Logistic Regression

Sigmoid

函数特性

$$\sigma(x)=rac{1}{1+e^{-x}}\;\sigma(x)^{'}=\sigma(x)*(1-\sigma(x))$$

损失函数

似然函数

$$\sigma(x\cdot heta), y_i = 1$$
 $1 - \sigma(x\cdot heta), y_i = 0$ $L(heta) = \prod_{i=1}^N \sigma(x\cdot heta)^{y_i} * (1 - \sigma(x\cdot heta))^{1-y_i}$

负对数似然函数

$$J(heta) = -\sum_{i=1}^N y_i * log(\sigma(x \cdot heta)) + (1-y_i) * log(1-\sigma(x \cdot heta))$$

梯度求解

$$egin{aligned} w_j &:= w_j + \eta * rac{\partial J(heta)}{\partial w_j} \ & rac{\partial J(heta)}{\partial w_j} = rac{\partial J(heta)}{\partial \sigma(x \cdot heta)} * rac{\partial \sigma(x \cdot heta)}{\partial w_j} \ & rac{\partial J(heta)}{\partial \sigma(x \cdot heta)} = \sum_{i=1}^N rac{y_i - \sigma(x \cdot heta)}{\sigma(x \cdot heta) * (1 - \sigma(x \cdot heta))} \ & rac{\partial \sigma(x \cdot heta)}{\partial w_j} = \sigma(x \cdot heta) * (1 - \sigma(x \cdot heta)) * x_{ij} \ & rac{\partial J(heta)}{\partial w_i} = \sum_{i=1}^N \left(y_i - \sigma(x \cdot heta) \right) * x_{ij} \end{aligned}$$