Control Systems Project

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Introduction to Project

A hard disk is a data storage device. It uses magnetic storage system with electronic hardware to access the data. The electronic circuit consists of a dc motor. A dc motor has the following state space

$$\begin{bmatrix} \dot{\theta} \\ \dot{\Theta} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{-b}{J} & \frac{k}{J} \\ 0 & \frac{k}{L} & \frac{-R}{L} \end{bmatrix} \begin{bmatrix} \theta \\ \Theta \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u \tag{1}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \Theta \\ i \end{bmatrix} \tag{2}$$

Introduction to Project

a. Use $J = 3.2$, $b = 3.5$, $k = 0.0274$, $R = 4$, and $L = 2.75$; Check the stability of the system using all methods that you know

b. Simulate the unstable system and show that its response is unstable

c. Compute the controllability matrix for the system. If the system is controllable, place the controller eigenvalues at (-14, -33, -33) and observer eigenvalues at a location which is faster than the controller eigenvalues.

d. Simulate the stable system and show its response

e. Design a PID Controller and compare it with response obtained from part d

f. Compute the steady state errors before and after designing controller

g. Design a tracking controller for step tracking of amplitude 5u(t) and ramp tracking of 5tu(t)

State-space Representation of the System

The state-space representation of the system can be written as follows:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\Theta} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1.0938 & 0.0086 \\ 0 & 0.01 & -1.4545 \end{bmatrix} \begin{bmatrix} \theta \\ \Theta \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.3636 \end{bmatrix} u \qquad (3)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \Theta \\ i \end{bmatrix} \qquad (4)$$

Stability Analysis of the System

The Transfer Function of the System is as follows:

$$\frac{\mathbf{0.003114}}{\boldsymbol{s^3 + 2.548s^2 + 1.591s}} \tag{5}$$

The Eigenvalues of the system are:

$$\boldsymbol{\lambda_1 = 0, \lambda_2 = -1.4545, \lambda_3 = -1.0935} \tag{6}$$

The eigenvalues of the system were computed using *eig(A)* matlab function

The poles of the system are:

$$\boldsymbol{p_1 = 0, p_2 = -1.0935, p_3 = -1.4545} \tag{7}$$

The poles of the system were computed using the *roots(denum)* matlab function.

Considering one of the eigenvalues and one of the poles is equal to zero, it indicates the system is marginally stable.

Stability Analysis of the System

Routh-Hurwitz table is shown below

s^3	1	1.591
s^2	2.548	0
s^1	$-\frac{1}{2.548} \times \begin{vmatrix} 1 & 1.591 \\ 2.548 & 0 \end{vmatrix} = 1.5910$	0
s^0	$-\frac{1}{-1.5910} \times \begin{vmatrix} 1.591 & 0 \\ 1.591 & 0 \end{vmatrix} = 0$	0

Sign has not changed in the first column; therefore, the system is stable.

Stability Analysis of the System

The step response of the system is as below: System is unbounded, which means it's marginally stable.

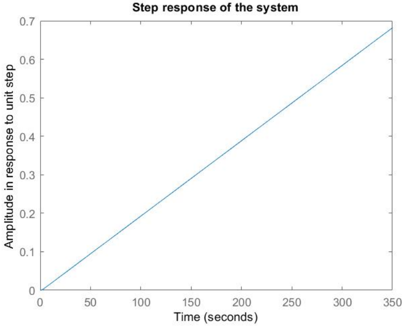

Figure: Plot of step response.

Stability Analysis of the System

The Root locus plot of the system is as below: At gain k = 0, system is marginally stable

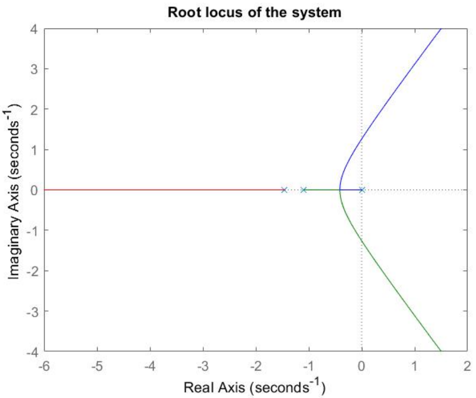

Figure: Plot of root locus.

Controllability Analysis

$$P = ctrb(A, B) \tag{8}$$

$$\begin{bmatrix} 0 & 0 & 0.0031 \\ 0 & 0.0031 & -0.0079 \\ 0.3636 & -0.5289 & 0.7694 \end{bmatrix} \tag{9}$$

$$Ctrbrank = rank(P) \tag{10}$$

$$Ctrbrank = 3 \tag{11}$$

- As rank of Matrix P is same as matrix A, we conclude that the system passes controllability test.
- Since the system is controllable, we place controller eigen values at (-14, -33, -33)

Observability Analysis

$$\boldsymbol{Q = obsv(A, C)} \tag{12}$$

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1.0938} & \mathbf{0.0086} \end{bmatrix} \tag{13}$$

$$\boldsymbol{Obsvrank = obsv(Q)} \tag{14}$$

$$\boldsymbol{Obsvrank = 3} \tag{15}$$

- As rank of Matrix Q is same as matrix A, we conclude that the system passes observability test.
- Since the system is observable, we place observer eigenvalues at (-15, -34, -34)

Controller Design - Results

$$\boldsymbol{L} = \begin{bmatrix} \mathbf{80.4517} \\ \mathbf{1.9694 * 10^3} \\ \mathbf{1.6756 * 10^6} \end{bmatrix} \tag{16}$$

$$\boldsymbol{K} = \begin{bmatrix} \mathbf{4.8965 * 10^6} & \mathbf{6.1879 * 10^5} & \mathbf{212.9922} \end{bmatrix} \tag{17}$$

$$\boldsymbol{B(obv)} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{L} \end{bmatrix} \tag{18}$$

$$\boldsymbol{B(obv)} = \begin{bmatrix} \mathbf{0} & \mathbf{80.4517} \\ \mathbf{0} & \mathbf{1.9694 * 10^3} \\ \mathbf{0.3636} & \mathbf{1.6756 * 10^6} \end{bmatrix} \tag{19}$$

Schematics - Marginally Stable System

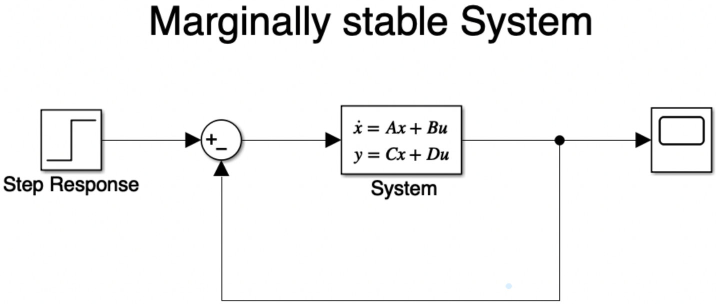

Figure: Schematic of Marginally Stable System

Step Response

(Simulink) The step response of the system is as below: System is unbounded, which means it's marginally stable.

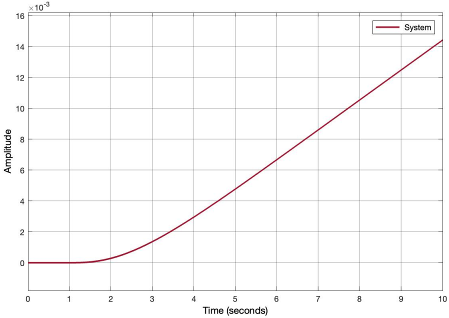

Figure: Plot of step response in simulink.

Schematics - Marginally Stable System with Controller

Schematic of Marginally Stable System after using observer-based feedback controller

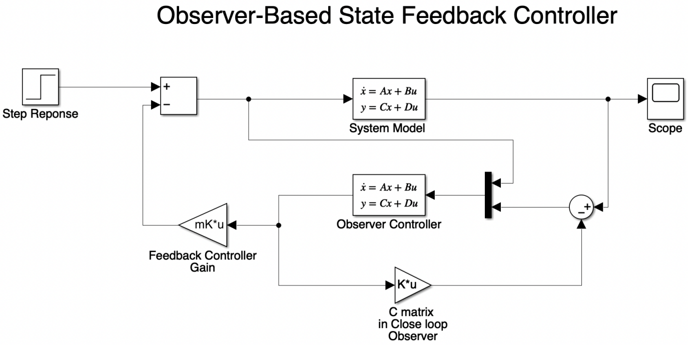

Figure: Schematic of Observer based State Feedback Controller.

Step Response

Step Response of Marginally Stable System after using observer-based feedback controller

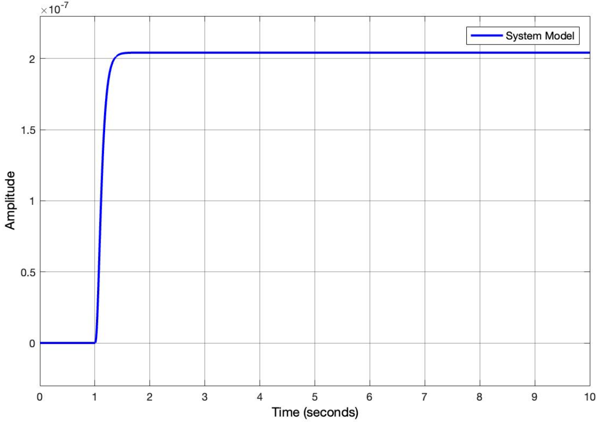

Figure: Plot of Step Response of Observer based State Feedback Controller.

Step Response Details

- Rise Time = 0.196 seconds
- Settling Time = 0.3580 seconds
- Overshoot = 0
- Undershoot = 0
- Final Value =

$$\frac{\mathbf{2.0417}}{\mathbf{10^7}} \tag{20}$$

Schematics - PID Controller with Controlled System

Schematic of PID Controller with Controlled System

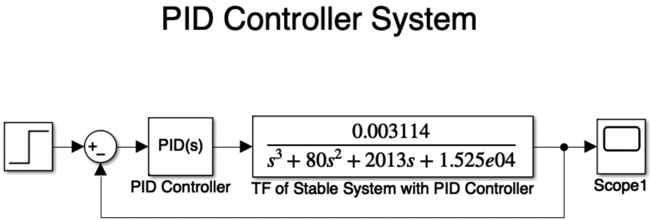

Figure: Schematic of PID Controller.

Step Response

Step Response of PID Controller with Controlled System

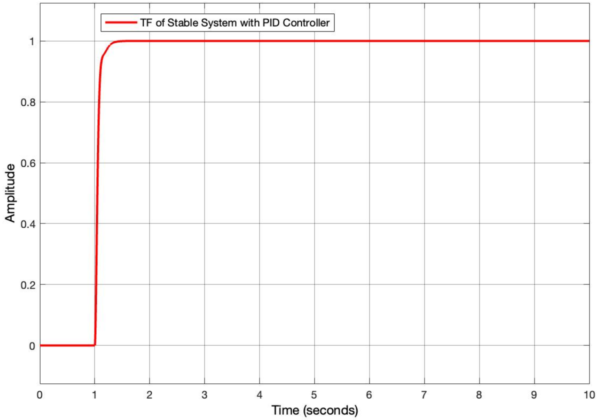

Figure: Plot of Step Response of PID Controller.

Step Response Details

- Rise Time $= 0.0844$ seconds
- Settling Time $= 0.2257$ seconds
- Overshoot $= 0$
- Undershoot $= 0$
- Final Value $= 1$

Comparison of PID Controller with Controlled System

Comparison of Step Responses from PID Controller and Controlled System

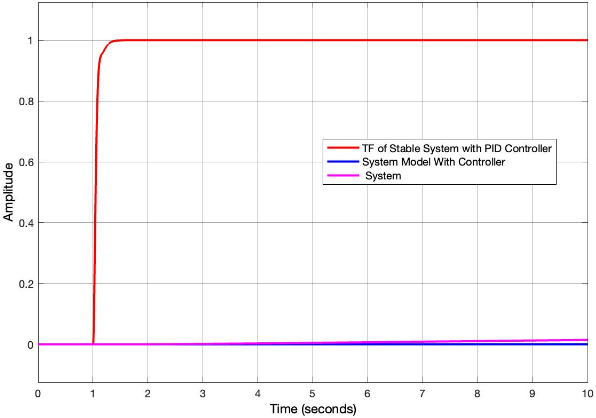

Figure: Comparison of Step Responses of Observer based State Feedback Controller, PID Controller, and Marginally Stable System.

Steady State Error

- Steady State Error before the controller:
  - Infinity or Undefined because system is unbounded.
- Steady State Error after Observer-based Feedback Controller
  - $$\mathbf{1 - \frac{2.0417}{10^7} \approx 1} \tag{21}$$
- Steady State Error after PID Controller
  - $$\mathbf{1 - 1 = 0} \tag{22}$$
- Steady State Error for Ramp Input after Controller
  - Infinity because system is Type 0
- Steady State Error for Parabolic Input after Controller
  - Infinity because system is Type 0

Tracking Controller

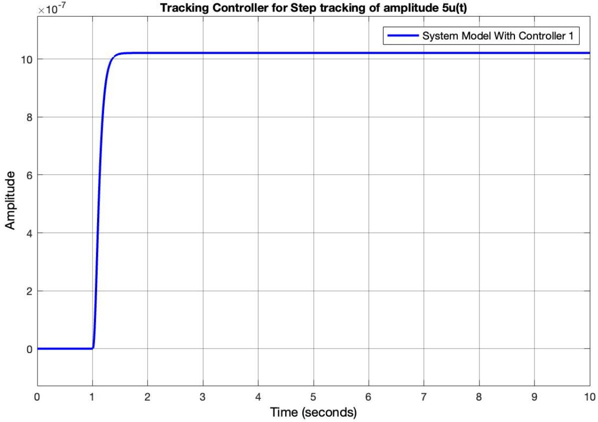

Figure: Plot of Step Tracking of amplitude 5u(t).

Tracking Controller

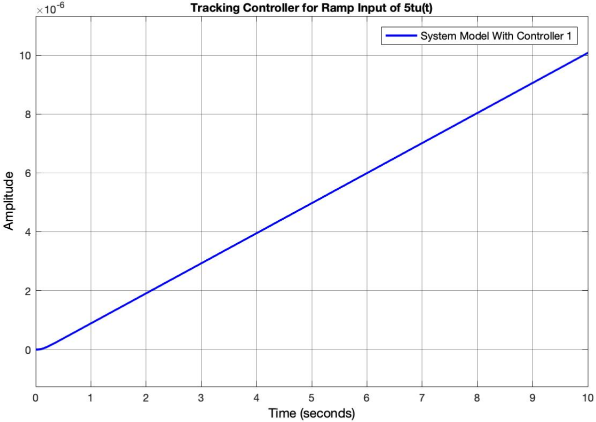

Figure: Plot of Ramp Tracking of amplitude 5tu(t).