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Abstract

1 Introduction

In this report, we are addressing the problem of variable stiffness control for robust grasping. As shown in our previous work (IROS), variable stiffness control can effectively stabilize the grasp under disturbance. However, the stability of such controller has never been addressed before. In this work, we are trying to address this problem starting from the simple test examples,see Fig. 1. To this end, first, a detailed formulation for the dynamics of the hand-object is derived, which taken rolling constraints and the soft fingertip into account Then, a variable stiffness controller is presented to grasp the object stable and the upper bound for the change rate of the stiffness is derived from the proof of stability. (We follow the same notation as in Arimoto's book, add ref.)
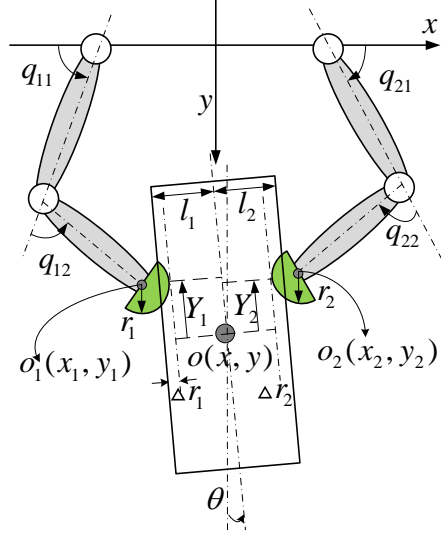

Figure 1: The hand-object system in 2D. Each finger has 2 DOFs with soft fingertips.

2 Dynamics

In this part, we first consider the contact model of the fingertips and the constraints involved in the hand-object system. Then the overall dynamics of the system is formulated.

2.1 Contact model of soft fingertip

$$f = c_1(\Delta r)^2 + c_2 \frac{d}{dt}\Delta r \tag{1}$$

where c_1 and c_2 are positive constant parameters which depends on the material of the fingertip.
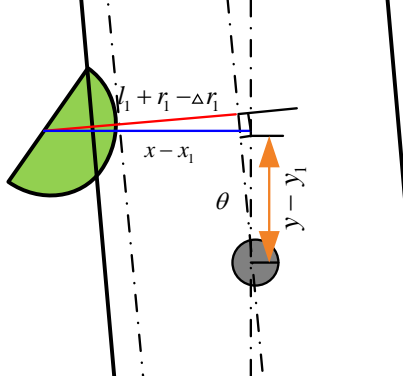
2.2 Contact constraints

Figure 2: The constraint that the fingertip should keep contact with the object surface.

The fingertip should keep contact with the object surface, as shown in Fig. 2, which can be expressed as follows:

$$l_1 + r_1 - \Delta r_1 = (x - x_1)\cos\theta - (y - y_1)\sin\theta \tag{2}$$

$$l_2 + r_2 - \Delta r_2 = -(x - x_2)\cos\theta + (y - y_2)\sin\theta \tag{3}$$

2.3 Rolling constraints

The rolling constraints on each fingertip can be represented as:

$$(r_i - \Delta r_i)\frac{d}{dt}\phi_i = -\frac{d}{dt}Y_i, \quad i = 1, 2 \tag{4}$$

where Y_i and ϕ_i are given by:

$$Y_i = (x_i - x)\sin\theta + (y_i - y)\cos\theta \tag{5}$$

$$q_{11} + q_{12} + \phi_1 = \pi + \theta \tag{6}$$

$$q_{21} + q_{22} + \phi_2 = \pi - \theta \tag{7}$$

2.4 Overall Dynamics

The total kinetic energy for the overall system can be described as follows:

$$K = \sum_{i=1,2} \frac{1}{2} \dot{\mathbf{q}}_i^T H_i \dot{\mathbf{q}}_i + \frac{1}{2} M(\dot{x}^2 + \dot{y}^2) + \frac{1}{2} I \dot{\theta}^2 \tag{8}$$

where $\mathbf{q}_i = [q_{i1}, q_{i2}]^T$.

The total potential energy can be given as:

$$P = \sum_{i=1,2} \int_0^{\Delta r_i} c_1 \eta^2 d\eta \tag{9}$$

Then from the Hamilton's principle, we have:

$$\int_{t_0}^{t_1} \left[\delta(K - P) - \sum_{i=1,2} c_2 \Delta \dot{r}_i \frac{\partial \Delta r_i}{\partial X^T} \delta X + \sum_{i=1,2} \lambda_i [(r_i - \Delta r_i) \frac{\partial \phi_i}{\partial X^T} + \frac{\partial Y_i}{\partial X^T}] \delta X + \sum_{i=1,2} \mathbf{u_i}^T \delta \mathbf{q}_i \right] dt = 0 \tag{10}$$

where $X = [\mathbf{q}_1^T, \mathbf{q}_2^T, x, y, \theta]^T$.

Then we have the overall dynamics for the object-hand system as follows:

$$H_i(\mathbf{q_i}) \ddot{\mathbf{q}}_i + (\frac{1}{2} \dot{H}_i + S_i) \dot{\mathbf{q}}_i + f_i \frac{\partial \Delta r_i}{\partial \mathbf{q}_i^T} - \lambda_i [(r_i - \Delta r_i) \frac{\partial \phi_i}{\partial \mathbf{q}_i^T} + \frac{\partial Y_i}{\partial \mathbf{q}_i^T}] = \mathbf{u}_i \tag{11}$$

$$M\ddot{x} + \sum_{i=1,2} [f_i \frac{\partial \Delta r_i}{\partial x} - \lambda_i \frac{\partial Y_i}{\partial x}] = 0 \tag{12}$$

$$M\ddot{y} + \sum_{i=1,2} [f_i \frac{\partial \Delta r_i}{\partial y} - \lambda_i \frac{\partial Y_i}{\partial y}] = 0 \tag{13}$$

$$I\ddot{\theta} + \sum_{i=1,2} [f_i \frac{\partial \Delta r_i}{\partial \theta} - \lambda_i ((r_i - \Delta r_i) \frac{\partial \phi_i}{\partial \theta} + \frac{\partial Y_i}{\partial \theta})] = 0 \tag{14}$$

With the identities in appendix A, the overall dynamics can be simplified as:

$$H_i(\mathbf{q_i}) \ddot{\mathbf{q}}_i + (\frac{1}{2} \dot{H}_i + S_i) \dot{\mathbf{q}}_i - (-1)^i f_i J_i^T \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} - \lambda_i [(r_i - \Delta r_i) \begin{bmatrix} -1 \\ -1 \end{bmatrix} + J_i^T \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}] = \mathbf{u}_i \tag{15}$$

$$M\ddot{x} - (f_1 - f_2)\cos\theta + (\lambda_1 + \lambda_2)\sin\theta = 0 \tag{16}$$

$$M\ddot{y} + (f_1 - f_2)\sin\theta + (\lambda_1 + \lambda_2)\cos\theta = 0 \tag{17}$$

$$I\ddot{\theta} - f_1 Y_1 + f_2 Y_2 + l_1 \lambda_1 - l_2 \lambda_2 = 0 \tag{18}$$

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3 Variable stiffness control

Motivated from the analysis of the overall system dynamics, the following control law is adopted for each finger to achieve stable grasp:

$$u_i = -D_i\dot{\mathbf{q}}_i + kJ_i^T(\mathbf{x}_i - \mathbf{x}_d) \tag{19}$$

$$\mathbf{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \qquad\qquad \mathbf{x}_d = \frac{1}{2}\begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix} \tag{20}$$

where D_i is a diagonal positive definite matrix representing the damping gain. k represents the *variable* stiffness for each fingertip.

4 Stability Proof–1

Taking the sum of inner product of (15) with $\dot{\mathbf{q}}_i, i = 1, 2$, (16) with $\dot{x}$, (17) with $\dot{y}$, (18) with $\dot{\theta}$, we have:

$$\frac{d}{dt}E = -\sum_{i=1,2}\{\dot{\mathbf{q}}_i^T D_i\dot{\mathbf{q}}_i + c_2\Delta\dot{r}_i^2\} + \frac{\dot{k}}{4}(\mathbf{x}_1 - \mathbf{x}_2)^T(\mathbf{x}_1 - \mathbf{x}_2) \tag{21}$$

$$E = K + V + P \tag{22}$$

$$K = \sum_{i=1,2}\frac{1}{2}\dot{\mathbf{q}}_i^T H_i\dot{\mathbf{q}}_i + \frac{1}{2}M(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}I\dot{\theta}^2 \tag{23}$$

$$P = \sum_{i=1,2}\int_0^{\Delta r_i} c_1\eta^2 d\eta \tag{24}$$

$$V = \frac{k}{4}(\mathbf{x}_1 - \mathbf{x}_2)^T(\mathbf{x}_1 - \mathbf{x}_2) \tag{25}$$

As proved in Arimoto's book, the closed-loop dynamics is asymptotically stable if:

$$\frac{d}{dt}E < 0 \tag{26}$$

which leads to

$$\dot{k} < \frac{\sum_{i=1,2}\{\dot{\mathbf{q}}_i^T D_i\dot{\mathbf{q}}_i + c_2\Delta\dot{r}_i^2\}}{(\mathbf{x}_1 - \mathbf{x}_2)^T(\mathbf{x}_1 - \mathbf{x}_2)} \tag{27}$$

From this, we can conclude the following comments Is this conclusion correct? Any example?:

- If the object is softer, c_2 is bigger, we can change the stiffness faster.

- If the object is smaller, $(\mathbf{x}_1 - \mathbf{x}_2)^T(\mathbf{x}_1 - \mathbf{x}_2)$, we can change the stiffness faster

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As noticed that the change rate of the stiffness is still depending on the state variable and we still don't know how wo realize the desired stiffness. Note: in our previous work of grasp adaptation, we simply change the k to k_d, which may lead to unstable behavior if $k_d - k$ is too big. This leads us to the following section.

5 Stability Proof–2

As noticed in Eq. 21, the dissipated energy $\sum\limits_{i=1,2} \{\dot{\mathbf{q}}_i^T D_i \dot{\mathbf{q}}_i + c_2 \Delta \dot{r}_i^2\}$ set an upper bound for the energy that we can inject into the system. In order to stably vary the stiffness, we keep track of the energy dissipated by the system using a virtual tank(add ref.). The control signal is designed as follows:

$$u_i = -D_i \dot{\mathbf{q}}_i + J_i^T(\omega z) \tag{28}$$

The new term $z \in \mathbb{R}$ is the state variable associated with the energy in the tank and,

$$T(z) = \frac{1}{2} z^2 \tag{29}$$

$$\dot{z} = \frac{\sigma}{z} \sum_{i=1,2} \{\dot{\mathbf{q}}_i^T D_i \dot{\mathbf{q}}_i + c_2 \Delta \dot{r}_i^2\} - \sum_{i=1,2} \omega^T J_i \dot{\mathbf{q}}_\mathbf{i} \tag{30}$$

$\omega(t) \in \mathbb{R}^3$ is the control variable through which it is possible to exchange the energy between the impedance components and the tank. σ is a design parameter that opens/closes the tank for energy exchange:

$$\sigma = \left\{ \begin{array}{ll} 1 & T(z) < T_{lim} \\ 0 & otherwise \end{array} \right.$$

Now with the new controller and tank energy, we can derive:

$$E = K + V + T \tag{31}$$

$$\frac{d}{dt} E = -\sum_{i=1,2} \{\dot{\mathbf{q}}_i^T D_i \dot{\mathbf{q}}_i + c_2 \Delta \dot{r}_i^2\} + \sigma \sum_{i=1,2} \{\dot{\mathbf{q}}_i^T D_i \dot{\mathbf{q}}_i + c_2 \Delta \dot{r}_i^2\} \leq 0 \tag{32}$$

Recalling Eq. 19, to get the desired stiffness k_d, we can exploit the energy stored in the tank by setting:

$$\omega(t) = \left\{ \begin{array}{ll} K_d(\mathbf{x}_i - \mathbf{x}_d)/z & T(z) > T_{low} \\ 0 & otherwise \end{array} \right.$$

The stability proof is similar by following Arimoto's method from Eq. 32.

6 simulation

In this section, we will use an 1D example to example the idea of virtual tank for implementing the varying stiffness.

7 appendix

A Identities

From equation $(2)(3)(5)(6)(7)$, we have the following identities:

$$\frac{\partial \Delta r_1}{\partial \mathbf{q}_1^T} = J_1^T \left[\cos\theta, -\sin\theta\right]^T \qquad\qquad \frac{\partial \Delta r_2}{\partial \mathbf{q}_2^T} = -J_2^T \left[\cos\theta, -\sin\theta\right]^T$$

$$\frac{\partial \phi_1}{\partial \mathbf{q}_1^T} = [-1, -1]^T \qquad\qquad \frac{\partial \phi_2}{\partial \mathbf{q}_2^T} = [-1, -1]^T$$

$$\frac{\partial Y_1}{\partial \mathbf{q}_1^T} = J_1^T [\sin\theta, \cos\theta]^T \qquad\qquad \frac{\partial Y_2}{\partial \mathbf{q}_2^T} = J_2^T [\sin\theta, \cos\theta]^T$$

$$\frac{\partial \Delta r_1}{\partial x} = -\cos\theta \qquad\qquad \frac{\partial \Delta r_2}{\partial x} = \cos\theta$$

$$\frac{\partial Y_1}{\partial x} = -\sin\theta \qquad\qquad \frac{\partial Y_2}{\partial x} = -\sin\theta$$

$$\frac{\partial \Delta r_1}{\partial y} = \sin\theta \qquad\qquad \frac{\partial \Delta r_2}{\partial y} = -\sin\theta$$

$$\frac{\partial Y_1}{\partial y} = -\cos\theta \qquad\qquad \frac{\partial Y_2}{\partial y} = -\cos\theta$$

$$\frac{\partial \Delta r_1}{\partial \theta} = -Y_1 \qquad\qquad \frac{\partial \Delta r_2}{\partial \theta} = Y_2$$

$$\frac{\partial \phi_1}{\partial \theta} = 1 \qquad\qquad \frac{\partial \phi_2}{\partial \theta} = -1$$

$$\frac{\partial Y_1}{\partial \theta} = (x_1 - x)\cos\theta - (y_1 - y)\sin\theta$$

$$\frac{\partial Y_2}{\partial \theta} = (x_2 - x)\cos\theta - (y_2 - y)\sin\theta$$