

Logistic Regression

Sigmoid

函数特性

$$\sigma(x) = \frac{1}{1+e^{-x}} \quad \sigma(x)' = \sigma(x) * (1 - \sigma(x))$$

损失函数

似然函数

$$\sigma(x \cdot \theta), y_i = 1 \quad 1 - \sigma(x \cdot \theta), y_i = 0 \quad L(\theta) = \prod_{i=1}^N \sigma(x \cdot \theta)^{y_i} * (1 - \sigma(x \cdot \theta))^{1-y_i}$$

负对数似然函数

$$J(\theta) = - \sum_{i=1}^N y_i * \log(\sigma(x \cdot \theta)) + (1 - y_i) * \log(1 - \sigma(x \cdot \theta))$$

梯度求解

$$w_j := w_j + \eta * \frac{\partial J(\theta)}{\partial w_j}$$

$$\frac{\partial J(\theta)}{\partial w_j} = \frac{\partial J(\theta)}{\partial \sigma(x \cdot \theta)} * \frac{\partial \sigma(x \cdot \theta)}{\partial w_j}$$

$$\frac{\partial J(\theta)}{\partial \sigma(x \cdot \theta)} = \sum_{i=1}^N \frac{y_i - \sigma(x \cdot \theta)}{\sigma(x \cdot \theta) * (1 - \sigma(x \cdot \theta))}$$

$$\frac{\partial \sigma(x \cdot \theta)}{\partial w_j} = \sigma(x \cdot \theta) * (1 - \sigma(x \cdot \theta)) * x_{ij}$$

$$\frac{\partial J(\theta)}{\partial w_j} = \sum_{i=1}^N (y_i - \sigma(x \cdot \theta)) * x_{ij}$$