SAMPLE NUMERICAL (ANN)

Q. FIND WEIGHT ADJUSTMENT ΔW_{35} & ERROR GRADIENT AT NODE '4' (δ_4) FOR THE FOLLOWING MLP. ASSUME THE FOLLOWING:

LEARNING CONSTANT 'c' = 0.1

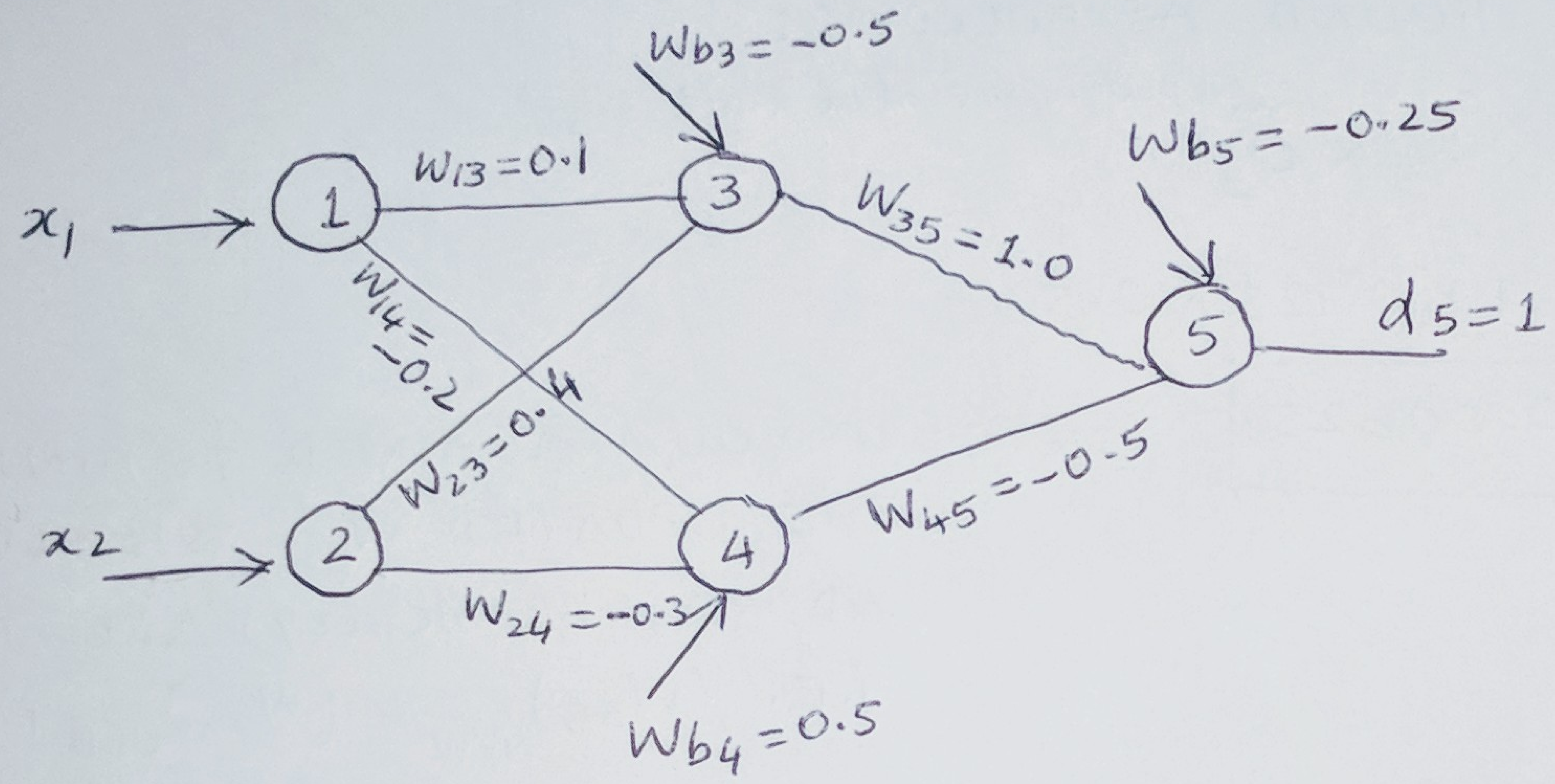
MOMENTUM FACTOR 'β' = 0.9

ERROR FUNCTION : MEAN SQUARED ERROR (MSE)

OUTPUT FUNCTION : LOG-SIGMOID (FOR ALL PROCESSING LAYERS)

INPUT VECTOR : $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; BIASED INPUT : 1

DESIRED OUTPUT : 1



SOLN. FIRST, FIND THE ACTUAL OUTPUT 'O_5'.

$$O_5 = \frac{1}{1+e^{-net_5}}$$

$$net_5 = O_3 \times W_{35} + O_4 \times W_{45} + W_{b5}$$

$$O_3 = \frac{1}{1+e^{-net_3}}$$

$$net_3 = x_1 \times W_{13} + x_2 W_{23} + W_{b3} = 0.1 + 0.4 + (-0.5) = 0$$

$$O_4 = \frac{1}{1+e^{-net_4}}$$

$$net_4 = x_1 \times W_{14} + x_2 \times W_{24} + W_{b4} = -0.2 - 0.3 + 0.5 = 0$$

$$\therefore \; O_3 = \frac{1}{1+e^0} = 0.5$$

$$\& \; O_4 = \frac{1}{1+e^0} = 0.5$$

$$\therefore \; net_5 = 0.5 \times 1 + 0.5 \times (-0.5) + (-0.25) = 0$$

$$\Rightarrow O_5 = \frac{1}{1+e^0} = 0.5$$

NOW WE CAN CALCULATE ERROR AT NODE '5'

$$e_5 = d_5 - O_5 = 1 - 0.5 = 0.5$$

TO BACK-PROPAGATE ERROR & ADJUST WEIGHTS, WE NEED TO FIND ERROR GRADIENT AT NODE 5, δ_5

$$\delta_5 = e_5 \times \underbrace{O_5(1 - O_5)}_{\text{gradient of output fn. at node 5}}$$

$$\delta_5 = 0.5(0.5(1 - 0.5)) = 0.125$$

NOW ΔW_{35} CAN BE FOUND AS FOLLOWS:

$$\Delta W_{35} = c \times \delta_5 \times \underset{\text{input associated with } 'W_{35}'}{O_3}$$

$$\Delta W_{35} = 0.1 \times 0.125 \times 0.5$$

$$\boxed{\Delta W_{35} = 0.00625}$$

NOTE: IF YOU ARE ASKED TO FIND THE UPDATED 'W_{35}', SIMPLY ADD (ALGEBRAICALLY) ΔW_{35} TO W_{35} i.e. $(W_{35})_{NEW} = (W_{35})_{OLD} + \Delta W_{35}$

NEXT, ERROR GRADIENT AT NODE '4', $\delta_4 = ?$

USING BACKPROPAGATION ALGO., WE CAN ESTIMATE ERROR CONTRIBUTION OF NODE '4' AND WORK OUT 'δ_4' AS FOLLOWS:

WE KNOW $$\delta_j = \underbrace{\sum_k \delta_k W_{jk}}_{\text{error contribution}} \cdot \underbrace{O_j(1 - O_j)}_{\text{gradient of output fn. at node (hidden) 'j'.}}$$

$$\therefore \delta_4 = \delta_5 W_{45} \times O_4(1 - O_4)$$

$$\delta_4 = 0.125 \times (-0.5) \times 0.5(1 - 0.5)$$

$$\boxed{\delta_4 = -0.0156}$$