\right) \times \left(\prod_j^{\text{Sons}} \left(\tfrac{1}{2}\,\text{Fact}_j + \tfrac{1}{2}\,\text{Not Fact}_j\right)\right)$$

$+$ 

$$M.G.P_{d,4,5} \times F.G.P_{m,4,6} \times \left(\prod_i^{\text{Daughters}} \text{Fact}_i\right) \times \left(\prod_j^{\text{Sons}} \text{Fact}_j\right)\Bigg)$$

$$F\_G\_P_{i,4,4} = M.G.P_{d,4,4} \times \left(F.G.P_{m,4,4} + \tfrac{1}{2} F.G.P_{m,4,5}\right) \times \left(\text{Not Fact}_i\right)$$

(d) X^RY — $X^RX^R / X^RX^r \times 1/2$ (m)

X^RX^R (i)

✗

$$F\_G\_P_{i,4,6} = M.G.P_{d,\ ,5} \times \left(F.G.P_{m,4,6} + \tfrac{1}{2} F.G.P_{m,4,5}\right) \times \text{Fact}_i$$

(d) X^rY — $X^RX^r \times 1/2 / X^rX^r$ (m)

X^rX^r (i)

✓

$$F\_G\_P_{i,4,5} = \left(1 - \left(F.G.P_{i,4,1} + F.G.P_{i,4,6}\right)\right) \times \left(\text{Not Fact}_i\right)$$

X^RX^r

✗

$$M\_G\_P_{j,4,4} = \left(F.G.P_{m,4,4} + \tfrac{1}{2} F.G.P_{m,4,5}\right) \times \left(\text{Not Fact}_j\right)$$

(d) X^rY / X^RY — $X^RX^R / X^RX^r \times 1/2$ (m)

X^RY (j)

✗

$$M\_G\_P_{j,4,5} = \left(F.G.P_{m,4,6} + \tfrac{1}{2} F.G.P_{m,4,5}\right) \times \text{Fact}_j$$

(d) X^RY / X^rY — $X^RX^r \times 1/2 / X^rX^r$ (m)

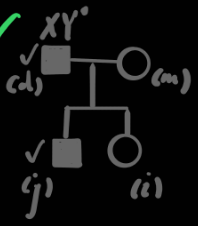
X^rY (j)

✓

End of node #4.

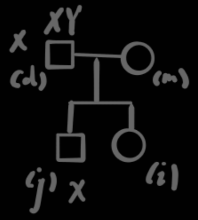
Mode V)

$$P_{k,5} = \Bigg($$



$$M.G.P_{d,5,6} \times \Big(\overset{\text{Sons}}{\prod_{j}} Fact_j \Big) \times \Big(\overset{\text{Daughters}}{\prod_{i}} (Not\ Fact_i) \Big) \times (Not\ Fact_m)$$

$$+$$



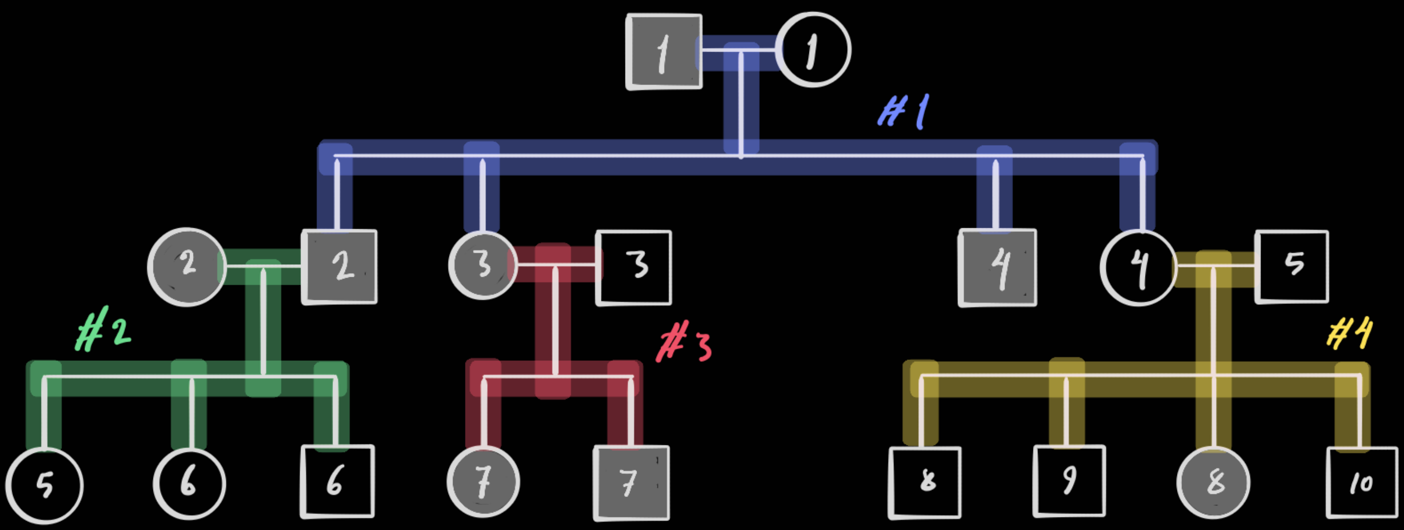
$$M.G.P_{d,5,7} \times \Big(\overset{\text{Sons}}{\prod_{j}} (Not\ Fact_j) \Big) \Big(\overset{\text{Daughters}}{\prod_{i}} (Not\ Fact_i) \Big) \times (Not\ Fact_m) \Bigg)$$

$$M.G.P_{j,5,6} = M.G.P_{d,5,6} \times Fact_j$$

$$M.G.P_{j,5,7} = M.G.P_{d,5,7} \times (Not\ Fact_j)$$

End of Mode #5!

- Since we need the product of all children Facts in a subtree, we calculate it once at the begining. It will cost $O(n)$. number of people.
- In reality, some of the children with specific disorders will not survive, which can be consider in further works.
- I used Dynamic Programming & the whole algorithm costs $O(10n)$ at max.



EOF.

Kimia Khodsiyani