Lab 3 Answer Sheet

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Lab 3 Answer...

ECE368: Probabilistic Reasoning

Lab 3: Hidden Markov Model

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You should hand in: 1) A scanned .pdf version of this sheet with your answers (file size should be under 2 MB); 2) one Python file inference.py that contains your code. The files should be uploaded to Quercus.

1. (a) Write down the formulas of the forward-backward algorithm to compute the marginal distribution $p(\mathbf{z}_i|(\hat{x}_0, \hat{y}_0), \ldots, (\hat{x}_{N-1}, \hat{y}_{N-1}))$ for $i = 0, 1, \ldots, N-1$. Your answer should contain the initializations of the forward and backward messages, the recursion relations of the messages, and the computation of the marginal distribution based on the messages. (1 **pt**)

Forward Message: $\alpha(z_0) = p(z_0)p((\tilde{x}_0, \tilde{y}_0|z_0))$ $(\tilde{x}, \tilde{y})$: observed location

Backword Message: $\beta(z_{N-1}) = 1$

Recursive Relations:

$\alpha(z_i) = p((\tilde{x}_i, \tilde{y}_i|z_i) \cdot \sum_{z_{i-1}} \alpha(z_{i-1}) p(z_i|z_{i-1})$

$\beta(z_i) = \sum_{z_{i+1}} \beta(z_{i+1}) \cdot p((\tilde{x}_{i+1}, \tilde{y}_{i+1}) | z_{i+1}) p(z_{i+1}|z_i)$

Marginal Distribution: $p(z_i|(x,y)) = \dfrac{\alpha(z_i)\beta(z_i)}{\sum \alpha(z_i)\beta(z_i)}$

(b) After you run the forward-backward algorithm on the data in test.txt, write down the obtained marginal distribution of the state at $i = 99$ (the last time step), i.e., $p(\mathbf{z}_{99}|(\hat{x}_0, \hat{y}_0), \ldots, (\hat{x}_{99}, \hat{y}_{99}))$. Only include states with non-zero probability in your answer. (2 **pt**)

$$p(z_{99}|(\hat{x}_0,\hat{y}_0), \cdots, (\hat{x}_{99}, \hat{y}_{99})) = \begin{cases} 0.8103 & z_{99} = (11, 0, \text{stay}) \\ 0.1796 & z_{99} = (11, 0, \text{right}) \\ 0.0101 & z_{99} = (11, 1, \text{down}) \end{cases}$$

2. Modify your forward-backward algorithm so that it can handle missing observations. After you run the modified forward-backward algorithm on the data in test_missing.txt, write down the obtained marginal distribution of the state at $i = 30$, i.e., $p(\mathbf{z}_{30}|(\hat{x}_0, \hat{y}_0), \ldots, (\hat{x}_{99}, \hat{y}_{99}))$. Only include states with non-zero probability in your answer. (1 **pt**)

$$p(z_{30}|(\hat{x}_0,\hat{y}_0) \cdots (\hat{x}_{99}, \hat{y}_{99})) = \begin{cases} 0.9130 & z_{30} = (6, 7, \text{right}) \\ 0.0435 & z_{30} = (5, 7, \text{right}) \\ 0.0435 & z_{30} = (5, 7, \text{stay}) \end{cases}$$

3. (a) Write down the formulas of the Viterbi algorithm using $\mathbf{z}_i$ and $(\hat{x}_i, \hat{y}_i), i = 0, 1, \ldots, N-1$. Your answer should contain the initialization of the messages and the recursion of the messages in the Viterbi algorithm. (1 **pt**)

$w_0 = \ln(P(z_0) \cdot P((\hat{x}_0, \hat{y}_0)|z_0))$ $(\hat{x}, \hat{y})$: observation

$w_i = \ln(P((\hat{x}_i, \hat{y}_i)|z_i)) + \underset{z_{i-1}}{\text{argmax}}\left(\ln(P(z_i|z_{i-1})) + w_{i-1}\right)$

(b) After you run the Viterbi algorithm on the data in test_missing.txt, write down the last 10 hidden states of the most likely sequence (i.e., $i = 90, 91, 92, \ldots, 99$) based on the MAP estimate. (3 **pt**)

MAP Estimate of last 10 hidden states:

(11, 5, down) (10, 7, left)
(11, 6, down) (9, 7, left)
(11, 7, down) (8, 7, left)
(11, 7, stay) (7, 7, left)
(11, 7, stay) (6, 7, left)

4. Compute and compare the error probabilities of $\{\tilde{\mathbf{z}}_i\}$ and $\{\check{\mathbf{z}}_i\}$ using the data in test_missing.txt. The error probability of $\{\tilde{\mathbf{z}}_i\}$ is $\boxed{3\%}$. The error probability of $\{\check{\mathbf{z}}_i\}$ is $\boxed{2\%}$. (1 **pt**)

5. Is sequence $\{\check{\mathbf{z}}_i\}$ a valid sequence? If not, please find a small segment $\check{\mathbf{z}}_i, \check{\mathbf{z}}_{i+1}$ that violates the transition model for some time step i. You answer should specify the value of i as well as the corresponding states $\check{\mathbf{z}}_i, \check{\mathbf{z}}_{i+1}$. (1 **pt**)

Sequence $\{\check{z}_i\}$ is not a valid sequence

$z_{64} = (3, 7, \text{stay})$

$z_{65} = (2, 7, \text{stay})$