Hannan Fareed (Automata) 27/03/2024

(21L-5392) 6C Assignment #3

Q1) (a)

$S \to TU \mid V$

$T \to aTb \mid \varepsilon$

$U \to cU \mid \varepsilon$

$V \to aVc \mid W$

$W \to bW \mid \varepsilon$

→ (i) Adding one more state

$S_0 \to S$

$S \to TU \mid V$

$T \to aTb \mid \varepsilon$

$U \to cU \mid \varepsilon$

$V \to aVc \mid W$

$W \to bW \mid \varepsilon$

→ (ii) Removing Null production

Nullable variable $= \{W, V, U, T, S, S_0\}$

$S_0 \to S \mid \varepsilon$

$S \to TU \mid V \mid T \mid U \mid$

$T \to aTb \mid ab \mid$

$U \to cU \mid c$

$V \to aVc \mid W \mid ac$

$W \to bW \mid b$

→ (iii) Removing unit production

Unit productions $= \{S_0 \to S,\ S \to V,\ S \to T,\ S \to U,\ V \to W\}$

$S_0 \to TU \mid aVc \mid bW \mid b \mid ac \mid aTb \mid ab \mid cU \mid c \mid \varepsilon$

$S \to TU \mid aVc \mid bW \mid b \mid ac \mid aTb \mid ab \mid cU \mid c$

$T \to aTb \mid ab$

$U \to cU \mid c$

$V \to aVc \mid bW \mid b \mid ac$

$W \to bW \mid b$

→ (iv) Removing useless states

useful = {a, c, b, S_0, S, T, U, V, W}

Hence, no state is useless.

→ (v) Final form of CNF:

$S_0 \to TU \mid LZ \mid YW \mid b \mid XZ \mid MY \mid XY \mid ZU \mid c \mid \varepsilon$

$S \to TU \mid LZ \mid YW \mid b \mid XZ \mid MY \mid XY \mid ZU \mid c$

$T \to MY \mid XY$

$U \to ZU \mid c$

$V \to NZ \mid YW \mid b \mid XZ$

$W \to YW \mid b$

$X \to a$

$Y \to b$

$Z \to c$

$L \to XV$

$M \to XT$

$N \to XV$

(1b) $S \to ASA \mid aB$

$A \to B \mid S$

$B \to b \mid \varepsilon$

$X \to A$

→ (i) Removing null production

Adding a starting position S_0

∵ $B \to \varepsilon$

$S_0 \to S$

$S \to ASA \mid aB \mid a$

$A \to B \mid S \mid \varepsilon$

$B \to b$

$X \to A$

∵ $A \to \varepsilon$

$S_0 \to S$

$S \to ASA \mid aB \mid a \mid AS \mid SA \mid S$

$A \to B \mid S$

$B \to b$

$X \to A \mid \varepsilon$

∵ $X \to \varepsilon$

$S_0 \to S$

$S \to ASA \mid aB \mid a \mid AS \mid SA \mid S$

$A \to B \mid S$

$B \to b$

$X \to A$

→ (ii) Removing unit productions and duplicates

∵ $S \to S$

∵ $S_0 \to S$

∵ $A \to B$

∵ $A \to S$

∵ $X \to A$

$S_0 \rightarrow ASA \mid aB \mid a \mid SA \mid AS$.

$S \rightarrow ASA \mid aB \mid a \mid SA \mid AS$

$A \rightarrow b \mid ASA \mid aB \mid a \mid SA \mid AS$

$B \rightarrow b$

→ (iii) Replacing terms with more than 2 terminals or non-terminals:

$S_0 \rightarrow XA \mid YB \mid a \mid SA \mid AS$

$S \rightarrow XA \mid YB \mid a \mid SA \mid AS$

$A \rightarrow b \mid XA \mid YB \mid a \mid SA \mid AS$

→ CNF

$X \rightarrow AS$

$Y \rightarrow a$

(1c) $S \rightarrow a \mid aA \mid aB$

$A \rightarrow aBB \mid \varepsilon$

$B \rightarrow Aa \mid b$

→ (i) Removing null production

Nullable variable $\{A\}$

$\because A \rightarrow \varepsilon$

$S \rightarrow a \mid aA \mid aB$

$A \rightarrow aBB$

$B \rightarrow Aa \mid b \mid a$

→ (ii) Removing unit production

There is no unit production as there is no variable → variable form

→ (iii) Removing symbols that are useless

Useful symbols = $\{a, b, S, B, A\}$

There is no useless symbol.

→ (iv) CNF form

$S \rightarrow a \mid XA \mid XB$

$A \rightarrow YB$ $(\because A \rightarrow XBB, Y \rightarrow XB, X \rightarrow a)$

$Y \rightarrow XB$

$X \rightarrow a$

$B \rightarrow AX \mid b \mid a$