Theory Of Automata (BCS-6C)
M. Azlan Shahid (21L-5494)

A) $S \to TU \mid V$
$T \to aTb \mid \varepsilon$
$U \to cU \mid \varepsilon$
$V \to aVc \mid W$
$W \to bW \mid \varepsilon$

Sol:

① $T \to \varepsilon$

$S \to TU \mid V \mid U$
$T \to aTb \mid ab$
$U \to cU \mid \varepsilon$
$V \to aVc \mid W$
$W \to bW \mid \varepsilon$

② $U \to \varepsilon$

$S \to TU \mid V \mid U \mid T \mid \varepsilon$
$T \to aTb \mid ab$
$U \to cU \mid c$
$V \to aVc \mid W$
$W \to bW \mid \varepsilon$

③ $W \to \varepsilon$

$S \to TU \mid V \mid U \mid T \mid \varepsilon$
$T \to aTb \mid ab$
$U \to cU \mid c$
$V \to aVc \mid W$
$W \to bW \mid b$

④ $V \to \varepsilon$

$S \to TU \mid \varepsilon \mid V \mid U$
$T \to aTb \mid ab$
$U \to cU \mid c$
$V \to aVc \mid ac \mid W$
$W \to bW \mid b$

Now, Unit Production

$S \to TU \mid cU \mid c \mid aTb \mid ab \mid aVc \mid ac \mid bW \mid b \mid \varepsilon$

$T \to aTb \mid ab$

$U \to cU \mid c$

$V \to aVc \mid ac \mid bW \mid b$

$W \to bw \mid b$

Useless Production

All included $\{a, b, c, T, U, V, W, S\}$

Evaluate

$S \to TU \mid cU \mid c \mid MB \mid AB \mid NC \mid AC \mid BW \mid b \mid \varepsilon$

$T \to MB \mid AB$

$U \to cU \mid c$

$V \to NC \mid AC \mid BW$

$W \to BW \mid b$

$A \to a$

$B \to b$

$C \to c$

$M \to AT$

$N \to AV$

$S_0 \to S$

(2) $S \to ASA \mid ab$
$A \to B \mid S$
$B \to b \mid \varepsilon$
$X \to A$

Add S_0
$S_0 \to S$

1 – NULL Production

① $B \to \varepsilon$, $S_0 \to S$
$S \to ASA \mid aB \mid a$
$A \to B \mid S \mid \varepsilon$
$B \to b$
$X \to A$

② $A \to \varepsilon$
$S_0 \to S$
$S \to ASA \mid aB \mid a \mid SA \mid AS \mid S$
$A \to B \mid S$
$B \to b$
$X \to A$

2 – Unit Production

$S_0 \to ASA \mid aB \mid a \mid SA \mid AS$
$S \to ASA \mid aB \mid a \mid SA \mid AS$
$A \to b \mid ASA \mid aB \mid a \mid SA \mid AS$
$B \to b$
$X \to b \mid ASA \mid aB \mid a \mid SA \mid AS$

All included

3 – Useless Production $\{a, b, S_0, S, A, B, X\}$
As X is not reachable, it is discarded

4- Evaluate

$S_0 \to XA|CB|SA|AS|a$

$S \to XA|CB|SA|AS|a$

$A \to XA|CB|.SA|AS|a|b$

$B \to b$

$X \to AS$

$C \to a$

(c) $S \to a|aA|aB$

$A \to aBB|\varepsilon$

$B \to Aa|b$

1- NULL Production

(1) . $\boxed{A \to \varepsilon}$

$S \to a|aA|aB$

$A \to aBB$

$B \to Aa|b|a$

2- Unit Production (Not any removal of unit Production)

3- Useless Production

All included $\{a, b, S, B, A\}$

4- Evaluate

$S \rightarrow T \mid TA \mid TB$

$A \rightarrow TU$

$B \rightarrow AT \mid b \mid a$

$B \rightarrow AT \mid b \mid a$

$T \rightarrow a$

$U \rightarrow BB$